Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2025 (43) 3** : 1–25.
ISSN-0037-8712
doi:10.5269/bspm.79085

Positive Impact of Waste Management Strategies and Decision Analysis with Intuitionistic Fuzzy Sugeno-Weber Aggregation Operators

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ABSTRACT: Waste management is a crucial and significant subject that has gained much attention globally because it has several environmental, social, financial and economic implications. Solid waste management is a very challenging task for clean urban and rural societies. We studied some reliable strategies for handling the waste materials and garbage produced by people. To serve this purpose, an intuitionistic fuzzy set (IFS) is a well-known model used for modeling and processing unpredictable information and providing accurate approximated results in the decision-making process. Power average operators allow the interrelationship of the input arguments and deal with uncertain information in complicated situations. This article expresses Sugeno-weber triangular norms under intuitionistic fuzzy (IF) information. We developed a class of new aggregation operators, including intuitionistic fuzzy Sugeno-Weber power-weighted average (IFSWPWA) and intuitionistic fuzzy Sugeno-Weber power-weighted geometric (IFSWPWG) operators. It is observed that both the newly proposed operators satisfy the properties of aggregation. The multi-criteria decision-making (MCDM) problem is proposed to evaluate real-life applications and numerical examples. An experimental case study under the system of waste materials is considered in the article to reveal the intensity and applicability of derived approaches. The comparison analysis and sensitivity analysis show the significance of our proposed work.

Key Words: Sugeno-Weber t- t-norms, intuitionistic fuzzy set, green supplier selection, multi-criteria decision-making process.

Contents

2010 *Mathematics Subject Classification:* 03E72.
Submitted September 21, 2025. Published December 20, 2025

1. Introduction

One major difficulty faced by several nations worldwide is the sustainable and appropriate handling of their generated solid wastes [1]. Even yet, emerging countries like Nigeria experience it more than developed ones. Due to their distraction from rapid industrial and economic growth, many emerging countries have neglected to manage the solid waste that they produce [2]. The six stages of solid waste management are creation, collection, storage, processing/recovery, transportation, and disposal [3]. These stages are generally separated into six categories. The process of collecting and transporting solid waste includes storing it temporarily at various locations such as dumps, gathering it, loading it into waste trucks or compactor vehicles, and then moving it to transfer stations, treatment facilities, or landfills for eventual disposal [4]. In contrast, processing and recovery include various treatment and recovery techniques that support recycling, reduction, and utilization efforts in accordance with the hierarchy of integrated solid waste management [7]. The characterization analysis of the generated solid waste is the cornerstone of any appropriate solid waste management strategy or procedure [5]. Since the formation of garbage has become an inherent aspect of life, it is imperative to conduct a thorough analysis of the many components that comprise the waste stream [[6], [8], [9]]. Two significant benefits are realized for the organization or municipality when the appropriate reduction/minimization, preventative, or recovery technique is used: decreasing institutional costs and extending the sanitary landfill's lifespan. Additionally, a decrease in the amount of solid waste generated in any given facility or city directly lessens the negative impacts on the surrounding environment and values of society. Higher education institutions are required to take the lead in the movement for environmental preservation and conservation as well as to uphold moral and ethical standards. Furthermore, colleges educate and teach those who are purportedly the society's experts in this field. Higher education institutions possess the necessary ability to initiate campaigns, transfer knowledge, create tools and technology, and encourage sustainable practices on campus and in the surrounding communities by virtue of their purported expertise in environmental management, including solid waste management. The aforementioned procedures will set a good example for the community and pupils and lower expenses across the board for the management process [10]. For this purpose, we initiated some reliable techniques and strategies under consideration in the decision-analysis process. The MCDM approach is used to handle complex problems where many elements must be considered [11]. The MCDM is essential in real-world decision-making situations, particularly when the stakes are high or the decision maker (DM) selects based on equally compelling possibilities. In this post, we'll look at the practical benefits of MCDM and how it may aid both individuals and companies in making better decisions [12], [13]. A variety of fields, including business, engineering, economics, healthcare and politics, may benefit from MCDM. To choose the finest supplier, acquire staff, make the most profitable investment, or develop the best marketing policy, for instance, businesses utilize MCDM. MCDM is used in engineering to find the best design, choose the best material, or raise a system's performance. The MCDM process also used in the healthcare sector to apportion resources, assess the quality of treatment and assess the effectiveness of different medicines. Politicians decide public policies based on the theory of the MCDM process such as prioritizing public projects and distributing public funds. The MCDM problem's key feature is to provide a systematic and rational approach to decision-making, helping decision-makers (DMs) avoid drawing judgments based on personal preferences, preconceptions, or instincts. The DM must identify the decision issue as part of MCDM, define the criteria, estimate the options and choose the best one utilizing a transparent process. This procedure ensures that all pertinent factors are considered and that the choice is made based on fair, impartial standards. The ability for DMs to take into account multiple factors at once, which can be challenging to achieve without a clear decision-making process, is another advantage of MCDM. For example, the DM might need to consider the applicant's background, skills, personality, and compatibility with the company culture. Using MCDM, the DM can assess each candidate based on each criterion, assigning a weight to each criterion based on its relative importance and selecting the candidate with the highest total score [14]. This method guarantees that all relevant data regarding every candidate will be considered during the selection process rather than just one or two factors. Decision-making is very straightforward when dealing with situations with a single criterion. In this case, we only need to select the option with the highest preference characteristics. However, a number of problems, including weighted criteria, preference dependence, and attribute conflicts, make things more difficult when dealing with alternatives that satisfy a large number of criteria. Complex and more sophisticated strategies must be created to address these problems. In order to solve MCDM challenges, one must first determine how many features or criteria are present in each case and how to recognize them. Most real-world situations are characterized by uncertainty, which may be due to the significant number of failures and strong opinions throughout the information-acquiring process. The information cannot be shown exactly due to the system's complexity. Numerous research scholars and mathematicians have recently developed a number of theories.

1.1. Literature Review

To eliminate the uncertainty of human opinions, Zadeh [15] introduced the theory of fuzzy set (FS) with membership term (MT) of an object range from 0 to 1 by generalizing the concepts of crips set theory. The crips set theory provides knowledge about limited or discrete circumstances and fails to express certain environments such as beauty, age, intelligence and many other scenarios. Several mathematicians utilized the theory of FSs in different circumstances and gained efficient results, which helped resolve unrealistic real-life challenges. Adlassnig [16] illustrated the properties of FSs for formalizing vague information of human opinions for improving medical diagnosis. Information on human opinion is not unidirectional. Atanassov [17] also extended the theory of FSs with two different components of MT $\vartheta(\zeta)$ and non-membership term (NMT) $\gamma(\zeta)$ range from 0 and 1. The mathematical expression of an IFS, such as $0 \leq \vartheta(\zeta) + \gamma(\zeta) \leq 1$. The concepts of an IFS demonstrated in several circumstances to estimate inconsistency among dubious information. By using the basic notion of an IFS, Atanassov [18] determined fuzzy relation and their operations to deduce certain new characteristics of an IFS. Different operational laws, unions and intersections under consideration of intuitionistic fuzzy information were developed by Atanassov [19]. The IFS has drawn much attention in recent decades due to its useful and understandable qualities compared to conventional FS. A robust generalization of an IFS in the form of an IF soft set was presented by Ahmmad et al. [20], Albaity et al. [21] applied the theory of IFS to enhance the reliability of machine leering to simulate human behavior, Imran et al. [22] build up decision making framework for industry robot selection under the system of an IF information. Due to their capacity for handling uncertainty and imprecision, AOs play an essential role in MCDM. Fuzzy logic and other mathematical methods may be utilized to address imprecise scores in AOs. Additionally, based on probability theory or other statistical approaches, they may be utilized to deal with ambiguous scores. The AOs also can manage competing criteria. In certain circumstances, the DM may have opposing preferences for various criteria. By using compromise programming or other methods, AOs may be utilized to address competing criteria [23]. Finding a solution that more fully fulfills certain requirements than others is the goal of compromise programming.

A sustainable mechanism for the development of a transportation system was presented by Sarkar et al. [24]. Senapati et al. [25] anticipated robust mathematical strategies by operational laws of Aczel Alsina t-norms under considering the theory of IFSs and gave a strong mechanism for sharing the practice of transportation. Hussain et al. [26] developed reliable strategies for solving real-life challenges based on pythagorean fuzzy models. By using the theory of entropy measure to express unknown degrees of weights based on IFVs, Garg [27] developed a series of new strategies. Zhou et al. [28] introduced a robust theory of divergence measures by using properties of belief function based on pythagorean fuzzy information. Garg and Chen [29] illustrated a series of new approaches using certain neutrality aggregation properties. Using the prioritization theory in decision-making problems, Arora and Garg [30] explained linguistic terms with IF information to handle fuzziness in human opinions. Akram and Bibi [31] established a series of improved decision-making techniques under the system of a 2-tuple linguistic farm teen fuzzy system. Sarkar et al. [32] gave some useful aggregation tools to express fuzziness in given information with Dual hesitant q-rung orthopair fuzzy information. Fei and Deng [33] generalized the theory of IFSs to resolve complex real-life situations with a multi-attribute decision-making process. Motivated by the theory of Dombi t-norms, Hussain et al. [34] illustrated some realistic mathematical methodologies by exploring the concepts of fuzziness. Mahmood et al. [35] utilized a reliable theory of an interval-valued picture fuzzy information to handle awkward and unrealistic human opinion information. Deng and Deng [36] discussed some appropriate properties of fuzzy membership function in the frameworks like volume of fuzzy membership terms. By combining two different concepts of unknown weight vectors and power Bonferroni operators, Gao [37] developed a class of new approaches to resolve real-life situations.

Ali and Naeem [38] presented reliable mathematical approaches to overcome the impact of uncertain information under the system of complex q-rung orthopair fuzzy information. Hayat et al. [39] proposed an advanced decision-making approach to resolve different real-life situations under the interval-valued q-rung orthopair fuzzy soft sets. Hussain et al. [40] resolved a complicated real life application by utilizing the properties of Hamy mean models and Aczel Alsina t-norms. To express correlation among different input arguments, Juan and Qiang [41] utilized linguistic terms in the interval-valued hesitant fuzzy sets framework.

1.2. Motivation for Proposed Research Work

In 1974, Sugeno [42] established a class of nilpotent t-conorms with certain properties in his Ph.D. thesis. Additionally, Weber [43] presented a class of nilpotent t-norms (as well as asymptotic members products and drastic products). Discussed t-norm and t-conorm are dual to each other, specifically Sugeno t-conorm with parameter $\Psi \in]-1,\infty[$ and Weber t-norm with the parameter $\Psi \in \frac{-1}{1+\Psi}]-1,\infty[$. Both derived families are known as Sugeno-Weber t-norm and t-conorm in honor of Sugeno and Weber. We suggest reading about basic concepts in Klement [44] paragraph 4.7 for further information. The Sugeno-Weber t-norms can be seen as reduced by 0 lineae using a probabilistic approach, taking into account the product for independent random events A and B, $P(A\bigcap B) = T_P(P(A), P(B))$, the product t-norm is given by $T_P(x, y) = x.y$ as well as the valuation-based boundary $P(A\bigcap B) \geq P(A)+P(B)-1$. Decision-makers (DM) have more freedom when using a parameter ψ in the operations of Sugeno-Weber t-norms since it allows them to adjust the parameter's value correctly. As a consequence, the SW t-N and t-CN seem suitable for specifying IFS operations and minimizing errors and redundant data. We studied proposed research work recently by different mathematicians. For instance, Ghodousian et al. [45] applied different properties of Sugeno Weber triangular norms to solve non-linear optimization. Murofushi and Sugeno [46] discussed some specific properties of the Choquet integral using Sugeno-weber triangular norms. They also studied different fuzzy measures based on algebraic t-norms and t-conorms. An innovative approach of the spherical fuzzy hypersoft set based on certain characteristics of the Sugeno-Weber triangular norms was developed by Sarkar et al. [47].

1.3. The Novelty of Proposed Research Work

An intuitionistic fuzzy aggregation model deals with vague and imprecision information to cope with awkward information of human opinions. An IFS is the modified version of the FS due to extensive information on two components of human opinions in the form of MT and NMT. However, the above-discussed methodologies have several benefits, which numerous mathematicians have developed. Different scholars propose many aggregation tools based on the generalization of triangular norms. Sugeno-Weber aggregation tools were introduced by generalizing the theory of nilpotent triangular norms. Considering the significance of two different theories, like power aggregation tools and basic operations of Sugeno-Weber aggregation tools, we introduced an innovation of new approaches to cope with imprecision and unpredictable human information. To serve this purpose, the main contribution of the proposed work is organized as follows:

1. To expose the theory of IF information to cope with uncertain information during decision-making.
2. We demonstrate some appropriate operations of Sugeno-weber aggregation tools in light of IF information.
3. We developed some robust aggregation approaches by combining two theories, power aggregation tools and Sugeno-Weber triangular norms, including IFSWPA, IFSWPG, IFSWPWA and IFSWPWG operators.
4. Some special cases and notable characteristics are also illustrated to express the effectiveness and consistency of our developed approaches.
5. The theory of the MCDM problem is utilized to find out the solution to real-life situations under consideration of IF information.

6. We gave a numerical example to demonstrate a suitable optimal solution under consideration of developed approaches.

7. A comprehensive comparative study is illustrated to contrast the results of existing methodologies with current approaches.

1.4. Structure of Article

The remaining part of this article is characterized as follows: Section 2 overviews the basic notion of Sugeno-Weber aggregation tools and IFSs with some basic rules. Basic operations of Sugeno-Weber triangular norms in the light of an IF information illustrated in section 3. We proposed new approaches to Sugeno-Weber t-norms, such as IFSWPA and IFSWPG operators in section 4. Section 5 also presents a reliable mathematical strategy of Sugeno-Weber t-norms based on intuitionistic fuzzy information like IFSWPWA and IFSWPWG operators. Section 6 explores the significance of the MCDM problem under considering derived mathematical approaches. With the help of numerical examples, a case study also reveals the intensity of discussed research work in section 7. The efficiency and validity of our proposed methodologies are seen by comparing the findings of existing approaches with currently developed aggregation techniques in section 8. Finally, a comprehensive overview of the whole article is presented in section 9.

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2. Preliminaries

In this section, we discuss some particular notions of Sugeno Weber's t-norm and t-conorm, IFSs, and their related fundamental rules necessary to improve current research work.

Definition 2.1 *[47] The Sugeno-weber t-norm and t-conorm are expressed as follows:*

$$T^{\psi}(\alpha,\eta)=\begin{cases}T_D(\alpha,\eta), & \text{if } \psi=-1,\\ \max\left(0,\dfrac{\alpha+\eta-1+\psi\alpha\eta}{1+\psi}\right), & \text{if } -1<\psi<+\infty,\\ T_P(\alpha,\eta), & \text{if } \psi=+\infty.\end{cases}$$

and

$$S^{\psi}(\alpha,\eta)=\begin{cases}S_D(\alpha,\eta), & \text{if } \psi=-1,\\ \min\left(1,\alpha+\eta-\dfrac{\psi}{1+\psi}\alpha\eta\right), & \text{if } -1<\psi<+\infty,\\ S_P(\alpha,\eta), & \text{if } \psi=+\infty.\end{cases}$$

Definition 2.2 *[17] Consider $\mathcal{W}$ be a non-empty set and an IFS $\mathcal{A}$ on $\mathcal{W}$ is given by:*

$$\mathcal{A}=\{\zeta,(\vartheta(\zeta),\gamma(\zeta))\mid\zeta\in\mathcal{W}\}$$

Where $\vartheta:\mathcal{W}\to[0,1]$ and $\gamma:\mathcal{W}\to[0,1]$ represent the membership term (MT) and non-membership term (NMT) of ζ in $\mathcal{A}$, respectively. An IFS is defined with the following conditions:

$$0\le\vartheta(\zeta)+\gamma(\zeta)\le1,\zeta\in\mathcal{W}$$

The hesitancy term of ζ in IFS is given by $r(\zeta)=1-(\vartheta(\zeta)+\gamma(\zeta))$. The pair $\mathcal{F}=(\vartheta(\zeta),\gamma(\zeta))$ is known as an intuitionistic fuzzy value (IFV).

Definition 2.3 *[48] For three IFVs $\mathcal{F} = (\vartheta(\zeta), \gamma(\zeta))$, $\mathcal{F}_1 = (\vartheta_1(\zeta), \gamma_1(\zeta))$ and $\mathcal{F}_2 = (\vartheta_2(\zeta), \gamma_2(\zeta)$. Some necessary operations are expressed as follows:*

1. $\mathcal{F}_1 \subseteq \mathcal{F}_2,$ *if* $\vartheta_1(\zeta) \leq \vartheta_2(\zeta),\ \gamma_1(\zeta) \geq \gamma_2(\zeta)$
2. $\mathcal{F}_1 = \mathcal{F}_2,$ *if* $\mathcal{F}_1 \subseteq \mathcal{F}_2,$ *and* $\mathcal{F}_1 \supseteq \mathcal{F}_2$
3. $\mathcal{F}_1 \cup \mathcal{F}_2 = \left(\max\left(\vartheta_1(\zeta), \vartheta_2(\zeta)\right),\ \min\left(\gamma_1(\zeta), \gamma_2(\zeta)\right)\right)$
4. $\mathcal{F}_1 \cap \mathcal{F}_2 = \left(\min\left(\vartheta_1(\zeta), \vartheta_2(\zeta)\right),\ \max\left(\gamma_1(\zeta), \gamma_2(\zeta)\right)\right)$
5. $\mathcal{F}^C = (\gamma(\zeta), \vartheta(\zeta)), \quad \forall\, \zeta \in \mathcal{W}$

Definition 2.4 *[48] For three IFVs $\mathcal{F} = (\vartheta(\zeta), \gamma(\zeta))$, $\mathcal{F}_1 = (\vartheta_1(\zeta), \gamma_1(\zeta))$ and $\mathcal{F}_2 = (\vartheta_2(\zeta), \gamma_2(\zeta)$, with $\mho > 0$ Then, we have:*

1. $\mathcal{F}_1 \oplus \mathcal{F}_2 = (\vartheta_1(\zeta) + \vartheta_2(\zeta) - \vartheta_1(\zeta)\,\vartheta_2(\zeta),\ \gamma_1(\zeta)\,\gamma_2(\zeta))$
2. $\mathcal{F}_1 \otimes \mathcal{F}_2 = (\vartheta_1(\zeta) \cdot \vartheta_2(\zeta),\ \gamma_1(\zeta) + \gamma_2(\zeta) - \gamma_1(\zeta) \cdot \gamma_2(\zeta))$
3. $\mho\mathcal{F} = \left(1 - (1 - \vartheta(\zeta))^{\mho}, \gamma^{\mho}(\zeta)\right)$
4. $\mathcal{F}^{\mho} = \left(\vartheta^{\mho}(\zeta),\, 1 - (1 - \gamma(\zeta))^{\mho}\right)$

Definition 2.5 *[49] For any IFV $\mathcal{F} = (\vartheta(\zeta), \gamma(\zeta))$, we can express the score function and accuracy function as follows:*

$$\mathcal{S}(\mathcal{F}) = (\vartheta(\zeta) - \gamma(\zeta)), \mathcal{S}(\mathcal{F}) \in [-1, 1]$$

and

$$\mathcal{A}(\mathcal{F}) = (\vartheta(\zeta) + \gamma(\zeta)), \mathcal{A}(\mathcal{F}) \in [0, 1]$$

For two IFVs $\mathcal{F}_1 = (\vartheta_1(\zeta), \gamma_1(\zeta))$ and $\mathcal{F}_2 = (\vartheta_2(\zeta), \gamma_2(\zeta)$. $\mathcal{F}_1$ is preferable over $\mathcal{F}_2$ if $\mathcal{S}(\mathcal{F}_1) > \mathcal{S}(\mathcal{F}_2)$ and $\mathcal{F}_2$ is preferable over $\mathcal{F}_1$ if $\mathcal{S}(\mathcal{F}_2) > \mathcal{S}(\mathcal{F}_1)$. Otherwise, we move on accuracty function $\mathcal{A}(\mathcal{F})$.

Definition 2.6 *[50] Suppose $\chi_i, i = 1, 2, ...\eta$ be a class of positive real numbers. Then the power average operator is expressed as follows:*

$$PA(\chi_1, \chi_2, \ldots, \chi_\eta) = \frac{(1 + A(\chi_i))\chi_i}{\sum_{i=1}^{\eta}(1 + A(\chi_i))}$$

Where $A(\chi_i) = \sum_{\substack{i=1 \\ i \neq \tau}}^{\bar{i}} Supp(\chi_i, \chi_\tau), \quad i, \tau = 1, 2, \ldots, \eta.$

3. Operations of Sugeno-Weber Triangular Norms Based on IF Information

Using basic notions of Sugeno-Weber triangular norm, some necessary operations are demonstrated under the IF information system as follows.

Definition 3.1 *For three IFVs $\mathcal{F} = (\vartheta(\zeta), \gamma(\zeta))$, $\mathcal{F}_1 = (\vartheta_1(\zeta), \gamma_1(\zeta))$ and $\mathcal{F}_2 = (\vartheta_2(\zeta), \gamma_2(\zeta)$, with $\Delta > 0$ Then, some basic operations of SW-TNM and SW-TCNM are expressed as follows:*

1. $\mathcal{F}_1 \oplus \mathcal{F}_2 = \left(\vartheta_1(\zeta) + \vartheta_2(\zeta) - \frac{\Psi}{1+\Psi}\,\vartheta_1(\zeta)\,\vartheta_2(\zeta),\ \frac{\gamma_1(\zeta)+\gamma_2(\zeta)-1+\Psi\,\gamma_1(\zeta)\,\gamma_2(\zeta)}{1+\Psi}\right)$

2. $\mathcal{F}_1 \otimes \mathcal{F}_2 = \left(\frac{\vartheta_1(\zeta)+\vartheta_2(\zeta)-1+\Psi\,\vartheta_1(\zeta)\,\vartheta_2(\zeta)}{1+\Psi},\ \frac{\gamma_1(\zeta)+\gamma_2(\zeta)-\Psi+\Psi\,\gamma_1(\zeta)\,\gamma_2(\zeta)}{1+\Psi} \right)$

3. $\Delta\mathcal{F} = \left(\frac{1+\psi}{\psi}\left(1-\left(1-\vartheta(\zeta)\,\frac{\psi}{1+\psi}\right)^{\Delta}\right),\ \frac{(1+\psi)\left(\frac{\psi\gamma(\zeta)+1}{1+\psi}\right)^{\Delta}-1}{\psi} \right)$

4. $\mathcal{F}^{\Delta} = \left(\frac{1}{\psi}\left((1+\psi)\left(\frac{\psi\vartheta(\zeta)+1}{1+\psi}\right)^{\Delta}-1\right),\ \frac{1+\psi}{\psi}\left(1-\left(1-\gamma(\zeta)\frac{\psi}{1+\psi}\right)^{\Delta}\right) \right)$

4. Intuitionistic Fuzzy Sugeno-Weber Power Aggregation Operators

We develop new approaches based on Sugeno-Weber t-norms in the light of intuitionistic fuzzy information, including IFSWPA and IFSWPA operators.

Definition 4.1 *Let $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., \eta$ be a class of IFVs. The IFSWPA operator is given by:*

$$IFSWPA(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_\eta) = \bigoplus_{i=1}^{\eta} \mathfrak{w}_i \mathcal{F}_i$$

Where $\mathfrak{w}_i = \frac{(1+A(\mathcal{F}_i)\big)}{\sum_{i=1}^{\eta}(1+A(\mathcal{F}_i))}, i = 1, 2, 3, ..., \eta$ *and* $A(\mathcal{F}_i) = \sum_{\substack{i=1\\ i\neq j}}^{\eta} Supp(\mathcal{F}_i, \mathcal{F}_j)$.

Theorem 4.1 *Let $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., n$ be a class of IFVs. Then, the aggregated outcome by the IFSWPA operator is still an IFV given by:*

$$IFSWPA(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_\eta) = \begin{pmatrix} \frac{1+\psi}{\psi}\left(1-\prod_{i=1}^{\eta}\left(1-\vartheta_i(\zeta)\,\frac{\psi}{1+\psi}\right)^{\mathfrak{w}_i}\right), \\ \frac{1}{\psi}\left((1+\psi)\prod_{i=1}^{\eta}\left(\frac{\psi\gamma_i(\zeta)+1}{1+\psi}\right)^{\mathfrak{w}_i}-1\right). \end{pmatrix}$$

Proof: We can prove the above expression by using the induction method. For $\eta = 2$, we can write:

$$\mathfrak{w}_1\mathcal{F}_1 = \left(\frac{1+\psi}{\psi}\left(1-\left(1-\vartheta_1(\zeta)\frac{\psi}{1+\psi}\right)^{\mathfrak{w}_1}\right),\ \frac{(1+\psi)\left(\frac{\psi\gamma_1(\zeta)+1}{1+\psi}\right)^{\mathfrak{w}_1}-1}{\psi} \right)$$

$$\mathfrak{w}_2\mathcal{F}_2 = \left(\frac{\frac{1+\psi}{\psi}\left(1-\left(1-\vartheta_2(\zeta)\frac{\psi}{1+\psi}\right)^{\mathfrak{w}_2}\right)}{},\ \frac{(1+\psi)\left(\frac{\psi\gamma_2(\zeta)+1}{1+\psi}\right)^{\mathfrak{w}_2}-1}{\psi} \right)$$

$$IFSWPA(\mathcal{F}_1, \mathcal{F}_2) = \bigoplus_{i=1}^{2} \mathfrak{w}_i\mathcal{F}_i = \mathfrak{w}_1\mathcal{F}_1 \bigoplus \mathfrak{w}_2\mathcal{F}_2$$

$$\text{IFSWPA}(\mathcal{F}_1, \mathcal{F}_2) = \begin{pmatrix} \frac{1+\psi}{\psi}\left(1-\left(1-\vartheta_1(\zeta)\,\frac{\psi}{1+\psi}\right)^{\mathfrak{w}_1}\right) \oplus \frac{1+\psi}{\psi}\left(1-\left(1-\vartheta_2(\zeta)\,\frac{\psi}{1+\psi}\right)^{\mathfrak{w}_2}\right) \\ \frac{1}{\psi}\left[(1+\psi)\left(\frac{\psi\,\gamma_1(\zeta)+1}{1+\psi}\right)^{\mathfrak{w}_1}-1\right] \oplus \frac{1}{\psi}\left[(1+\psi)\left(\frac{\psi\,\gamma_2(\zeta)+1}{1+\psi}\right)^{\mathfrak{w}_2}-1\right] \end{pmatrix}$$

$$\mathrm{IFSWPA}(\mathcal{F}_1,\mathcal{F}_2)=\left(\begin{array}{c}\frac{1+\psi}{\psi}\Big(1-\big(1-\vartheta_1(\zeta)\frac{\psi}{1+\psi}\big)^{\mathfrak{w}_1}\Big)+\frac{1+\psi}{\psi}\Big(1-\big(1-\vartheta_2(\zeta)\frac{\psi}{1+\psi}\big)^{\mathfrak{w}_2}\Big)\\ -\frac{\psi}{1+\psi}\left[\frac{1+\psi}{\psi}\Big(1-\big(1-\vartheta_1(\zeta)\frac{\psi}{1+\psi}\big)^{\mathfrak{w}_1}\Big)\right]\left[\frac{1+\psi}{\psi}\Big(1-\big(1-\vartheta_2(\zeta)\frac{\psi}{1+\psi}\big)^{\mathfrak{w}_2}\Big)\right]\end{array}\right.,$$

$$\left.\begin{array}{c}\frac{1}{1+\psi}\left(\Big(\frac{(1+\psi)\left(\frac{\psi\gamma_1(\zeta)+1}{1+\psi}\right)^{\mathfrak{w}_1}-1}{\psi}\Big)+\Big(\frac{(1+\psi)\left(\frac{\psi\gamma_2(\zeta)+1}{1+\psi}\right)^{\mathfrak{w}_2}-1}{\psi}\Big)-1\right.\\ \left.+\psi\left[\frac{(1+\psi)\left(\frac{\psi\gamma_1(\zeta)+1}{1+\psi}\right)^{\mathfrak{w}_1}-1}{\psi}\right]\left[\frac{(1+\psi)\left(\frac{\psi\gamma_2(\zeta)+1}{1+\psi}\right)^{\mathfrak{w}_2}-1}{\psi}\right]\right)\end{array}\right)$$

$$\mathrm{IFSWPA}(\mathcal{F}_1,\mathcal{F}_2)=\left(\begin{array}{c}\frac{1+\psi}{\psi}\left(1-\prod_{i=1}^{2}\left(1-\vartheta_i(\zeta)\Big(\frac{\psi}{1+\psi}\Big)\right)^{\mathfrak{w}_i}\right)\\ \frac{1}{\psi}\left((1+\psi)\prod_{i=1}^{2}\Big(\frac{\psi\,\gamma_i(\zeta)+1}{1+\psi}\Big)^{\mathfrak{w}_i}-1\right)\end{array}\right)$$

We see Eq. 3 holds true for $\eta=2$. Let it also be true for $\eta=k$. i.e.,

$$IFSWPA(\mathcal{F}_1,\mathcal{F}_2,...,\mathcal{F}_k)=\bigoplus_{i=1}^{k}\mathfrak{w}_i\mathcal{F}_i$$

$$=\left(\begin{array}{c}\frac{1+\psi}{\psi}\left(1-\prod_{i=1}^{k}\left(1-\vartheta_i(\zeta)\left(\frac{\psi}{1+\psi}\right)\right)^{\mathfrak{w}_i}\right),\\ \frac{1}{\psi}\left((1+\psi)\prod_{i=1}^{k}\left(\frac{\psi\gamma_i(\zeta)+1}{1+\psi}\right)^{\mathfrak{w}_i}-1\right)\end{array}\right)$$

For further process, we have to show that Eq. 3 holds for $\eta=k+1$.

$$IFSWPA(\mathcal{F}_1,\mathcal{F}_2,...,\mathcal{F}_{k+1})=\bigoplus_{i=1}^{k}\mathfrak{w}_i\mathcal{F}_i\bigoplus\mathfrak{w}_{k+1}\mathcal{F}_{k+1}$$

$$=\left(\begin{array}{c}\frac{1+\psi}{\psi}\left(1-\prod_{i=1}^{k}\left(1-\vartheta_i(\zeta)\left(\frac{\psi}{1+\psi}\right)\right)^{\mathfrak{w}_i}\right)\\ \frac{1}{\psi}\left((1+\psi)\prod_{i=1}^{k}\left(\frac{\psi\gamma_i(\zeta)+1}{1+\psi}\right)^{\mathfrak{w}_i}-1\right)\end{array}\right)\oplus\left(\begin{array}{c}\frac{1+\psi}{\psi}\left(1-\left(1-\vartheta_{k+1}(\zeta)\left(\frac{\psi}{1+\psi}\right)\right)^{\mathfrak{w}_{k+1}}\right)\\ \frac{1}{\psi}\left((1+\psi)\left(\frac{\psi\gamma_{k+1}(\zeta)+1}{1+\psi}\right)^{\mathfrak{w}_{k+1}}-1\right)\end{array}\right)$$

$$\mathrm{IFSWPA}(\mathcal{F}_1,\mathcal{F}_2,\ldots,\mathcal{F}_{k+1})=\left(\begin{array}{c}\frac{1+\psi}{\psi}\left(1-\prod_{i=1}^{k+1}\left(1-\vartheta_i(\zeta)\frac{\psi}{1+\psi}\right)^{\mathfrak{w}_i}\right),\\ \frac{1}{\psi}\left((1+\psi)\prod_{i=1}^{k+1}\left(\frac{\psi\gamma_i(\zeta)+1}{1+\psi}\right)^{\mathfrak{w}_i}-1\right)\end{array}\right)$$

Now, we discuss some appropriate characteristics to show the validation and applicability of derived strategies.

Theorem 4.2 *Let $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., n$ be a class of IFVs,implies that $\mathcal{F}_i$=$\mathcal{F}$. Then we have:*

$$IFSWPA(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_\eta) = \mathcal{F}$$

Proof: Since all IFVs $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., n$ are identical $\mathcal{F}_i = \mathcal{F}$. Then we have:

$$\text{IFSWPA}(\mathcal{F}_1, \mathcal{F}_2, \dots, \mathcal{F}_\eta) = \begin{pmatrix} \frac{1+\psi}{\psi}\left(1 - \prod_{i=1}^{\eta}\left(1 - \vartheta_i(\zeta)\,\frac{\psi}{1+\psi}\right)^{\mathfrak{w}_i}\right), \\ \frac{1}{\psi}\left((1+\psi)\prod_{i=1}^{\eta}\left(\frac{\psi\gamma_i(\zeta)+1}{1+\psi}\right)^{\mathfrak{w}_i} - 1\right). \end{pmatrix}$$

$$= \begin{pmatrix} \frac{(1+\psi)}{\psi}\left(1 - \left(1 - \vartheta(\zeta)\frac{\psi}{1+\psi}\right)^{\sum_{i=1}^{\eta}\mathfrak{w}_i}\right), \\ \frac{1}{\psi}\left((1+\psi)\left(\frac{\psi\gamma(\zeta)+1}{1+\psi}\right)^{\sum_{i=1}^{\eta}\mathfrak{w}_i} - 1\right) \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1+\psi}{\psi}\left(1 - \left(1 - \vartheta(\zeta)\left(\frac{\psi}{1+\psi}\right)\right)\right), \\ \frac{1}{\psi}\left((1+\psi)\left(\frac{\psi\gamma(\zeta)+1}{1+\psi}\right) - 1\right) \end{pmatrix}$$

$$= \begin{pmatrix} \frac{(1+\psi)}{\psi}\left(1 - 1 + \vartheta(\zeta)\left(\frac{\psi}{1+\psi}\right)\right), \\ \frac{1}{\psi}\left(\psi\gamma(\zeta) + 1 - 1\right) \end{pmatrix}$$

$$= \begin{pmatrix} \frac{(1+\Psi)}{\Psi}\left(\vartheta(\zeta)\left(\frac{\Psi}{1+\Psi}\right)\right), \\ \frac{1}{\Psi}\left(\Psi\,\gamma(\zeta)\right), \end{pmatrix}$$

$$= (\vartheta(\zeta), \gamma(\zeta)) = \mathcal{F}$$

Theorem 4.3 *For any two sets of IFVs $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta))$ and $\mathcal{F}'_i = (\vartheta'_i(\zeta), \gamma'_i(\zeta))$, $i = 1, 2, ..., n$*

$$IFSWPA(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_\eta) \leq IFSWPA(\mathcal{F}'_1, \mathcal{F}'_2, ..., \mathcal{F}'_\eta)$$

Proof: Proof is straightforward.

Theorem 4.4 *Let $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., n$ be a class of IFVs. If $\mathcal{F}^- = (min\{\vartheta_i(\zeta)\}, max\{\gamma_i(\zeta)\})$ and $\mathcal{F}^+ = (max\{\vartheta_i(\zeta)\}, min\{\gamma_i(\zeta)\})$. Then we have:*

$$\mathcal{F}^- \leq IFSWPA(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_\eta) \leq \mathcal{F}^+$$

Proof: Let

$$\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., \eta$$

be a class of IFVs. Since we have:

$$\min \vartheta_i(\zeta) \leq \max \vartheta_i(\zeta)$$

$$\min\left(\vartheta_i(\zeta)\left(\frac{\Psi}{1+\Psi}\right)\right) \leq \vartheta_i(\zeta)\left(\frac{\Psi}{1+\Psi}\right) \leq \max\left(\vartheta_i(\zeta)\left(\frac{\Psi}{1+\Psi}\right)\right)$$

$$1-\min\left[\vartheta_i(\zeta)\left(\frac{\Psi}{1+\Psi}\right)\right]\geq 1-\vartheta_i(\zeta)\left(\frac{\Psi}{1+\Psi}\right)\geq 1-\max\left[\vartheta_i(\zeta)\left(\frac{\Psi}{1+\Psi}\right)\right]$$

$$\left(1-\min\vartheta_i(\zeta)\left(\frac{\Psi}{1+\Psi}\right)\right)^{\mathfrak{w}_i}\geq\left(1-\vartheta_i(\zeta)\left(\frac{\Psi}{1+\Psi}\right)\right)^{\mathfrak{w}_i}\geq\left(1-\max\vartheta_i(\zeta)\left(\frac{\Psi}{1+\Psi}\right)\right)^{\mathfrak{w}_i}$$

$$\left(1-\min_i\left[\vartheta_i(\zeta)\left(\frac{\Psi}{1+\Psi}\right)\right]\right)^{\sum_{i=1}^{\eta}\mathfrak{w}_i}\geq\prod_{i=1}^{\eta}\left(1-\vartheta_i(\zeta)\left(\frac{\Psi}{1+\Psi}\right)\right)^{\mathfrak{w}_i}\geq\left(1-\max_i\left[\vartheta_i(\zeta)\left(\frac{\Psi}{1+\Psi}\right)\right]\right)^{\sum_{i=1}^{\eta}\mathfrak{w}_i}$$

$$(1-\min[\vartheta_i(\zeta)\left(\frac{\psi}{1+\psi}\right)])\geq\prod_{i=1}^{\eta}\left(1-\vartheta_i(\zeta)\left(\frac{\psi}{1+\psi}\right)\right)^{\mathfrak{w}_i}\geq(1-\max[\vartheta_i(\zeta)\left(\frac{\psi}{1+\psi}\right)])$$

$$\min\vartheta_i(\zeta)\left(\frac{\Psi}{1+\Psi}\right)\leq 1-\prod_{i=1}^{\eta}\left(1-\vartheta_i(\zeta)\left(\frac{\Psi}{1+\Psi}\right)\right)^{\mathfrak{w}_i}\leq\max\vartheta_i(\zeta)\left(\frac{\Psi}{1+\Psi}\right)$$

$$\min\vartheta_i(\zeta)\leq\frac{(1+\Psi)}{\Psi}\left(1-\prod_{i=1}^{\eta}\left(1-\vartheta_i(\zeta)\left(\frac{\Psi}{1+\Psi}\right)\right)^{\mathfrak{w}_i}\right)\leq\max\vartheta_i(\zeta)$$

Similarly, we can write:

$$\min\gamma_i(\zeta)\geq\frac{1}{\psi}\left[(1+\psi)\prod_{i=1}^{\eta}\left(\frac{\psi\gamma_i(\zeta)+1}{1+\psi}\right)^{\mathfrak{w}_i}-1\right]\geq\max\gamma_i(\zeta)$$

Therefore:

$$\mathcal{F}^-\leq IFSWPA(\mathcal{F}_1,\mathcal{F}_2,...,\mathcal{F}_\eta)\leq\mathcal{F}^+$$

Definition 4.2 *Let $\mathcal{F}_i=(\vartheta_i(\zeta),\gamma_i(\zeta)), i=1,2,...,\eta$ be a class of IFVs. Then, the IFSWPOA operator is given by:*

$$IFSWPOA(\mathcal{F}_1,\mathcal{F}_2,...,\mathcal{F}_\eta)=\bigoplus_{i=1}^{\eta}\mathfrak{w}_i\mathcal{F}_i$$

Where $\mathfrak{w}_i=\frac{(1+A(\mathcal{F}_i))}{\sum_{i=1}^{\eta}(1+A(\mathcal{F}_i))}, i=1,2,3,...,\eta$ *and* $A(\mathcal{F}_i)=\sum_{\substack{i=1\\i\neq j}}^{\eta}$ *Supp*$(\mathcal{F}_i,\mathcal{F}_j)$.

Theorem 4.5 *Let $\mathcal{F}_i=(\vartheta_i(\zeta),\gamma_i(\zeta)), i=1,2,...,n$ be a class of IFVs. Then, the aggregated outcome by the IFSWPOA operator is still an IFV given by:*

$$IFSWPOA(\mathcal{F}_1,\mathcal{F}_2,\ldots,\mathcal{F}_\eta)=\begin{pmatrix}\frac{1+\psi}{\psi}\left(1-\prod_{i=1}^{\eta}\left(1-\vartheta_{g(\iota)}(\zeta)\frac{\psi}{1+\psi}\right)^{\mathfrak{w}_i}\right),\\ \frac{1}{\psi}\left((1+\psi)\prod_{i=1}^{\eta}\left(\frac{\psi\gamma_{g(\iota)}(\zeta)+1}{1+\psi}\right)^{\mathfrak{w}_i}-1\right)\end{pmatrix}$$

Where $\breve{w}_i=\frac{\Xi_i(1+A(\mathcal{F}_i))}{\sum_{i=1}^{\eta}\Xi_i(1+A(\mathcal{F}_i))}, i=1,2,3,...,\eta$ *and* $A(\mathcal{F}_i)=\sum_{\substack{i=1\\i\neq j}}^{\eta}\Xi_i$ *Supp*$(\mathcal{F}_i,\mathcal{F}_j)$, $\Xi=(\Xi_1,\Xi_2,...,\Xi_\eta)$, $\Xi_i>0$ *and* $\sum_{i=1}^{\eta}\Xi_i=1$.

Definition 4.3 *Let $\mathcal{F}_i=(\vartheta_i(\zeta),\gamma_i(\zeta)), i=1,2,...,\eta$ be a class of IFVs. Then, the IFSWPG operator is given by:*

$$IFSWPG(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_\eta) = \bigotimes_{i=1}^{\eta} \mathfrak{w}_i \mathcal{F}_i$$

Where $\breve{w}_i = \frac{\Xi_i(1+A(\mathcal{F}_i))}{\sum_{i=1}^{\eta} \Xi_i(1+A(\mathcal{F}_i))}, i = 1, 2, 3, ..., \eta$ *and* $A(\mathcal{F}_i) = \sum_{\substack{i=1 \\ i \neq j}}^{\eta} \Xi_i \, Supp(\mathcal{F}_i, \mathcal{F}_j)$, $\Xi = (\Xi_1, \Xi_2, ..., \Xi_\eta)$, $\Xi_i \geqslant 0$ *and* $\sum_{i=1}^{\eta} \Xi_i = 1$.

Theorem 4.6 *Let* $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., n$ *be a class of IFVs. Then, the aggregated outcome by the IFSWPG operator is still an IFV given by:*

$$IFSWPG(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_\eta) = \begin{pmatrix} \frac{1}{\psi}\left((1+\psi)\prod_{i=1}^{\eta}\left(\frac{\psi\vartheta_i(\zeta)+1}{1+\psi}\right)^{\mathfrak{w}_i} - 1\right), \\ \frac{1+\psi}{\psi}\left(1 - \prod_{i=1}^{\eta}\left(1 - \gamma_i(\zeta)\frac{\psi}{1+\psi}\right)^{\mathfrak{w}_i}\right) \end{pmatrix}$$

Where $\breve{w}_i = \frac{\Xi_i(1+A(\mathcal{F}_i))}{\sum_{i=1}^{\eta} \Xi_i(1+A(\mathcal{F}_i))}, i = 1, 2, 3, ..., \eta$ *and* $A(\mathcal{F}_i) = \sum_{\substack{i=1 \\ i \neq j}}^{\eta} \Xi_i \, Supp(\mathcal{F}_i, \mathcal{F}_j)$, $\Xi = (\Xi_1, \Xi_2, ..., \Xi_\eta)$, $\Xi_i \geqslant 0$ *and* $\sum_{i=1}^{\eta} \Xi_i = 1$.

Theorem 4.7 *Let* $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., n$ *be a class of IFVs,implies that* $\mathcal{F}_i = \mathcal{F}$. *Then we have:*

$$IFSWPG(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_\eta) = \mathcal{F}$$

Theorem 4.8 *For any two sets of IFVs* $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta))$ *and* $\mathcal{F}'_i = (\vartheta'_i(\zeta), \gamma'_i(\zeta))$, $i = 1, 2, ..., n$

$$IFSWPG(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_\eta) \leq IFSWPG(\mathcal{F}'_1, \mathcal{F}'_2, ..., \mathcal{F}'_\eta)$$

Theorem 4.9 *Let* $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., n$ *be a class of IFVs. If* $\mathcal{F}^- = (min\{\vartheta_i(\zeta)\}, max\{\gamma_i(\zeta)\})$ *and* $\mathcal{F}^+ = (max\{\vartheta_i(\zeta)\}, min\{\gamma_i(\zeta)\})$. *Then we have:*

$$\mathcal{F}^- \leq IFSWPG(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_\eta) \leq \mathcal{F}^+$$

Definition 4.4 *Let* $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., \eta$ *be a class of IFVs. Then, the IFSWPOG operator is given by:*

$$IFSWPOG(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_\eta) = \bigotimes_{i=1}^{\eta} \mathfrak{w}_i \mathcal{F}_i$$

Where $\mathfrak{w}_i = \frac{(1+A(\mathcal{F}_i))}{\sum_{i=1}^{\eta}(1+A(\mathcal{F}_i))}, i = 1, 2, 3, ..., \eta$ *and* $A(\mathcal{F}_i) = \sum_{\substack{i=1 \\ i \neq j}}^{\eta} Supp(\mathcal{F}_i, \mathcal{F}_j)$.

Theorem 4.10 *Let* $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., n$ *be a class of IFVs. Then, the aggregated outcome by the IFSWPOG operator is still an IFV given by:*

$$IFSWPOG(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_\eta) = \begin{pmatrix} \frac{1}{\psi}\left((1+\psi)\prod_{i=1}^{\eta}\left(\frac{\psi\,\vartheta_{g(i)}(\zeta)+1}{1+\psi}\right)^{\mathfrak{w}_i} - 1\right), \\ \frac{1+\psi}{\psi}\left(1 - \prod_{i=1}^{\eta}\left(1 - \gamma_{g(i)}(\zeta)\frac{\psi}{1+\psi}\right)^{\mathfrak{w}_i}\right) \end{pmatrix}$$

Where $\mathfrak{w}_i = \frac{(1+A(\mathcal{F}_i))}{\sum_{i=1}^{\eta}(1+A(\mathcal{F}_i))}, i = 1, 2, 3, ..., \eta$ *and* $A(\mathcal{F}_i) = \sum_{\substack{i=1 \\ i \neq j}}^{\eta} Supp(\mathcal{F}_i, \mathcal{F}_j)$.

5. Intuitionistic Fuzzy Sugeno-Weber Power Weighted Aggregation Operators

In his section, we illustrated some particular approaches to intuitionistic fuzzy information based on Sugeno-Weber triangular norms.

Definition 5.1 *Let $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., \eta$ be a class of IFVs. Then, the IFSWPWA operator is given by:*

$$IFSWPWA(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_\eta) = \bigoplus_{i=1}^{\eta} \breve{w}_i \mathcal{F}_i$$

Where $\breve{w}_i = \frac{\Xi_i(1+A(\mathcal{F}_i)\big)}{\sum_{i=1}^{\eta}\Xi_i(1+A(\mathcal{F}_i))}, i = 1, 2, 3, ..., \eta$ *and* $A(\mathcal{F}_i) = \sum_{\substack{i=1\\ i\neq j}}^{\eta} \Xi_i \, Supp(\mathcal{F}_i, \mathcal{F}_j)$, Ξ=(Ξ_1,Ξ_2,...,Ξ_η), Ξ_i⩾0 *and* $\sum_{i=1}^{\eta}\Xi_i = 1$.

Theorem 5.1 *Let $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., n$ be a class of IFVs. Then, the aggregated outcome by the IFSWPWA operator is still an IFV given by:*

$$IFSWPWA(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_\eta) = \begin{pmatrix} \frac{1+\psi}{\psi}\left(1 - \prod_{i=1}^{\eta}\left(1 - \vartheta_i(\zeta)\left(\frac{\psi}{1+\psi}\right)\right)^{\breve{w}_i}\right) \\ \frac{1}{\psi}\left((1+\psi)\prod_{\imath=1}^{\eta}\left(\frac{\psi\,\gamma_i(\zeta)+1}{1+\psi}\right)^{\breve{w}_i} - 1\right) \end{pmatrix}$$

Where $\breve{w}_i = \frac{\Xi_i(1+A(\mathcal{F}_i)\big)}{\sum_{i=1}^{\eta}\Xi_i(1+A(\mathcal{F}_i))}, i = 1, 2, 3, ..., \eta$ *and* $A(\mathcal{F}_i) = \sum_{\substack{i=1\\ i\neq j}}^{\eta} \Xi_i \, Supp(\mathcal{F}_i, \mathcal{F}_j)$, Ξ=(Ξ_1,Ξ_2,...,Ξ_η), Ξ_i⩾0 *and* $\sum_{i=1}^{\eta}\Xi_i = 1$.

Theorem 5.2 *Let $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., n$ be a class of IFVs,implies that $\mathcal{F}_i$=$\mathcal{F}$. Then we have:*

$$IFSWWA(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_\eta) = \mathcal{F}$$

Theorem 5.3 *For any two sets of IFVs $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta))$ and $\mathcal{F}'_i = (\vartheta'_i(\zeta), \gamma'_i(\zeta))$, $i = 1, 2, ..., n$*

$$IFSWPWA(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_\eta) \leq IFSWPWA(\mathcal{F}'_1, \mathcal{F}'_2, ..., \mathcal{F}'_\eta)$$

Theorem 5.4 *Let $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., n$ be a class of IFVs. If $\mathcal{F}^- = (min\{\vartheta_i(\zeta)\}, max\{\gamma_i(\zeta)\})$ and $\mathcal{F}^+ = (max\{\vartheta_i(\zeta)\}, min\{\gamma_i(\zeta)\})$. Then we have:*

$$\mathcal{F}^- \leq IFSWPWA(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_\eta) \leq \mathcal{F}^+$$

Definition 5.2 *Let $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., \eta$ be a class of IFVs. Then, the IFSWPOWA operator is given by:*

$$IFSWPOWA(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_\eta) = \bigoplus_{i=1}^{\eta} \breve{w}_i \mathcal{F}_i$$

Where $\breve{w}_i = \frac{\Xi_i(1+A(\mathcal{F}_i)\big)}{\sum_{i=1}^{\eta}\Xi_i(1+A(\mathcal{F}_i))}, i = 1, 2, 3, ..., \eta$ *and* $A(\mathcal{F}_i) = \sum_{\substack{i=1\\ i\neq j}}^{\eta} \Xi_i \, Supp(\mathcal{F}_i, \mathcal{F}_j)$, Ξ=(Ξ_1,Ξ_2,...,Ξ_η), Ξ_i⩾0 *and* $\sum_{i=1}^{\eta}\Xi_i = 1$.

Theorem 5.5 *Let $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., n$ be a class of IFVs. Then, the aggregated outcome by the IFSWPOWA operator is still an IFV given by:*

$$IFSWPOWA(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_\eta) = \begin{pmatrix} \frac{1+\psi}{\psi}\left(1-\prod_{i=1}^{\eta}\left(1-\vartheta_{g(i)}(\zeta)^{\frac{\psi}{1+\psi}}\right)^{\breve{w}_i}\right) \\ \frac{1}{\psi}\left((1+\psi)\prod_{i=1}^{\eta}\left(\frac{\psi\,\gamma_{g(i)}(\zeta)+1}{1+\psi}\right)^{\breve{w}_i}-1\right) \end{pmatrix}$$

Definition 5.3 *Let $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., \eta$ be a class of IFVs. Then, the IFSWPWG operator is given by:*

$$IFSWPWG(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_\eta) = \bigotimes_{i=1}^{\eta} \breve{w}_i \mathcal{F}_i$$

Where $\breve{w}_i = \frac{\Xi_i(1+A(\mathcal{F}_i))}{\sum_{i=1}^{\eta}\Xi_i(1+A(\mathcal{F}_i))}, i = 1, 2, 3, ..., \eta$ *and* $A(\mathcal{F}_i) = \sum_{\substack{i=1\\ i\neq j}}^{\eta}\Xi_i\, Supp(\mathcal{F}_i, \mathcal{F}_j)$, $\Xi=(\Xi_1,\Xi_2,...,\Xi_\eta)$, $\Xi_i > 0$ *and* $\sum_{i=1}^{\eta}\Xi_i = 1$.

Theorem 5.6 *Let $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., n$ be a class of IFVs. Then, the aggregated outcome by the IFSWPWG operator is still an IFV given by:*

$$IFSWPWG(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_\eta) = \begin{pmatrix} \frac{1}{\psi}\left((1+\psi)\prod_{i=1}^{\eta}\left(\frac{\psi\vartheta_i(\zeta)+1}{1+\psi}\right)^{\breve{w}_i}-1\right) \\ \frac{1+\psi}{\psi}\left(1-\prod_{i=1}^{\eta}\left(1-\gamma_i(\zeta)\frac{\psi}{1+\psi}\right)^{\breve{w}_i}\right) \end{pmatrix}$$

Theorem 5.7 *Let $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., n$ be a class of IFVs,implies that $\mathcal{F}_i$=$\mathcal{F}$. Then we have:*

$$IFSWWG(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_\eta) = \mathcal{F}$$

Theorem 5.8 *For any two sets of IFVs $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta))$ and $\mathcal{F}'_i = (\vartheta'_i(\zeta), \gamma'_i(\zeta))$,$i = 1, 2, ..., n$*

$$IFSWPWG(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_\eta) \leq IFSWPWG(\mathcal{F}'_1, \mathcal{F}'_2, ..., \mathcal{F}'_\eta)$$

Theorem 5.9 *Let $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., n$ be a class of IFVs. If $\mathcal{F}^- = (min\{\vartheta_i(\zeta)\}, max\{\gamma_i(\zeta)\})$ and $\mathcal{F}^+ = (max\{\vartheta_i(\zeta)\}, min\{\gamma_i(\zeta)\})$. Then we have:*

$$\mathcal{F}^- \leq IFSWPWG(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_\eta) \leq \mathcal{F}^+$$

Definition 5.4 *Let $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., \eta$ be a class of IFVs. Then, the IFSWPOWG operator is given by:*

$$IFSWPOWG(\mathcal{F}_1, \mathcal{F}_2, ..., \mathcal{F}_\eta) = \bigotimes_{i=1}^{\eta} \breve{w}_i \mathcal{F}_i$$

Where $\breve{w}_i = \frac{\Xi_i(1+A(\mathcal{F}_i))}{\sum_{i=1}^{\eta}\Xi_i(1+A(\mathcal{F}_i))}, i = 1, 2, 3, ..., \eta$ *and* $A(\mathcal{F}_i) = \sum_{\substack{i=1\\ i\neq j}}^{\eta}\Xi_i\, Supp(\mathcal{F}_i, \mathcal{F}_j)$, $\Xi=(\Xi_1,\Xi_2,...,\Xi_\eta)$, $\Xi_i > 0$ *and* $\sum_{i=1}^{\eta}\Xi_i = 1$.

Theorem 5.10 *Let $\mathcal{F}_i = (\vartheta_i(\zeta), \gamma_i(\zeta)), i = 1, 2, ..., n$ be a class of IFVs. Then, the aggregated outcome by the IFSWPOWG operator is still an IFV given by:*

$$IFSWPOWG(\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_\eta) = \left(\begin{array}{c} \frac{1}{\psi}\left((1+\psi) \prod_{i=1}^{\eta} \left(\frac{\psi \vartheta_{g(i)}(\zeta)+1}{1+\psi} \right)^{\breve{w}_i} - 1 \right) \\ \frac{1+\psi}{\psi}\left(1 - \prod_{i=1}^{\eta} \left(1 - \gamma_{g(i)}(\zeta) \left(\frac{\psi}{1+\psi} \right) \right)^{\breve{w}_i} \right) \end{array} \right)$$

Where $\breve{w}_i = \frac{\Xi_i(1+A(\mathcal{F}_i)))}{\sum_{i=1}^{\eta} \Xi_i(1+A(\mathcal{F}_i))}, i = 1, 2, 3, ..., \eta$ *and* $A(\mathcal{F}_i) = \sum_{\substack{i=1 \\ i \neq j}}^{\eta} \Xi_i \, Supp(\mathcal{F}_i, \mathcal{F}_j)$, Ξ=$(\Xi_1$,Ξ_2,...,$\Xi_\eta)$, Ξ_i>0 *and* $\sum_{i=1}^{\eta} \Xi_i = 1$.

6. Assessment of the MCDM Problem Based on Intuitionistic Fuzzy Information

This section evaluates uncertainty in human opinions by applying the structure of the IFSWPWA and IFSWPWG operators [51]. The IF information covers extensive information about any object in the form of membership terms (MTs) and non-membership terms (NMTs). Consider a finite collection of an alternative denoted by $\hat{\hat{O}} = (\hat{\hat{O}}_1, \hat{\hat{O}}_2, ..., \hat{\hat{O}}_t)$ and set of finite attributes $\hat{J} = (\hat{J}_1, \hat{J}_2, \hat{J}_s)$ associate degree of weight to each attribute $\hat{w} = (\hat{w}_1, \hat{w}_2, \hat{w}_s)$ such that $\hat{w}_i > 0$ and $\sum_{i=1}^{s} \hat{w}_i = 1$ [52]. The decision maker takes an IF information $0 \leq \vartheta_{fi}(\zeta) + \gamma_{fi}(\zeta) \leq 1$ and listed this given information $\mathcal{F}_{fi} = (\vartheta_{fi}(\zeta), \gamma_{fi}(\zeta)), f, i = 1, 2, ..., t, s$ in a standard decision matrix $\bar{\bar{R}} = [\mathcal{F}_{fi}]_{t,s}$. The decision maker proposed an innovative algorithm for the MCDM technique. The flowchart also explores the steps of an algorithm in Figure 1.

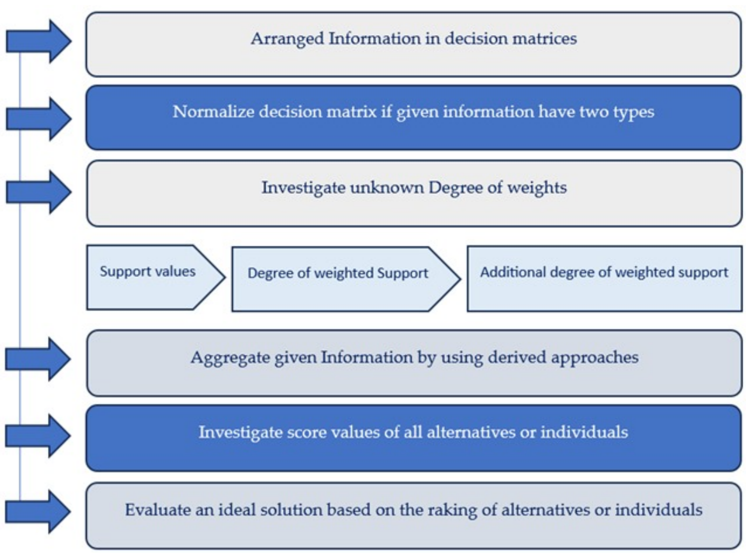

Figure 1: Flowchart for the MCDM problem.

Step 1: A decision matrix $\bar{\bar{R}}$=$[\mathcal{F}_{fi}]_{t,s}$ construct by a decision maker, which packs intuitionistic fuzzy information in the form of attributes corresponding to each individual or alternative.
Step 2: Collected information has two types of attributes, such as beneficial and non-beneficial. If the given information contains more than one type of attribute, then we need to transform the standard decision matrix into a normalized one; otherwise, this technique is unnecessary.
Step 3: Compute support by using the following expression:

$$A(\mathcal{R}_{j\iota}) = \sum_{\substack{\iota=1 \\ \iota\neq\tau}}^{\eta} \mathrm{Supp}(\mathcal{R}_{j\iota}, \mathcal{R}_{j\tau}), \quad j = 1,2,3,\ldots,\lambda,\ \iota,\tau = 1,2,\ldots,\tau$$

Where

$$D(\mathcal{R}_{j\iota}, \mathcal{R}_{j\tau}) = \tfrac{1}{2}\Big(|\vartheta_{j\iota} - \vartheta_{j\tau}| + |\Upsilon_{j\iota} - \Upsilon_{j\tau}|\Big)$$

Step 4: Calculate the degree of weighted support:

$$A(\mathfrak{R}_{j\iota}) = \sum_{\substack{\iota=1 \\ \iota\neq\tau}}^{\eta} \Xi_\iota\ \mathrm{Supp}(\mathfrak{R}_{j\iota}, \mathfrak{R}_{j\tau}), \qquad \Xi = (\Xi_1, \Xi_2, \ldots, \Xi_\eta),\ \Xi_\iota > 0,\ \sum_{\iota=1}^{\eta} \Xi_\iota = 1.$$

Step 5: Compute the degree of support:

$$C_{j\iota} = \frac{\Xi_\iota\big(1 + A(\mathcal{R}_{j\iota})\big)}{\sum_{\iota=1}^{\eta} \Xi_\iota\big(1 + A(\mathcal{R}_{j\iota})\big)}, \qquad j = 1,2,3,\ldots,\lambda,\ \iota = 1,2,\ldots,\tau \tag{6.1}$$

Step 6: Apply the proposed strategies of the IFSWPWA and IFSWPWG operators to evaluate the given information.
Step 7: Investigate the score values of all individuals or alternatives by using the Definition 5.
Step 8: To determine an ideal solution, rank the score values of all alternatives or individuals.

7. A Case Study of Solid Waste Management

In general, selecting and putting the right procedures, techniques, technology, and infrastructure are required to create a sustainable system. Every sustainable waste management schedule must incorporate the following tactics, Integrated Solid Waste Management (ISWM) principles shown in Figure 2: preventing the generation of avoidable wastes, reducing generated waste through recovery, reusing recovered wastes, recycling recyclables, composting organic wastes for the generation of energy and electricity, and final disposal at sanitary landfills. This approach helps waste managers construct a sustainable system by taking into account the unique environmental, economic, and social characteristics of the given site.

Figure 2: Flowchart for the MCDM problem.

Additionally, Ogwueleka [53] discussed a few solid waste management techniques, such as composting, recycling, resource recovery, and treatment and disposal systems. He went on to say that some management strategies, such as garbage recovery/recycling, reuse, and composting, are now being carried out by the unofficial sector. The recovery/recycling process improves the economy by helping to recover, convert, and reuse valuable resources like paper, plastic, polythene, organic wastes, etc. These procedures cut down on the amount of solid trash that is dumped at the landfills, prolonging their lifespan, lowering the number of toxic pollutants they release, and saving the authority a respectable sum of money. In order to preserve green spaces on campus and in the surrounding neighborhood, organic wastes must be composted. The composts that are created are used as nutrients for soil enrichment. In this article, we study some robust strategies for reducing the impact of solid waste materials and reliable management technology to achieve a sustainable system. Consider there are five different strategies $\hat{\hat{A}} = (\hat{\hat{A}}_1, \hat{\hat{A}}_2, \ldots, \hat{\hat{A}}_5)$ available for reducing the drawbacks of waste material and also improving the environmental ecosystem. The key features and characteristics of the above-discussed strategies are given as follows:

Friendly environment and less waste in landfills $\hat{\hat{J}}_1$

Reduce the impact of harmful gases and emissions from manufacturing $\hat{\hat{J}}_2$

Conserving resources $\hat{\hat{J}}_3$

Conserve Wildlife and species $\hat{\hat{J}}_4$

The decision maker evaluates a more realistic strategy under-discussed characteristics with a certain degree of weight. For this purpose, experts evaluate the degree of unknown weights from the given information listed in Table 1. Experts may also assign particular weights to the discussed attributes or characteristics. In this case study, the decision maker assigned particular weights to the discussed characteristics (0.30,0.35,0.15,0.20) as hypothecally. The decision maker investigates the best strategy by using invented approaches in light of the proposed algorithm for the MCDM problem.

7.1. Decision-Making Procedure

The Decision maker evaluates given information based on certain criteria about green supplier enterprises under consideration of IF information.

Step 1: The decision maker acquires information in the form of alternatives associated with some specific criteria and lists this information in Table 1.

Table 1: Decision matrix of intuitionistic fuzzy information.

Alternatives	$\hat{\hat{J}}_1$	$\hat{\hat{J}}_2$	$\hat{\hat{J}}_3$	$\hat{\hat{J}}_4$
$\hat{\hat{A}}_1$	(0.57,0.36)	(0.11,0.45)	(0.12,0.46)	(0.13,0.41)
$\hat{\hat{A}}_2$	(0.56,0.15)	(0.43,0.51)	(0.43,0.38)	(0.15,0.38)
$\hat{\hat{A}}_3$	(0.33,0.46)	(0.51,0.46)	(0.37,0.46)	(0.17,0.37)
$\hat{\hat{A}}_4$	(0.16,0.45)	(0.31,0.62)	(0.16,0.57)	(0.31,0.45)
$\hat{\hat{A}}_5$	(0.28,0.61)	(0.18,0.51)	(0.61,0.23)	(0.11,0.44)

Step 2: Given that information has only one benefited type of attribute, there is no need to transform the standard decision matrix into a normalized one.

Step 3: compute the support of individuals by using expressions.

Step 4: Calculate the degree of weighted support:

Step 5: Compute the degree of support:

From steps 3-5, we compute the following results:

$$\mathfrak{C}_{ij} = \begin{pmatrix} 0.2313 & 0.2384 & 0.2687 & 0.2616 \\ 0.2378 & 0.2382 & 0.2725 & 0.2515 \\ 0.2461 & 0.2354 & 0.2686 & 0.2498 \\ 0.2422 & 0.2348 & 0.2647 & 0.2583 \\ 0.2485 & 0.2445 & 0.2438 & 0.2632 \end{pmatrix}$$

Step 6: Utilized discussed methodologies like IFSWPA, IFSWPG, IFSWPWA and IFSWPWG operators to evaluate the finest individual under consideration of given information. All computed results are stated in Table 2.

Table 2: Decision matrix of intuitionistic fuzzy information.

Alternatives	IFSWPA	IFSWPG	IFPSWPWA	IFSWPWG
$\hat{\hat{A}}_1$	(0.2321,0.4215)	(0.2082,0,4225)	(0.2355,0.4209)	(0.2112,0.4219)
$\hat{\hat{A}}_2$	(0.3992,0.3521)	(0.3844,0.3633)	(0.3971,0.3501)	(0.3820,0.3610)
$\hat{\hat{A}}_3$	(0.3507,0.4379)	(0.3410,0.4389)	(0.3475,0.4370)	(0.3378,0.4380)
$\hat{\hat{A}}_4$	(0.2363,05204)	(0.2325,0.5241)	(0.2356,0.5199)	(0.2317,0.5235)
$\hat{\hat{A}}_5$	(0.2940,0.4486)	(0.2708,0.4609)	(0.3207,0.4413)	(0.2781,0.4541)

Step 7: Using the computed results of Table 2, demonstrate the score values of all individuals and display them in Table 3.

Table 3: Score values of all individuals.

Alternatives	IFSWPA	IFSWPG	IFPSWPWA	IFSWPWG
$\hat{\hat{A}}_1$	-0.1894	-0.2142	-0.1853	-0.2106
$\hat{\hat{A}}_2$	0.0471	0.0211	0.0470	0.0210
$\hat{\hat{A}}_3$	-0.0872	-0.0979	-0.0895	-0.1002
$\hat{\hat{A}}_4$	-0.2840	-0.2916	-0.2843	-0.2918
$\hat{\hat{A}}_5$	-0.1546	-0.1901	-0.1385	-0.1760

Step 8: To assess a notable individual or alternative form group of hired optimal options after rearranging the results of all score values in Table 4. We examined that $\hat{\hat{A}}_2$ is an appropriate alternative or individual captured from the proposed mathematical approaches.

Table 4: Ranking of score values.

Aggregation operators	Ranking of alternatives
IFSWPA	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$
IFSWPG	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$
IFSWPWA	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$
IFSWPWG	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$

For a better understanding, we explore all computed results by the invented approaches in a bar chart in Figure 3. The graphical representation shows the structural behavior of aggregated outcomes by diagnosed research work. It is easier to show the credibility and effectiveness of derived mathematical strategies in decision analysis.

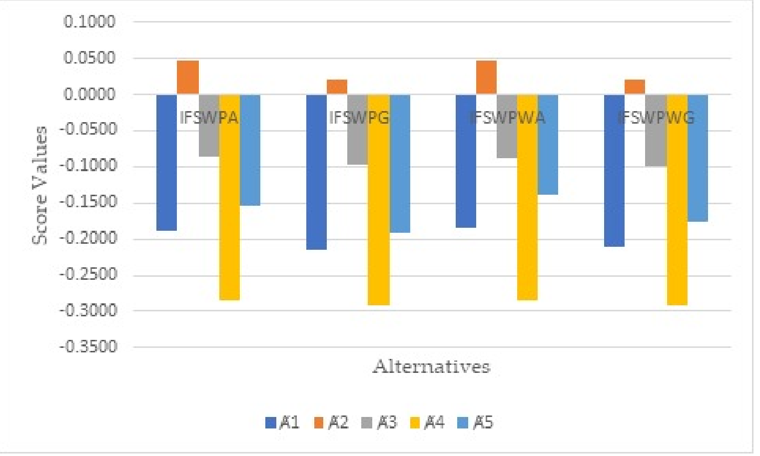

Figure 3: shows the graphical behavior of obtained score values by diagnosed mathematical approaches.

7.2. Sensitivity Analysis

In this section, we examine the validity of the proposed methodologies by setting different parametric values of ψ in Sugeno-Weber triangular norms. The decision-maker uses this technique to acquire consistency in the results of score values. The decision maker may get results of score values by changing the value of ψ in step 3 of an algorithm. Table 5 covered the results of score values by setting parametric values in the IFSWPWA operator. All obtained results from the IFSWPWA operator are listed in Table 5. After the overview of acquired results, we see that score values gradually increase by increasing the value of ψ. Ranking of score values $\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$ remain unchanged throughout this process.

Table 5: Results of score values by the IFSWPWA and IFSWPWG operators at different parametric values of Ψ.

Parametric values	Ranking and Ordering	Parametric values	Ranking and Ordering
$\Psi = 1$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$	$\Psi = 1$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$
$\Psi = 5$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$	$\Psi = 5$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$
$\Psi = 25$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$	$\Psi = 25$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$
$\Psi = 40$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$	$\Psi = 40$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$
$\Psi = 75$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$	$\Psi = 75$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$
$\Psi = 100$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$	$\Psi = 100$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$
$\Psi = 135$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$	$\Psi = 135$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$
$\Psi = 195$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$	$\Psi = 195$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$
$\Psi = 225$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$	$\Psi = 225$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$
$\Psi = 285$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$	$\Psi = 285$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$
$\Psi = 330$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$	$\Psi = 330$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$
$\Psi = 400$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$	$\Psi = 400$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$

Table 5 also covered the results of score values, which are acquired by setting different parametric values of ψ in the IFSWPWG operator. If we increase the parametric value of ψ in the IFSWPWG operator, aggregated outcomes of the score values begin to decrease and the ranking of score values begin to decrease. $\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$ remain unchanged. To better understand this section, the decision maker explores all aggregated results in the graphical representations of Figure 4 and Figure 5. These geometrical shapes show the consistency of invented approaches during decision analysis and the aggregating process. 4 and Figure 5 illustrate the aggregated outcomes of the IFSWPWA and IFSWPWG operators, respectively.

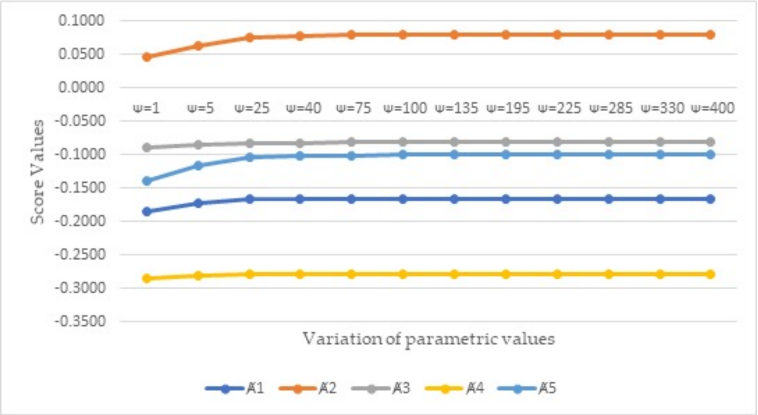

Figure 4: Results of score values by the IFSWPWA operator at different values of Ψ.

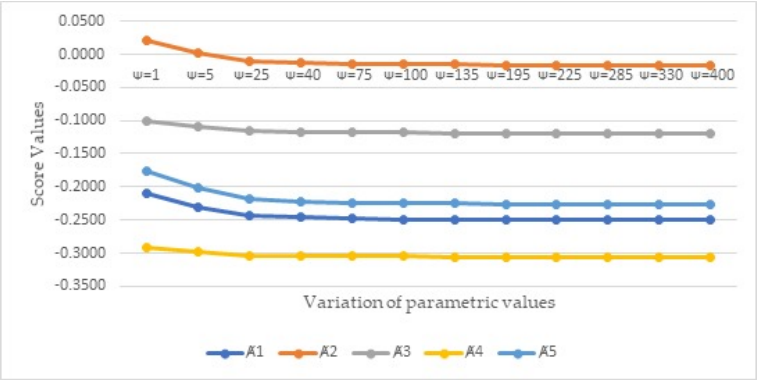

Figure 5: Results of score values by the IFSWPWG operator at different values of Ψ.

8. Comparative Study

In this section, the efficiency and reliability of proposed methodologies are characterized by contrasting results of existing approaches, which are presented in the literature review [49], [54], [55], [56], [57]. These presented approaches applied to the given information listed in Table 1 under consideration discussed algorithm of the MCDM problem. Senapati et al. [56] developed a class of new approaches of an intuitionistic fuzzy Aczel Alsina weighted average (IFAAWA) and intuitionistic fuzzy Aczel Alsina weighted geometric (IFAAWG) operators in the light of basic operations of Aczel Alsina aggregation tools. Xu [54] enhanced the compatibility of simple weighted average and weighted geometric operators under the system of IF information. Another generalization of the Dombi aggregation tools was presented by Seikh and Mandal [55] and they also developed a series of new approaches such as intuitionistic fuzzy Dombi weighted average (IFDWA) and intuitionistic fuzzy Dombi weighted geometric (IFDWG) operators. The theory of Hamy means aggregation models explored by Hussain et al. [57]. All presented results from the existing aggregation approaches are listed in Tables 6 and 7.

Table 6: Results of weighted average operators that exist in literature.

IFSWPWA	**Score Values**	**Ranking Order**
IFSWPWA	$S(\hat{\hat{A}}_1) = -0.1853,\ S(\hat{\hat{A}}_2) = 0.0470,$ $S(\hat{\hat{A}}_3) = -0.0895,$ $S(\hat{\hat{A}}_4) = -0.2843,\ S(\hat{\hat{A}}_5) = -0.1385$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$
IFAAWA by Senapati et al. [56]	$S(\hat{\hat{A}}_1) = -0.0346,\ S(\hat{\hat{A}}_2) = 0.1676,$ $S(\hat{\hat{A}}_3) = -0.0359,$ $S(\hat{\hat{A}}_4) = -0.2543,\ S(\hat{\hat{A}}_5) = -0.0884$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_4$
IFWA by Xu [54]	$S(\hat{\hat{A}}_1) = -0.1255,\ S(\hat{\hat{A}}_2) = 0.1100,$ $S(\hat{\hat{A}}_3) = -0.0614,$ $S(\hat{\hat{A}}_4) = -0.2754,\ S(\hat{\hat{A}}_5) = -0.1807$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_4$
IFDWA by Seikh and Mandal [55]	$S(\hat{\hat{A}}_1) = 0.0147,\ S(\hat{\hat{A}}_2) = 0.2359,$ $S(\hat{\hat{A}}_3) = -0.0182,$ $S(\hat{\hat{A}}_4) = -0.2416,\ S(\hat{\hat{A}}_5) = 0.0069$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_4$
CIFHM by Hussain et al. [57]	Not applicable	

Table 7: Results of weighted geometric operators that exist in literature.

IFSWPWA	**Score Values**	**Ranking Order**
IFSWPWG	$S(\hat{\hat{A}}_1) = -0.2106,\ S(\hat{\hat{A}}_2) = 0.0210,$ $S(\hat{\hat{A}}_3) = -0.1002,$ $S(\hat{\hat{A}}_4) = -0.2918,\ S(\hat{\hat{A}}_5) = -0.1760$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4$
IFAAWG by Senapati et al. [56]	$S(\hat{\hat{A}}_1) = -0.2581,\ S(\hat{\hat{A}}_2) = -0.0650,$ $S(\hat{\hat{A}}_3) = -0.1256,$ $S(\hat{\hat{A}}_4) = -0.3210,\ S(\hat{\hat{A}}_5) = -0.3103$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_4$
IFWG by Xu [54]	$S(\hat{\hat{A}}_1) = -0.2291,\ S(\hat{\hat{A}}_2) = 0.0047,$ $S(\hat{\hat{A}}_3) = -0.1007,$ $S(\hat{\hat{A}}_4) = -0.3041,\ S(\hat{\hat{A}}_5) = -0.2734$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_5 \succ \hat{\hat{A}}_4$
IFDWA by Seikh and Mandal [55]	$S(\hat{\hat{A}}_1) = -0.2860,\ S(\hat{\hat{A}}_2) = -0.1513,$ $S(\hat{\hat{A}}_3) = -0.1685,$ $S(\hat{\hat{A}}_4) = -0.3484,\ S(\hat{\hat{A}}_5) = -0.3534$	$\hat{\hat{A}}_2 \succ \hat{\hat{A}}_3 \succ \hat{\hat{A}}_1 \succ \hat{\hat{A}}_4 \succ \hat{\hat{A}}_5$
CIFDHM by Hussain et al. [57]	Not applicable	

Tables 6 and 7 summarized ranks of all score values computed by the pioneered strategies and existing mathematical approaches based on weighted average and weighted geometric operators. By the analysis of wide comparison, we provide a convincing indication of the applicability and superiority of derived approaches. We concluded that derived approaches of the Sugeno-weber t-norms offered a unique procedure for handling ambiguous and vague type information of human opinion. To provide a clearer understanding of the comparative study for the reader, we have displayed the results of all aggregation operators in two bar graphs. We can easily understand information about any object when data is presented in geometrical representations. For this purpose, Figures 6 and 7 show the ranking of computed results by different strategies listed in Tables 6 and 7, respectively.

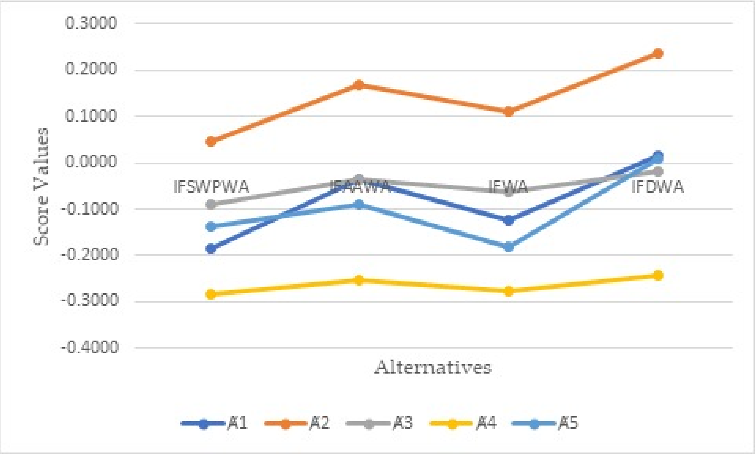

Figure 6: Comparison among existing weighted average operators in the comparative study.

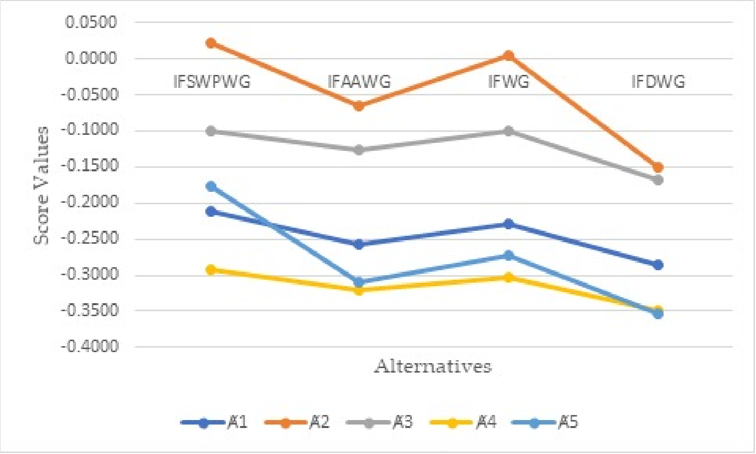

Figure 7: Comparison among existing weighted geometric operators in the comparative study.

9. Conclusion and Future Work

The MCDM technique is widely used in numerous fields, including artificial intelligence, environmental and social science, game theory and many other decision-making processes. Mathematical strategies are essential for the aggregation process and play an important role in decision-making. The main theme of this research work is to expose the theory of Sugeno-Weber t-norms in light of intuitionistic fuzzy information. We also derived a list of reliable mathematical approaches based on the Sugeno-Weber t-norm, such as IFSWPA and IFSWPG operators. Furthermore, another innovation is also explored with the help of weight vectors, namely IFSWPWA and IFSWPWG operators. Some notable characteristics are also discussed to show the validity and consistency of presented approaches. An algorithm of the MCDM technique is used to handle vague type and ambiguous information about human opinion. An experimental case study is also designed to evaluate a suitable strategy for the waste materials under consideration of our derived mathematical strategies. The advantages of currently presented strategies and the impact of different parametric values are examined in the sensitivity analysis. To describe the validity of the proposed approaches, a comparison method was developed to contrast the results of existing AOs with the presented approaches. We examined a variety of advantages of pioneered research work and mathematical approaches. However, we can also point out some drawbacks and limitations of the proposed research work. Sometimes, decision-makers face crucial challenges during decision analysis due to incomplete or redundant information about human opinion. We may also explore the proposed research work in different fuzzy frameworks to handle such situations and real-life dilemmas. Next, we will try to find solutions to different real-life applications such as renewable energy, social and environmental science, game theory, etc.

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