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The Incredible Impact of Nanodecagonal Fuzzy Number and its Arithmetic Operations

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ABSTRACT: Mathematical modeling under uncertainty requires robust fuzzy representations to handle imprecise data effectively. Traditional fuzzy numbers, such as triangular and trapezoidal, may be inadequate in cases of heightened uncertainty. To overcome these limitations, we introduce nanodecagonal fuzzy numbers (NDFNs), an advanced extension of higher-order fuzzy numbers, offering greater precision and adaptability. This paper formally defines NDFNs, investigates their structural properties, and establishes arithmetic operations, including addition, subtraction, and multiplication, supported by illustrative examples. Furthermore, the study explores the defuzzification process, which converts fuzzy values into crisp numerical outputs, enhancing their practical applicability. The proposed framework expands the utility of fuzzy numbers in complex decision-making and computational models, making them more suitable for handling real-world uncertainty.

Keywords: Nanodecagonal fuzzy number, fuzzy arithmetic operations, defuzzification, graded mean integration method.

Contents

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1. Introduction

In 1965, Zadeh [1] introduced the fuzzy set theory, which is expanded upon by fuzzy numbers, which offer a framework for expressing imprecision and uncertainty in quantitative data. In contrast to conventional numbers, fuzzy numbers support a range of values, each of which corresponds to a level of membership. When modeling real-world issues where precise numbers are insufficient, this flexibility is beneficial. In many real-world scenarios, we dealt with imprecision and uncertainty that are difficult for classical mathematics to handle. For instance, phrases such as "roughly 10", "around 5" minutes", or "moderate risk" are essentially imprecise. Fuzzy logic and fuzzy numbers offer a helpful mathematical foundation for dealing with such imprecision. Fuzzy numbers are one type of fuzzy set that represents unknown quantities. A fuzzy number is not a single precise value, but rather a range of possible values. Each value is given a membership degree, which ranges from 0 to 1, indicating how strongly that value is regarded as a part of the number. Depending on the complexity of the data being modeled, common fuzzy number types, including triangular, trapezoidal, and some other fuzzy numbers, are used to get a precise value. Numerous fields, including artificial intelligence, psychology, physics, chemistry, engineering, operation research, computer science, robotics, and medical science, have explored and used the idea of uncertainty in great detail. We are progressively exposed to a variety of fuzzy set types, such as type-2 fuzzy sets [2], interval valued fuzzy sets [3], intuitionistic fuzzy sets [4], neutrophilic fuzzy sets [7], hesitant fuzzy sets [9], spherical fuzzy sets [11], and others, based on the discoveries of different scholars.

Evalution of fuzzy set

A generalization of the standard fuzzy set and is distinguished by a three-dimensional membership function, was introduced by L. A. Zadeh [2] in 1975. Expansion of the fuzzy set called the interval valued fuzzy set was first presented by Grattan-Guinness, I. [3] in 1976. Additions to the field of fuzzy sets were made in 1986: the fuzzy multi-set was introduced by Yager, R. R. [5] and the intuitionistic fuzzy set was introduced by Atanassov, K. T. et al. [4]. Area was further improved in 1989 by Atanassov, K. T. et al. [6], who introduced the idea of intuitionistic fuzzy sets of second type and revised the concept of intuitionistic fuzzy sets. Idea of the neutrosophic set, which is an extension of the intuitionistic fuzzy set. Set was first presented by Garibaldi, J. M. et al. [8] in 2007. Set was created in 2010 by Torra, V. [9] and depends on the type of group decision-making task. Functions are involved in the context of intuitionistic fuzzy sets: the membership function (μ) and the non-membership function (ϑ), provided that $0 \leq \mu + \vartheta \leq 1$. [10] Transformative notion in 2013, creating a new concept known as pythagorean fuzzy sets by substituting $0 \leq \mu^2 + \vartheta^2 \leq 1$ for the previous constraint. Developments in fuzzy sets have surfaced in the modern era: the Interval-valued Spherical fuzzy set by Aydın, S. et al. [12] in 2021 and the Spherical fuzzy set, which was introduced by Gundogdu, F. K. et al. [11] in 2019. Evolution of fuzzy sets over time is seen in Fig.1

An overview of fuzzy numbers and how they are calculated

These fuzzy sets help us solve a number of challenging instances of decision-making and extension of the premise of typical fuzzy sets. Thus, using various fuzzy sets, multiple fuzzy numbers have been obtained. Lofti A. Zadeh [1] designed the fuzzy number, a unique kind of fuzzy set, which is essential for managing uncertain amounts in a useful way. Earlier publications have shown the diverse applications of fuzzy numbers in multiple domains, necessitating the use of fuzzy arithmetic for estimation, approximation, and other computations. A Crucial component of studying fuzzy set theory is fuzzy arithmetic. In 1978, Dubois, D., & Prade, H. [13] presented a few arithmetic procedures on fuzzy integers. Fuzzy number operations are performed using a variety of methods, including the vertex method, extension principle, and the α-cut method, which is essentially interval arithmetic. Many scholars have recently shown interest in fuzzy mathematics, and the literature on fuzzy numbers has grown in terms of its contribution to fuzzy arithmetic operations. Operations are performed using a variety of methods, including the vertex method, extension principle, and the α-cut method, which is essentially interval arithmetic. When we operate fuzzy numbers, the form of their membership functions has a substantial effect on the results of our calculations. Take the interval arithmetic operations on triangular fuzzy integers, for example. Two such ambiguous integers cannot be multiplied to have the same outcome; that is, they lose their form. Frequently, the approximation multiplication of fuzzy integers is used to overcome the issue. The Arithmetic operations of pentagonal fuzzy numbers have attracted the attention of Mondal, S. P. et al. [14], who have successfully used this knowledge to solve fuzzy differential equations. A malaria model

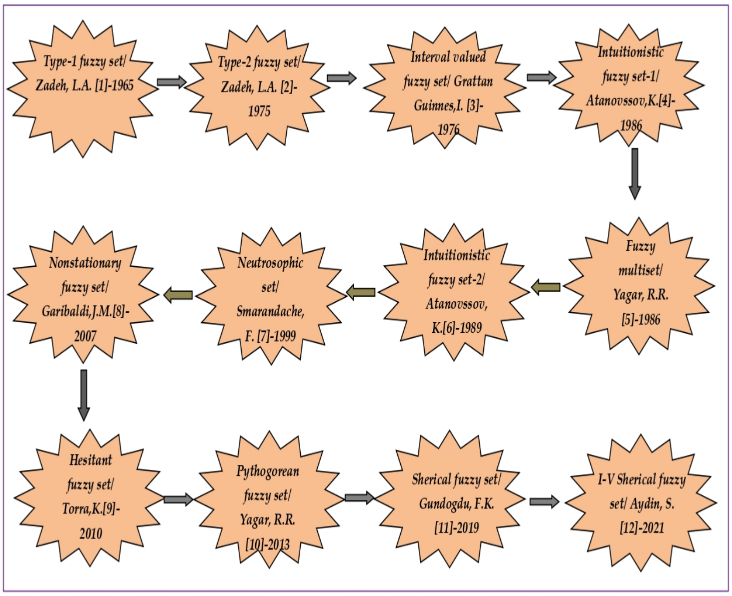

Figure 1: Evaluation of fuzzy set

and solution utilizing fuzzy differential equations are developed by Singh, P. et al. [15] employing fuzzy numbers. In order to solve the Transportation issue, Jana, D. K. [16], Chakraborty, D. et al. [17] carried out a thorough investigation on arithmetic operations employing intuitionistic fuzzy numbers. Fuzzy number arithmetic has benefited greatly from the work of the researchers mentioned above as well as Shaw, A. K. et al. [18], Naveena, N. R. [19], Gani, A. N. et al. [20], Deschrijver, G. et al. [21], Gao, S. et al. [22], Thiripurasundari, K. et al. [23], and Ranjbar, M.et al. [24].

Adrian et al. (2011) proposed the discontinuity of the trapezoidal fuzzy number- valued operators preserving core [25], Asesh et al. (2021) coined a brief analysis and interpretation on arithmetic operations of fuzzy numbers [26], Bartlomiej Kizielewicz et al.(2023) proposed the stochastic triangular fuzzy number (S-TFN) normalization: a new approach for non-monotonic normalization [27], Dan Wanget al., (2024) define the optimal solutions to granular fuzzy relation equations with fuzzy logic operations [28], Ezhilarasan Natarajan et al. (2023) described the various defuzzification and ranking techniques for the heptagonal fuzzy number to prioritize the vulnerable countries of stroke disease [29], Rouhparvar & Panahi (2015) introduced a new definition for defuzzification of generalized fuzzy numbers and its application[30], Jorge de Andres & Sanchez(2023) proposed a systematic review of the interactions of fuzzy set theory and option pricing [31], Kokila A and Deepa G (2024) coined improved fuzzy multi-objective transportation problem with Triangular fuzzy numbers [32], Sangeetha et al. (2021) coined on solving a fuzzy game problem using hexagonal fuzzy numbers [33], Sankar Prasad Mandal et al. (2017) Pentagonal fuzzy number, its properties and application in fuzzy equation [34], Muthukumar et al. (2021) defined an optimal solution of unbalanced octagonal fuzzy transportation problem [35], Nima Gerami Seresht et al.(2019) described the Computational method for fuzzy arithmetic operations on triangular fuzzy numbers by extension principle [36], Nivetha Martin et al. (2020) proposed Risk factors of lifestyle Diseases-Analysis by decagonal linguistic neutrosophic fuzzy cognitive map [37].

Literature survey of fuzzy numbers and its application

Reference	year	Author	Arithmetic operation	Application area
[20]	2012	Gani, A. N. et al.	Subtraction and division of Triangular Fuzzy Numbers (TFN)	Linear Programming Problem
[18]	2013	Shaw, A. K. et al.	Scalar multiplication, addition, multiplication and division of Trapezoidal Intuitionistic Fuzzy Numbers (TrIFN)	System failure
[40]	2013	Seikh, M. R. et al.	Addition, subtraction, multiplication, division, scalar multiplication, inverse, kth power, exponential, logarithm, ordering and interval arithmetic of TIFN	Optimization
[17]	2015	Chakraborty, D. et al.	Addition, subtraction, multiplication and division of Generalized intuitionistic fuzzy numbers (GIFN)	Transportation problem
[41]	2015	Sudha, A. S. et al.	Addition, subtraction, multiplication and division of fuzzy numbers	Linear Programming Problem
[16]	2016	Jana, D. K.	Addition, subtraction, multiplication and division of Generalized trapezoidal intuitionistic fuzzy numbers	Transportation problem
[14]	2017	Mandal, S. P. et al.	Addition, subtraction and scalar multiplication of pentagonal fuzzy numbers	Solution of fuzzy equation
[42]	2017	Sahayasudha, A. et al.	Addition and Subtraction of Symmetric Hexagonal Fuzzy Numbers	Transportation problem
[30]	2018	Yeganehma- nesh, S. et al.	Addition, subtraction, multiplication of general fuzzy semi-numbers	Medical case study
[34]	2019	Abbasi, F.	Addition, subtraction, multiplication and division of developed parabolic fuzzy numbers	Fuzzy system reliability
[43]	2019	Vanitha, V.	Addition, subtraction, multiplication and division of fuzzy numbers	Arithmetic operation
[44]	2020	Tamilarasi, G. et al.	Addition, subtraction, multiplication and measure of Icosikaipentagonal fuzzy number	Fuzzy linear programming problem
[24]	2022	Ranjbar, M. et al.	Addition, subtraction, multiplication and division of hesitant fuzzy numbers	Optimization problem
[45]	2023	Khajuria, R.	Addition, subtraction, multiplication & compliment	Electronic device control system
[46]	2023	Wang, H. et al.	Addition, subtraction & scalar multiplication	Numerical example
[47]	2023	Kumar, M. et al.	Addition, subtraction, multiplication & division	Reliability analysis of weapon system
[48]	2023	Raj, M. E. A. et al.	Addition, subtraction & multiplication	Fuzzy transformation problem
[49]	2023	Fateminia, S. H. et al.	Addition, subtraction & multiplication	Wind farm installation project

Motivation of this study

L. A. Zadeh [1] created fuzzy set theory in 1965, and it is a study strategy that may address issues related to inconsistency and undefined perception. Hence, the fuzzy set theory offers a strong foundation for managing unreliable data and is useful in modeling scenarios involving ambiguity or vagueness. Fuzzy arithmetic is thus crucial to forecasts, algorithms, approximations, and other associated operations. Arithmetic operations between two fuzzy integers come in a variety of forms. All numerical procedure requires a distinct methodology. For linear manifestations of arithmetic operations, we get outcomes that are mostly comparable to those found in previously published research. For non-linear operations like multiplication, division, and inverse, we get multiple results. The introduction is addressed in chapter one, subsequent to basic definitions in section two, nanodecagonal fuzzy numbers and arithmetic operations insection three, fuzzy number defuzzification in section four, and a conclusion on nanodecagonal fuzzy numbers in the fifth subsection.

2. Preliminaries

Definition 2.1 Given χ a collection of all the values. Then the fuzzy set $\widetilde{A} = \{(x, \mu_A(x))/x \in \chi\}$ of χ is established by its function within the membership $\mu_A : \chi \to [0, 1]$.

2.1. Fuzzy Numbers

A fuzzy number $\widetilde{A}$ is a fuzzy set on a real line R such that

1. $\mu_A(x_0)$ is piecewise continuous
2. There exists at least one $x_0 \in$ R with $\mu_{\widetilde{A}}(x_0) = 1$
3. A must be normal and convex

2.2. Triangular Fuzzy Numbers

Triangular fuzzy number (fig.1) define $\widetilde{A}$ = {a, b, c}, where all a, b, c are real numbers and its membership function $\mu_{\widetilde{A}}(x)$ is given below.

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)} & ; for\ a \le x \le b \\ \frac{(c-x)}{(c-b)} & ; for\ b \le x \le c \\ 0 & ; for\ x > d \end{cases}$$

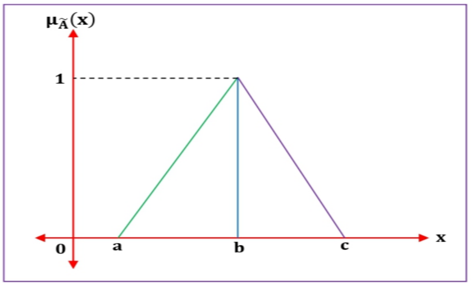

Figure 2: Triangular fuzzy numbers

2.3. Trapezoidal Fuzzy Numbers

$\widetilde{A}$ = {a, b, c, d} is defined by trapezoidal fuzzy numbers, where a, b, c, and d are real numbers. The membership function $\mu_{\widetilde{A}}(x)$ of these numbers is as follows:

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & ; for\ x < a \\ \frac{(x-a)}{(b-c)} & ; for\ a \le x \le b \\ 1 & ; for\ b \le x \le c \\ \frac{(d-x)}{(d-c)} & ; for\ c \le x \le d \\ 0 & ; for\ x > d \end{cases}$$

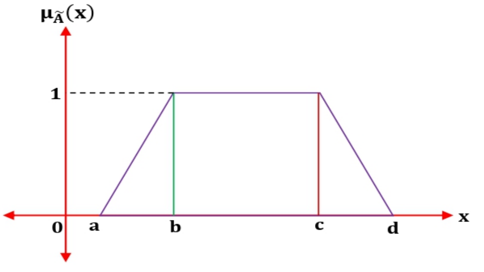

Figure 3: Trapezoidal fuzzy numbers

2.4. Pentagonal Fuzzy Numbers

The pentagonal fuzzy number is defined as $\widetilde{A}$ = {a, b, c, d, e}, where all a, b, c, d, e are real numbers, and its membership function $\mu_{\widetilde{A}}(x)$ is given below:

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & ; for\ x < a \\ \frac{(x-a)}{(b-a)} & ; for\ a \le x \le b \\ \frac{(x-b)}{(c-b)} & ; for\ b \le x \le c \\ 1 & ; for\ x = c \\ \frac{(d-x)}{(d-c)} & ; for\ c \le x \le d \\ \frac{(e-x)}{(e-d)} & ; for\ d \le x \le e \\ 0 & ; for\ x > e \end{cases}$$

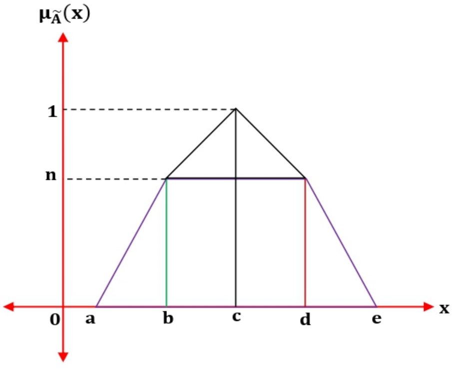

Figure 4: Pentagonal Fuzzy Numbers

2.5. Hexagonal Fuzzy Numbers

Hexagonal fuzzy numbers are defined as $\widetilde{A}$= { a_1, a_2, a_3, a_4, a_5, a_6}, where all a_1, a_2, a_3, a_4, a_5, a_6 are real numbers, and their membership function $\mu_{\widetilde{A}}(x)$ is given below:

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & ; x < a_1 \\ \frac{1}{2}\left(\frac{x-a_1}{a_2-a_1}\right) & ; for\ a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-a_2}{a_3-a_2}\right) & ; for\ a_2 \le x \le a_3 \\ 1 & ; for\ a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\left(\frac{a_5-x}{a_5-a_4}\right) & ; for\ a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6-x}{a_6-a_5}\right) & ; for\ a_5 \le x \le a_6 \\ 0 & ; for\ x > a_6 \end{cases}$$

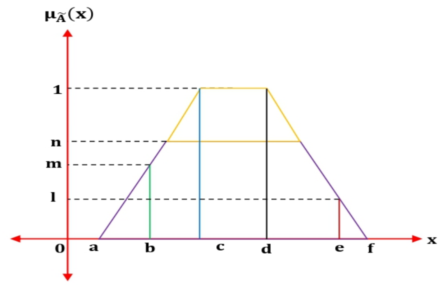

Figure 5: Hexagonal Fuzzy Numbers

2.6. Heptagonal Fuzzy Numbers

Heptagonal fuzzy number (fig.5) defined as $\widetilde{A} = \{a_1, a_2, a_3, a_4, a_5, a_6 a_7\}$ where all $\{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ are real numbers and its membership function $\mu_{\widetilde{A}}(x)$ is given below,

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & ; for x < a \\ \frac{1}{2}\left(\frac{x-a}{b-a}\right) & ; for a \le x \le b \\ \frac{1}{2} & ; for b \le x \le c \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-c}{d-c}\right) & ; for c \le x \le d \\ 1 - \frac{1}{2}\left(\frac{d-x}{e-f}\right) & ; for d \le x \le e \\ \frac{1}{2} & ; for e \le x \le f \\ \frac{1}{2}\left(\frac{f-x}{g-f}\right) & ; for f \le x \le g \\ 0 & ; for x > g \end{cases}$$

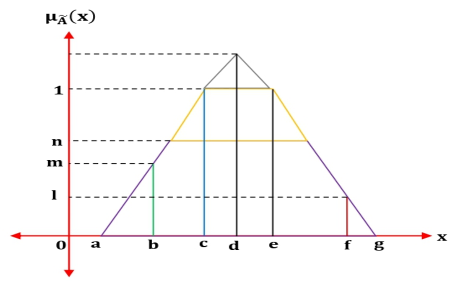

Figure 6: Heptagonal Fuzzy Numbers

2.7. Octagonal Fuzzy Numbers

A fuzzy number $\widetilde{A}$ is a normal octagonal fuzzy numbers denoted by A $= \{a_1, a_2, a_3, a_4, a_5, a_6 a_7, a_8\}$, where all $\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8\}$ are real numbers and its membership function $\mu_{\widetilde{A}}(x)$ is given below,

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & ; for x < a \\ k\left(\frac{x-a}{b-c}\right) & ; for a \le x \le b \\ k & ; for b \le x \le c \\ k + (1-k)\left(\frac{x-c}{d-c}\right) & ; for c \le x \le d \\ 1 & ; for d \le x \le e \\ k + (1-k)\left(\frac{f-x}{f-e}\right) & ; for e \le x \le f \\ k & ; for f \le x \le g \\ k\left(\frac{h-x}{h-g}\right) & ; for g \le x \le h \\ 0 & ; for x > h \end{cases}$$

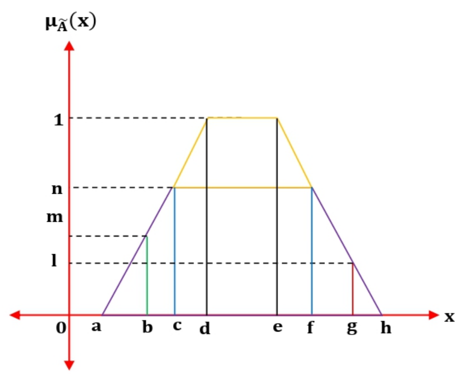

Figure 7: Octagonal Fuzzy Numbers

2.8. Nanogonal Fuzzy Numbers

A fuzzy number $\widetilde{A}$ is a nanogonal fuzzy numbers denoted by A $= a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$, where all $\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9\}$ are real numbers and its membership function is given below,

$$\mu_{\widetilde{A}}(x)=\begin{cases}0 & ;for \wedge x<a\\ \frac{1}{4}\left(\frac{x-a}{b-a}\right) & ;for a\le x\le b\\ \frac{1}{4}+\frac{1}{4}\left(\frac{x-b}{c-b}\right) & ;for b\le x\le c\\ \frac{1}{2}+\frac{1}{4}\left(\frac{x-c}{d-c}\right) & ;for c\le x\le d\\ \frac{3}{4}+\frac{1}{4}\left(\frac{x-d}{e-d}\right) & ;for d\le x\le e\\ 1-\frac{1}{4}\left(\frac{x-e}{f-e}\right) & ;for e\le x\le f\\ \frac{3}{4}-\frac{1}{4}\left(\frac{x-f}{g-f}\right) & ;for f\le x\le g\\ \frac{1}{2}-\frac{1}{4}\left(\frac{x-g}{h-g}\right) & ;for g\le x\le h\\ \frac{1}{4}\left(\frac{i-x}{i-h}\right) & ;for h\le x\le i\\ 0 & ;for x\ge i\end{cases}$$

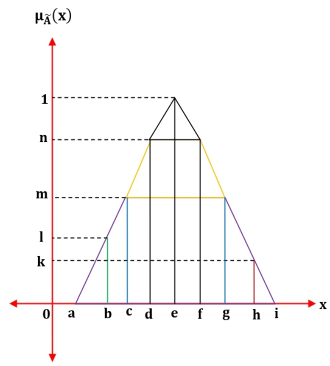

Figure 8: Octagonal Fuzzy Numbers

2.9. Decagonal fuzzy number

A fuzzy number $\widetilde{D}$ is a decagonal fuzzy numbers denoted by $\widetilde{D}=\{a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10}\}$, where all $\{a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10}\}$ are real number and its membership function $\mu_{\widetilde{D}}(x)$ is given below,

$$\mu_{\widetilde{D}}(x)=\begin{cases}0 & ;for x<a_1\\ \frac{1}{4}\left(\frac{x-a_1}{a_2-a_1}\right) & ;for a_1\le x\le a_2\\ \frac{1}{4}+\frac{1}{4}\left(\frac{x-a_2}{a_3-a_2}\right) & ;for a_2\le x\le a_3\\ \frac{1}{2}+\frac{1}{4}\left(\frac{x-a_3}{a_4-a_3}\right) & ;for a_3\le x\le a_4\\ \frac{3}{4}+\frac{1}{4}\left(\frac{x-a_4}{a_5-a_4}\right) & ;for a_4\le x\le a_5\\ 1 & ;for a_5\le x\le a_6\\ 1-\frac{1}{4}\left(\frac{x-a_6}{a_7-a_6}\right) & ;for a_6\le x\le a_7\\ \frac{3}{4}-\frac{1}{4}\left(\frac{x-a_7}{a_8-a_7}\right) & ;for a_7\le x\le a_8\\ \frac{1}{2}-\frac{1}{4}\left(\frac{x-a_8}{a_9-a_8}\right) & ;for a_8\le x\le a_9\\ \frac{1}{4}\left(\frac{a_{10}-x}{a_{10}-a_9}\right) & ;for a_9\le x\le a_{10}\\ 0 & ;for x\ge a_{10}\end{cases}$$

3. Nanodecagonal Fuzzy Numbers

A Nanodecagonal fuzzy number (fig.8) of a fuzzy set A is defined as $A_{nd}=\{a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,$ $a_9,a_{10},a_{11},a_{12},a_{13},a_{14},a_{15},a_{16},a_{17},a_{18},a_{19}\}$ where all $\{a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12},a_{13},$ $a_{14},a_{15},a_{16},a_{17},a_{18},a_{19}\}$ are real number and its membership function $\mu_{\widetilde{D}}(x)$ is provided by,

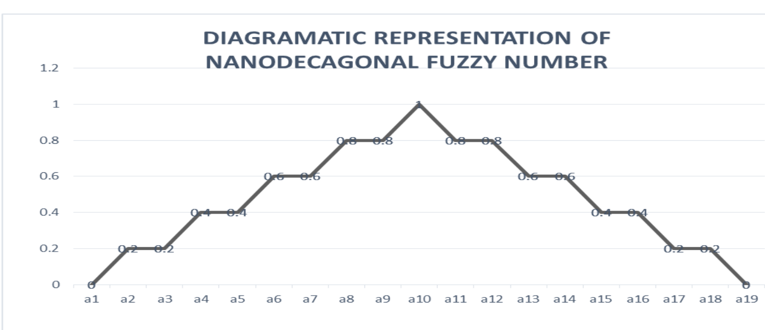

Figure 9: Nanodecagonal Fuzzy Numbers

$$\mu_{\widetilde{A}_n d}(x) = \begin{cases} 0 & ; for x < a_1 \\ \frac{1}{5}\left(\frac{x-a_1}{a_2-a_1}\right) & ; for a_1 \leq x \leq a_2 \\ \frac{1}{5} & ; for a_2 \leq x \leq a_3 \\ \frac{1}{5}+\frac{1}{5}\left(\frac{x-a_3}{a_4-a_3}\right) & ; for a_3 \leq x \leq a_4 \\ \frac{2}{5} & ; for a_4 \leq x \leq a_5 \\ \frac{2}{5}+\frac{1}{5}\left(\frac{x-a_5}{a_6-a_5}\right) & ; for a_5 \leq x \leq a_6 \\ \frac{3}{5} & ; for a_6 \leq x \leq a_7 \\ \frac{3}{5}-\frac{1}{5}\left(\frac{x-a_7}{a_8-a_7}\right) & ; for a_7 \leq x \leq a_8 \\ \frac{4}{5} & ; for a_8 \leq x \leq a_9 \\ \frac{4}{5}+\frac{1}{5}\left(\frac{x-a_9}{a_{10}-a_9}\right) & ; for a_9 \leq x \leq a_{10} \\ 1-\frac{1}{5}\left(\frac{a_{10}-x}{a_{11}-a_{10}}\right) & ; for a_{10} \leq x \leq a_{11} \\ \frac{4}{5} & ; for a_{11} \leq x \leq a_{12} \\ \frac{4}{5}-\frac{1}{5}\left(\frac{a_{12}-x}{a_{13}-a_{12}}\right) & ; for a_{12} \leq x \leq a_{13} \\ \frac{3}{5} & ; for a_{13} \leq x \leq a_{14} \\ \frac{3}{5}-\frac{1}{5}\left(\frac{a_{14}-x}{a_{15}-a_{14}}\right) & ; for a_{14} \leq x \leq a_{15} \\ \frac{2}{5} & ; for a_{15} \leq x \leq a_{16} \\ \frac{2}{5}-\frac{1}{5}\left(\frac{a_{16}-x}{a_{17}-a_{16}}\right) & ; for a_{16} \leq x \leq a_{17} \\ \frac{1}{5} & ; for a_{17} \leq x \leq a_{18} \\ \frac{1}{5}\left(\frac{a_{18}-x}{a_{19}-a_{18}}\right) & ; for a_{18} \leq x \leq a_{19} \\ 0 & ; for x \geq a_{19} \end{cases}$$

3.1. Arithmetic Operations on Nanodecagonal Fuzzy Number

3.1.1. Addition on nanodecagonal fuzzy number. Let $\widetilde{A} = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}\}$ and $\widetilde{B} = \{b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16}, b_{17}, b_{18}, b_{19}\}$ be two nanodecagonal fuzzy number. The addition of nanodecagonal fuzzy number is

$$\begin{aligned}\left(\widetilde{A}_{nd} + \widetilde{B}_{nd}\right) &= \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}\} \\ &+ \{b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16}, b_{17}, b_{18}, b_{19}\} \\ &= \{a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4, a_5+b_5, a_6+b_6, a_7+b_7, a_8+b_8, a_9+b_9, a_{10}+b_{10} \\ &a_{11}+b_{11}, a_{12}+b_{12}, a_{13}+b_{13}, a_{14}+b_{14}, a_{15}+b_{15}, a_{16}+b_{16}, a_{17}+b_{17}, a_{18}+b_{18}, a_{19}+b_{19}\end{aligned}$$

Example 3.1 *If,* $\widetilde{A}_{nd} = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20)$ *and*
$\widetilde{B}_{nd} = (2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38)$ *then*
$\left(\widetilde{A}_{nd} + \widetilde{B}_{nd}\right) = (3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 55, 58)$

3.1.2. Subtraction on nanodecagonal fuzzy number. Let $\widetilde{A} = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}\}$ and $\widetilde{B} = \{b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16}, b_{17}, b_{18}, b_{19}\}$ be two nanodecagonal fuzzy number. The Subtraction of nanodecagonal fuzzy number is

$$\begin{aligned}\left(\widetilde{A}_{nd} - \widetilde{B}_{nd}\right) &= \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}, a_{17}, a_{18}, a_{19}\} \\ &- \{b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8, b_9, b_{10}, b_{11}, b_{12}, b_{13}, b_{14}, b_{15}, b_{16}, b_{17}, b_{18}, b_{19}\} \\ &= \{a_1-b_1, a_2-b_2, a_3-b_3, a_4-b_4, a_5-b_5, a_6-b_6, a_7-b_7, a_8-b_8, a_9-b_9, a_{10}-b_{10} \\ &a_{11}-b_{11}, a_{12}-b_{12}, a_{13}-b_{13}, a_{14}-b_{14}, a_{15}-b_{15}, a_{16}-b_{16}, a_{17}-b_{17}, a_{18}-b_{18}, a_{19}-b_{19}\end{aligned}$$

Example 3.2 *If,* $\widetilde{A}_{nd} = (1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,19,20)$ *and* $\widetilde{B}_{nd} = (2,4,6,8,10,12,14,16,18,20,22,24,26,28,30,32,34,36,38)$ *then* $\left(\widetilde{A}_{nd} - \widetilde{B}_{nd}\right) = (3,6,9,12,15,18,21,24,27,30,33,36,39,42,45,48,51,55,58)$

3.1.3. Scalar Multiplication of Nanodecagonal Fuzzy Numbers. If, $\widetilde{A}_{nd}$ =$(a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12},a_{13},a_{14},a_{15},a_{16},a_{17},a_{18},a_{19})$ then k $(\widetilde{A}_{nd})$ defined by k.$\widetilde{A}_{nd}$ =$(ka_1,ka_2,ka_3,ka_4,ka_5,ka_6,ka_7,ka_8,ka_9,ka_{10},ka_{11},ka_{12},ka_{13},ka_{14},ka_{15},ka_{16},ka_{17},ka_{18},ka_{19})$

Example 3.3 *If* $\widetilde{A}_{nd} = (2,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,22)$,
Then $2(\widetilde{A}_{nd}) = (4,8,10,12,14,16,18,20,22,24,26,28,30,32,34,36,38,40,44)$.

3.1.4. Multiplication Nanodecagonal Fuzzy number. Let $\widetilde{A} = \{a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12},a_{13},a_{14},a_{15},a_{16},a_{17},a_{18},a_{19}\}$ and
$\widetilde{B} = \{b_1,b_2,b_3,b_4,b_5,b_6,b_7,b_8,b_9,b_{10},b_{11},b_{12},b_{13},b_{14},b_{15},b_{16},b_{17},b_{18},b_{19}\}$ be two nanodecagonal fuzzy number. The Multiplication of nanodecagonal fuzzy numbers

$$\begin{aligned}\left(\widetilde{A}_{nd} * \widetilde{B}_{nd}\right) &= (a_1,a_2,a_3,a_4,a_5,a_6,a_7,a_8,a_9,a_{10},a_{11},a_{12},a_{13},a_{14},a_{15},a_{16},a_{17},a_{18},a_{19})\\ &* (b_1,b_2,b_3,b_4,b_5,b_6,b_7,b_8,b_9,b_{10},b_{11},b_{12},b_{13},b_{14},b_{15},b_{16},b_{17},b_{18},b_{19})\\ &= a_1*b_1,a_2*b_2,a_3*b_3,a_4*b_4,a_5*b_5,a_6*b_6,a_7*b_7,a_8*b_8,a_9*b_9,a_{10}*b_{10}\\ &,a_{11}*b_{11},a_{12}*b_{12},a_{13}*b_{13},a_{14}*b_{14},a_{15}*b_{15},a_{16}*b_{16},a_{17}*b_{17},a_{18}*b_{18},a_{19}*b_{19}\end{aligned}$$

Example 3.4 *If* $\widetilde{A}_{nd}$ = *(2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20)* $\widetilde{B}_{nd}$ = *(2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20) Then,* $\left(\widetilde{A}_{nd} * \widetilde{B}_{nd}\right)$ = *(4, 9, 16, 25, 36, 42, 64, 72, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400)*

3.1.5. Positive Nanodecagonal Fuzzy number. A positive nanodecagonal fuzzy number (p - *nd* FN) is defined as the $\widetilde{A}_{od}$ = $(a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s)$, where $a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s$ for all > 0.

Example 3.5 $\widetilde{A}_{nd}$ = *(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16,17,18, 19)*

3.1.6. Negative nanodecagonal fuzzy number. A Negative nanodecagonal fuzzy number (n - *nd* FN) is defined as the $\widetilde{A}_{nd}$ = $(a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s)$, where a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s for all > 0.

Example 3.6 $\widetilde{A}_{nd}$ = *(-1, -2, -3, -4, -5, -6, -7, -8, -9, -10, -11, -12, -13, -14, -15, -16, -17, -18, -19).*

4. Defuzzification of Fuzzy Numbers

4.1. Graded Mean Method

The graded mean defuzzification method (GMDM) is used to convert a fuzzy number into a crisp representative value. It does so by assigning graded weights to the defining points (vertices) of the fuzzy number and then computing a weighted average. This method provides a more accurate defuzzification, especially for polygonal fuzzy numbers, by considering the geometry and symmetry of their structure. For a fuzzy number A = {a, b, c,. . . ,z} the defuzzified value is calculated as:

$$GM(A) = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

Where x_i are the vertices of the fuzzy number and w_i are the respective weights assigned according to the fuzzy shape (e.g., triangular, trapezoidal, octadecagonal, etc.

4.2. Defuzzification of fuzzy numbers

Fuzzy Numbers	Example	Graded Mean Integration Representation	Defuzzification Value
Triangular Fuzzy Numbers	{23, 25, 27}	(a+4b+c)/ 6	3
Trapezoidal Fuzzy Numbers	{21, 24, 26, 29}	(a+2b+2c+d)/6	4
Pentagonal Fuzzy Numbers	{19, 23, 25, 27, 31}	(a+3b+4c+3d+e)/12	5
Hexagonal Fuzzy Numbers	{17, 22, 24, 26, 28, 33}	(a+3b+2c+2d+3e +f)/12	6
Heptagonal Fuzzy Numbers	{15, 20, 23, 25, 27, 30,35}	(a+3b+3c+4d+3e +3f+g)/18	7
Octagonal Fuzzy Numbers	{13, 19, 23, 25, 27, 30,35}	(a+3b+3c+2d+2e +3f+3g+h)/18	8
Nanogonal Fuzzy Numbers	{13, 19, 23, 25, 27, 29,31, 37}	(a+3b+3c+3d+4e+ 3f+3g+3h+I)/24	9
Decagonal Fuzzy Numbers	{11, 17, 21,23, 25, 27,29, 33, 35}	(a+3b+3c+3d+2e+ 2f+3g+3h+3i+j)/24	10
Hendecagonal Fuzzy Numbers	{9, 15, 19, 21, 23, 27, 29, 31, ,33, 35, 37}	(a+3b+3c+3d+ 3e+4f+3g+3h+ 3i+3j+k)/30	11
Undecagonal Fuzzy Numbers	{7, 13, 17, 18, 21, 25, 27, 29, 31, 35, 37}	(a+3b+3c+3d+ 3e+2f+2g+3h+ 3i+3j+3k+l)/30	12
Tridecagonal Fuzzy Numbers	{5, 11, 15,17, 19, 23, 25,27, 29, 31, 35, 37}	(a+3b+3c+3d+3e +3f+4g+3h+3i+ 3j+3k+3l+m)/36	13
Tetradecagonal Fuzzy Numbers	{5, 11, 15, 17, 19, 23, 27, 29, 31, 33, 37, 39}	(a+3b+3c+3d+3e+ 3f+2g+2h+3i+3j+ 3k+3l+3m+n)/36	14
Pentadecagonal Fuzzy Numbers	{3, 9, 13, 15, 17, 21, 23, 27, 29, 31, 33, 35, 39, 41}	(a+3b+3c+3d+3e+ 3f+3g+4h+3i+3j+ 3k+3l+3m+3n+o)/42	15
hexadecagonal Fuzzy Numbers	{1, 7, 11, 13, 15, 19, 21, 25, 27, 29, 31, 33, 35, 39, 41}	(a+3b+3c+3d+3e+3f+ 3g+2h+2i+3j+3k+3l+ 3m+3n+3o+p)/42	16
Heptadecagonal Fuzzy Numbers	{1, 7, 11,13, 15, 19, 21, 23, 27, 29, 31, 33, 35, 37, 39, 43}	(a+3b+3c+3d+3e+3f+ 3g+3h+4i+3j+3k+3l+ 3m+3n+3o+3p+q)/48	17
Octadecagonal Fuzzy Numbers	{1, 7, 11, 13, 15, 19, 21, 23, 25, 29, 31, 33, 35, 37, 39, 41, 45}	(a+3b+3c+3d+3e+3f+ 3g+3h+2i+2j+3k+3l+ 3m+3n+3o+3p+3q+r)/48	18
Nanodecagonal Fuzzy Numbers	{1, 7, 11, 13,15, 19, 21, 23,25, 27, 31, 33,35, 37, 39, 41, 43, 47}	(a+3b+3c+3d+3e+ 3f+3g+3h+3i+4j+ 3k+3l+3m+3n+3o +3p+3q+3r+s)/54	19

5. Conclusion

In this paper, the concept of nanodecagonal fuzzy numbers is newly introduced, accompanied by an explanation of arithmetic operations such as addition, subtraction, and multiplication, complete with relevant examples. Traditional forms of fuzzy numbers may prove insufficient when faced with heightened uncertainty; under such circumstances, nanodecagonal Fuzzy Numbers become particularly applicable. Furthermore, the process of defuzzification is employed to convert a fuzzy number into a precise, crisp value.

5.1. Major finding

This paper mainly talks about fuzzy arithmetic operations using cut methods. This study conducts all operations on triangular fuzzy numbers (TFN). The focus is specifically on multiplication and division operations. From previously published articles, it is observed that in most instances, multiplication and division operations are carried out with two positive fuzzy numbers. However, in this paper, we extensively explore various multiplication and division operations scenarios involving different combinations of positive, negative and partial negative fuzzy numbers. To enhance understanding of the operations, we have included relevant numerical examples.

5.2. Future research scope

This paper thoroughly explores arithmetic operations on triangular fuzzy numbers (TFN), including addition, subtraction, multiplication and division. Instead of TFN, trapezoidal fuzzy numbers, pentagonal fuzzy numbers, or hexagonal fuzzy numbers may be considered in future research. This arithmetic operation may be applied to intuitionistic fuzzy numbers or neutrosophic numbers in the future. Some real-life applications may take for fuzzy arithmetic. This future direction holds promise for further enriching the field of fuzzy mathematics.

Data availability
No datasets were employed in the study discussed in the article.

Declaration of competing interest
The authors confirm that they have no known financial or personal affiliations that could have impacted the work reported in this research.

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