Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2026 (44) 10** : 1–9.
ISSN-0037-8712
doi:10.5269/bspm.79076

Fixed Point Results in Generalized Branciari b-Metric Spaces with Applications

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ABSTRACT: In 2000, A. Branciari introduced rectangular metric space replacing the triangular inequality with more general inequality, namely, rectangular inequality (involving four points instead of three) in metric space. The rectangular inequality was generalized by adding two more terms to the right hand of rectangular inequality to introduce generalized Branciari metric space by A. Kostic [2] in 2025. In this paper, we extend this further by multiplying it with a variable in order to introduce the generalized Branciari b-metric space. We establish fixed point results for Banach contractions and Kannan contractive mappings in order to improve existing literature. The validity of the result is evidenced by an example. We construct an example in support of our theorem. Furthermore, we establish the existence and uniqueness of the Fredholm integral equation.

Keywords: Fixed point, generalized Branciari b-metric space, Kannan mappings.

Contents

1. Introduction

One of the most thoroughly examined topics of mathematics is metric fixed point theory. As the topic is expertise in solving differential and integral equations, non linear problems, etc,. Along with this, it also provides guidance in solving problems of economic, physics, engineering and so forth. Theory was originated in 1922 by S. Banach [13]. Banach contraction principle can be generalized either by improving the contractive condition or by adopting another metric space. At first contraction mapping was generalized by R. Kannan [12] in 1968 and after that, there were many genralizations have been proposed; for instance, one can see ([3], [4], [6], [7], [15], [17]).

Metric space was first introduced by M. Frechet [9] in 1906. In 1989, I. A. Bakhtin [5] introduced the b-metric space in order to improve the standard metric space and the chain of generalization continued. After that, the b-metric has been generalized in many ways. For instance, one can see ([14], [16], [11], [8], [18]). In 2000, A. Branciari [1] generalized the standard metric space by replacing triangular inequality to rectangular inequality. In 2025, A. Kostic [2] generalized rectangular metric space by adding two more terms in the right-hand side of the rectangular inequality. He introduced this metric space as generalized Branciari metric space. As the rectangular inequality does not include all the complete set of distance between u, v, a, b. So, he generalized rectangular metric space by adding all the possible set of distance between them.

The paper includes the introduction of a new metric space, generalized Branciari b-metric space by generalizing generalized Branciari metric space. We use contraction and Kannan contractive mapping to prove fixed point results in generalized Branciari b-metric space. We also use our result in order to examine the existence and uniqueness of a solution of integral equation.

2020 *Mathematics Subject Classification:* 47H10, 54H25.
Submitted September 20, 2025. Published March 12, 2026

2. Preliminaries

At first, we revise the definition of rectangular metric space.

Definition 2.1 *[1] Let $U \neq \emptyset$. A mapping $r_\theta : U \times U \to \mathbb{R}^+$ is referred to be a rectangular metric space if upcoming conditions hold ($\forall u, v \in U$ and all distinct $a, b \in U \setminus \{u, v\}$) $a \neq b$:*

(1) $r_\theta(u, v) = 0$ *iff* $u = v$,

(2) $r_\theta(u, v) = r_\theta(v, u)$,

(3) $r_\theta(u, v) \leq r_\theta(u, a) + r_\theta(a, b) + r_\theta(b, v)$.

Then, we assure that the pair (U, r_θ) is rectangular metric space.

Now, we recall the definition of generalized Branciari metric space.

Definition 2.2 *[2] Let $U \neq \emptyset$. A mapping $r_\theta : U \times U \to \mathbb{R}^+$ is referred to be a generalized Branciari metric space if upcoming conditions hold ($\forall u, v \in U$ and all distinct $a, b \in U \setminus \{u, v\}$), $a \neq b$:*

(1) $r_\theta(u, v) = 0$ *iff* $u = v$,

(2) $r_\theta(u, v) = r_\theta(v, u)$,

(3) $r_\theta(u, v) \leq r_\theta(u, a) + r_\theta(a, b) + r_\theta(b, v) + r_\theta(u, b) + r_\theta(v, a)$.

Then, we assure that the pair (U, r_θ) is generalized Branciari metric space.

Remark 2.1 *Every rectangular metric space is a generalized Branciari metric space but converse may not be true.*

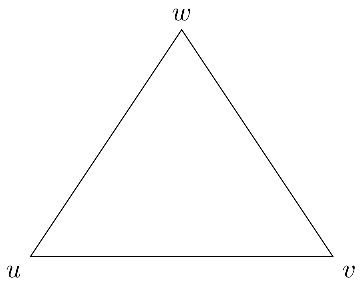


Figure 1: Graphical representation of metric space.

Figure 2: Graphical representation of rectangular metric space.

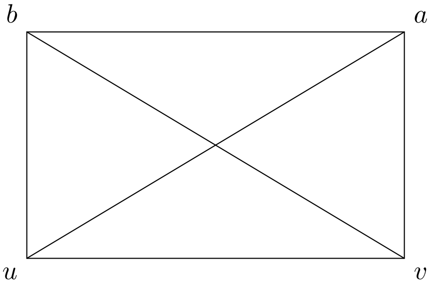


Figure 3: Graphical representation of generalized Branciari metric space.

Now, in standard metric space, we only take one point between two points i.e, w between u and v. But in rectangular metric space, we take two points i.e, a and b between u and v, but we are not taking here diagonals of rectangle implies we are not taking all the possible distance of u, v, a and b i.e, $r_\theta(a, v)$ and $r_\theta(u, b)$. In generalized Branciari metric space, we also take the diagonals of rectangle.

3. Main Results

First of all, we define generalized Branciari b-metric space.

Definition 3.1 *Let $U \neq \emptyset$. A mapping $r_\theta : U \times U \to \mathbb{R}^+$ and $s \geq 1$ is referred to be a generalized Branciari b-metric space if upcoming conditions hold ($\forall u, v \in U$ and all distinct $a, b \in U \setminus \{u, v\}$), $a \neq b$:*

(1) $r_\theta(u, v) = 0$ iff $u = v$,

(2) $r_\theta(u, v) = r_\theta(v, u)$,

(3) $r_\theta(u, v) \leq s[r_\theta(u, a) + r_\theta(a, b) + r_\theta(b, v) + r_\theta(u, b) + r_\theta(v, a)]$.

Then, we assure that the pair (U, r_θ) is generalized Branciari b-metric space.

Now, we construct the example of generalized Branciari b-metric space.

Example 3.1 *Let $U = \{1, 2, 3, 4, 5\}$ and $r_\theta : U \times U \to \mathbb{R}^+$ and $s = 2$ be defined as $\forall u, v \in U$*

$$r_\theta(u, v) = \begin{cases} 0, & \text{if } u = v \\ 4, & \text{if } u = 1, v \neq 1 \\ 1, & \text{if otherwise.} \end{cases}$$

Then, (U, r_θ) is generalized Branciari b-metric space.

Now, we define Cauchy, convergent and complete sequence of generalized Branciari b-metric space.

Definition 3.2 *Assume (U, r_θ) be a generalized Branciari b-metric space and $\{u_n\}$ be any sequence in U.*

(1) Then, $\{u_n\}$ is called Cauchy if for each positive number ϵ, $\exists$, a natural number N in order that $r_\theta(u_n, u_m) < \epsilon$, $\forall\ n > m > N$.

(2) Then, $\{u_n\}$ is called convergent if for each positive number ϵ, $\exists$, a natural number N in order that $r_\theta(u_n, u) < \epsilon$, $\forall\ n > N$.

(3) Then, (U, r_θ) is called complete if convergent property followed by each and every Cauchy sequence in (U, r_θ).

Now, we state a lemma that will be useful to our subsequent work.

Lemma 3.1 *Suppose (U, r_θ) be a generalized Branciari b-metric space and $\{u_n\}$ is a convergent sequence in U. Now, if for all $n, m \in \mathbb{N}$ $m \neq n$ $\{u_n\}$ is a Cauchy sequence and $u_n \neq u_m$, then $\{u_n\}$ has a unique limit.*

Proof: Let $\lim_{n\to\infty} r_\theta(u_n,u) = \lim_{n\to\infty} r_\theta(u_n,v) = 0$ for some $u,v\in U$. As $\{u_n\}\neq\{u_m\}$ for all $n,m\in\mathbb{N}$ $m\neq n$ then there exists $q\in\mathbb{N}$ in order that $u_n\in U\setminus\{u,v\}$ $\forall n\geq q$. Then for $m>n\geq q$ we have:

$$r_\theta(u,v)\leq s[r_\theta(u,u_n)+r_\theta(u_n,u_m)+r_\theta(u_m,v)+r_\theta(u,u_m)+r_\theta(u_n,v)]\to 0$$

as $n,m\to+\infty$, implies $r_\theta(u,v)=0$ also $u=v$. □

Now, we state and prove fixed point theorem on generalized Branciari b-metric space.

Theorem 3.1 *Let (U,r_θ) be a complete generalized Branciari b-metric space and $T:U\to U$ satisfy the following condition:*

$$r_\theta(Tu,Tv)\leq Lr_\theta(u,v)\ \forall u,v\in U \tag{3.1}$$

and $L\in[0,1)$. Then, the mapping T has a unique fixed point $w\in U$, in order that $\lim_{n\to\infty}T^nu=w$ $\forall u\in U$.

Proof: *Let $u\in U$ be an arbitrary point and let $\forall n\in\mathbb{N}_0$ $u_n=T^nu$. Then by frequent use of condition 3.1, we get*

$$r_\theta(u_n,u_{n+1})\leq Lr_\theta(u_{n-1},u_n)\leq\cdots\leq L^nr_\theta(u_0,u_1)\qquad\forall n\in\mathbb{N}_0.$$

Hence,

$$\lim_{n\to\infty}r_\theta(u_n,u_{n+1})=0.$$

If $u_n=u_{n+1}$ for some $n\in\mathbb{N}_0$ then $u_n=Tu_n$, implies u_n is a fixed point of mapping T. Therefore assume that $u_n\neq u_{n+1}$ $\forall n\in\mathbb{N}_0$.
Suppose $\{u_n\}\neq\{u_m\}$ for all $n,m\in\mathbb{N}_0$ with $m\neq n$. Indeed let $\{u_n\}=\{u_m\}$ for some $n,m\in\mathbb{N}_0$ and $m>n$. Then

$$0<r_\theta(u_n,u_{n+1})=r_\theta(u_m,u_{m+1}).$$

By using condition 3.1, we get

$$0<r_\theta(u_m,u_{m+1})\leq Lr_\theta(u_{m-1},u_m)\leq\cdots\leq L^{m-n}r_\theta(u_n,u_{n+1})<r_\theta(u_n,u_{n+1}),$$

which is contradiction. By using induction method, we will prove that

$$r_\theta(u_0,u_n)\leq sb\sum_{k=0}^{n-1}F_{k+1}L^k\quad\forall n\in\mathbb{N} \tag{3.2}$$

where $b=r_\theta(u_0,u_1)+r_\theta(u_1,u_2)+r_\theta(u_0,u_2)$ and s is constant such that $s\geq1$. Here, $\{F_k\}_{k\in\mathbb{N}}$ is the Fibonacci sequence, defined as $F_1=F_2=1$ and $F_n=F_{n-1}+F_{n-2}\quad n\geq3$. For $n=1$ and $n=2$ condition 3.2 holds trivially. Now, for $n\geq3$ assume that condition 3.2 is true for $n-1$ and $n-2$. Then we get:

$$\begin{aligned}
r_\theta(u_0,u_n) &\leq s[r_\theta(u_0,u_1)+r_\theta(u_1,u_2)+r_\theta(u_2,u_n)+r_\theta(u_0,u_2)+r_\theta(u_1,u_n)]\\
&\leq s[r_\theta(u_0,u_1)+r_\theta(u_1,u_2)+r_\theta(u_0,u_2)+L^2r_\theta(u_0,u_{n-2})+Lr_\theta(u_0,u_{n-1})]\\
&\leq s[b+bL^2\sum_{k=0}^{n-3}F_{k+1}L^k+bL\sum_{k=0}^{n-2}F_{k+1}L^k]\\
&= s[b+b\sum_{k=0}^{n-3}F_{k+1}L^{k+2}+b\sum_{k=0}^{n-2}F_{k+1}L^{k+1}]\\
&= s[b+bL+b\sum_{k=2}^{n-1}(F_{k-1}+F_k)L^k]\\
&= s[b\sum_{k=0}^{n-1}F_{k+1}L^k]
\end{aligned}$$

Now, we know that

$$\sum_{k=0}^{+\infty} F_{k+1}L^k = \frac{1}{1-L-L^2} \ \forall |L| \leq \psi^{-1} \tag{3.3}$$

where $\psi^{-1} = \frac{\sqrt{5}+1}{2}$ *[known as "golden ratio"] constant ([10]).*
Now, we will show that $\{u_n\}$ *is Cauchy sequence. We have two cases:*
Case 1 $L \in [0, \psi^{-1}]$. *Then by using 3.1, 3.2 and 3.3,* $\forall m, n \in \mathbb{N}$ *in order that* $m > n$*, we get*

$$\begin{aligned} r_\theta(u_n, u_m) &\leq L^n r_\theta(u_0, u_{m-n}) \leq bL^n \sum_{k=0}^{m-n-1} F_{k+1}L^k \leq bL^n \sum_{k=0}^{+\infty} F_{k+1}L^k \\ &= b\frac{L^n}{1-L-L^2} \to 0 \text{ as } m, n \to +\infty \end{aligned}$$

Case 2 $L \in [\psi^{-1}, 1]$. *Then* $\exists\ L^{n_0} < \psi^{-1}$ *for large* $n_0 \in \mathbb{N}$. *By using 3.1, we get*

$$r_\theta(T^{n_0}u, T^{n_0}v) \leq L^{n_0} r_\theta(u, v)$$

forall $u, v \in U$. *By applying same procedure to mapping* T^{n_0}*, this is also be turn down into first one. As* (U, r_θ) *is complete generalized Branciari b-metric space, then* $\exists\ w \in U$ *such that*

$$\lim_{n\to+\infty} r_\theta(u_n, w) = 0.$$

By using Lemma, we are able to say that limit is unique implies that $\{u_n\}$ *is Cauchy sequence of pairwise distinct points. Then from*

$$r_\theta(u_{n+1}, Tw) \leq L r_\theta(u_n, w) \to 0 \text{ as } n \to \infty.$$

Implies that $Tw = w$. *Hence,* w *is a fixed point of mapping* T. *One can easily show its uniqueness by contradiction, using contractive condition 3.1.* □

In 1968, R. Kannan improved the result of the Banach contraction principle by removing the condition of continuity. Now, we state and prove Kannan fixed point theorem on generalized Branciari b-metric space.

Theorem 3.2 *Let* (U, r_θ) *be a complete generalized Branciari b-metric space and* $T : U \to U$ *satisfy the following condition:*

$$r_\theta(Tu, Tv) \leq L[r_\theta(u, Tu) + r_\theta(v, Tv)] \ \forall u, v \in U \tag{3.4}$$

and $L \in [0, \frac{1}{2})$. *Then the mapping* T *has a unique fixed point* $w \in U$*, in order that* $\lim_{n\to\infty} r_\theta(T^n u, w) = 0\ \forall u \in U$.

Proof: *Let* $u \in U$ *be an arbitrary point and let* $\forall n \in \mathbb{N}_0\ u_n = T^n u$. *Then by frequent use of condition 3.4, we get*

$$r_\theta(u_n, u_{n+1}) \leq L[r_\theta(u_{n-1}, u_n) + r_\theta(u_n, u_{n+1})]$$

implies

$$r_\theta(u_n, u_{n+1}) \leq \frac{L}{1-L} r_\theta(u_{n-1}, u_n).$$

As $L \in [0, \frac{1}{2})$*, then* $\frac{L}{1-L} \in [0, 1)$. *Then, we are able to use the same method as in the proof of Theorem in order to show* $\{u_n\}$ *is a Cauchy sequence in* U. *Also as* T *is Kannan mapping then* T^n *satisfy the same condition having contractive constant* $\frac{L^n}{(1-L)^{n-1}}\ \forall n \in \mathbb{N}$. *Now, we have two cases* $\frac{L}{1-L} \in [0, \psi^{-1})$ *and* $\frac{L}{1-L} \in [\psi^{-1}, 1)$.

As (U, r_θ) *is complete generalized Branciari b-metric space, then* $\exists\ w \in U$ *such that* $\{u_n\}$ *converges to* w *uniquely. By using condition 3.4, we have*

$$\begin{aligned} r_\theta(w, Tw) &\leq s[r_\theta(w, u_n) + r_\theta(u_n, u_{n+1}) + r_\theta(u_{n+1}, Tw) + r_\theta(w, u_{n+1}) + r_\theta(u_n, Tw)] \\ &\leq s[r_\theta(w, u_n) + r_\theta(u_n, u_{n+1}) + Lr_\theta(u_n, u_{n+1}) + r_\theta(w, Tw) \\ &+ r_\theta(w, u_{n+1}) + Lr_\theta(u_{n-1}, u_n) + r_\theta(w, Tw)] \\ &\leq \frac{s}{1-2L}[r_\theta(w, u_n) + (1+L)r_\theta(u_n, u_{n+1}) + r_\theta(w, u_{n+1}) + Lr_\theta(u_{n-1}, u_n)] \end{aligned}$$

tends to 0 *as* $n \to +\infty$*. This implies that* $r_\theta(w, Tw) = 0$*. One can find the uniqueness of fixed point in a similar way of Theorem 3.1.* □

Example 3.2 *Let* $U = \{1, 2, 3, 4, 5\}$ *and* $r_\theta : U \times U \to \mathbb{R}^+$ *and* $s = 2$ *be defined as* $\forall u, v \in U$

$$r_\theta(u, v) = \begin{cases} 0, & \text{if } u = v \\ 4, & \text{if } u = 1, v \neq 1 \\ 1, & \text{if otherwise.} \end{cases}$$

By example 3.1, we are able to say that (U, r_θ) *is generalized Branciari b-metric space. Now, let* $T : U \to U$ *be defined as*

$$T(U) = \begin{cases} 2, & \text{if } u = 1, \\ 4, & \text{if otherwise,} \end{cases}$$

with $L = \frac{1}{2}$.

(1) $r_\theta(T(u), T(u)) = 0 \leq \frac{1}{2}r_\theta(u, u) = 0$ *forall* $u \in U$.

(2) $r_\theta(T(1), T(u)) = 1 \leq \frac{1}{2}r_\theta(1, u) = 2$ *forall* $u \in U \setminus \{1\}$.

(3) $r_\theta(T(2), T(u)) = 0 \leq \frac{1}{2}r_\theta(2, u) = \frac{1}{2}$ *forall* $u \in U \setminus \{1, 2\}$.

(4) $r_\theta(T(3), T(u)) = 0 \leq \frac{1}{2}r_\theta(3, u) = \frac{1}{2}$ *forall* $u \in U \setminus \{1, 2, 3\}$.

(5) $r_\theta(T(4), T(5)) = 0 \leq \frac{1}{2}r_\theta(4, 5) = \frac{1}{2}$.

Then the mapping T *satisfies the contraction condition. Hence,* T *has a unique fixed point* $w = 4 \in U$.

Furthermore, we apply our Theorem 3.2, that is, we will verify it for Kannan contraction. Now, taking $L = \frac{3}{4}$.

(1) $r_\theta(T(u), T(u)) = 0 \leq \frac{3}{4}r_\theta(u, Tu) + r_\theta(u, Tu) = \frac{3}{2}$ *forall* $u \in U \setminus \{1, 4\}$.

(2) $r_\theta(T(1), T(1)) = 0 \leq \frac{3}{4}r_\theta(1, T1) + r_\theta(1, T1) = 6$.

(3) $r_\theta(T(4), T(4)) = 0 \leq \frac{3}{4}r_\theta(4, T4) + r_\theta(4, T4) = 0$.

(4) $r_\theta(T(1), T(u)) = 1 \leq \frac{3}{4}r_\theta(1, T1) + r_\theta(u, Tu) = \frac{15}{4}$ *forall* $u \in U \setminus \{1, 4\}$.

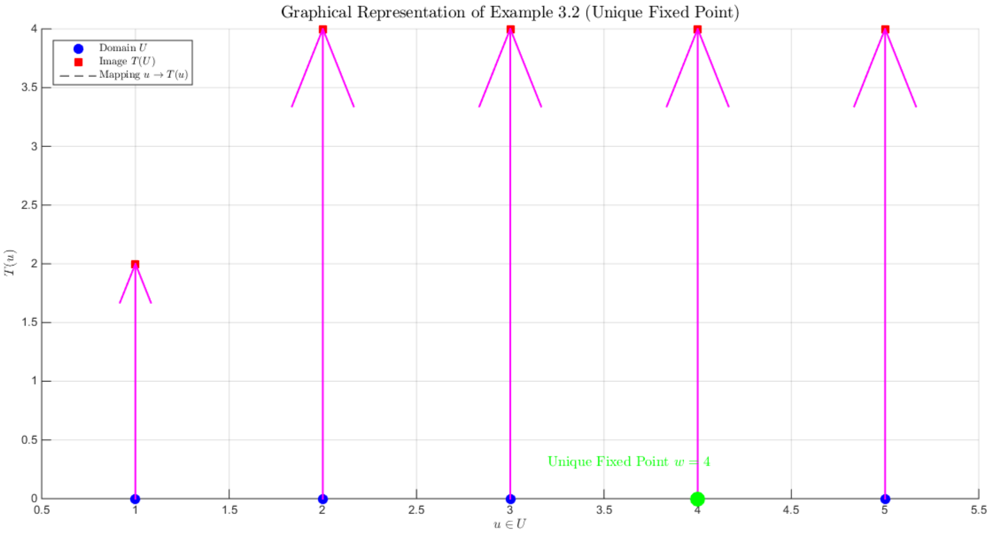
(5) $r_\theta(T(1), T(4)) = 1 \leq \frac{3}{4}r_\theta(1, T1) + r_\theta(4, T4) = 3$.

(6) $r_\theta(T(u), T(v)) = 0 \leq \frac{3}{4}r_\theta(u, Tu) + r_\theta(v, Tv) = \frac{3}{2}$ *forall* $u, v \in U \setminus \{1, 4\}$.

(7) $r_\theta(T(u), T(4)) = 0 \leq \frac{3}{4}r_\theta(u, Tu) + r_\theta(4, T4) = \frac{3}{2}$ *forall* $u \in U \setminus \{1, 4\}$.

The mapping T *also satisfies the Kannan contraction condition.* T *has a unique fixed point* $w = 4 \in U$ *(as shown in (??))*

On setting $s = 1$ in Theorem 3.1, we have the following corollary due to A. Kostic [2].



Corollary 3.1 *Let* (U, r_θ) *be a complete generalized Branciari metric space and* $T : U \to U$ *satisfy the following condition:*

$$r_\theta(Tu, Tv) \leq L r_\theta(u, v) \ \forall u, v \in U$$

and $L \in [0, 1)$. *Then the mapping* T *has a unique fixed point* $w \in U$, *in order that* $\lim_{n\to\infty} T^n u = w \ \forall u \in U$.

On setting $s = 1$ and $r_\theta(u, b) + r_\theta(v, a) = 0$ in Theorem 3.1, we have the following corollary due to A. Branciari [1], in 2000.

Corollary 3.2 *Let* (U, r_θ) *be a complete rectangular metric space and* $T : U \to U$ *satisfy the following condition:*

$$r_\theta(Tu, Tv) \leq L r_\theta(u, v) \ \forall u, v \in U$$

and $L \in [0, 1)$. *Then the mapping* T *has a unique fixed point* $w \in U$, *in order that* $\lim_{n\to\infty} T^n u = w \ \forall u \in U$.

On setting $s = 1$ in Theorem 3.2, we have the following corollary due to A. Kostic [2].

Corollary 3.3 *Let* (U, r_θ) *be a complete generalized Branciari metric space and* $T : U \to U$ *satisfy the following condition:*

$$r_\theta(Tu, Tv) \leq L[r_\theta(u, Tu) + r_\theta(v, Tv)] \ \forall u, v \in U$$

and $L \in [0, \frac{1}{2})$. *Then the mapping* T *has a unique fixed point* $w \in U$, *in order that* $\lim_{n\to\infty} r_\theta(T^n u, w) = 0 \ \forall u \in U$.

4. Application

We determine the existence and uniqueness of following type of integral equation as an application of the theorem 3.1.

$$u(\phi) = \int_\alpha^\beta \Gamma(\phi, \xi, u(\xi)) d\xi + h(\phi), \ \forall \phi, \xi \in [\alpha, \beta]. \tag{4.1}$$

Let $U = C([\alpha, \beta]], \mathbb{R})$ and $\Gamma, h \in C([\alpha, \beta], \mathbb{R})$. Assume $r_\theta : U \times U \to \mathbb{R}^+$ such that $r_\theta(u, v) = \sup_{\phi \in [\alpha, \beta]} |(u(\phi) - v(\phi))|, \ \forall u, v \in U$. As a result (U, r_θ) is a complete generalized Branciari b-metric space.

Theorem 4.1 *Suppose*

$$|\Gamma(\phi,\xi,u(\xi))-\Gamma(\phi,\xi,v(\xi))|\leq \frac{1}{2(\beta-\alpha)}|u(\phi)-v(\phi)|,\ \forall\phi,\xi\in[\alpha,\beta].$$

This implies that integral equation 4.1 has a solution which is unique.

Proof: *Assume* $T:U\to U$ *by* $Tu(\phi)=\int_{\alpha}^{\beta}\Gamma(\phi,\xi,u(\xi))d\xi+h(\phi),\ \forall\phi,\xi\in[\alpha,\beta]$. *As it is obvious that,* u *is the fixed point iff it is a solution of the integral equation 4.1.* $\forall\ u,v\in U$, *we get*

$$\begin{aligned}
r_\theta(Tu,Tv) &= \sup_{\phi\in[\alpha,\beta]}|T(u(\phi))-T(v(\phi))|\\
&= \sup_{\phi\in[\alpha,\beta]}\Big|\int_{\alpha}^{\beta}\Gamma(\phi,\xi,u(\xi))-\Gamma(\phi,\xi,v(\xi))d\xi\Big|\\
&\leq \sup_{\phi\in[\alpha,\beta]}\int_{\alpha}^{\beta}\frac{1}{2(\beta-\alpha)}|u(\phi)-v(\phi)|d\xi\\
&\leq \frac{1}{2(\beta-\alpha)}\sup_{\phi\in[\alpha,\beta]}|u(\phi)-v(\phi)|\int_{\alpha}^{\beta}d\xi\\
&\leq \frac{1}{2}r_\theta(u,v)\\
&= Lr_\theta(u,v).
\end{aligned}$$

Hence, the condition of theorem 3.1 is satisfied. So, we are able to say that T *has a solution which is unique.* □

5. Conclusion

The paper included the theory of newly introduced generalized Branciari b-metric space and also provided the topological properties like Cauchy, convergent and completeness. We also proved some fixed point results using contraction mapping and Kannan contractive mapping. An example is constructed to demonstrate the validity of results. Finally, we applied our main results to find the existence and uniqueness of a solution of the system of Fredholm integral equation.

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