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Topological Insights into Weighted Statistical Convergence for Triple Sequence

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ABSTRACT: This research explores the concept of weighted statistical convergence for triple sequences under a topological perspective, an extension of weighted statistical convergence by incorporating three weight functions g, h and i into the framework, refining the notion of convergence to accommodate variations in density and distribution of sequence elements. The study examines the properties and behaviors of these sequences, emphasizing their applications in topology. We provide characterizations, theorems, and examples to illustrate the nuanced convergence criteria under different topological settings, highlighting the impact of weights on convergence rates and patterns. This generalized approach offers deeper insights and more flexible tools for analyzing convergence in complex structures. The work also discusses the product spaces, which play a critical role in understanding the interaction and convergence properties of multiple sequences simultaneously. By leveraging the structure of product spaces, the study provides a comprehensive view of how triple sequences converge under three weight functions and topological features.

Key Words: Asymptotic density, weighted density, weight functions, product space.

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1. Introduction

Schoenberg [15] and Fast [10] explored the idea as a summability technique and offered a few fundamental characteristics of statistical convergence. The asymptotic density of any set $S \subseteq \mathbb{N}$ is represented by the symbol $\delta(S)$, where

$$\delta(S) = \lim_{n\to\infty} \frac{|\{k \le n : k \in S\}|}{n}$$

The concept of convergence was broadened in 1951 by Fast [10] to encompass statistical convergence for real number sequences. The notion of statistical convergence in a topological space was first developed by Maio and Kočinac in 2008. In a topological space (X, τ), a sequence $\{a_n : n \in \mathbb{N}\}$ is said to be statistically convergent to $a \in X$ if the natural density of the set $\{n \in \mathbb{N} : a_n \notin U\}$ is 0 for each neighborhood U of a.

The authors of [5] suggested a modified version of asymptotic density in which $n^\alpha, 0 < \alpha < 1$ was used instead of n, which is the statistical convergence of order α. The notation $\delta^\alpha(S)$, which is as follows, represents the natural density of the set S of order α.

$$\delta^\alpha(S) = \lim_{n\to\infty} \frac{|\{k \le n : k \in S\}|}{n^\alpha}.$$

A more broader type of natural density was described by the authors of [1] by substituting a weight function g for n^α, such that $g : \mathbb{N} \longrightarrow [0, \infty)$, such that $\lim_{n\to\infty} g(n) = \infty$ and $\lim_{n\to\infty} \frac{n}{g(n)} \neq 0$. The following formula is used to define the weighted density as $\delta_g(S)$ of the same set $S \subset \mathbb{N}$ which is given by,

$$\delta_g(S) = \lim_{n\to\infty} \frac{|\{k \le n : k \in S\}|}{g(n)}.$$

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Alfred Pringsheim, in the year 1900, was the first German mathematician to give the concept of convergence for double sequence. Early in the year 2000, Mursaleen and Edely [13] made contributions to the development of double statistical convergence. When a $n_0 \in \mathbb{N}$ such that $|x_{m,n} - x_0| < \epsilon$ for all $m, n \geq n_0$, then a double sequence $\{x_{m,n} : (m,n) \in \mathbb{N} \times \mathbb{N}\}$ is said to have double statistical convergence and converges to x_0. Eventually, restricting the work of [13] with the two parameters α and β, where $0 < \alpha, \beta < 1$, Rakshit et al. present the concept of double statistical convergence of order (α, β) for double sequences.

The concept of double weighted statistical convergence of double sequence with two weight functions g and h is adopted by the author in accordance with the work of [1,8,13].

Later in 2007, Sahiner et al. [14] extended the work of statistical convergence for triple sequence, which has been studied to understand the behavior of more complex settings in higher dimensions.

Now in this paper we follow up the work of [1,2,3,4,6,11,12] to extend the notion of weighted statistical convergence for triple sequence adopted with three weight functions g, h, and i that are defined with a mapping $g : \mathbb{N} \longrightarrow [0, \infty)$ such that $\lim_{m\to\infty} \frac{m}{g(m)} \neq 0$ and $\lim_{m\to\infty} g(m) = \infty$; $h : \mathbb{N} \longrightarrow [0, \infty)$ such that $\lim_{n\to\infty} \frac{n}{h(n)} \neq 0$ and $\lim_{n\to\infty} h(n) = \infty$; and $i : \mathbb{N} \longrightarrow [0, \infty)$ such that $\lim_{i\to\infty} \frac{i}{i(p)} \neq 0$ and $\lim_{i\to\infty} i(p) = \infty$.

2. Preliminaries

This portion sets the stage for the reader's convenience. For symbols, terms, and concepts, we follow King et al. [9]. Rather, no separated axioms have been adopted, otherwise stated.

Definition 2.1 *[13] A real double sequence $x = \{x_{mn}\}$ is said to be statistically convergent to x_0 if for each $\epsilon > 0$ the set $\{(m,n), k \leq m, l \leq n : |x_{mn} - x_0| \leq \epsilon\}$ has double density zero.*

Definition 2.2 *[5,7] Let $x = \{x_k : k \in \mathbb{N}\}$ be a real sequence and $0 < \alpha < 1$ be given. The sequence $\{x_k : k \in \mathbb{N}\}$ is said to be statistically convergent of order α if there is a real number L such that*

$$\lim_{n\to\infty} \frac{|\{k \leq n : |x_k - L| \geq \varepsilon\}|}{n^\alpha} = 0.$$

and we write $s^\alpha\text{-}\lim_{n\to\infty} x_k = L$.

Definition 2.3 *[3] A sequence $\{a_n : n \in \mathbb{N}\}$ in a topological space X is said to be statistically convergence of order α $(0 < \alpha < 1)$ to $a \in X$, if for every neighborhood U of a, $\delta^\alpha(\{n \in \mathbb{N} : a_n \notin U\})=0$.*

Definition 2.4 *[1] Let $P \subset \mathbb{N}$ and $P_n = \{k \leq n : k \in P\}$. Then the notation $\delta_g(P)$ defines the weight density of the set P is given by*

$$\delta_g(P) = \lim_{n\to\infty} \frac{|\{k \leq n : k \in P\}|}{g(n)}.$$

Definition 2.5 *[8] Let g be a weight function. A sequence $\{x_n : n \in \mathbb{N}\}$ is said to be g-statistical convergence to x_0 in a topological space (X, τ), if for every neighborhood U of x_0, $\delta_g(\{n \in \mathbb{N} : x_n \notin U\}) = 0$.*

Definition 2.6 *[8] A sub sequence $\{x_{n_k} : k \in \mathbb{N}\}$ of a sequence $\{x_n : n \in \mathbb{N}\}$ is said to be s_g-dense if $\{n \in \mathbb{N} : x_n \in \{x_{n_k} : k \in \mathbb{N}\}\}$ is s_g-dense.*

3. $s_{g,h,i}$ convergence

In order to create a novel theory of weighted density for a three-dimensional set of natural numbers $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$, we incorporated three readily verifiable weight functions g, h, and i that maps from natural numbers to positive real numbers (i.e., $g, h, i : \mathbb{N} \longrightarrow [0, \infty)$) defined as $g(m) = \ln(1+m)$ and $g(m) = m^\alpha$ where $0 < \alpha < 1$; $h(n) = \ln(1+n)$ and $h(n) = n^\alpha$ where $0 < \alpha < 1$ as well as $i(p) = \ln(1+p)$ and $i(p) = p^\alpha$ where $0 < \alpha < 1$. In this research, a number of examples will be developed using these weight functions.

Definition 3.1 *Let $T \subset \mathbb{N}\times\mathbb{N}\times\mathbb{N}$, where $\mathbb{N}$ is the set of all natural numbers. Then the triple weighted density of the set T is denoted as $\delta_{g,h,i}(T)$ and given by*

$$\delta_{g,h,i}(T) = \lim_{m,n,p\to\infty} \frac{|\{(j,k,l)\le(m,n,p):(j,k,l)\in T\}|}{g(m)\,h(n)\,i(p)}$$

Definition 3.2 *In a topological space (X,τ), a triple sequence $\{x_{m,n,p}:(m,n,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ is considered triple weighted statistical convergence (or shortly $s_{g,h,i}$ convergence) to x_0 if for each neighborhood of W of x_0, $\delta_{g,h,i}(\{(m,n,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:x_{m,n,p}\notin W\})=0$. Under this scenario, we can write $s_{g,h,i}-\lim_{m,n,p\to\infty} x_{m,n,p}=x_0$.*

Theorem 3.1 *If $\lim_{m,n,p\to\infty}\frac{m.n.p}{g(m)h(n)i(p)}$ exists then every $s_{g,h,i}$ - convergence sequence is s-triple convergence.*

Proof: Let $\{x_{m,n,p}:(m,n,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ be an $s_{g,h,i}$ - convergent sequence in (X,τ) and converges to x_0. Then for each neighborhood W of x_0, $\delta_{g,h,i}(\{(m,n,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:x_{m,n,p}\notin W\})=0$.
Now, this can be written as

$$\lim_{m,n,p\to\infty}\frac{|\{(j,k,l)\in T:(j,k,l)\le(m,n,p)\}|}{g(m)\,h(n)\,i(p)}=0$$

where $T=\{(m,n,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:x_{m,n,p}\notin W\}$ and $\delta_{g,h,i}(T)=0$.
i.e.,

$$\lim_{m,n,p\to\infty}\frac{|\{(j,k,l)\in T:(j,k,l)\le(m,n,p)\}|}{m.n.p}\times\lim_{m,n,p\to\infty}\frac{m.n.p}{g(m).h(n).i(p)}=0.$$

As $\lim_{m,n,p\to\infty}\frac{m.n.p}{g(m)h(n)i(p)}$ exists. So, it will not be equal to zero. So,

$$\lim_{m,n,p\to\infty}\frac{|\{(j,k,l)\in T:(j,k,l)\le(m,n,p)\}|}{m.n.p}=0.$$

Thus, $\delta(\{(m,n,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:x_{m,n,p}\notin W\})=0$.
Hence the sequence $\{x_{m,n,p}:(m,n,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ is s-triple convergent. □

When the weight functions are $g(m)=m, h(n)=n$, and $i(p)=p$, then every triple weighted statistical convergence and triple statistical convergence are equivalent to each other. However, the preceding theorem's opposite might not be true. So, we will construct a counterexample.

Example 3.1 *Let $X=\{a,b,c,d\}$ and $\tau=\{\emptyset,X,\{a,b\},\{c,d\}\}$. Then (X,τ) is a topological space.*
Let $g(m)=\ln(1+m); h(n)=\ln(1+n)$ and $i(p)=\ln(1+p)$ be three weight functions. Considering, $\{x_{m,n,p}:(m,n,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ be the triple sequence where,

$$x_{m,n,p}=\begin{cases}\{a,b\}, & \text{if } m=j^2, n=k^3 \text{ and } p=l^4 \text{ for some } (j,k,l)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}\\ \{c,d\}, & \text{otherwise}\end{cases}$$

Here open neighborhood of $\{a,b\}$ are $W_1=X$ and $W_2=\{a,b\}$.
So, for the neighborhood W_2 of $\{a,b\}, \delta_{g,h,i}(\{(m,n,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:x_{m,n,p}\notin W_2\})=\delta_{g,h,i}(\{(j^2,k^3,l^4):(j,k,l)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}\})$ that implies

$$\lim_{m,n,p\to\infty}\frac{m.n.p}{ln(1+m^2).ln(1+n^3).ln(1+p^4)}=\infty\neq 0.$$

Therefore, $x_{m,n,p}\xrightarrow{s_{g,h,i}-\lim}\!\!\!\!\!\!\!\!/\;\;\;\{a,b\}$.
Similarly, open neighborhood of $\{c,d\}$ are $V_1=X$ and $V_2=\{c,d\}$. So, for the neighborhood V_2 of $\{c,d\}$,

$$\begin{aligned}&=\delta_{g,h,i}(\{(m,n,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:x_{m,n,p}\notin V_2\})\\&=\delta_{g,h,i}(\{(1,1,2),(1,1,3),(1,1,5),\cdots,(2,2,1),(2,2,2),\cdots,(3,3,2),(3,3,3),\cdots\})\\&=\infty\neq 0.\end{aligned}$$

Therefore $x_{m,n,p} \xrightarrow{s_{g,h,i}-\lim}\!\!\!\!\!\!\!\!/\;\;\;\; \{c,d\}$.
Thus the triple sequence $\{x_{m,n,p}\}$ *is not* $s_{g,h,i}$ *- convergent.*
But for the neighborhood W_1 *of* X *and* W_2 *of* $\{a,b\}$, $\delta(\{(m,n,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N} : x_{m,n,p}\notin W_1\}) = 0$ *and* $\delta(\{(m,n,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N} : x_{m,n,p}\notin W_2\}) = \delta(\{(j^2,k^3,l^4) : (j,k,l)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}\}) = 0$ *respectively.*
So, for every open neighborhood W *of* $\{a,b\}$, $\delta(\{(m,n,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N} : x_{m,n,p}\notin W\}) = 0$. *Therefore* $x_{m,n,p} \xrightarrow{s-\lim} \{a,b\}$. *Thus the triple sequence* $\{x_{m,n,p}\}$ *is s-triple convergent.*
Hence the triple sequence $\{x_{m,n,p} : (m,n,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ *is s-triple convergent but not* $s_{g,h,i}$*-convergent.*

Theorem 3.2 *If a sequence* $\{x_{m,n,p} : (m,n,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ *is* $s_{g,h,i}$*-convergent sequence and converges to* x_0 *in a first countable space* X. *Then there exists* $\mu = \{(Q_1,R_1,S_1)\leq(Q_2,R_2,S_2)\leq\cdots\}\subseteq\mathbb{N}\times\mathbb{N}\times\mathbb{N}$ *such that*

$$\delta_{g,h,i}(\mu) = \lim_{m,n,p\to\infty}\frac{m.n.p}{g(m).h(n).i(p)} \text{ and } \lim_{m,n,p\to\infty, r\in\mathbb{N}} x_{m,n,p} = x_0.$$

Proof: Let $\{x_{m,n,p} : (m,n,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ be an $s_{g,h,i}$-convergent sequence in a first countable space X and converges to x_0.
Let $\cdots\subseteq B_{3,x_0}\subseteq B_{2,x_0}\subseteq B_{1,x_0}$ be countable decreasing local base of the limit of x_0.
For each $r\in\mathbb{N}$, we build $\mu_r = \{(m,n,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N} : x_{m,n,p}\in B_{r,x_0}\}$. Clearly, $\cdots\subseteq\mu_3\subseteq\mu_2\subseteq\mu_1$. But for any neighborhood W of x_0, $\delta_{g,h,i}(\{(m,n,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N} : x_{m,n,p}\notin W\}) = 0$. So for all $r\in\mathbb{N}$, $\delta_{g,h,i}(\{(m,n,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N} : x_{m,n,p}\notin B_{r,x_0}\}) = 0$. i.e., $\delta_{g,h,i}(\mu'_r) = 0$ for all $r\in\mathbb{N}$ i.e., $\delta_{g,h,i}(\mu_r) = \lim_{m,n,p\to\infty}\frac{m.n.p}{g(m).h(n).i(p)}$ for all $r\in\mathbb{N}$. There are now two distinct cases, either for all $r\in\mathbb{N}$,

$$\delta_{g,h,i}(\mu_r) = \lim_{m,n,p\to\infty}\frac{m.n.p}{g(m).h(n).i(p)} = b\neq 0 \text{ or } \delta_{g,h,i}(\mu_r) = \lim_{m,n,p\to\infty}\frac{m.n.p}{g(m).h(n).i(p)} = \infty$$

Suppose $\{b-\frac{b}{\alpha^3} : \alpha\in\mathbb{N}\}$ be an increasing sequence of positive real numbers.
Case-1: When $\lim_{m,n,p\to\infty}\frac{m.n.p}{g(m).h(n).i(p)} = b\neq 0$;
Take $(C_1,D_1,E_1)\in\mu_1$ arbitrarily, as $\delta_{g,h,i}(\mu_2) = \lim_{m,n,p\to\infty}\frac{m.n.p}{g(m).h(n).i(p)} = b$, so there exist one $(C_2,D_2,E_2)\in\mu_2$ such that $(C_2,D_2,E_2) > (C_1,D_1,E_1)$ and for all $m > C_2$, $n > D_2$ and $p > E_2$ that implies

$$\frac{|\{(j,k,l)\in\mu_2 : j\leq m, k\leq n, l\leq p\}|}{g(m).h(n).i(p)} > b-\frac{b}{2^3}.$$

As $\delta_{g,h,i}(\mu_3) = \lim_{m,n,p\to\infty}\frac{m.n.p}{g(m).h(n).i(p)} = b$ so there exist one $(C_3,D_3,E_3)\in\mu_3$ such that $(C_3,D_3,E_3) > (C_2,D_2,E_2)$ and for all $m > C_3$, $n > D_3$ and $p > E_3$ that implies

$$\frac{|\{(j,k,l)\in\mu_3 : j\leq m, k\leq n, l\leq p\}|}{g(m).h(n).i(p)} > b-\frac{b}{3^3}.$$

Continuing in this way, we will get a sequence $(C_1,D_1,E_1) < (C_2,D_2,E_2) < \cdots$
$(C_s,D_s,E_s) < \cdots$ of positive numbers such that $(C_s,D_s,E_s)\in\mu_s$ and for all $m > C_s$, $n > D_s$ and $p > E_s$ that implies

$$\frac{|\{(j,k,l)\in\mu_s : j\leq m, k\leq n, l\leq p\}|}{g(m).h(n).i(p)} > b-\frac{b}{s^3}.$$

Now, let μ be the set of all three-dimensional set of natural numbers of the interval $[C_s,C_{s+1}]\times[D_s,Ds+1]\times[E_s,E_{s+1}]$ which belongs to μ_s where $(s=1,2,\cdots)$
Now,

$$\frac{|\{(j,k,l)\in\mu : j\leq m, k\leq n, l\leq p\}|}{g(m).h(n).i(p)} \geq \frac{|\{(j,k,l)\in\mu_s : j\leq m, k\leq n, l\leq p\}|}{g(m).h(n).i(p)} > b-\frac{b}{s^3}$$

for all $(m,n,p)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}$ and $C_s\leq m\leq C_{s+1}$, $D_s\leq n\leq D_{s+1}$ and $E_s\leq p\leq E_{s+1}$.

$$\lim_{m,n,p\to\infty}\frac{|\{(j,k,l)\in\mu : j\leq m, k\leq n, l\leq p\}|}{g(m).h(n).i(p)} \geq \lim_{m,n,p\to\infty}(b-\frac{b}{s^3}).$$

Thus, $\delta_{g,h,i}(\mu) = \lim_{m,n,p\to\infty}(b - \frac{b}{s^3}) = \lim_{s\to\infty}(b - \frac{b}{s^3}) = b$. As when $m, n, p \to \infty$, $s \to \infty$.
Case-2: When $\lim_{m,n,p\to\infty} \frac{m.n.p}{g(m).h(n).i(p)} = \infty$;
Take $(C_1, D_1, E_1) \in \mu_1$ arbitrarily, as $\delta_{g,h,i}(\mu_2) = \lim_{m,n,p\to\infty} \frac{m.n.p}{g(m).h(n).i(p)} = \infty$, so there exist one $(C_2, D_2, E_2) \in \mu_2$ such that $(C_2, D_2, E_2) > (C_1, D_1, E_1)$ and for all $m > C_2$, $n > D_2$ and $p > E_2$ that implies

$$\frac{|\{(j,k,l) \in \mu_2 : j \leq m, k \leq n, l \leq p\}|}{g(m).h(n).i(p)} > 2^3.$$

As $\delta_{g,h,i}(\mu_3) = \lim_{m,n,p\to\infty} \frac{m.n.p}{g(m).h(n).i(p)} = \infty$ so there exist one $(C_3, D_3, E_3) \in \mu_3$ such that $(C_3, D_3, E_3) > (C_2, D_2, E_2)$ and for all $m > C_3$, $n > D_3$ and $p > E_3$ that implies

$$\frac{|\{(j,k,l) \in \mu_3 : j \leq m, k \leq n, l \leq p\}|}{g(m).h(n).i(p)} > 3^3.$$

Continuing in this way, we will get a sequence $(C_1, D_1, E_1) < (C_2, D_2, E_2) < \cdots (C_s, D_s, E_s)$ $< \cdots$ of positive numbers such that $(C_s, D_s, E_s) \in \mu_s$ and for all $m > C_s$, $n > D_s$ and $p > E_s$ that implies

$$\frac{|\{(j,k,l) \in \mu_s : j \leq m, k \leq n, l \leq p\}|}{g(m).h(n).i(p)} > s^3.$$

Now, let μ be the set of all three-dimensional set of natural numbers of the interval $[C_s, C_{s+1}] \times [D_s, Ds + 1] \times [E_s, E_{s+1}]$ which belongs to μ_s where $(s = 1, 2, \cdots)$
Now,

$$\frac{|\{(j,k,l) \in \mu : j \leq m, k \leq n, l \leq p\}|}{g(m).h(n).i(p)} \geq \frac{|\{(j,k,l) \in \mu_s : j \leq m, k \leq n, l \leq p\}|}{g(m).h(n).i(p)} > s^3$$

for all $(m, n, p) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N}$ and $C_s \leq m \leq C_{s+1}$, $D_s \leq n \leq D_{s+1}$ and $E_s \leq p \leq E_{s+1}$.

$$\lim_{m,n,p\to\infty} \frac{|\{(j,k,l) \in \mu : j \leq m, k \leq n, l \leq p\}|}{g(m).h(n).i(p)} \geq \lim_{m,n,p\to\infty} s^3.$$

Thus, $\delta_{g,h,i}(\mu) = \lim_{m,n,p\to\infty} s^3 = \lim_{s\to\infty} s^3 = \infty$. As when $m, n, p \to \infty$, $s \to \infty$.
So, for both the cases, we will have $\delta_{g,h,i}(\mu) = \lim_{m,n,p\to\infty} \frac{m.n.p}{g(m).h(n).i(p)}$.
Let Z be a neighborhood of x_0 and $B_{s,x_0} \subset Z$. If $(j, k, l) \in \mu$ such that $j \geq C_s$, $k \geq D_s$ and $l \geq E_s$, then there exist a $v > r$ such that $C_v \leq j \leq C_{v+1}$, $D_v \leq k \leq D_{v+1}$, $E_v \leq l \leq E_{v+1}$. So, $(j, k, l) \in \mu_v$. Thus for every $(j, k, l) \in \mu$, $j \geq C_s$, $k \geq D_s$ and $l \geq E_s$. Therefore, $x_{j,k,l} \in B_{t,x_0} \subset B_{s,x_0} \subset Z$.
Thus,

$$\lim_{m,n,p\to\infty, r\in\mathbb{N}} x_{m,n,p} = x_0.$$

Hence the theorem.

□

Example 3.2 *Limit of an $s_{g,h,i}$-convergent sequence may not be unique.*
Let $X = \{a_1, a_2\}$ and $\tau = \{\emptyset, X, \{a_1\}, \{a_2\}\}$. Then clearly (X, τ) is a topological space. Suppose $\{x_{m,n,p} : (m, n, p) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N}\}$ be the triple sequence in X given by

$$x_{m,n,p} = \begin{cases} \{a_1\}, & \text{if } m = j^j, n = k^k \text{ and } p = l^l \text{ for some } (j,k,l) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} \\ \{a_2\}, & \text{otherwise} \end{cases}$$

Here open neighborhood of $\{a_1\}$ are $W_1 = \{a_1\}$ and $W_2 = X$.
For the neighborhood W_1 of $\{a_1\}$, $\delta_{g,h,i}(\{(m, n, p) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : x_{m,n,p} \notin W_1\}) = \delta_{g,h,i}(\{(j^j, k^k, l^l) : (j, k, l) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N}\}) = lim_{m,n,p\to\infty} \frac{m.n.p}{ln(1+m^m).ln(1+n^n).ln(1+p^p)} = 0$.
Also for the neighborhood W_2 of $\{a_1\}$, $\delta_{g,h,i}(\{(m, n, p) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : x_{m,n,p} \notin W_2\}) = \delta_{g,h,i}(\{\emptyset\}) = 0$.
So, for every open neighborhood W of $\{a_1\}$, $x_{m,n,p} \xrightarrow{s_{g,h,i}-lim} \{a_1\}$.
Here the neighborhoods of $\{a_1\}$ are just similar to the neighborhoods of $\{a_2\}$. So for every neighborhood V of $\{a_2\}$, $x_{m,n,p} \xrightarrow{s_{g,h,i}-lim} \{a_2\}$.
Thus the limit of the $s_{g,h,i}$-convergent sequence $\{x_{m,n,p}\}$ is not unique.

Theorem 3.3 *In a Hausdörff space, the limit of an $s_{g,h,i}$-convergence sequence is unique.*

Proof: Let $\{\beta_{m,n,p} : (m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ be an $s_{g,h,i}$-convergent sequence in Hausdörff space X and converges to limit points β_1 and β_2 with $\beta_1 \neq \beta_2$.
As, the space X is Hausdörff space so there exist two distinct open neighborhoods $A, B \in \tau$ such that $\beta_1 \in A$ and $\beta_2 \in B$. Now for every neighborhood A of β_1, $\delta_{g,h,i}(\{(m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \beta_{m,n,p} \notin A\}) = 0$ i.e., $\delta_{g,h,i}(\{(m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \beta_{m,n,p} \in X \setminus A\}) = 0$. But $B \subseteq X \setminus A$; also $A \cap B = \emptyset$. So, $\delta_{g,h,i}(\{(m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \beta_{m,n,p} \in B\}) \leq \delta_{g,h,i}(\{(m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \beta_{m,n,p} \in X \setminus A\}) = 0$. that implies $\delta_{g,h,i}(\{(m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \beta_{m,n,p} \in B\}) = 0$.
So, $\delta_{g,h,i}(\{(m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \beta_{m,n,p} \notin B\}) = \lim_{m,n,p\to\infty} \frac{m.n.p}{g(m).h(n).i(p)} \neq 0$ which contradicts the fact that the sequence $\{\beta_{m,n,p}\}_{\mathbb{N}\times\mathbb{N}\times\mathbb{N}}$ is $s_{g,h,i}$-convergent to β_2.
Thus the limit point of the sequence $\{\beta_{m,n,p}\}$ is unique, i.e., $\beta_1 = \beta_2$.
Hence the theorem.

□

Example 3.3 *A subsequence of an $s_{g,h,i}$-convergent sequence may not be an $s_{g,h,i}$-convergent sequence. Let (X,τ) be a topological space where $X = \{b_1, b_2\}$ and $\tau = \{\emptyset, X, \{b_1\}\}$. Let us consider the triple sequence $\{B_{i,j,k} : (i,j,k) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ such that*

$$B_{i,j,k} = \begin{cases} \{b_1\}, & \text{if } i = m^m, j = n^n \text{ and } k = p^p \text{ for some } (m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} \\ \{b_2\}, & \text{otherwise} \end{cases}$$

Here open neighborhoods of $\{b_1\}$ are $W_1 = X$ and $W_2 = \{b_1\}$. So for every open neighborhood W of $\{b_1\}, \delta_{g,h,i}(\{(i,j,k) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : B_{i,j,k} \notin W\}) = 0$. Therefore $B_{i,j,jk}$ is an $s_{g,h,i}$-convergent sequence. Now considering the subsequence $\{B_{i_t,j_u,k_v} : (t,u,v) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ of $\{B_{i,j,k}\}$ such that

$$B_{i_t,j_u,k_v} = \begin{cases} B_{t^t,u^u,v^v}, & \text{if } t,u,v \text{ are odd.} \\ B_{t^t+1,u^u+1,v^v+1}, & \text{if } t,u,v \text{ are even.} \end{cases}$$

Thus for every open neighborhood S of $\{b_1\}$, $\delta_{g,h,i}(\{(t,u,v) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : B_{i_t,j_u,k_v} \notin S\})$

$$= \lim_{t,u,v\to\infty} \frac{|\{(t,u,v) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : B_{i_t,j_u,k_v} \notin S\}|}{g(t).h(u).i(v)}$$
$$= \lim_{t,u,v\to\infty} \frac{t.u.v}{\ln(1+2t).\ln(1+2u).\ln(1+2v)} = \infty \neq 0.$$

Thus the subsequence $\{B_{i_t,j_u,k_v}\}$ is not a $s_{g,h,i}$-convergent sequence.

Notations:

In a topological space (X,τ), the following symbols stated below are implemented:
M_0: The collection comprising all statistically convergent sequences in a space (X,τ).
${}_gM_0$: The collection comprising all weighted statistically convergent sequences in a space (X,τ).
${}_{g,h}M_0$: The collection comprising all double weighted statistically convergent sequences in a space (X,τ).
${}_{g,h,i}M_0$: The collection comprising all triple weighted statistically convergent sequences in a space (X,τ).

Theorem 3.4 *Let $g_1(x), g_2(x), h_1(x), h_2(x), i_1(x)$ and $i_2(x)$ are weight functions. If $g_1(n) < g_2(n)$; $h_1(n) < h_2(n)$ and $i_1(n) < i_2(n)$ for all $n \in \mathbb{N}$. Then ${}_{g_1,h_1,i_1}M_0 \subseteq {}_{g_2,h_2,i_2}M_0$.*

Proof: Let $\{\alpha_{p,q,r} : (p,q,r) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\} \in_{g_1,h_1,i_1} M_0$ in a topological space (X,τ).
But $g_1(n) < g_2(n)$; $h_1(n) < h_2(n)$; and $i_1(n) < i_2(n)$ for all $n \in \mathbb{N}$. Then there exists a limit point α_0 such that for each neighborhood S of α_0, $\delta_{g_1,h_1,i_1}(\{(p,q,r) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \alpha_{p,q,r} \notin S\}) = 0$.
i.e.,

$$\lim_{p,q,r\to\infty} \frac{|\{(p,q,r) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \alpha_{p,q,r} \notin S\}|}{g_1(n).h_1(n).i_1(n)} = 0.$$

Now, $g_1\, h_1\, i_1 < g_2\, h_2\, i_2$ implies

$$\begin{aligned} 0 &= \lim_{p,q,r\to\infty} \frac{|\{(p,,q,r) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \alpha_{p,q,r} \notin S\}|}{g_1(n).h_1(n).i_1(n)} \\ &\geq \lim_{p,q,r\to\infty} \frac{|\{(p,,q,r) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \alpha_{p,q,r} \notin S\}|}{g_2(n).h_2(n).i_2(n)}. \end{aligned}$$

Therefore, $\delta_{g_2,h_2,i_2}(\{(p,q,r) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \alpha_{p,q,r} \notin S\}) = 0$.
Thus the sequence $\{\alpha_{p,q,r} : (p,q,r) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\} \in_{g_2,h_2,i_2} M_0$.
Hence ${}_{g_1,h_1,i_1}M_0 \subseteq {}_{g_2,h_2,i_2}M_0$. □

Definition 3.3 *A subset $T \subseteq \mathbb{N}\times\mathbb{N}\times\mathbb{N}$ will be called triple weighted statistically g,h,i-dense (or shortly $s_{g,h,i}$-dense) if*

$$\delta_{g,h,i}(T) = \lim_{m,n,p\to\infty} \frac{m.n.p}{g(m).h(n).i(p)}.$$

Obviously, union of two $s_{g,h,i}$-dense sets results in another $s_{g,h,i}$-dense set. But $(\mathbb{N}\setminus 2\mathbb{N}) \cap 2\mathbb{N} = \emptyset$ which results in the intersection of two $s_{g,h,i}$-dense sets is not an $s_{g,h,i}$-dense set, even if both $2\mathbb{N}$ and $\mathbb{N}\setminus 2\mathbb{N}$ are $s_{g,h,i}$-dense sets.

Definition 3.4 *In a topological space (X,τ), a subsequence $\{x_{m_q,n_r,p_s} : (q,r,s) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ of $\{x_{m,n,p} : (m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ will be called triple weighted statistically g,h,i-dense (or simply $s_{g,h,i}$-dense) if $\{(m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : x_{m,n,p} \in \{x_{m_q,n_r,p_s} : (q,r,s) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\}\}$ is $s_{g,h,i}$-dense.*

Theorem 3.5 *In a topological space (X,τ), a triple sequence is $s_{g,h,i}$-convergent if and only if its $s_{g,h,i}$-dense subsequence is $s_{g,h,i}$-convergent.*

Proof: Let $\{x_{m,n,p} : (m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ be a triple sequence in space X and every $s_{g,h,i}$-dense subsequence $\{x_{m_q,n_r,p_s} : (q,r,s) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ of the sequence $\{x_{m,n,p} : (m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ is $s_{g,h,i}$-convergent. Also, $\{x_{m,n,p} : (m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ is a subsequence to itself. Now,

$$\begin{aligned} &= \lim_{m,n,p\to\infty} \frac{|\{(q,r,s) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : x_{m_q,n_r,p_s} \in \{x_{m,n,p} : (m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\}\}|}{g(m).h(n).i(p)} \\ &= \lim_{m,n,p\to\infty} \frac{m.n.p}{g(m).h(n).i(p)} \neq 0. \end{aligned}$$

Therefore, the triple sequence $\{x_{m,n,p} : (m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ is $s_{g,h,i}$-dense in itself. So, the triple sequence $\{x_{m,n,p} : (m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ is an $s_{g,h,i}$-convergent.
Conversely, Suppose $\{x_{m,n,p} : (m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ be an $s_{g,h,i}$-convergent sequence and the subsequence $\{x_{m_q,n_r,p_s} : (q,r,s) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ be an $s_{g,h,i}$-dense subsequence of $\{x_{m,n,p} : (m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$.
If possible, let for every $x \in X$, there exist one neighborhood U of x such that $\delta_{g,h,i}(\{(m_q,n_r,p_s) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : x_{m_q,n_r,p_s} \notin U\}) \neq 0$. But $\{(m_q,n_r,p_s) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : x_{m_q,n_r,p_s} \notin U\} \subseteq \{(m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : x_{m,n,p} \notin U\}$. i.e., $0 \neq \delta_{g,h,i}(\{(m_q,n_r,p_s) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : x_{m_q,n_r,p_s} \notin U\}) \leq \delta_{g,h,i}(\{(m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : x_{m,n,p} \notin U\})$. Thus, $\delta_{g,h,i}(\{(m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : x_{m,n,p} \notin U\}) \neq 0$ which contradicts the fact that $\{x_{m,n,p} : (m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ is an $s_{g,h,i}$-convergent.
Hence the theorem. □

4. Triple weighted statistical convergence in product space

Theorem 4.1 *If $x_m \xrightarrow{s_g-lim} x_0$, $y_n \xrightarrow{s_h-lim} y_0$ and $z_p \xrightarrow{s_i-lim} z_0$ with $\lim_{m\to\infty} \frac{m}{g(m)}$, $\lim_{n\to\infty} \frac{n}{h(n)}$ and $\lim_{p\to\infty} \frac{p}{i(p)}$ are finite in three topological spaces $(X,\tau), (Y,\sigma)$ and (Z,η) respectively. Then $\{(x_m, y_n, z_p) : (m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ is an $s_{g,h,i}$-convergent to (x_0,y_0,z_0) in the product space $X \times Y \times Z$.*

Proof: $\mathcal{B} = \{(U, V, W) : U \in \tau, V \in \sigma, W \in \eta\}$ is the base of the product space $X \times Y \times Z$.
Suppose S be an arbitrary open neighborhood of the limit point (x_0, y_0, z_0) so there exist $U \times V \times W \in \mathcal{B}$ such that $(x_0, y_0, z_0) \in U \times V \times W \subset S$ where $x_0 \in U \in \tau$, $y_0 \in V \in \sigma$, and $z_0 \in W \in \eta$. But $x_m \xrightarrow{s_g - lim} x_0$, $y_n \xrightarrow{s_h - lim} y_0$, and $z_p \xrightarrow{s_i - lim} z_0$ with $\lim_{m\to\infty} \frac{m}{g(m)}$, $\lim_{n\to\infty} \frac{n}{h(n)}$ and $\lim_{p\to\infty} \frac{p}{i(p)}$ are finite. Now,

$$\begin{aligned}
&= \lim_{m,n,p\to\infty} \frac{|\{(m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : (x_m, y_n, z_p) \notin U \times V \times W\}|}{g(m).h(n).i(p)} \\
&\leq \lim_{m,n,p\to\infty} \frac{|\{m \in \mathbb{N} : x_m \notin U\}|np + |\{n \in \mathbb{N} : y_n \notin V\}|mp + |\{p \in \mathbb{N} : z_p \notin W\}|nm}{g(m).h(n).i(P)} \\
&\leq \lim_{m\to\infty} \frac{|\{m \in \mathbb{N} : x_m \notin U\}|}{g(m)} \times \lim_{n\to\infty} \frac{n}{h(n)} \times \lim_{p\to\infty} \frac{p}{i(p)} + \lim_{n\to\infty} \frac{|\{n \in \mathbb{N} : y_n \notin V\}|}{h(n)} \times \\
&\lim_{m\to\infty} \frac{m}{g(m)} \times \lim_{p\to\infty} \frac{p}{i(p)} + \lim_{p\to\infty} \frac{|\{p \in \mathbb{N} : z_p \notin W\}|}{i(p)} \times \lim_{m\to\infty} \frac{m}{g(m)} \times \lim_{n\to\infty} \frac{n}{h(n)}.
\end{aligned}$$

Thus, $\delta_{g,h,i}(\{(m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : (x_m, y_n, z_p) \notin U \times V \times W\}) = 0$.
Hence the theorem. □

Example 4.1 *The triple weighted statistical convergence of a sequence in the product space $X \times Y \times Z$ does not guarantee that their projections are weighted statistical convergence in their respective topological spaces.*
Let (X, τ) be a topological space where $X = \{a_1, a_2\}$ and $\tau = \{\emptyset, X, \{a_1\}, \{a_2\}\}$.
Let $\{(x_m, y_n, z_p) : (m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ be the triple sequence in the product space $X \times X \times X$ where

$$x_m = \begin{cases} a_1, & \text{if } m = q^q \text{ for some } q \in \mathbb{N} \\ a_2, & \text{otherwise} \end{cases}$$

and

$$y_n = \begin{cases} a_1, & \text{if } m = q^2 \text{ for some } q \in \mathbb{N} \\ a_2, & \text{otherwise} \end{cases}$$

and

$$z_p = \begin{cases} a_1, & \text{if } m = q^3 \text{ for some } q \in \mathbb{N} \\ a_2, & \text{otherwise} \end{cases}$$

Choosing the weight functions $g(m) = \ln(1+m)$; $h(n) = \sqrt{n}$ and $i(p) = \sqrt[3]{p}$ for the sequences $\{x_m : m \in \mathbb{N}\}$; $\{y_n : n \in \mathbb{N}\}$ and $\{z_p : p \in \mathbb{N}\}$ respectively. Now the triple sequence $\{(x_m, y_n, z_p) : (m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\}$ in the product space having the smallest open set $\{a_2\} \times \{a_2\} \times \{a_2\} = \{a_2, a_2, a_2\} = W$. Then for each open neighborhood W of $\{a_2, a_2, a_2\}$,

$$\lim_{m,n,p\to\infty} \frac{|\{(m,n,p) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : x_{m,n,p} \notin W\}|}{g(m).h(n).i(p)} = \lim_{m,n,p\to\infty} \frac{m.n.p}{\ln(1+m^m).\sqrt{n}.\sqrt[3]{p}} = 0.$$

Thus $\delta_{g,h,i}(\{(m,n,p) : x_{m,n,p} \notin W\}) = 0$. But the sequences $\{y_n : n \in \mathbb{N}\}$ and $\{z_p : p \in \mathbb{N}\}$ are not weighted statistical convergence with respect to the weight functions $h(n) = \sqrt{n}$ and $i(p) = \sqrt[3]{p}$.

5. Conclusion remarks

Triple weighted statistical convergence in topological fields enhances the classical convergence concept by introducing three weights g, h and i and considering triple sequences, making it more complex and versatile in the nature of statistical convergence settings. This approach allows for a deeper and more flexible analysis of convergence behaviors in three-dimensional set of natural numbers $\mathbb{N}\times\mathbb{N}\times\mathbb{N}$, fitting well with the intrinsic structure of topological fields.

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