



A Class of Modified Pathway-Type Integral Transforms

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ABSTRACT: Pathway integral transforms were studied and introduced by Kumar in 2011, which is obtained in the sense of the pathway model given by Mathai in 2005. This paper introduces and presents the study of a class of modified pathway integral transforms. It is a generalization of the pathway model given by Mathai. The Mellin and Laplace transforms of the modified pathway transforms and the composition formulae for the various fractional operators with the modified pathway transform are provided. The presented results related to the modified pathway transforms are generalizations of pathway transform given by Kumar and many other existing results in the literature.

Keywords: Pathway model, Mellin transform, H-function, modified pathway integral transforms.

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1. Introduction

The pathway model was established by Mathai ([6], [7]) in a systematic way to understand complex systems characterized by random variations and evolving states, particularly in the field of mathematics, physics, and economics. Tsallis ([9], [10]), Beck and Coben ([8]) and many other researchers have used this model in statistics and super-statistics models. In the theory of thermonuclear reaction rate theory, Kumar, Haubold, and Kilbas have applied the pathway model in the following research articles ([14], [1], [11], [13], [15], [16]). Srivastava [17] has presented the relation between the Riemann-Liouville FC operators and the Pathway FC operators.

Definition 1.1 Kumar [12] presented the pathway transform as a trans-formative extension of the pathway model, building upon Mathai’s basic work, and defined the pathway integral transform in the following manner:

$$(P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h} f)(y) = \int_0^\infty D_{\alpha_h, \beta_h}^{\eta_h, \gamma_h}(yt) f(t) dt, y > 0, \quad (1.1)$$

where $D_{\alpha_h, \beta_h}^{\eta_h, \gamma_h}(y)$ denotes the kernel function.

$$D_{\alpha_h, \beta_h}^{\eta_h, \gamma_h}(y) = \int_0^{[\frac{1}{a(1-\gamma_h)}]^{1/\alpha_h}} x^{\eta_h-1} [1 - a(1-\gamma_h)x^{\alpha_h}]^{\frac{1}{1-\gamma_h}} e^{-yx^{-\beta_h}} dx, y > 0, \quad (1.2)$$

2020 *Mathematics Subject Classification:* 33C60, 44A20, 26A33, 44A10.

Submitted September 17, 2025. Published March 28, 2026

for $\eta_h \in C, \beta_h > 0, \alpha_h > 0, a > 0, \gamma_h < 1$.

In view of the kernel (1.2), (1.1) is the type-1 P-transform. If

$$D_{\alpha_h, \beta_h}^{\eta_h, \gamma_h}(y) = \int_0^\infty x^{\eta_h-1} [1 + a(\gamma_h - 1)x^{\alpha_h}]^{-\frac{1}{\gamma_h-1}} e^{-yx^{-\beta_h}} dx, y > 0, \quad (1.3)$$

for $\eta_h \in C, \alpha_h \in R, \alpha_h \neq 0, \beta_h, \sigma \in R, a > 0, \gamma_h > 1$,

then (1.1) is known as the type-2 P-transform in the case of kernel (1.3). Pathway transforms with different composition formulae are mentioned in Kumar [12], Kumar and Kilbas [13], Ghiya [19] and Ghiya and Patil [20]. Pathway transforms can be seen in many domains, including biology, engineering, finance, and more. Because of its adaptability and effectiveness, it has established itself as a useful tool for scholars attempting to understand the complexities of contemporary science and technology.

In this paper we provide the modified pathway integral transforms involving the Mittag-Leffler function since in recent years researchers are interested in the Mittag-Leffler function as a model in various applied fields for multiple reasons; one is, it yields a thicker tail in the pathway model compared to the exponential model. Mittag-Leffler function and its generalized functions occur as solutions of fractional differential equations, particularly when solving fractional equations in the context of reaction-diffusion problems. Several instances of this kind are described by Kilbas, Srivastava, Trujillo [2] and Kumar, Haubold [14]. For more details about the fractional calculus see [23]-[25].

2. Modified Pathway Integral Transforms

Definition 2.1 The modified pathway integral transforms are defined in the following form:

$$(P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f)(y) = \int_0^\infty D_{\alpha_h, \beta_h}^{\eta_h, \gamma_h, \xi_h}(yt) f(t) dt, y > 0, \quad (2.1)$$

where

$$D_{\alpha_h, \beta_h}^{\eta_h, \gamma_h, \xi_h}(y) = \int_0^{[\frac{1}{a(1-\gamma_h)}]^{\frac{1}{\alpha_h}}} x^{\eta_h-1} [1 - a(1-\gamma_h)x^{\alpha_h}]^{\frac{1}{1-\gamma_h}} E_{\xi_h}(-yx^{-\beta_h}) dx, y > 0, \quad (2.2)$$

with $\eta_h \in C, \beta_h > 0, \alpha_h > 0, \xi_h > 0, a > 0, \gamma_h < 1$.

In the case of (2.2), (2.1) is the type-1 modified pathway transform. If the kernel of transform is defined by

$$D_{\alpha_h, \beta_h}^{\eta_h, \gamma_h, \xi_h}(y) = \int_0^\infty x^{\eta_h-1} [1 + a(\gamma_h - 1)x^{\alpha_h}]^{\frac{-1}{\gamma_h-1}} E_{\xi_h}(-yx^{-\beta_h}) dx, y > 0, \quad (2.3)$$

with $\eta_h \in C, \beta_h > 0, \alpha_h \in R, \alpha_h \neq 0, \xi_h > 0, a > 0, \gamma_h > 1$,

then (2.1) is type-2 modified pathway integral transform and both types of pathway transforms are described in space $L_{\eta_h, s}(0, \infty)$. These consist Lebesgue measurable type of complex valued functions for that

$$\|f\|_{\eta_h, s} = \left(\int_0^\infty \|t^{\eta_h} f(t)\|^s \frac{dt}{t} \right)^{\frac{1}{s}} < \infty.$$

In (2.2) and (2.3), $E_{\xi_h}(-yx^{-\beta_h})$ is Mittag-Leffler function and is defined as:

$$E_{\xi_h}(-yx^{-\beta_h}) = \frac{1}{2\pi i} \int_L \frac{\Gamma(s)\Gamma(1-s)}{\Gamma(1-\xi_h s)} (-yx^{-\beta_h})^{-s} ds, \quad (2.4)$$

we can observe that for $\xi_h = 1$, both types of modified pathway transforms reduce to type-1 and type-2 Pathway transform respectively. It is a versatile operator bridging classical transforms with fractional and generalized systems. Its applications span fractional kinetic equations, special functions, probability distributions, and computational modeling in physics and energy systems.

3. Mellin Transform (MT) of Modified Pathway Transforms (MPT)

Theorem 3.1 *If $f \in L_{\eta_h, s}(0, \infty)$, $s, \eta_h \in C$, $\beta_h > 0$, $y > 0$ and $\alpha_h > 0$ then the image of the modified type-1 pathway transform under the Mellin transform is:*

$$(MP_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f)(s) = \frac{\Gamma(s)\Gamma(1-s)\Gamma(\frac{\eta_h + \beta_h s}{\alpha_h})\Gamma(\frac{1}{1-\gamma_h} + 1)}{\alpha_h [a(1-\gamma_h)]^{\frac{\eta_h + \beta_h s}{\alpha_h}} \Gamma(1-\xi_h s)\Gamma(1 + \frac{1}{1-\gamma_h} + \frac{\eta_h + \beta_h s}{\alpha_h})} M_f(1-s), \quad (3.1)$$

where $R(\frac{\eta_h + \beta_h s}{\alpha_h}) > 0$.

If $f \in L_{\eta_h, s}(0, \infty)$, $s, \eta_h \in C$, $\beta_h > 0$, $y > 0$ and $\alpha_h \in R$, $\alpha_h \neq 0$, then the image of the modified type-2 pathway transform under the Mellin transform is:

$$(MP_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f)(s) = \frac{\Gamma(s)\Gamma(1-s)\Gamma(\frac{\eta_h + \beta_h s}{\alpha_h})\Gamma(\frac{1}{\gamma_h - 1} - \frac{\eta_h + \beta_h s}{\alpha_h})}{\alpha_h [a(\gamma_h - 1)]^{\frac{\eta_h + \beta_h s}{\alpha_h}} \Gamma(1-\xi_h s)\Gamma(\frac{1}{\gamma_h - 1})} M_f(1-s), \quad (3.2)$$

where $R(\frac{\eta_h + \beta_h s}{\alpha_h}) > 0$ and $R(\frac{1}{1-\gamma_h} - \frac{\eta_h + \beta_h s}{\alpha_h}) > 0$.

Proof: For (3.1) use the definition of Mellin transform from [5], modified type-1 pathway transform (2.1) and (2.2), then interchanging the integrals, we have

$$(MP_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f)(s) = \int_0^\infty f(t) \int_0^{[\frac{1}{a(1-\gamma_h)}]^{1/\alpha_h}} x^{\eta_h - 1} [1 - a(1-\gamma_h)x^{\alpha_h}]^{\frac{1}{1-\gamma_h}} \\ \times \int_0^\infty y^{s-1} E_{\xi_h}(-ytx^{-\beta_h}) dy dx dt,$$

using Mellin transform (MT) of Mittag-Leffler function(MLF), we get

$$= \frac{\Gamma(s)\Gamma(1-s)}{\Gamma(1-\xi_h s)} \int_0^\infty t^{-s} f(t) \left[\int_0^{[\frac{1}{a(1-\gamma_h)}]^{1/\alpha_h}} x^{\eta_h + \beta_h s - 1} [1 - a(1-\gamma_h)x^{\alpha_h}]^{\frac{1}{1-\gamma_h}} dx \right] dt,$$

solving the inner integral, we obtain

$$(MP_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f)(s) = \frac{\Gamma(s)\Gamma(1-s)\Gamma(\frac{\eta_h + \beta_h s}{\alpha_h})\Gamma(\frac{1}{1-\gamma_h} + 1)}{\alpha_h [a(1-\gamma_h)]^{\frac{\eta_h + \beta_h s}{\alpha_h}} \Gamma(1-\xi_h s)\Gamma(1 + \frac{1}{1-\gamma_h} + \frac{\eta_h + \beta_h s}{\alpha_h})} \\ \times \int_0^\infty t^{-s} f(t) dt \\ = \frac{\Gamma(s)\Gamma(1-s)\Gamma(\frac{\eta_h + \beta_h s}{\alpha_h})\Gamma(\frac{1}{1-\gamma_h} + 1)}{\alpha_h [a(1-\gamma_h)]^{\frac{\eta_h + \beta_h s}{\alpha_h}} \Gamma(1-\xi_h s)\Gamma(1 + \frac{1}{1-\gamma_h} + \frac{\eta_h + \beta_h s}{\alpha_h})} M_f(1-s).$$

The result (3.2) can be proved on similar lines of the proof given for the result (3.1). \square

Corollary 3.1 : *Let $\alpha_h \in R$, $\alpha_h \neq 0$, $a > 0$, $\beta_h > 0$, $\gamma_h > 1$ and $\eta_h \in C$, $R(s) > 0$ then the Mellin transform(MT) of $D_{\alpha_h, \beta_h}^{\eta_h, \gamma_h, \xi_h}(y)$ presented in (2.2) is given by:*

$$(MD_{\alpha_h, \beta_h}^{\eta_h, \gamma_h, \xi_h})(s) = \frac{\Gamma(s)\Gamma(1-s)\Gamma(\frac{\eta_h + \beta_h s}{\alpha_h})\Gamma(\frac{1}{1-\gamma_h} + 1)}{\alpha_h [a(1-\gamma_h)]^{\frac{\eta_h + \beta_h s}{\alpha_h}} \Gamma(1-\xi_h s)\Gamma(1 + \frac{1}{1-\gamma_h} + \frac{\eta_h + \beta_h s}{\alpha_h})}, \quad (3.3)$$

where $\alpha_h > 0$ and $R(\eta_h + \beta_h s) > 0$.

If $D_{\alpha_h, \beta_h}^{\eta_h, \gamma_h, \xi_h}(y)$ is defined as mentioned in (2.3), then

$$\left(MD_{\alpha_h, \beta_h}^{\eta_h, \gamma_h, \xi_h} \right) (s) = \frac{\Gamma(s)\Gamma(1-s)\Gamma\left(\frac{\eta_h + \beta_h s}{\alpha_h}\right)\Gamma\left(\frac{1}{\gamma_h - 1} - \frac{\eta_h + \beta_h s}{\alpha_h}\right)}{\alpha_h [a(\gamma_h - 1)]^{\frac{\eta_h + \beta_h s}{\alpha_h}} \Gamma(1 - \xi_h s)\Gamma\left(\frac{1}{\gamma_h - 1}\right)}, \quad (3.4)$$

where $\alpha_h < 0$, $R(\eta_h + \beta_h s) > 0$. and $R\left(\frac{1}{\gamma_h - 1} - \frac{\eta_h + \beta_h s}{\alpha_h}\right) > 0$.

Proof: These results can be derived from the similar lines of the proofs (3.1) and (3.2) given in Theorem 3.1. \square

Theorem 3.2 For $\eta_h, z \in C$, $a > 0$, $\alpha_h > 0$, $\beta_h > 0$ and $\gamma_h < 1$,

$$D_{\alpha_h, \beta_h}^{\eta_h, \gamma_h, \xi_h}(z) = \frac{\Gamma\left(1 + \frac{1}{1 - \gamma_h}\right)}{\alpha_h [a(1 - \gamma_h)]^{\frac{\eta_h}{\alpha_h}}} H_{2,3}^{2,1} \left[z [a(1 - \gamma_h)]^{\frac{\beta_h}{\alpha_h}} \left| \begin{matrix} (1 + \frac{1}{1 - \gamma_h} + \frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}), (0, 1) \\ (0, 1), (0, \xi_h), (\frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}) \end{matrix} \right. \right], \quad (3.5)$$

where $D_{\alpha_h, \beta_h}^{\eta_h, \gamma_h, \xi_h}(z)$ is mentioned as (2.2).

Proof: This result can be derived applying the inverse Mellin transform on (3.3) in Corollary (3.1.A). \square

Theorem 3.3 For $\alpha_h \in R$, $\eta_h \in C$, $a > 0$, $\beta_h > 0$ and $\gamma_h > 1$,

$$D_{\alpha_h, \beta_h}^{\eta_h, \gamma_h, \xi_h}(z) = \frac{1}{\alpha_h [a(\gamma_h - 1)]^{\frac{\eta_h}{\alpha_h}} \Gamma\left(\frac{1}{\gamma_h - 1}\right)} H_{2,3}^{2,2} \left[z [a(\gamma_h - 1)]^{\frac{\beta_h}{\alpha_h}} \left| \begin{matrix} (1 - \frac{1}{\gamma_h - 1} + \frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}), (0, 1) \\ (0, 1), (0, \xi_h), (\frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}) \end{matrix} \right. \right], \quad (3.6)$$

when $\alpha_h > 0$.

$$D_{\alpha_h, \beta_h}^{\eta_h, \gamma_h, \xi_h}(z) = \frac{-1}{\alpha_h [a(\gamma_h - 1)] \Gamma\left(\frac{1}{\gamma_h - 1}\right)} H_{2,3}^{2,2} \left[z [a(\gamma_h - 1)]^{\frac{\beta_h}{\alpha_h}} \left| \begin{matrix} (0, 1), (0, \xi_h), (\frac{1}{\gamma_h - 1} + \frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}) \\ (0, 1), (0, \xi_h), (\frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}) \end{matrix} \right. \right], \quad (3.7)$$

when $\alpha_h < 0$.

$D_{\alpha_h, \beta_h}^{\eta_h, \gamma_h, \xi_h}$ is given in (2.3).

Proof: These results can be derived applying the inverse Mellin transform on (3.4) in Corollary (3.1A). \square

4. Laplace Transform(LT) of Modified Pathway Transforms(MPT)

Theorem 4.1 If $f \in L_{\eta_h, s}(0, \infty)$, $\theta_h, \eta_h \in C$ and $\beta_h > 0$, then the image of the modified type-1 pathway transform under Laplace transform is:

$$\begin{aligned} L(P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f)(\theta_h) &= \frac{\Gamma\left(1 + \frac{1}{1 - \gamma_h}\right)}{\alpha_h \theta_h [a(1 - \gamma_h)]^{\frac{\eta_h}{\alpha_h}}} \\ &\times \int_0^\infty H_{3,3}^{2,2} \left[\frac{[a(1 - \gamma_h)]^{\frac{\beta_h}{\alpha_h}} \omega_h}{\theta_h} \left| \begin{matrix} (1 + \frac{1}{1 - \gamma_h} + \frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}), (0, 1), (0, 1) \\ (0, 1), (0, \xi_h), (\frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}) \end{matrix} \right. \right] f(\omega_h) d\omega_h, \end{aligned} \quad (4.1)$$

for $\alpha_h > 0$,

and the image of the type-2 pathway transform under (LT) is

$$\begin{aligned} L(P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f)(\theta_h) &= \frac{1}{\alpha_h \theta_h [a(\gamma_h - 1)]^{\frac{\eta_h}{\alpha_h}} \Gamma\left(\frac{1}{\gamma_h - 1}\right)} \\ &\times \int_0^\infty H_{3,3}^{2,3} \left[\frac{[a(\gamma_h - 1)]^{\frac{\beta_h}{\alpha_h}} \omega_h}{\theta_h} \left| \begin{matrix} (1 - \frac{1}{\gamma_h - 1} + \frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}), (0, 1), (0, 1) \\ (0, 1), (0, \xi_h), (\frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}) \end{matrix} \right. \right] f(\omega_h) d\omega_h, \end{aligned} \quad (4.2)$$

for $\alpha_h > 0$, and

$$\begin{aligned} L(P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f)(\theta_h) &= \frac{-1}{\alpha_h \theta_h [a(\gamma_h - 1)]^{\frac{\eta_h}{\alpha_h}} \Gamma(\frac{1}{\gamma_h - 1})} \\ &\times \int_0^\infty H_{3,3}^{2,3} \left[\frac{a(\gamma_h - 1)]^{\frac{\beta_h}{\alpha_h}} \omega_h}{\theta_h} \middle| \begin{matrix} (1 - \frac{\eta_h}{\alpha_h}, -\frac{\beta_h}{\alpha_h}), (0, 1), (0, 1) \\ (0, 1), (0, \xi_h), (\frac{1}{\gamma_h - 1} - \frac{\eta_h}{\alpha_h}, -\frac{\beta_h}{\alpha_h}) \end{matrix} \right] f(\omega_h) d\omega_h, \end{aligned} \quad (4.3)$$

for $\alpha_h < 0$.

Proof: For (4.1), using the basic definition of Laplace Transform (LT) on Modified type-1 pathway transform (2.1) and (2.2), then applying the inverse Mellin transform of (3.3), we get

$$L(P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f)(\theta_h) = \int_0^\infty e^{-\theta_h t} \int_0^\infty D_{\alpha_h, \beta_h}^{\eta_h, \gamma_h, \xi_h} f(\omega_h t) d\omega_h dt \quad (4.4)$$

$$\begin{aligned} &= \frac{\Gamma(1 + \frac{1}{1-\gamma_h})}{\alpha_h [a(1-\gamma_h)]^{\frac{\eta_h}{\alpha_h}}} \frac{1}{2\pi i} \int_0^\infty e^{-\theta_h t} \\ &\times \int_0^\infty \int_L \frac{\Gamma(s)\Gamma(1-s)\Gamma(\frac{\eta_h + \beta_h s}{\alpha_h})}{\Gamma(1-\xi_h s)\Gamma(1 + \frac{1}{1-\gamma_h} + \frac{\eta_h + \beta_h s}{\alpha_h})} [a(1-\gamma_h)]^{\frac{\beta_h}{\alpha_h}} (\omega_h t)^{-s} ds f(\omega_h) d\omega_h dt, \end{aligned} \quad (4.5)$$

on interchanging the integrals and then applying Laplace transform, the following expression occurs

$$\begin{aligned} L(P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f)(\theta_h) &= \frac{\Gamma(1 + \frac{1}{1-\gamma_h})}{\alpha_h [a(1-\gamma_h)]^{\frac{\eta_h}{\alpha_h}}} \frac{1}{2\pi i} \int_0^\infty \int_L \frac{\Gamma(s)\Gamma(1-s)\Gamma(\frac{\eta_h + \beta_h s}{\alpha_h})}{\Gamma(1-\xi_h s)\Gamma(1 + \frac{1}{1-\gamma_h} + \frac{\eta_h + \beta_h s}{\alpha_h})} \\ &\times [a(1-\gamma_h)]^{\frac{\beta_h}{\alpha_h}} (\omega_h)^{-s} \int_0^\infty e^{-\theta_h t} t^{-s} dt ds f(\omega_h) d\omega_h \end{aligned} \quad (4.6)$$

$$\begin{aligned} &= \frac{\Gamma(1 + \frac{1}{1-\gamma_h})}{\alpha_h [a(1-\gamma_h)]^{\frac{\eta_h}{\alpha_h}}} \frac{1}{2\pi i} \int_0^\infty \int_L \frac{\Gamma(s)\Gamma(1-s)\Gamma(\frac{\eta_h + \beta_h s}{\alpha_h})}{\Gamma(1-\xi_h s)\Gamma(1 + \frac{1}{1-\gamma_h} + \frac{\eta_h + \beta_h s}{\alpha_h})} \\ &\times [a(1-\gamma_h)]^{\frac{\beta_h}{\alpha_h}} (\omega_h)^{-s} ds f(\omega_h) d\omega_h, \end{aligned} \quad (4.7)$$

provided $R(s) > 0$, $R(1-s) > 0$ and $R(\frac{\eta_h + \beta_h s}{\alpha_h}) > 0$. \square

By interpretation, it gives the result mentioned in (4.1). Similarly, the same steps are used to find the results (4.2) and (4.3).

5. Modified Pathway Transform and Fractional Integral Operator

Definition 5.1 Saigo fractional integral operator is described by Saigo [18] for $R(\lambda) > 0$ with $\lambda, \eta, \nu \in C$ in the following form:

$$\left(I_{0+}^{\lambda_A, \eta_A, \nu_A} f \right) (y) = \frac{y^{-\lambda_A - \eta_A}}{\Gamma(\lambda_A)} \int_0^y (y-t)^{\lambda_A - 1} {}_2F_1(\lambda_A + \eta_A, -\nu_A; \lambda_A; 1 - \frac{t}{y}) f(t) dt, \quad (5.1)$$

and

$$\left(I_{0-}^{\lambda_A, \eta_A, \nu_A} f \right) (y) = \frac{1}{\Gamma(\lambda_A)} \int_y^\infty (t-y)^{\lambda_A - 1} {}_2F_1(\lambda_A + \eta_A, -\nu_A; \lambda_A; 1 - \frac{y}{t}) f(t) dt, \quad (5.2)$$

where ${}_2F_1(\beta_k, \sigma_l; \lambda_A; z)$ represents the Gauss hypergeometric function which is defined for $\beta_k, \sigma_l, \lambda_A \in C$, $\lambda_A \neq 0, -1, -2, \dots$ in ([4], 2.1(2)) as:

$${}_2F_1(\beta_k, \sigma_l; \lambda_A; z) = \sum_{n=0}^{\infty} \frac{(\beta_k)_n (\sigma_l)_n z^n}{(\lambda_A)_n n!}, \quad (5.3)$$

where $(\beta_k)_n, (\sigma_l)_n$ and $(\lambda_A)_n$ represent the Pochhammer symbol. For $\lambda_A \in C$ by $(\lambda_A)_0 = 1$, $(\lambda_A)_n = \lambda_A(\lambda_A + 1) \cdots (\lambda_A + n - 1) = \frac{\Gamma(\lambda_A + n)}{\Gamma(\lambda_A)}$, $n = 1, 2, 3, \dots$, $\lambda_A \neq 0$ whenever $\Gamma(\lambda_A)$ exists. The series given in (5.3) is absolutely convergent for $|z| < 1$ and $R(\lambda_A - \beta_k - \sigma_l) > 0$,

and

$${}_2F_1(\beta_k, \sigma_l; \lambda_A; 1) = \frac{\Gamma(\lambda_A)\Gamma(\lambda_A - \beta_k - \sigma_l)}{\Gamma(\lambda_A - \beta_k)\Gamma(\lambda_A - \sigma_l)}, \quad R(\lambda_A - \beta_k - \sigma_l) > 0. \quad (5.4)$$

When $\eta_A = -\lambda_A$, the operators (5.1) and (5.2) reduce to the Liouville fractional integrals [21] as:

$$\left(I_{0+}^{\lambda_A, -\lambda_A, \nu_A} f\right)(y) = \left(I_{0+}^{\lambda_A} f\right)(y) = \frac{1}{\Gamma(\lambda_A)} \int_0^y (y-t)^{\lambda_A-1} f(t) dt, \quad R(\lambda_A) > 0 \quad (5.5)$$

and

$$\left(I_{-}^{\lambda_A, -\lambda_A, \nu_A} f\right)(y) = \left(I_{-}^{\lambda_A} f\right)(y) = \frac{1}{\Gamma(\lambda_A)} \int_y^{\infty} (t-y)^{\lambda_A-1} f(t) dt, \quad R(\lambda_A) > 0. \quad (5.6)$$

When $\eta_A = 0$, (5.1) and (5.2) convert to Erdelyi-Kober fractional operators in the following form:

$$\left(I_{0+}^{\lambda_A, 0, \nu_A} f\right)(y) = \left(I_{\nu_A, \lambda_A}^+ f\right)(y) = \frac{y^{\lambda_A - \nu_A}}{\Gamma(\lambda_A)} \int_0^y (y-t)^{\lambda_A-1} t^{\nu_A} f(t) dt, \quad y \in R_+ \quad (5.7)$$

and

$$\begin{aligned} \left(I_{-}^{\lambda_A, 0, \nu_A} f\right)(y) &= \left(K_{\nu_A, \lambda_A}^- f\right)(y) = \left(I_{-, 1, \nu_A}^{\lambda_A} f\right)(y) \frac{y^{\nu_A}}{\Gamma(\lambda_A)} \\ &\times \int_y^{\infty} (t-y)^{\lambda_A-1} t^{-\lambda_A - \nu_A} f(t) dt, \quad y \in R_+. \end{aligned} \quad (5.8)$$

The composition formulae of generalized fractional integral operators with first kind of Bessel function were studied by Kilbas and Sebastian [3]. The theory of generalized fractional integrals and differential operators, special cases, and properties are mentioned in Kiryakova [22].

The Power function under the left hand side Saigo integrals is:

$$I_{0+}^{\lambda_A, \eta_A, \nu_A} (y^{\phi_A}) = \frac{\Gamma(\phi_A + 1)\Gamma(\phi_A + \nu_A - \eta_A + 1)}{\Gamma(\phi_A - \eta_A + 1)\Gamma(\phi_A + \lambda_A + \nu_A + 1)} y^{\phi_A - \eta_A}, \quad (5.9)$$

for $\lambda_A, \eta_A, \nu_A, \phi_A \in C$ and $R(\lambda_A) > 0$, $R(\phi_A + 1) > 0$ and $R(\phi_A + \nu_A - \eta_A + 1) > 0$.

when $\eta_A = 0$, in (5.9) then the image of the power function under the left hand side Erdelyi-Kober fractional integral operator is

$$I_{0+}^{\lambda_A, 0, \nu_A} (y^{\phi_A}) = I_{\nu_A, \lambda_A} y^{\phi_A} = \frac{\Gamma(\phi_A + \nu_A + 1)}{\Gamma(\phi_A + \lambda_A + \nu_A + 1)} y^{\phi_A}. \quad (5.10)$$

If $\eta_A = -\lambda_A$, in (5.9) then the composition of the Riemann-Liouville fractional integral and power function is:

$$I_{0+}^{\lambda_A, -\lambda_A, \nu_A} (y^{\phi_A}) = I_{0+}^{\lambda_A} y^{\phi_A} = \frac{\Gamma(\phi_A + 1)}{\Gamma(\phi_A + \lambda_A + 1)} y^{\phi_A + \lambda_A}. \quad (5.11)$$

Results (5.9), (5.10) and (5.11) are useful to obtain the results of the next theorem.

Theorem 5.1 Let $f \in L_{\eta_h, s}(0, \infty)$, $\lambda_A, \eta_h, \nu_A \in C$ and $R(\lambda_A) > 0$, then the image of modified type-1 pathway transform under the left hand side Saigo operator is:

$$\begin{aligned} \left(I_{0+}^{\lambda_A, \eta_A, \nu_A} P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f\right)(y) &= \frac{\Gamma\left(\frac{1}{1-\gamma_h} + 1\right)}{\alpha_h y^{\eta_A} [a(1-\gamma_h)]^{\frac{\eta_h}{\alpha_h}}} \\ &\times \int_0^{\infty} H_{4,5}^{2,3} \left[[a(1-\gamma_h)]^{\frac{\beta_h}{\alpha_h}} \omega_h y \left| \begin{array}{l} \left(1 + \frac{1}{1-\gamma_h} + \frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}\right), (-\nu_A + \eta_A, 1), (0, 1), (0, 1) \\ (0, 1), (0, \xi_h), \left(\frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}\right), (-\eta_A, 1), (-\lambda_A - \nu_A, 1) \end{array} \right. \right] \\ &\times f(\omega_h) d\omega_h, \end{aligned} \quad (5.12)$$

where $\alpha_h > 0$ and the image of modified pathway type-2 transform under Saigo-operator is:

$$\begin{aligned} \left(I_{0+}^{\lambda_A, \eta_A, \nu_A} P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f \right) (y) &= \frac{1}{\alpha_h y^{\eta_A} [a(\gamma_h - 1)]^{\frac{\eta_h}{\alpha_h}} \Gamma\left(\frac{1}{\gamma_h - 1}\right)} \\ &\times \int_0^\infty H_{4,5}^{2,4} \left[[a(\gamma_h - 1)]^{\frac{\beta_h}{\alpha_h}} \omega_h y \left| \begin{array}{l} \left(1 - \frac{1}{\gamma_h - 1} + \frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}\right), (-\nu_A + \eta_A, 1), (0, 1), (0, 1) \\ (0, 1), (0, \xi_h), \left(\frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}\right), (\eta_A, 1), (-\lambda_A - \nu_A, 1) \end{array} \right. \right] \\ &\times f(\omega_h) d\omega_h, \end{aligned} \quad (5.13)$$

$$\begin{aligned} \left(I_{0+}^{\lambda_A, \eta_A, \nu_A} P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f \right) (y) &= \frac{-1}{\alpha_h y^{\eta_A} [a(\gamma_h - 1)]^{\frac{\eta_h}{\alpha_h}} \Gamma\left(\frac{1}{\gamma_h - 1}\right)} \\ &\times \int_0^\infty H_{4,5}^{2,4} \left[a(1 - \gamma_h) \left| \begin{array}{l} \left(1 - \frac{\eta_h}{\alpha_h}, -\frac{\beta_h}{\alpha_h}\right), (-\nu_A + \eta_A, 1), (0, 1), (0, 1) \\ (0, 1), (0, \xi_h), \left(\frac{1}{\gamma_h - 1} - \frac{\eta_h}{\alpha_h}, -\frac{\beta_h}{\alpha_h}\right), (\eta_A, 1), (-\lambda_A - \nu_A, 1) \end{array} \right. \right] \\ &\times f(\omega_h) d\omega_h, \end{aligned} \quad (5.14)$$

for $\alpha_h < 0$.

Proof: Using the Saigo operator (5.1) and the modified pathway transform (2.1) with (2.2), then making use of the inverse Mellin transform of (3.3), we obtain

$$\begin{aligned} \left(I_{0+}^{\lambda_A, \eta_A, \nu_A} P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f \right) (y) &= \frac{y^{-\lambda_A - \eta_A}}{\Gamma(\lambda_A)} \int_0^y (y - t)^{\lambda_A - 1} {}_2F_1(\lambda_A + \eta_A, -\nu_A; \lambda_A; 1 - \frac{t}{y}) \\ &\times \int_0^\infty D_{\alpha_h, \beta_h}^{\eta_h, \gamma_h, \xi_h} f(\omega_h t) d\omega_h dt \end{aligned} \quad (5.15)$$

$$\begin{aligned} &= \frac{y^{-\lambda_A - \eta_A} \Gamma\left(1 + \frac{1}{1 - \gamma_h}\right)}{\alpha_h \Gamma(\lambda_A) [a(1 - \gamma_h)]^{\frac{\eta_h}{\alpha_h}}} \int_0^y (y - t)^{\lambda_A - 1} {}_2F_1(\lambda_A + \eta_A, -\nu_A; \lambda_A; 1 - \frac{t}{y}) \\ &\times \int_0^\infty \frac{1}{2\pi i} \int_L \frac{\Gamma(s) \Gamma(1 - s) \Gamma\left(\frac{\eta_h + \beta_h s}{\alpha_h}\right)}{\Gamma(1 - \xi_h s) \Gamma\left(1 + \frac{1}{1 - \gamma_h} + \frac{\eta_h + \beta_h s}{\alpha_h}\right)} \\ &\times [a(1 - \gamma_h)]^{\frac{\beta_h}{\alpha_h}} (\omega_h t)^{-s} ds f(\omega_h) d\omega_h dt, \end{aligned} \quad (5.16)$$

for $R(s) > 0$ and $R\left(\frac{\eta_h + \beta_h s}{\alpha_h}\right) > 0$.

Now, interchanging the integrals and using the result (5.9), we get

$$\begin{aligned} &= \frac{\Gamma\left(1 + \frac{1}{1 - \gamma_h}\right)}{\alpha_h [a(1 - \gamma_h)]^{\frac{\eta_h}{\alpha_h}}} \int_0^\infty \frac{1}{2\pi i} \int_L \frac{\Gamma(s) \Gamma(1 - s) \Gamma\left(\frac{\eta_h + \beta_h s}{\alpha_h}\right)}{\Gamma(1 - \xi_h s) \Gamma\left(1 + \frac{1}{1 - \gamma_h} + \frac{\eta_h + \beta_h s}{\alpha_h}\right)} \\ &\times [a(1 - \gamma_h)]^{\frac{\beta_h}{\alpha_h}} (\omega_h)^{-s} f(\omega_h) d\omega_h \left(I_{0+}^{\lambda_A, \eta_A, \nu_A} t^{-s} \right) (y) ds \end{aligned} \quad (5.17)$$

$$\begin{aligned} &= \frac{\Gamma\left(1 + \frac{1}{1 - \gamma_h}\right)}{\alpha_h y^{\eta_A} [a(1 - \gamma_h)]^{\frac{\eta_h}{\alpha_h}}} \int_0^\infty \frac{1}{2\pi i} \int_L \frac{\Gamma(s) \Gamma(1 - s) \Gamma\left(\frac{\eta_h + \beta_h s}{\alpha_h}\right)}{\Gamma(1 - \xi_h s) \Gamma\left(1 + \frac{1}{1 - \gamma_h} + \frac{\eta_h + \beta_h s}{\alpha_h}\right)} \\ &\times \frac{\Gamma(1 - s)}{\Gamma(-s - \eta_A + 1)} \frac{\Gamma(-s + \nu_A - \eta_A + 1)}{\Gamma(-s + \lambda_A + \nu_A + 1)} [a(1 - \gamma_h)]^{\frac{\beta_h}{\alpha_h}} (\omega_h x)^{-s} f(\omega_h) d\omega_h ds, \end{aligned} \quad (5.18)$$

for $R(s) > 0$, $R(1 - s) > 0$, $R(\eta_h + \beta_h s) > 0$ and $R(1 + \nu_A - s - \eta_A) > 0$. \square

This leads to the result (5.12).

The image of the modified type-2 pathway transform can also be found by applying the inversion formula of the Mellin transform on (5.12) and using the H-function.

Corollary 5.1 *Under the same conditions given for theorem 5.1 with $\eta_A = 0$, the composition formula for the left-sided Erdélyi-Kober operator with the modified type-1 pathway transform is:*

$$\begin{aligned} \left(I_{0+}^{\lambda_A, 0, \nu_A} P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f \right) (y) &= \left(I_{0+}^{\nu_A, \lambda_A} P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f \right) = \frac{\Gamma\left(\frac{1}{1-\gamma_h} + 1\right)}{\alpha_h [a(1-\gamma_h)]^{\frac{\eta_h}{\alpha_h}}} \\ &\times \int_0^\infty H_{3,4}^{2,2} \left[[a(1-\gamma_h)]^{\frac{\beta_h}{\alpha_h}} \omega_h y \left| \begin{array}{l} \left(1 + \frac{1}{1-\gamma_h} + \frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}\right), (-\nu_A, 1), (0, 1) \\ (0, 1), (0, \xi_h), \left(\frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}\right), (-\lambda_A - \nu_A, 1) \end{array} \right. \right] \\ &\times f(\omega_h) d\omega_h, \end{aligned} \quad (5.19)$$

and the composition formula for the left-sided Erdélyi-Kober operator with modified type-2 pathway transform are:

$$\begin{aligned} \left(I_{0+}^{\lambda_A, 0, \nu_A} P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f \right) (y) &= \left(I_{0+}^{\nu_A, \lambda_A} P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f \right) = \frac{1}{\alpha_h [a(\gamma_h - 1)]^{\frac{\eta_h}{\alpha_h}} \Gamma\left(\frac{1}{\gamma_h - 1}\right)} \\ &\times \int_0^\infty H_{3,4}^{2,3} \left[[a(\gamma_h - 1)]^{\frac{\beta_h}{\alpha_h}} \omega_h y \left| \begin{array}{l} \left(1 + \frac{1}{1-\gamma_h} + \frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}\right), (-\nu_A, 1), (0, 1) \\ (0, 1), (0, \xi_h), \left(\frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}\right), (-\lambda_A - \nu_A, 1) \end{array} \right. \right] \\ &\times f(\omega_h) d\omega_h, \end{aligned} \quad (5.20)$$

for $\alpha_h > 0$

$$\begin{aligned} \left(I_{0+}^{\lambda_A, 0, \nu_A} P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f \right) (y) &= \left(I_{0+}^{\nu_A, \lambda_A} P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f \right) = \frac{-1}{\alpha_h [a(1-\gamma_h)]^{\frac{\eta_h}{\alpha_h}} \Gamma\left(\frac{1}{\gamma_h - 1}\right)} \\ &\times \int_0^\infty H_{3,4}^{2,3} \left[[a(\gamma_h - 1)]^{\frac{\beta_h}{\alpha_h}} \omega_h y \left| \begin{array}{l} \left(1 - \frac{\eta_h}{\alpha_h}, -\frac{\beta_h}{\alpha_h}\right), (-\nu_A, 1), (0, 1) \\ (0, 1), (0, \xi_h), \left(\frac{1}{\gamma_h - 1} - \frac{\eta_h}{\alpha_h}, -\frac{\beta_h}{\alpha_h}\right), (-\lambda_A - \nu_A, 1) \end{array} \right. \right] \\ &\times f(\omega_h) d\omega_h, \end{aligned} \quad (5.21)$$

for $\alpha_h < 0$.

Corollary 5.2 *Under the same conditions of theorem 5.1 with $\eta_A = -\lambda_A$, the composition formula for the left hand sided Riemann-Liouville fractional operator with modified type-1 pathway transform is:*

$$\begin{aligned} \left(I_{0+}^{\lambda_A, -\lambda_A, \nu_A} P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f \right) (y) &= \left(I_{0+}^{\lambda_A} P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f \right) = \frac{y^{\lambda_A} \Gamma\left(\frac{1}{1-\gamma_h} + 1\right)}{\alpha_h [a(1-\gamma_h)]^{\frac{\eta_h}{\alpha_h}}} \\ &\times \int_0^\infty H_{3,4}^{2,2} \left[[a(1-\gamma_h)]^{\frac{\beta_h}{\alpha_h}} \omega_h y \left| \begin{array}{l} \left(1 + \frac{1}{1-\gamma_h} + \frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}\right), (0, 1), (0, 1) \\ (0, 1), (0, \xi_h), \left(\frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}\right), (-\lambda_A, 1) \end{array} \right. \right] f(\omega_h) d\omega_h, \end{aligned} \quad (5.22)$$

and the composition formula for the left hand sided Riemann-Liouville fractional operator with modified type-2 pathway transform are:

$$\begin{aligned} \left(I_{0+}^{\lambda_A, -\lambda_A, \nu_A} P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f \right) (y) &= \left(I_{0+}^{\lambda_A} P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f \right) = \frac{y^{\lambda_A}}{\alpha_h [a(\gamma_h - 1)]^{\frac{\eta_h}{\alpha_h}} \Gamma\left(\frac{1}{\gamma_h - 1}\right)} \\ &\times \int_0^\infty H_{3,4}^{2,3} \left[[a(\gamma_h - 1)]^{\frac{\beta_h}{\alpha_h}} \omega_h y \left| \begin{array}{l} \left(1 - \frac{1}{\gamma_h - 1} + \frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}\right), (0, 1), (0, 1) \\ (0, 1), (0, \xi_h), \left(\frac{\eta_h}{\alpha_h}, \frac{\beta_h}{\alpha_h}\right), (-\lambda_A, 1) \end{array} \right. \right] f(\omega_h) d\omega_h, \end{aligned} \quad (5.23)$$

for $\alpha_h > 0$ and

$$\begin{aligned} \left(I_{0+}^{\lambda_A, -\lambda_A, \nu_A} P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f \right) (y) &= \left(I_{0+}^{\lambda_A} P_{\eta_h}^{\alpha_h, \beta_h, \gamma_h, \xi_h} f \right) = \frac{-y^{\lambda_A}}{\alpha_h [a(1-\gamma_h)]^{\frac{\eta_h}{\alpha_h}} \Gamma\left(\frac{1}{\gamma_h - 1}\right)} \\ &\times \int_0^\infty H_{3,4}^{2,3} \left[[a(\gamma_h - 1)]^{\frac{\beta_h}{\alpha_h}} \omega_h y \left| \begin{array}{l} \left(1 - \frac{\eta_h}{\alpha_h}, -\frac{\beta_h}{\alpha_h}\right), (0, 1), (0, 1) \\ (0, 1), (0, \xi_h), \left(\frac{1}{\gamma_h - 1} - \frac{\eta_h}{\alpha_h}, -\frac{\beta_h}{\alpha_h}\right), (-\lambda_A, 1) \end{array} \right. \right] \\ &\times f(\omega_h) d\omega_h, \end{aligned} \quad (5.24)$$

for $\alpha_h < 0$.

6. Conclusion

The main purpose of this research is to study a class of Modified Pathway Transform taking the Mittag-Leffler function in the kernel. We have discussed some intriguing results and corollaries related to the composition of the modified pathway transform with Mellin and Laplace transform. We explored the behaviour of the modified transform under fractional integral operators. Modified Pathway transform employing Mittag-Leffler function is the most prevalent type. Thus, the results of our work are expected to have some potential applications in a variety of domains, including engineering, astrophysics, and thermonuclear integrals.

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