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Impact of Punitive Sanctions on the Spread of Academic Fraud

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ABSTRACT: School fraud has become a serious issue that has increasingly affected the education system in recent years. In this study, school fraud is treated as an infectious phenomenon that spreads within a school community through interactions between individuals who commit fraud and the broader student population. To identify the key factors influencing this undesirable behavior, we propose a mathematical model composed of three strongly coupled ordinary differential equations (ODEs) that describe the interactions among the three components of a school group. We establish the existence and uniqueness of the solution to the model, demonstrate the existence of a unique equilibrium point, and prove that this equilibrium is globally asymptotically stable. Finally, we present numerical simulations that support our theoretical findings.

Keywords: Ordinary differential equations, modelling, stability.

Contents

1. Introduction

School fraud has become an increasingly concerning issue in recent years, particularly within educational systems facing social and institutional challenges. To better understand this phenomenon, it is helpful to draw from various criminological theories each offering a different perspective on the roots and dynamics of fraudulent behavior in schools. From a person-centered perspective, students who engage in fraud may do so because of individual traits or learned behaviors. These individuals might possess a predisposition toward dishonest conduct, or they may have developed such tendencies through repeated exposure to peers who normalize or encourage cheating and academic dishonesty. In contrast, structural theories emphasize the influence of broader systemic forces. Under this lens, school fraud is not simply the result of individual choice but a response to rigid academic systems, intense pressure to succeed, and limited support structures. In such contexts, students may see dishonest practices as necessary or justifiable strategies for achieving academic success in an unforgiving environment. Environmental theories bring attention to how the educational setting itself can facilitate fraud. When classroom supervision is weak, exam procedures are poorly enforced, or digital tools are misused without oversight, opportunities for fraud multiply. These opportunities are not evenly distributed certain schools, regions, or contexts may present more vulnerabilities than others, creating hotspots for academic dishonesty. Modeling academic fraud can be seen as a special case of criminology in society. To model it, we draw inspiration from existing models of infection spread or models describing the phenomenon of crime in societies. There are several mathematical models that model crime in societies. These models come mainly from the fields of mathematical physics, epidemiology, dynamic systems, and stochastic processes. For modeling criminal activity in society, agent-based models are explored in [4,5]. In [14], the authors propose a mathematical

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framework based on reaction-diffusion partial differential equations to analyze the dynamics of crime hotspots. Their model, grounded in empirical observations of offender movement and interaction with potential targets, demonstrates how localized feedback mechanisms can lead to the emergence of crime hotspots through either supercritical or subcritical bifurcations. The analysis reveals that subcritical hotspots can be permanently eliminated through police suppression, whereas supercritical ones tend to spatially displace following predictable patterns. This mechanistic insight provides a theoretical explanation for the limited displacement effects observed in empirical evaluations of hotspot policing strategies. Self-exciting point process models [8,13] have been increasingly applied to the analysis of spatio-temporal patterns of crime, especially to capture the phenomenon of repeat and near-repeat victimization. In [8], Mohler et al. introduce a nonparametric self-exciting point process framework to model burglary events in Los Angeles. Their model distinguishes between background crime rates and triggered events, allowing for localized crime propagation through a Hawkes-type triggering kernel. Building on this, Reinhart and Greenhouse in [11] extend the model by incorporating spatial covariates and evaluating the effects of socio-environmental factors on the crime intensity function. Their approach improves predictive performance and interpretability, particularly in urban settings. A broader overview of self-exciting models and related mathematical approaches is provided by D'Orsogna and Perc in [3], who review a wide array of statistical physics models applied to crime dynamics, including reaction-diffusion equations, agent-based simulations, and point process models. Together, these works provide both the theoretical foundation and empirical support for modeling crime as a contagious process influenced by space, time, and social context. Epidemiological models, particularly those inspired by the classical SIR (Susceptible-Infectious-Removed) framework, have been adapted to describe the spread of criminal behavior within populations [10,16]. In this analogy, individuals transition from a non-criminal (susceptible) state to an active offender (infectious) state, and eventually to a removed or rehabilitated state through deterrence, punishment, or behavioral change. In [6], Felson and Boba introduce foundational ideas on routine activity theory, which align conceptually with epidemiological transitions by highlighting the role of opportunities and interactions in facilitating criminal acts. Subsequently, in [9], Nuno et al. propose a formal SIR-type compartmental model to study gang crime dynamics, where recruitment and law enforcement interventions are explicitly modeled as transition mechanisms. The model demonstrates how crime can spread and persist in ways analogous to contagious diseases. More generally, D'Orsogna and Perc's review [3] provides a comprehensive overview of crime modeling using methods from statistical physics, including epidemiological models, emphasizing their value in capturing systemic behavioral patterns over time. Drawing inspiration from these works including dynamic and epidemiological approaches, we aim to apply similar modeling frameworks to the study of scool fraud. While each of these perspectives contributes valuable insight, they differ in how actionable they are when it comes to intervention. Structural and individual factors are often deep-rooted and difficult to change in the short term. However, environmental factors, such as monitoring practices, classroom design, and exam protocols, can be modified more easily. Therefore, modeling short-term, localized fraud processes such as when and where cheating is most likely to occur can offer practical pathways to reduce school fraud effectively. By using tools such as ordinary differential equations to simulate how school fraud evolves and spreads in specific conditions, researchers and policymakers can identify key parameters influencing the phenomenon. This approach not only allows for a theoretical understanding but also guides the implementation of targeted, evidence-based interventions that disrupt fraudulent behavior before it becomes entrenched. The paper is organized as follows. In the next section, the model and underlying assumptions are presented and discussed. Then, the existence and stability of the equilibrium points are analyzed. In the subsequent section, we establish sufficient conditions to reduce the rate of fraud within the student population. Finally, we present numerical simulations that are consistent with the theoretical results and determine procedures that can reduce this phenomenon.

2. Mathematical model

Prior research has applied the infection disease to other social phenomena [7,15,12]. Inspired by SIR (Susceptible-Infectious-Recovered) models, we model the spread of academic fraud in the school environment. At time t, let $S(t)$ be number of the susceptible students, who are non-fraudulent individuals but likely to become fraudsters, $N(t)$ the number of students not likely to be among the cheating students and $F(t)$ the number of major students engaged in fraud due to insufficient acquired skills or laziness. Then the student population A is the sum of these components so we can write

$$A = N(t) + S(t) + F(t).$$

Naturally, there are interactions between these different compartments. Specifically, non-susceptible individuals can become susceptible at a rate denoted by a_1, while susceptible individuals may revert to the non-susceptible state at a rate a_2. Moreover, susceptible students can become engaged in fraudulent behavior at a rate b_1, and engaged individuals can return to the susceptible state at a rate b_2. Likewise, engaged students may transition to the non-susceptible group, particularly when exposed to strict measures or reeducation sessions, at a rate c. These dynamics are illustrated in the following diagram.

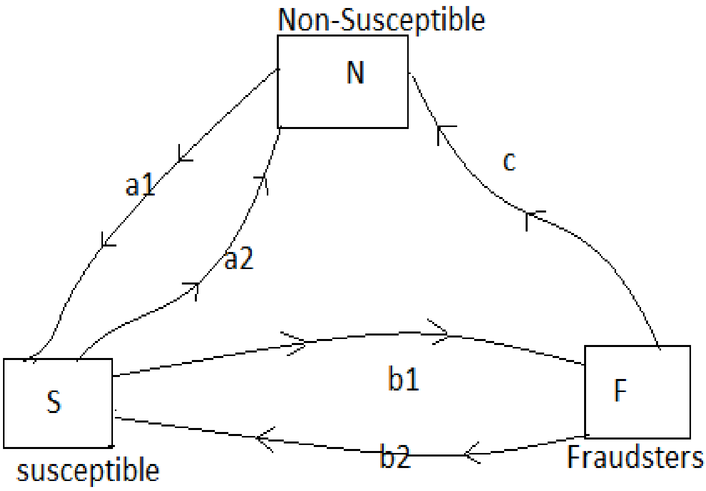

Figure 1: The school fraud model.

The attractiveness of the area, which represents the likelihood that a fraudster will act at a given time is influenced by two key factors: the first, denoted by $\alpha(t)$, represents the characteristics of the space in which the exam takes place and the personality traits of the teachers assigned to the exam room, the second denoted by $\beta(t)$ is relative to the student himself. Respect the above the evolution of N, S and F over time is given by the following system

$$P1 : \begin{cases} \dfrac{dN(t)}{dt} = -a_1(\alpha(t), \beta(t))N(t) + a_2(\alpha(t), \beta(t))S(t) + c(\alpha(t), \beta(t))F(t) \quad \text{in} \quad [0, T], \\ \dfrac{dS(t)}{dt} = -(a_2(\alpha(t), \beta(t)) + b_1(\alpha(t), \beta(t)))S(t) + a_1(\alpha(t), \beta(t))N(t) \\ +b_2(\alpha(t), \beta(t))F(t) \quad \text{in} \quad [0, T], \\ \dfrac{dF(t)}{dt} = -(b_2(\alpha(t), \beta(t)) + c(\alpha(t), \beta(t)))F(t) + b_1(\alpha(t), \beta(t))S(t) \quad \text{in} \quad [0, T], \end{cases} \tag{2.1}$$

where $T \in \mathbb{R}^+$ and $N(0) \geq 0, S(0) \geq 0$ and $F(0) \geq 0$. Since the total population A is constant, we may consider a system of three equations. By rescaling, we work with the proportions of each subgroup within the population, where

$$n = \frac{N}{A}, s = \frac{S}{A}, f = \frac{F}{A}, \tag{2.2}$$

then the system (2.1) becomes

$$P2 : \begin{cases} \dfrac{dn(t)}{dt} = -a_1(\alpha(t),\beta(t))n(t) + a_2(\alpha(t),\beta(t))s(t) + c(\alpha(t),\beta(t))f(t) \quad \text{in} \quad [0,T], \\ \dfrac{ds(t)}{dt} = -(a_2(\alpha(t),\beta(t)) + b_1(\alpha(t),\beta(t)))s(t) + a_1(\alpha(t),\beta(t))n(t) \\ +b_2(\alpha(t),\beta(t))f(t) \quad \text{in} \quad [0,T], \\ \dfrac{df(t)}{dt} = -(b_2(\alpha(t),\beta(t)) + c(\alpha(t),\beta(t)))f(t) + b_1(\alpha(t),\beta(t))s(t) \quad \text{in} \quad [0,T], \end{cases} \tag{2.3}$$

with the initial conditions

$$n(0) \geq 0, s(0) \geq 0, f(0) \geq 0,$$

and

$$n(t) + s(t) + f(t) = 1 \ \forall t \geq 0. \tag{2.4}$$

Remark 1. *In a society where parents and the surrounding environment view obtaining a below-average grade as a disadvantage and also if the number of fraudsters in the room is significant, the values of the functions a_1 and b_1 become significant. This means that both the rate at which non-susceptible students become susceptible and the rate at which susceptible students engage in fraud increase. Consequently, these functions are also influenced by the student's own characteristics.*

The question of whether fraud remains a transient behavior or evolves into a scourge, spreading throughout a student population and becoming persistent or endemic, requires introducing a threshold: the basic spread rate, denoted as R_0. This rate represents the average number of secondary cases generated by a single fraudulent individual. In this context, R_0 indicates the average number of new fraudsters recruited by one existing fraudster. There are various ways to compute R_0, here, we use the next-generation operator method [2], It is a mathematical method used to calculate the basic reproduction rate, denoted here as R_0, in dynamic compartmental models, particularly in epidemiology or in analogous models such as those applied to crime. In applying this method, s and f are treated as the infectious compartments, n represents the non-infectious compartment, and $c(.,.)f$ denotes individuals who are infected but do not contribute to the spread of the scourge. We obtain

$$R_0 = \frac{a_1(.,.)}{a_2(.,.)} \times \frac{b_1(.,.)}{b_2(.,.)}, \tag{2.5}$$

if $R_0 < 1$, then fraud does not spread and thus remains a transient behavior. Conversely, if $R_0 > 1$, fraud spreads and may become an endemic or epidemic behavior. According to the above, the functions a_1 and a_2 depend on $\alpha(t)$ and $\beta(t)$, respectively. This implies that, in order to control the value of R_0, it is necessary to optimize $\alpha(t)$ and $\beta(t)$. The first parameter $\alpha(t)$, which does not depend on the fraudster directly, can be optimized by improving exam conditions. As for $\beta(t)$, its optimization requires organizing re-education sessions led by psychology experts, in collaboration with the fraudster's family, the school, and their social environment.

Remark 2. *If $R_0 = 1$ there are three cases*

- $\dfrac{a_1(.,.)}{a_2(.,.)} = \dfrac{b_1(.,.)}{b_2(.,.)} = 1$, *in this case, there is no transfer between compartments n and s, nor between compartments s and f. Consequently, there will be a change in the rate of non-susceptible individuals.*
- $\dfrac{a_1(.,.)}{a_2(.,.)} > 1$ *and* $\dfrac{b_1(.,.)}{b_2(.,.)} < 1$, *in this case, the proportion of non-susceptible individuals will dominate, and the value of b_1 will control the evolution of the system.*
- $\dfrac{a_1(.,.)}{a_2(.,.)} < 1$ *and* $\dfrac{b_1(.,.)}{b_2(.,.)} > 1$, *in this case, the non-susceptible population becomes small and may disappear, consequently, this will negatively affect the fraudster category.*

3. Mathematical analysis

Furthermore, we define:

$$\begin{cases} F_1(t,n,s,f) = -a_1(\alpha,\beta)n(t) + a_2(\alpha,\beta)s(t) + c(\alpha,\beta)f(t) \quad \text{in} \quad [0,T], \\ F_2(t,n,s,f) = -(a_2(\alpha,\beta) + b_1(\alpha,\beta))s(t) + a_1(\alpha,\beta)n(t) + b_2(\alpha,\beta)f(t) \quad \text{in} \quad [0,T], \\ F_3(t,n,s,f) = -(b_2(\alpha,\beta) + c(\alpha,\beta))f(t) + b_1(\alpha,\beta)s(t) \quad \text{in} \quad [0,T], \end{cases} \tag{3.1}$$

then the system (2.3) becames

$$P3 : \begin{cases} \dfrac{dn(t)}{dt} = F_1(t,n,s,f) \quad \text{in} \quad [0,T], \\ \dfrac{ds(t)}{dt} = F_2(t,n,s,f) \quad \text{in} \quad [0,T], \\ \dfrac{df(t)}{dt} = F_3(t,n,s,f) \quad \text{in} \quad [0,T]. \end{cases} \tag{3.2}$$

3.1. Existence and uniqueness of the solution

Assumptions

Assumption 1. *The functions a_1, a_2, b_1, b_2 and c are time-dependent, strictly positive, and take values less than* 1.

Assumption 2. *We assume that $\alpha(t)$ and $\beta(t)$ are constant over time, which is a natural assumption in the context described above. Therefore, we will subsequently denote them simply as α and β.*

Remark 3. *Thanks to the normalization condition (2.4) and Assumption (1) the functions F_1, F_2 and F_3 are bounded.*

Definition 3.1. *The function $\mathcal{F}$ such that $\mathcal{F} := (\mathcal{F}_1, \mathcal{F}_2, .., \mathcal{F}_p) \in C^1(\mathbb{R}_+ \times \mathbb{R}^p, \mathbb{R}^p)$ with $p \in \mathbb{N}^*$ is quasi-positive if*

$$\mathcal{F}_i(t,y) \geq 0 \quad \textit{whenever} \quad (t,y) \in (0,+\infty) \times \mathbb{R}^p_+ \quad \textit{is such that} \quad y_i = 0.$$

Proposition 3.1. *Under Assumption (1), Assumption (2) and (2.4) the system (3.2) admits a nonnegative global solution.*

Proof: The functions F_i for $i = 1, 2, 3$ are Lipschitz bounded then according to the Cauchy-Lipschitz theorem, the system (3.2) admits a maximal solution which is also a global solution thanks to Remark (3). In addition, the function $F = (F_1, F_2, F_3)$ is quasi-positive and consequently the solutions are nonnegative (see [1] page 3). □

To combat academic fraud, we aim to establish sufficient conditions that reduce the number of fraudulent students and increase the number of non-susceptible students. In this context, we present the following proposition:

Proposition 3.2. *A sufficient set of conditions on the constants $a_1, a_2, b_1, b_2,$ and c to ensure that $n(t)$ increases and $f(t)$ decreases is*

- $a_2 + c > a_1$, *strong inflow to n*
- $b_1 < \dfrac{1}{2}(b_2 + c)$, *weak inflow to f.*

Proof: $n(t)$ increase and $f(t)$ decrease implies $\dfrac{dn}{dt} > 0$ and $\dfrac{df}{dt} < 0$ which implies

$$\begin{cases} -a_1 n + a_2 s + cf > 0, \\ -(b_2 + c)f + b_1 s < 0, \end{cases}$$

which implies

$$\begin{cases} \dfrac{a_2 s + cf}{n} > a_1, \\ \dfrac{s}{f} < \dfrac{b_2 + c}{b_1}. \end{cases}$$

To guarantee the first condition for all time, we need to ensure that the inflows from s and f are strong enough compared to the loss term. For the second condition s must be small relative to f, or equivalently, b_1 must be small compared to $b_2 + c$. Then under the normalization condition (2.4):

- To insure the first condition $\frac{dn}{dt} > 0$ we want that the minimum value of the inflow $a_2 s + cf$ to always exceed the maximum of $a_1 n$ in the limiting case $n = 1$ and $s + f = 0$ but to avoid degenerate cases, we assume all variables are positive and normalized then let's consider $s + f \geq \varepsilon > 0$ then to ensure growth of n, we choose: $a_2 >> a_1$ and/or $c >> a_1$. More precisely, a sufficient condition is given by $a_2 + c > a_1$.
- In the same way as before $\frac{df}{dt} < 0$ is guaranteed if $b_1 << b_2 + c$ which can be replaced by the condition without loss of meaning $b_1 < \frac{1}{2}(b_2 + c)$.

□

Remark 4. *The first inequality ensures that the net inflow into the non-susceptible population $n(t)$ dominates the loss term $-a_1 n$, while the second inequality guarantees that the natural decay of the fraudulent population $f(t)$ exceeds its reinforcement through transition from the susceptible class $s(t)$.*

3.2. Equilibrium points and stability

Proposition 3.3. *Under the normalization condition (2.4) and Assumption (1) the system (3.2) admits a unique equilibrium point. Moreover, this equilibrium point is asymptotically globally stable.*

Proof: a_1, a_2, b_1, b_2, c are functions of $\alpha(t)$ and $\beta(t)$. However, to find the equilibrium points, we assume these functions are constant (i.e., we consider a steady-state case or fix a time t). An equilibrium point (n^*, s^*, f^*) is a constant solution such that:

$$\frac{dn}{dt} = \frac{ds}{dt} = \frac{df}{dt} = 0,$$

which implies

$$\begin{cases} 0 = -a_1 n^* + a_2 s^* + cf^*, \\ 0 = -(a_2 + b_1)s^* + a_1 n^* + b_2 f^*, \\ 0 = -(b_2 + c)f^* + b_1 s^*, \end{cases} \tag{3.3}$$

to find the equilibrium points we express f^* in terms of s^* from the third equation to obtain

$$f^* = \frac{b_1}{b_2 + c} s^*,$$

replacing f^* by its expression in the first equation we obtain

$$n^* = \frac{1}{a_1}(a_2 + \frac{cb_1}{b_2 + c})s^*.$$

Then the equilibrium points are given by:

$$\begin{cases} n^* = \dfrac{1}{a_1}(a_2 + \dfrac{cb_1}{b_2 + c})s^*, \\ f^* = \dfrac{b_1}{b_2 + c} s^*, \\ s^* \quad \text{is a positive given value,} \end{cases} \tag{3.4}$$

so, the set of equilibrium points forms a straight line parameterized by s^*, to obtain a unique point. Under the normalization condition

$$n^* + s^* + f^* = 1,$$

let's define:

$$B_1 = \frac{1}{a_1}(a_2 + \frac{cb_1}{b_2 + c}) \quad \text{and} \quad B_2 = \frac{b_1}{b_2 + c},$$

then $n^* = B_1 s^*$ and $f^* = B_2 s^*$, now substitute into the normalization condition we obtain

$$1 = n^* + s^* + f^* = B_1 s^* + s^* + B_2 s^* = (B_1 + B_2 + 1)s^*,$$

which imlies

$$s^* = \frac{1}{B_1 + B_2 + 1},$$

then the unique equilibrium point is given by

$$(n^*, s^*, f^*) = (\frac{B_1}{B_1 + B_2 + 1}, \frac{1}{B_1 + B_2 + 1}, \frac{B_2}{B_1 + B_2 + 1}).$$

On the other hand the equilibrium point (n^*, s^*, f^*) is unique under the constraint $n + s + f = 1$. To analyze local stability, we linearize the system near the equilibrium point by computing J the Jacobian matrix of the system which is constant and given by

$$J = \begin{pmatrix} -a_1 & a_2 & c \\ a_1 & -(a_2 + b_1) & b_2 \\ 0 & b_2 & -(b_2 + c) \end{pmatrix},$$

□

J is a Metzler matrix. For such systems, stability can be ensured due to the existence of a unique equilibrium, the negativity of the trace, and the irreducibility of the matrix. Therefore, the unique equilibrium (n^*, s^*, f^*) is locally asymptotically stable in the domain

$$\Delta = \{(n, s, f) \in [0, 1]^3 / n + s + f = 1\}.$$

Moreover, for linear systems, local asymptotic stability implies global asymptotic stability. Hence, the equilibrium is globally asymptotically stable.

3.3. Conditions for Maximizing n^* and Minimizing f^* at Equilibrium

Proposition 3.4. *Necessary conditions that maximize n^* and minimize f^* at equilibrium are:*

1. *a_2 be large (i.e promotes s to n)*
2. *a_1 small (limits loss from n)*
3. *b_1 small (limits s to f transition)*
4. *b_2 large (enhances f to s recovery)*
5. *c large (promotes both f to n and decay of f.*

Proof: We consider the equilibrium values of the system defined by:

$$n^* = \frac{B_1}{B_1 + B_2 + 1}, \ f^* = \frac{B_2}{B_1 + B_2 + 1},$$

with $B_1 = \frac{1}{a_1}(a_2 + \frac{cb_1}{b_2 + c})$ and $B_2 = \frac{b_1}{b_2 + c}$. The objective is to identify parameter conditions on $a_1, a_2, b_1, b_2, c > 0$ that maximize n^* (the proportion of non-susceptible individuals) and minimize f^* (the proportion of fraudsters) at equilibrium. Note that n^* is an increasing function of B_1 and a decreasing function of B_2 and f^* is an increasing function of B_2 and a decreasing function of B_1, to achieve the desired behavior, it is sufficient to maximize B_1 and minimize B_2. To reduce B_2, one must Decrease b_1, which controls the transition from susceptibles s to fraudsters f and Increase b_2+c, which accelerates the exit from the fraudulent state (toward s and n.) On the other hand B_1 increases with a_2, which governs the transition from susceptibles to non-susceptibles, also increases with c, provided b_1 is not negligible and if a_1 is small. $\square$

4. Numerical results

In this section, we present numerical experiments illustrating the solution of problem (3.2). The implementation was carried out using Python. We provide two test cases:

The first test:

In the first test, we study the impact of the parameters a_1, a_2, b_1, b_2 and c, which may vary depending on exam conditions and the structure of the educational system, as discussed earlier. This analysis focuses on how these parameters influence the equilibrium point. For $a_1 = 0.1, a_2 = 0.5, b_1 = 0.1, b_2 = 0.5$ and $c = 0.5$ Figure (2) shows an increase in the non-susceptible population accompanied by a decline in fraudsters. In contrast, for $a_1 = 0.5, a_2 = 0.1, b_1 = 0.5, b_2 = 0.1$ and $c = 0.1$ as illustrated in Figure (3), the fraudster population grows while the non-susceptible population declines. For $a_1 = 0.01, a_2 = 1.0, b_1 = 0.01, b_2 = 0.497$ and $c = 1.0$ we have $(n^*, f^*) = (0.987, 0.10)$ (see Figure (4)) let us note here that the basic spread rate given by (2.5) equal to $0.0002012 < 1$, so the number of fraudsters decreases over time while the number of non-susceptible likely increases. For $a_1 = 1.00, a_2 = 0.01, b_1 = 1.00, b_2 = 0.01$ and $c = 0.01$ we have $(n^*, f^*) = (0.0106, 0.971)$ (see Figure (5)) and $R_0 = 10000 > 1$, so the number of fraudsters increases over time while that of the non-susceptible likely decreases. These phase portraits illustrating how the values of the constants a_1, a_2, b_1, b_2 and c influence the equilibrium point, and how the values of n^* and f^* become optimal.

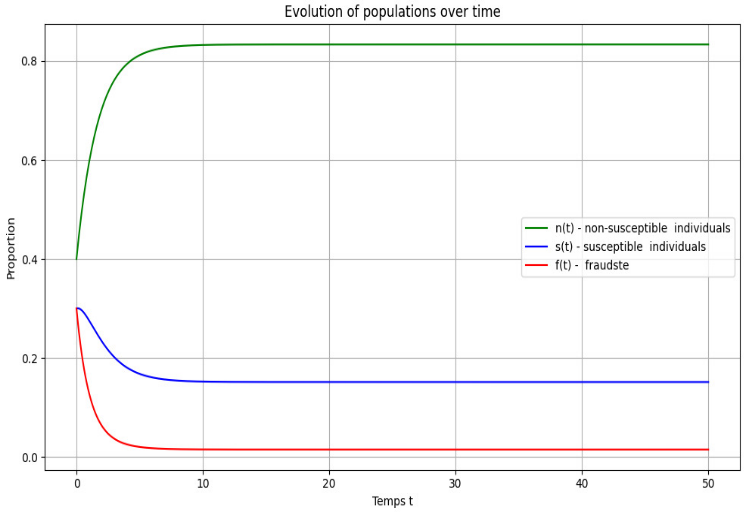

Figure 2: Evolution of n, s and f with $a_1 = 0.1, a_2 = 0.5, b_1 = 0.1, b_2 = 0.5, c = 0.5$.

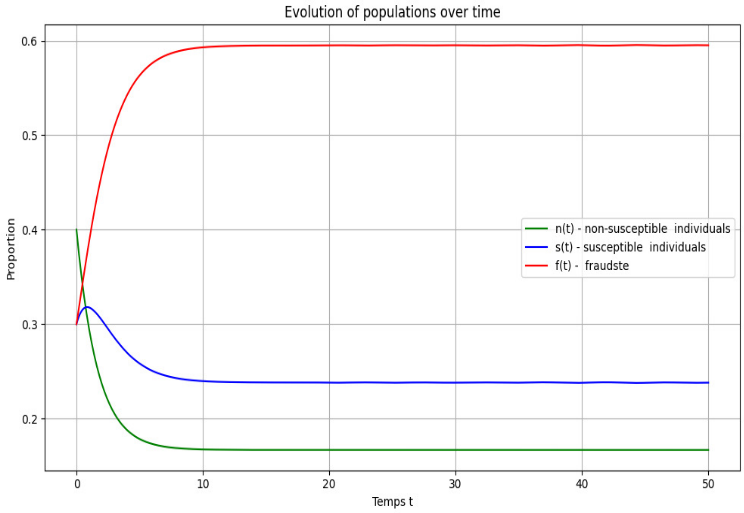

Figure 3: Evolution of n, s and f with $a_1 = 0.5, a_2 = 0.1, b_1 = 0.5, b_2 = 0.1, c = 0.1$.

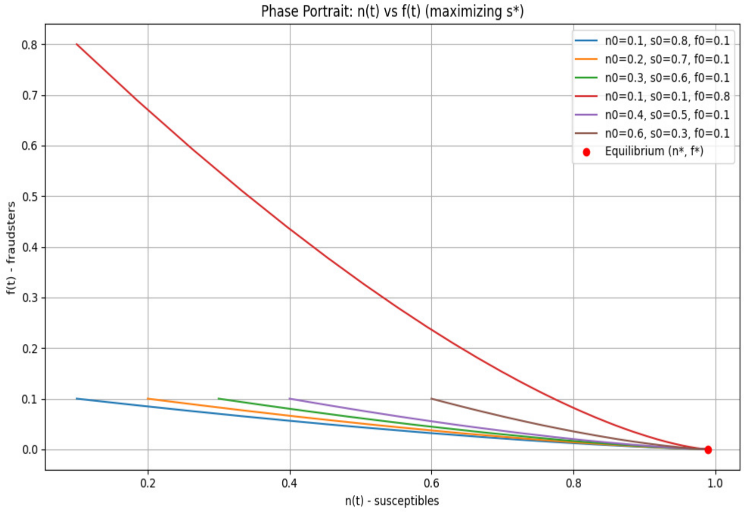

Figure 4: Equilibrium point (n^*, f^*) with $a_1 = 0.01, a_2 = 1.0, b_1 = 0.01, b_2 = 0.497, c = 1.0$.

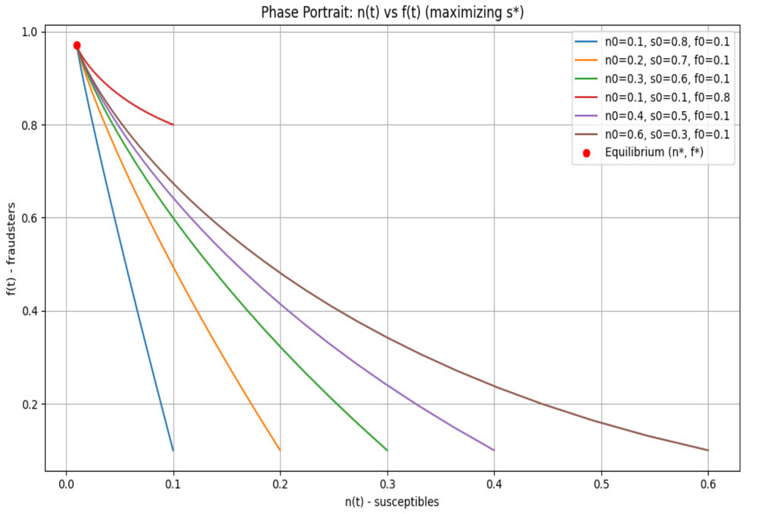

Figure 5: Equilibrium point (n^*, f^*) with $a_1 = 1.00, a_2 = 0.01, b_1 = 1.00, b_2 = 0.01, c = 0.01$.

The second test:

The decrease in the fraudster compartment is due either to a transition toward the non-susceptible group, governed by the coefficient c, or toward the susceptible group, governed by the coefficient b_2. This transition results from either strict punitive actions or re-education efforts. Given the effectiveness of punitive measures in reducing criminal behavior in civil societies, we focus on studying the impact of c and b_2 on the equilibrium values n^* and f^*. In the second test, we examine the effect of the parameters c and b_2 on the non-susceptible and fraudster compartments. For $a_1 = 0.01, a_2 = 1.0, b_1 = 0.01$ and $b_2 = 0.497$, Figure (6) present the evolution of n^* and f^* as functions of c, and observe that as c increases, the value of n^* rises while that of f^* decreases. For $a_1 = 0.01, a_2 = 1.0, b_1 = 0.01$ and $c = 0.0497$, Figure (7) present the evolution of n^* and f^* as functions of b_2, and observe that as b_2 increases, the value of n^* rises while that of f^* decreases. This implies that increasing c reduces the number of fraudsters. Such an increase could be achieved by requiring the administration to strictly enforce sanctions against fraudsters.

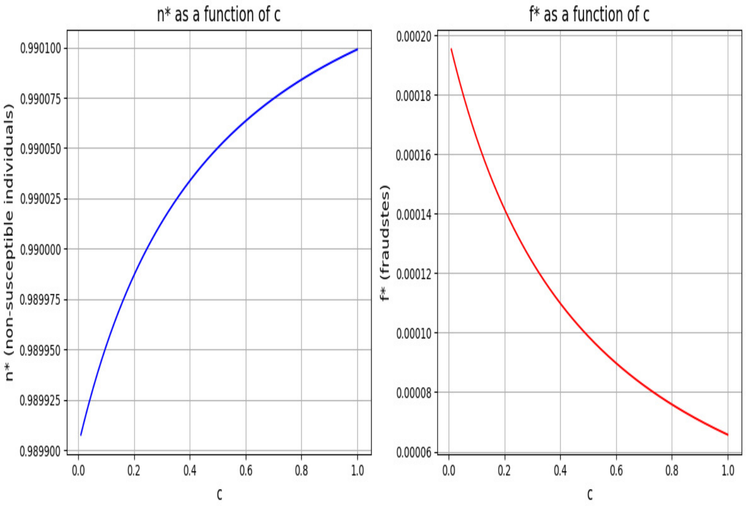

Figure 6: The evolution of the n^* and f^* as functions of c.

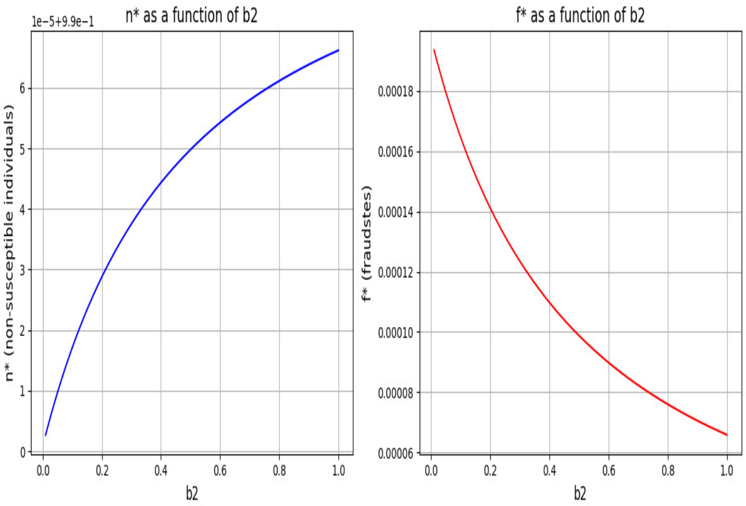

Figure 7: The evolution of the n^* and f^* as functions of b_2.

5. Conclusion

In this work, we developed a mathematical model composed of three strongly coupled ordinary differential equations (ODEs) that describe the interactions among the three components of a school group: individuals who are not susceptible to fraud, those who are susceptible, and those who engage in fraud. We showed that the system admits a unique equilibrium point and that this point is globally asymptotically stable. We then examined the influence of the system's parameters on the equilibrium state, establishing sufficient conditions on these coefficients to minimize the number of fraudsters and increase the proportion of non-susceptible individuals. This analysis led us to identify punitive sanctions as an effective means of reducing fraud. Finally, we presented numerical simulations that support and validate the theoretical results.

There are no relevant competing interests to declare

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