



Novel Classes of Fuzzy Nano Open Sets and Their Medical Applications: A Topological Approach to Diabetes Analysis and Prodrome Elimination

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ABSTRACT: In this paper, we introduce a new class of fuzzy nano open sets, including fuzzy nano ϑ -preopen sets, fuzzy nano ϑe -open sets, fuzzy nano ϑe^* -open sets, fuzzy nano ϑa -open sets, and fuzzy nano ϑ -semiopen sets. We explore the relationships among these sets within fuzzy nano topological spaces, providing counterexamples where specific relationships do not hold. Additionally, we examine key characterizations and properties of these sets in fuzzy nano topological spaces. Furthermore, we apply these findings to the medical domain, particularly in analyzing conditions such as diabetes. Using fuzzy nano topology, we address the elimination of prodromes within the system and identify essential factors contributing to diabetes. This is achieved by constructing a basis in fuzzy nano topology, offering new insights and practical applications in medical science.

Keywords: fuzzy nano ϑ -preopen, fuzzy nano ϑe -open, fuzzy nano ϑe^* -open, fuzzy nano ϑa -open.

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1. Introduction

Nano topological science and nanostructures have emerged as pivotal areas of research, impacting both engineering and medical sciences. The continuous advancements in nanoscience and nanotechnology have enabled the design, development, and control of sophisticated systems, from medical devices to engineering applications. In particular, the integration of topology with nanoscience has opened new dimensions for analyzing and managing complex systems characterized by vagueness and uncertainty. In 1965, Lotfi Zadeh [19] revolutionized the mathematical treatment of uncertainty by introducing fuzzy set theory, laying the groundwork for countless innovations. Building on this, Chang [1] introduced the concept of fuzzy topology, expanding the capacity of mathematical models to represent imprecision. Later, Pawlak [13,14] developed rough set theory, using topological constructs like interior and closure to handle vagueness through approximations. In 2013, Lellis Thivagar [8] introduced nano topology, offering a framework for dealing with small-scale phenomena by defining topological spaces through equivalence-based approximations. In 2024 P. Rajwade, R. Navalakhe [20] introduced Nano Fuzzy Z - Open Sets and its Application in Nano Fuzzy Topological Space. Despite these advances, the intersection of fuzzy logic, rough sets, and nano topology remains underexplored, particularly in the classification of nano open sets. Most existing studies address specific types of open sets but fail to comprehensively handle the diverse forms of uncertainty that arise in medical data analysis. This gap limits the applicability of current models when tackling real-world medical challenges such as disease diagnosis and prodrome identification. Notably, bipolar fuzzy circuits introduced by Pazar Varol, B.; Malkoç [15], positively polarized fuzzy concept lattices, soft rough approximations on graphs and hypergraphs, and type-2 fuzzy soft graphs for decision-making have gained attention for their ability to capture nuanced relationships and dynamic information structures. While these methods offer valuable tools in specific contexts, they

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often exhibit certain limitations. And while beneficial for knowledge representation, it often lacks dynamic approximation capabilities when confronted with overlapping, uncertain boundaries in medical or biological systems. That the proposed ϑ -fuzzy nano topological framework aims to overcome. Such that ϑ -fuzzy nano open sets provide a hierarchical and spatially-aware mechanism to classify uncertainty. Unlike bipolar circuits that focus on dual-polar information, our framework handles graded vagueness using nuanced topological relationships, which is crucial for applications like prodrome elimination in medical diagnostics. And to bridge this gap, the concept of fuzzy nano topology has evolved, providing robust mathematical tools for combinatorial problems across domains such as algebra, optimization, computer science, and more importantly, the medical field. Fuzzy nano topological structures possess a unique ability to model vagueness, ambiguity, and imprecision, offering a significant advantage over classical (non-fuzzy) topologies in practical applications. Several researchers have contributed to this growing field. For example, Rachna Navlakhe [16] explored fuzzy nano relations, while R. Thangammal et al. [17] introduced fuzzy nano Z-open sets. More recently, in 2025, J. M. Jamil and W. A. Hussein [2] introduced nano ic-open sets, applying them to anemia etiology analysis. Similarly, K. Kavithamani et al. developed c^*g -closed sets, applying them in medical contexts such as rheumatoid arthritis to manage prodromal symptoms. This paper addresses the lack of a comprehensive classification of fuzzy nano open sets by introducing an innovative new family called ϑ -preopen sets, ϑe -open sets, ϑe^* -open sets, ϑa -open sets, and ϑ -semiopen sets, these sets and their interrelationships are systematically studied, providing counterexamples where classical assumptions fail. By developing this framework, the study offers a deeper theoretical understanding of fuzzy nano topology, particularly in how it can be applied to model medical phenomena. The implications of this work are not purely theoretical. The proposed fuzzy nano ϑ -open sets are applied to analyze diabetes, focusing on the elimination of prodromal indicators and identifying critical factors contributing to disease onset. By constructing mathematically rigorous models, the study introduces new methodologies for early diagnosis and medical decision-making, which are vital for managing chronic diseases. The primary objectives of this study are to introduce and define new classes of fuzzy nano open sets and examine their properties. To develop counterexamples that highlight limitations of existing definitions. Also to apply the new theoretical models to medical data, particularly in the analysis of diabetes and anemia. Moreover to offer a systematic framework that unifies various types of fuzzy nano open sets, thus advancing the field of fuzzy nano topology.

2. Preliminaries

Definition 1.1 [13]

Let $X \neq \emptyset$, a fuzzy set $\widehat{\mathbb{S}}$ in X is defined by a function $\mu_{\widehat{\mathbb{S}}} : X \rightarrow \mathfrak{S}$, where \mathfrak{S} is closed interval between one and zero and written as

$$\widehat{\mathbb{S}} = \{(x, \mu_{\widehat{\mathbb{S}}}(x)) : x \in X, 0 \leq \mu_{\widehat{\mathbb{S}}}(x) \leq 1\},$$

the family of fuzzy sets in X briefly I^X , that is

$$I^X = \{\widehat{\mathbb{S}} : \widehat{\mathbb{S}} \text{ is a fuzzy sets } (\mathfrak{S}^*) \text{ in } X\}$$

where $\mu_{\widehat{\mathbb{S}}}$ is called the membership functions.

Definition 1.2 [13]

Let $\widehat{\mathbb{C}}$ and $\widehat{\mathbb{E}}$ be a (\mathfrak{S}^*) in X with memberships $\mu_{\widehat{\mathbb{C}}}$ and $\mu_{\widehat{\mathbb{E}}}$ respectively. Then $\forall x \in X$:

1. $\widehat{\mathbb{C}} \subseteq \widehat{\mathbb{E}} \iff \mu_{\widehat{\mathbb{C}}}(x) \leq \mu_{\widehat{\mathbb{E}}}(x)$.
2. $\widehat{\mathbb{C}} = \widehat{\mathbb{E}} \iff \mu_{\widehat{\mathbb{C}}}(x) = \mu_{\widehat{\mathbb{E}}}(x)$.
3. $\widehat{\mathbb{S}} = \widehat{\mathbb{C}} \cap \widehat{\mathbb{E}} \iff \mu_{\widehat{\mathbb{S}}}(x) = \min\{\mu_{\widehat{\mathbb{C}}}(x), \mu_{\widehat{\mathbb{E}}}(x)\}$.
4. $\widehat{\mathbb{E}} = \widehat{\mathbb{C}} \cup \widehat{\mathbb{E}} \iff \mu_{\widehat{\mathbb{E}}}(x) = \max\{\mu_{\widehat{\mathbb{C}}}(x), \mu_{\widehat{\mathbb{E}}}(x)\}$.

Definition 1.3 [6]

A fuzzy point x_p is a (\mathfrak{S}^*) and:

$$\mu_{x_p}(y) = \begin{cases} p > 0 & \text{if } x = y, \text{ for all } y \in X \\ 0 & \text{if } x \neq y, \text{ for all } y \in X \end{cases}$$

Definition 1.4 [8]

The fuzzy relation \mathbb{R} on a non-empty set X is said to be:

1. Fuzzy reflexive on X iff $\mu_{\mathbb{R}}(x, x) = 1$, for all $(x, x) \in X \times X$.
2. Fuzzy symmetric on X iff $\mu_{\mathbb{R}}(x, y) = \mu_{\mathbb{R}}(y, x)$, for all $(x, y), (y, x) \in X \times X$.
3. Fuzzy transitive on X iff $\mu_{\mathbb{R}}(x, z) \geq \max \{ \min \{ \mu_{\mathbb{R}}(x, y), \mu_{\mathbb{R}}(y, z) \} \}$, for all $(x, y), (y, z), (x, z) \in X \times X$.

Definition 1.5 [8]

Let $\mathcal{U} \neq \emptyset$ be a finite set of objects said to be universe and \mathbb{R} be fuzzy equivalent relations on \mathcal{U} said to be fuzzy indiscernibility relation. Then $(\mathcal{U}, \mathbb{R})$ is called fuzzy approximation space.

Remark 1.6:

Element belonging to the same fuzzy equivalence class are said to be fuzzy indiscernibility with one another.

Definition 1.7

Let \mathfrak{X} be a fuzzy set and $\mathbb{R}[x]$ be the fuzzy equivalence class determined by $x \in \mathcal{U}$, then

1. The fuzzy lower approximation of \mathfrak{X} is defined by

$$\mathcal{L}_{\mathbb{R}}(\mathfrak{X}) = \max_{x \in \mathcal{U}} \{ \mu_{\mathbb{R}[x]} : \mu_{\mathbb{R}[x]} \leq \mu_{\mathfrak{X}}(x) \}.$$

2. The fuzzy upper approximation of \mathfrak{X} is defined by

$$\mathcal{U}_{\mathbb{R}}(\mathfrak{X}) = \max_{x \in \mathcal{U}} \{ \mathbb{R}[x] : \min \{ \mu_{\mathbb{R}[x]}, \mu_{\mathfrak{X}}(x) \} \neq 0 \}.$$

3. The fuzzy boundary region of \mathfrak{X} is defined by

$$\mathcal{B}_{\mathbb{R}}(\mathfrak{X}) = \mathcal{U}_{\mathbb{R}}(\mathfrak{X}) - \mathcal{L}_{\mathbb{R}}(\mathfrak{X}).$$

Definition 1.8

Let \mathcal{U} be a non-empty finite set, \mathbb{R} be a fuzzy equivalence relation on \mathcal{U} , $\mathfrak{X} \leq \mathcal{U}$ be a fuzzy subset then

$$\mathcal{T}_{\mathbb{R}}(\mathfrak{X}) = \{ \emptyset, \hat{\mathfrak{X}}, \mathcal{L}_{\mathbb{R}}(\mathfrak{X}), \mathcal{U}_{\mathbb{R}}(\mathfrak{X}), \mathcal{B}_{\mathbb{R}}(\mathfrak{X}) \}$$

is fuzzy nano topology on \mathfrak{X} if:

1. $\emptyset, \hat{\mathfrak{X}} \in \mathcal{T}_{\mathbb{R}}(\mathfrak{X})$.
2. If $\mathfrak{B}, \mathfrak{C} \in \mathcal{T}_{\mathbb{R}}(\mathfrak{X})$, then $\mathfrak{B} \cap \mathfrak{C} \in \mathcal{T}_{\mathbb{R}}(\mathfrak{X})$.
3. If $\mathfrak{B}_{\alpha} \in \mathcal{T}_{\mathbb{R}}(\mathfrak{X})$, then $\bigcup_{\alpha} \mathfrak{B}_{\alpha} \in \mathcal{T}_{\mathbb{R}}(\mathfrak{X})$, $\alpha \in A$.

$(\mathfrak{X}, \mathcal{T}_{\mathbb{R}}(\mathfrak{X}))$ is called **fuzzy nano topological space** (briefly **fnts**) and every element of $\mathcal{T}_{\mathbb{R}}(\mathfrak{X})$ is said to be a **fuzzy nano open set** (briefly **fnos**) in \mathfrak{X} and its complement is a **fuzzy nano closed set** (briefly **fncs**).

Definition 1.9

Let \mathfrak{S} be a fuzzy nano set in a fnts $(\mathfrak{X}, \mathcal{T}_{\mathbb{R}}(\mathfrak{X}))$, then:

1. The nanointerior of \mathfrak{S} is $\text{NanoInt}(\mathfrak{S}) = \max\{\mu_{\mathcal{V}}(x) : \mathcal{V} \text{ is a fnos in } \mathfrak{X}, \mu_{\mathcal{V}}(x) \leq \mu_{\mathfrak{S}}(x)\}$ (briefly **nano Int**(\mathfrak{S})).
2. The nano closure of \mathfrak{S} is $\text{NanoCl}(\mathfrak{S}) = \min\{\mu_{\mathcal{R}}(x) : \mathcal{R} \text{ is a fnos in } \mathfrak{X}, \mu_{\mathfrak{S}}(x) \leq \mu_{\mathcal{R}}(x)\}$ (briefly **nano Cl**(\mathfrak{S})).

Definition 1.10

Let $(\mathfrak{X}, \mathcal{T}_{\mathbb{R}}(\mathfrak{X}))$ be a fnts, the fuzzy nano set $\bar{\beta} = \{\mathcal{U}, \mathcal{L}_{\mathbb{R}}(\mathfrak{X}), \mathcal{B}_{\mathbb{R}}(\mathfrak{X})\}$ is called fuzzy nano bases for the fnts $\mathcal{T}_{\mathbb{R}}(\mathfrak{X})$ on \mathcal{U} to \mathfrak{X} .

3. Some Types of Fuzzy Nano Open Sets

Definition 2.1

A fuzzy nano \mathfrak{D} of a fnts $(\mathfrak{X}, \mathcal{T}_{\mathbb{R}}(\mathfrak{X}))$ is called fuzzy nano θ -open set (briefly **fn θ -o**) if for each $\mu_{\mathfrak{X}}(x) < \mu_{\mathfrak{S}}(x)$ there exists an fnos, \mathfrak{G} such that $\mu_{\mathfrak{X}}(x) < \mu_{\mathfrak{G}}(x)$ and $\mu_{\text{NanoCl}(\mathfrak{G})}(x) \leq \mu_{\mathfrak{S}}(x)$.

Definition 2.2

A fuzzy set \mathfrak{J} in a fnts $(\mathfrak{X}, \mathcal{T}_{\mathbb{R}}(\mathfrak{X}))$ is said to be fuzzy nano θ -boundary (briefly **fn θ -Bdr**) iff

$$\mu_{\mathfrak{J}}(x) = \mu_{\text{nano } \theta \text{ Cl}(\mathfrak{J})}(x) - \mu_{\text{nano } \theta \text{ Int}(\mathfrak{J})}(x).$$

Definition 2.3

A fuzzy set \mathfrak{J} in a fnts $(\mathfrak{X}, \mathcal{T}_{\mathbb{R}}(\mathfrak{X}))$ is called fuzzy nano θ -dense iff

$$\mu_{\mathfrak{X}}(x) = \mu_{\text{nano } \theta \text{ Cl}(\mathfrak{J})}(x).$$

Definition 2.4

A fuzzy nano \mathfrak{A} of a fnts $(\mathfrak{X}, \mathcal{T}_{\mathbb{R}}(\mathfrak{X}))$ is called fuzzy nano regular open set (briefly **fnR-o**) iff

$$\mu_{\mathfrak{A}}(x) = \mu_{\text{nano Int}(\text{nano Cl}(\mathfrak{A}))}(x).$$

Definition 2.5

A fuzzy nano set \mathfrak{Q} of a fnts $(\mathfrak{X}, \mathcal{T}_{\mathbb{R}}(\mathfrak{X}))$ is called:

1. **Fuzzy nano θ -preopen** (briefly **fn θ -po**) if

$$\mu_{\mathfrak{Q}}(x) \leq \mu_{\text{nano Int}(\text{nano } \theta\text{-Cl}(\mathfrak{Q}))}(x).$$

2. **Fuzzy nano θ -preclosed** (briefly **fn θ -pc**) if

$$\mu_{\text{nano Cl}(\text{nano } \theta\text{-Int}(\mathfrak{Q}))}(x) \leq \mu_{\mathfrak{Q}}(x).$$

3. **Fuzzy nano θ e-open** (briefly **fn θ -eO**) if

$$\mu_{\mathfrak{Q}}(x) \leq \max\{\mu_{\text{nano Int}(\text{nano } \theta\text{-Cl}(\mathfrak{Q}))}(x), \mu_{\text{nano Cl}(\text{nano } \theta\text{-Int}(\mathfrak{Q}))}(x)\}.$$

4. **Fuzzy nano θ e-closed** (briefly **fn θ -eC**) if

$$\min\{\mu_{\text{nano Int}(\text{nano } \theta\text{-Cl}(\mathfrak{Q}))}(x), \mu_{\text{nano Cl}(\text{nano } \theta\text{-Int}(\mathfrak{Q}))}(x)\} \leq \mu_{\mathfrak{Q}}(x).$$

5. **Fuzzy nano θe^* -open** (briefly $\text{fn}\theta\text{-}e^*\text{o}$) if

$$\mu_{\Omega}(x) \leq \mu_{\text{nano Int}(\text{nano Int}(\text{nano } \theta\text{-Cl}(\Omega)))}(x).$$

6. **Fuzzy nano θe^* -closed** (briefly $\text{fn}\theta\text{-}e^*\text{c}$) if

$$\mu_{\text{nano Int}(\text{nano Int}(\text{nano } \theta\text{-Int}(\Omega)))}(x) \leq \mu_{\Omega}(x).$$

7. **Fuzzy nano θa -open** (briefly $\text{fn}\theta\text{-aO}$) if

$$\mu_{\Omega}(x) \leq \mu_{\text{nano Int}(\text{nano Cl}(\text{nano } \theta\text{-Int}(\Omega)))}(x).$$

8. **Fuzzy nano θa -closed** (briefly $\text{fn}\theta\text{-aC}$) if

$$\mu_{\text{nano Cl}(\text{nano Int}(\text{nano } \theta\text{-Cl}(\Omega)))}(x) \leq \mu_{\Omega}(x).$$

9. **Fuzzy nano θ -semiopen** (briefly $\text{fn}\theta\text{-sO}$) if

$$\mu_{\Omega}(x) \leq \mu_{\text{nano Cl}(\text{nano } \theta\text{-Int}(\Omega))}(x).$$

10. **Fuzzy nano θ -semiclosed** (briefly $\text{fn}\theta\text{-sC}$) if

$$\mu_{\text{nano Int}(\text{nano } \theta\text{-Cl}(\Omega))}(x) \leq \mu_{\Omega}(x).$$

Proposition 2.6

Every $\text{fn}\theta\text{-o}$ is $\text{fn}\theta\text{-ao}$.

Proof

Let \mathfrak{S} be a $\text{fn}\theta\text{-o}$ in a fnts $(\mathfrak{X}, \mathcal{T}_{\mathbb{R}}(\mathfrak{X}))$. Since $\mu_{\mathfrak{S}}(x) \leq \mu_{\text{nano Cl}(\mathfrak{S})}(x) \Rightarrow \mu_{\text{nano Int}(\mathfrak{S})}(x) \leq \mu_{\text{nano Int}(\text{nano Cl}(\mathfrak{S}))}(x)$, then $\mu_{\mathfrak{S}}(x) \leq \mu_{\text{nano Int}(\text{nano Cl}(\mathfrak{S}))}(x)$. Since $\mu_{\mathfrak{S}}(x) = \mu_{\text{nano } \theta\text{-Int}(\mathfrak{S})}(x)$, then $\mu_{\mathfrak{S}}(x) \leq \mu_{\text{nano Int}(\text{nano Cl}(\text{nano } \theta\text{-Int}(\mathfrak{S}))}(x)$, hence \mathfrak{S} is $\text{fn}\theta\text{-ao}$. ■

Remark 2.7

The opposite of the proposition above is not true, this example shows.

Example 2.8

Let $\mathcal{U} = \{a, b, c, d\}$ and $\mathfrak{X} = \{a_1, b_1, c_1, d_1\}$ with $\mathbb{R} = \{\{a_0.6, b_0, c_0, d_0\}, \{a_0, b_0, c_0.9, d_0\}, \{a_0, b_0.5, c_0, d_0.4\}\}$ and $\mathfrak{S} = \{a_0.7, b_0.8, c_0, d_0\}$ then the fuzzy nano topology is defined as: $\mathcal{T}_{\mathbb{R}}(\mathfrak{S}) = \{\emptyset, \mathfrak{X}, \{a_0.6, b_0, c_0, d_0\}, \{a_0, b_0.5, c_0, d_0.4\}, \{a_0.6, b_0.5, c_0, d_0.4\}\}$. Then the fuzzy nano set $\mathfrak{A} = \{a_0.7, b_0.7, c_0, d_0.5\}$ is $\text{fn}\theta\text{-ao}$ but not $\text{fn}\theta\text{-o}$.

Proposition 2.9

Every $\text{fn}\theta\text{-ao}$ is $\text{fn}\theta\text{-so}$.

Proof

Let \mathfrak{S} be a $\text{fn}\theta\text{-ao}$ in a fnts $(\mathfrak{X}, \mathcal{T}_{\mathbb{R}}(\mathfrak{X}))$, then $\mu_{\mathfrak{S}}(x) \leq \mu_{\text{nano Int}(\text{nano Cl}(\text{nano } \theta\text{-Int}(\mathfrak{S}))}(x)$, since $\mu_{\text{nano Int}(\text{nano Cl}(\text{nano } \theta\text{-Int}(\mathfrak{S}))}(x) \leq \mu_{\text{nano Cl}(\text{nano } \theta\text{-Int}(\mathfrak{S}))}(x)$, so $\mu_{\mathfrak{S}}(x) \leq \mu_{\text{nano Cl}(\text{nano } \theta\text{-Int}(\mathfrak{S}))}(x)$, hence \mathfrak{S} is $\text{fn}\theta\text{-so}$. ■

Remark 2.10

The converse of the proposition 2.9 is not true, this example shows.

Example 2.11

Let $\mathcal{U} = \{q, r, s, d\}$ and $\mathfrak{X} = \{q_1, r_1, s_1, d_1\}$ with

$\mathbb{R} = \{\{q_0.5, r_0.5, s_0.5, d_0.5\}, \{q_0.1, r_0.2, s_0.3, d_0.4\}, \{q_0.9, r_0.7, s_0.2, d_0.3\}, \{q_0.9, r_0.9, s_0.8, d_0.3\}\}$ and

$\mathfrak{S} = \{q_0.6, r_0.7, s_0.7, d_0.6\}$ then the fnt is defined as

$\mathcal{T}_{\mathbb{R}}(\mathfrak{S}) = \{\emptyset, \mathfrak{X}, \{q_0.5, r_0.5, s_0.5, d_0.5\}, \{q_0.9, r_0.9, s_0.8, d_0.4\}, \{q_0.4, r_0.4, s_0.3, d_0.1\}\}$. The $\text{fn}\theta$ -open sets of $\mathfrak{Y} = \{q_0.5, r_0.5, s_0.7, d_0.9\}$ is $\text{fn}\theta\text{-o}(\mathfrak{Y}) = \{\emptyset, \mathfrak{X}, \{q_0.5, r_0.5, s_0.7, d_0.9\}, \{q_0.5, r_0.5, s_0.5, d_0.5\}\}$, hence the fuzzy nano set $\mathfrak{S} = \{q_0.5, r_0.5, s_0.7, d_0.9\}$ is $\text{fn}\theta$ -so but not $\text{fn}\theta\text{ao}$.

Proposition 2.12

Every $\text{fn}\theta$ -po is $\text{fn}\theta\text{-e}^*\text{o}$.

Proof

Let \mathfrak{S} be a $\text{fn}\theta$ -po set in a fnts $(\mathfrak{X}, \mathcal{T}_{\mathbb{R}}(\mathfrak{X}))$, then $\mu_{\mathfrak{S}}(x) \leq \mu_{\text{nanoInt}(\text{nano}\theta\text{-Cl}(\mathfrak{S}))}(x) \Rightarrow \mu_{\text{nanoInt}(\text{nano}\theta\text{-Cl}(\mathfrak{S}))}(x) \leq \mu_{\theta\text{-Cl}(\mathfrak{S})}(x)$. So $\mu_{\mathfrak{S}}(x) \leq \mu_{\theta\text{nanoCl}(\mathfrak{S})}(x) \Rightarrow \mu_{\theta\text{nanoInt}(\mathfrak{S})}(x) \leq \mu_{\text{nanoInt}(\text{nano}\theta\text{-Cl}(\mathfrak{S}))}(x) \Rightarrow \mu_{\theta\text{nanoCl}(\text{nanoInt}(\mathfrak{S}))}(x) \leq \mu_{\text{nanoCl}(\text{nanoInt}(\text{nano}\theta\text{-Cl}(\mathfrak{S}))}(x)$. Since $\mu_{\text{nanoInt}(\mathfrak{S})}(x) \leq \mu_{\theta\text{nanoCl}(\text{nanoInt}(\mathfrak{S}))}(x)$, then we get $\mu_{\text{nanoInt}(\mathfrak{S})}(x) \leq \mu_{\text{nanoCl}(\text{nanoInt}(\text{nano}\theta\text{-Cl}(\mathfrak{S}))}(x)$, and $\mu_{\mathfrak{S}}(x) \leq \mu_{\text{nanoCl}(\text{nanoInt}(\text{nano}\theta\text{-Cl}(\mathfrak{S}))}(x)$, hence \mathfrak{S} is $\text{fn}\theta\text{-e}^*\text{o}$.

Remark 2.13

The converse of the proposition 2.12 is not true, this example shown.

Example 2.14

In the example 2.11 the fuzzy nano set $\mathfrak{S} = \{q_0.5, r_0.5, s_0.3, d_0.1\}$ is $\text{fn}\theta\text{-e}^*\text{o}$ but not $\text{fn}\theta$ -po

Proposition 2.15

Let $(\mathfrak{X}, \mathcal{T}_{\mathbb{R}}(\mathfrak{X}))$, be a fnts then

- 1- Every fnR-o is $\text{fn}\theta\text{-o}$.
- 2- Every $\text{fn}\theta\text{-ao}$ is $\text{fn}\theta\text{-po}$.
- 3- Every $\text{fn}\theta\text{-eo}$ is $\text{fn}\theta\text{-e}^*\text{o}$.
- 4- Every $\text{fn}\theta\text{-so}$ is $\text{fn}\theta\text{-eo}$.

Proof

Obvious.

Example 2.16

The opposite of the proposition above is not true in general.

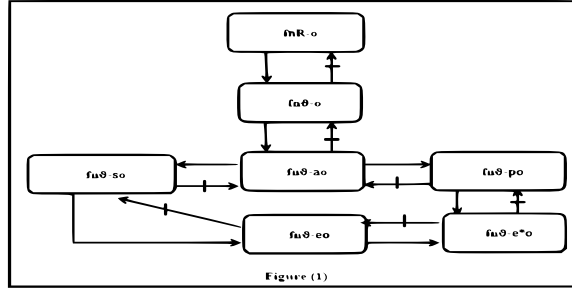
Example 2.17

figure (1) explain the relations between fnos.

Theorem 2.18

Let \hat{Q} be a fns of fnts $(\mathfrak{X}, \mathcal{T}_{\mathbb{R}}(\mathfrak{X}))$, then the following are equivalent

- i \hat{Q} is fnR-o .
- ii \hat{Q} is $\text{fn}\theta\text{-ao}$ and $\text{fn}\theta\text{-e}^*\text{o}$.
- iii \hat{Q} is $\text{fn}\theta\text{-Po}$ and $\text{fn}\theta\text{-sc}$.



Proof

(i \Rightarrow ii) Obvious

(ii \Rightarrow i) Let \hat{Q} be $\text{fn}\theta\text{-ao}$ and $\text{fn}\theta\text{-e}^*\text{o}$ sets, we have

$$\mu_{\hat{Q}}(x) \leq \mu_{\text{nano Int}(\text{nano Cl}(\text{nano } \theta\text{-Int}(\hat{Q})))(x)} \text{ and}$$

$$\mu_{\text{nano Int}(\text{nano Cl}(\text{nano } \theta\text{-Int}(\hat{Q})))(x)} \leq \mu_{\hat{Q}}(x) \text{ and hence}$$

$$\mu_{\text{nano Int}(\text{nano Cl}(\text{nano } \theta\text{-Int}(\hat{Q})))(x)} = \mu_{\hat{Q}}(x), \text{ thus } \hat{Q} \text{ is fnR-o.}$$

(i \Leftrightarrow iii) Let \hat{Q} be $\text{fn}\theta\text{-po}$ and $\text{fn}\theta\text{-sc}$, then $\mu_{\hat{Q}}(x) \leq \mu_{\text{nano Int}(\text{nano } \theta\text{-Cl}(\hat{Q}))}(x)$ and

$$\mu_{\text{nano Int}(\text{nano } \theta\text{-Cl}(\hat{Q}))}(x) \leq \mu_{\hat{Q}}(x), \text{ thus } \mu_{\hat{Q}}(x) = \mu_{\text{nano Int}(\text{nano } \theta\text{-Cl}(\hat{Q}))}(x) = \mu_{\text{nano Int}(\text{nano Cl}(\hat{Q}))}(x),$$

hence \hat{Q} is fnR-o set, the converse is similar.

Theorem 2.19

Let \hat{Q} be a fns of fnts $(\mathfrak{X}, \mathcal{T}_{\mathbb{R}}(\mathfrak{X}))$, then the following are equivalent:

i \hat{Q} is $\text{fn}\theta\text{-so}$.

ii \hat{Q} is $\text{fn}\theta\text{-e}^*\text{o}$ and $\mu_{\text{nano } \theta\text{-Int}(\text{nano } \theta\text{-Bdr}(\hat{Q}))}(x) = 0$.

Proof

(i \Rightarrow ii) Let \hat{Q} be $\text{fn}\theta\text{-so}$, then we have

$$\mu_{\text{nano Int}(\text{nano } \theta\text{-cl}(\hat{Q}))}(x) \leq \mu_{\text{nano } \theta\text{-cl}(\hat{Q})}(x) \leq \mu_{\text{nano cl}(\text{nano } \theta\text{-Int}(\hat{Q}))}(x), \text{ since } \mu_{\text{nano } \theta\text{-Int}(\text{nano cl}(\hat{Q}))} =$$

$$\mu_{\text{nano } \theta\text{-Int}(\mu_{\text{nano } \theta\text{-cl}(\hat{Q})})} \text{ and } \mu_{\text{nano } \theta\text{-Int}(\mu_{\text{nano } \theta\text{-cl}(\hat{Q})})} = \mu_{\text{nano } \theta\text{-Int}(\mu_{\text{nano cl}(\hat{Q})})}, \text{ then}$$

$$\mu_{\text{nano } \theta\text{-Int}(\text{nano } \theta\text{-Bdr}(\hat{Q}))}(x) = 0, \text{ since } \hat{Q} \text{ is } \text{fn}\theta\text{-so}, \text{ then } \hat{Q} \leq \mu_{\text{nano cl}(\text{nano Int}(\text{nano } \theta\text{-cl}(\hat{Q})))(x)} \leq$$

$$\mu_{\text{nano cl}(\text{nano Int}(\hat{Q}))}(x), \text{ thus } \hat{Q} \text{ is } \text{fn}\theta\text{-so.}$$

Theorem 2.20

Let $(\mathfrak{X}, \mathcal{T}_{\mathbb{R}}(\mathfrak{X}))$ be a fnts, then the $\text{fn}\theta\text{-ao}$ is a $\text{fn}\theta\text{-so}$ intersection $\text{fn}\theta\text{-po}$.

Proof

Let \hat{Q} be $\text{fn}\theta\text{-ao}$, then by Proposition 2.9 and Proposition 2.15

$$\mu_{\text{nano } \theta\text{-a}(\hat{Q})}(x) \leq \mu_{\text{nano } \theta\text{-semiopen}(\hat{Q})}(x) \cap \mu_{\text{nano } \theta\text{-preopen}(\hat{Q})}(x),$$

Conversely, let \hat{Q} be a $\text{fn}\theta\text{-so}$ intersection $\text{fn}\theta\text{-po}$, then by Theorem 2.19 $\mu_{\text{nano } \theta\text{-Int}(\text{nano } \theta\text{-Bdr}(\hat{Q}))}(x) = 0$, since $\mu_{\text{nano } \theta\text{-Int}(\text{nano } \theta\text{-cl}(\hat{Q}))} = 0$, and so \hat{Q} is $\text{fn}\theta\text{-ao}$.

Theorem 2.21

Let \hat{Q} be a fns of fnts $(\mathfrak{X}, \mathcal{T}_{\mathbb{R}}(\mathfrak{X}))$, then the following are equivalent:

- i.* \widehat{Q} is $\text{fn}\theta$ -po.
- ii.* There exist a fnR -o $\widetilde{M} \subset \mathfrak{X}$ such that

$$\mu_{\text{nano } \theta}(\widehat{Q})(x) \leq \mu_{\text{nano}}(\widetilde{M})(x) \quad \text{and} \quad \mu_{\text{nano } \theta \text{ Cl}(\widehat{Q})}(x) = \mu_{\text{nano } \theta \text{ Cl}(\widetilde{M})}(x).$$
- iii.* \widehat{Q} is the intersection of fnR -o and $\text{fn}\theta$ -dense.
- iv.* \widehat{Q} is the intersection of $\text{fn}\theta$ -o and $\text{fn}\theta$ -dense.

Proof

(*i* \Rightarrow *ii*) Let \widehat{Q} is $\text{fn}\theta$ -po, then we have $\mu_{\widehat{Q}}(x) \leq \mu_{\text{nano Int}(\text{nano } \theta\text{-Cl}(\widehat{Q}))(x)}$,
 Take $\mu_{\widetilde{M}}(x) = \mu_{\text{nano Int}(\text{nano } \theta\text{-Cl}(\widehat{Q}))(x)}$, then \widetilde{M} is fnR -o such that $\mu_{\widehat{Q}}(x) \leq \mu_{\widetilde{M}}(x)$, and
 $\mu_{\text{nano } \theta \text{ Cl}(\widehat{Q})}(x) = \mu_{\text{nano } \theta \text{ Cl}(\widetilde{M})}(x)$.

(*ii* \Rightarrow *iii*) Suppose that \exists a fnR -o $\widetilde{M} \subset \mathfrak{X}$ such that

$$\mu_{\text{nano } \theta}(\widehat{Q})(x) \leq \mu_{\text{nano}}(\widetilde{M})(x) \quad \text{and} \quad \mu_{\text{nano } \theta \text{ Cl}(\widehat{Q})}(x) = \mu_{\text{nano } \theta \text{ Cl}(\widetilde{M})}(x).$$

Put

$$\mu_{\text{nano } \mathcal{H}}(x) = \max\{\mu_{\text{nano } \widehat{Q}}(x), 1 - \mu_{\text{nano } \widehat{Q}}(x)\},$$

thus \mathcal{H} is $\text{fn}\theta$ -dense, and

$$\mu_{\text{nano } \widehat{Q}}(x) = \min\{\mu_{\text{nano}}(\widetilde{M})(x), \mu_{\text{nano } \mathcal{H}}(x)\}.$$

(*iii* \Rightarrow *iv*) Obvious.

(*iv* \Rightarrow *i*) Let $\mu_{\text{nano } \widehat{Q}}(x) = \min\{\mu_{\text{nano}}(\widetilde{M})(x), \mu_{\text{nano } \mathcal{H}}(x)\}$, such that \widetilde{M} is $\text{fn}\theta$ -o, and \mathcal{H} is $\text{fn}\theta$ -dense, we have

$$\mu_{\text{nano } \theta \text{ Cl}(\widehat{Q})}(x) = \mu_{\text{nano } \theta \text{ Cl}(\mathcal{H})}(x),$$

thus

$$\mu_{\text{nano } \theta}(\widehat{Q})(x) \leq \mu_{\text{nano}}(\widetilde{M})(x) \leq \mu_{\text{nano } \theta \text{ Cl}(\widetilde{M})}(x) = \mu_{\text{nano } \theta \text{ Cl}(\widehat{Q})}(x),$$

thus

$$\mu_{\text{nano } \widehat{Q}}(x) \leq \mu_{\text{nano Int}(\text{nano } \theta\text{-Cl}(\widehat{Q}))(x)},$$

And hence \widehat{Q} is $\text{fn}\theta$ -po set.

Theorem 2.22

Let $(\mathfrak{X}, \mathcal{T}_{\mathbb{R}}(\mathfrak{X}))$ be a fnts, Then the intersection of $\text{fn}\theta$ -po and $\text{fn}\theta$ -so is $\text{fn}\theta$ -e*o.

Proof

Let \widehat{Q} and \widetilde{K} be a $\text{fn}\theta$ -po and $\text{fn}\theta$ -so respectively, then

$$\min\{\mu_{\widehat{Q}}(x), \mu_{\widetilde{K}}(x)\} \leq \min\{\mu_{\text{nano Cl}(\text{nano } \theta\text{-Int}(\widehat{Q}))(x)}, \mu_{\text{nano Int}(\text{nano } \theta\text{-Cl}(\widetilde{K}))(x)}\}.$$

$$\leq \mu_{\text{nano Cl}(\text{nano } \theta\text{-Int}(\text{nano } \theta\text{-Cl}(\widehat{Q}))), \text{ nano Int}(\text{nano } \theta\text{-Cl}(\widetilde{K}))}$$

$$= \mu_{\text{nano Cl nano } \theta\text{-Int} \min\{\text{nano } \theta\text{-Int}(\text{nano } \theta\text{-Int}(\widehat{Q})), \text{ nano } \theta\text{-Cl}(\widetilde{K})\}}$$

$$\leq \mu_{\text{nano Cl nano } \theta\text{-Int nano } \theta\text{-Cl} \min\{(\text{nano } \theta\text{-Int}(\widehat{Q}))(\widetilde{K})\}} \leq$$

$$\mu_{\text{nano Cl nano Int nano } \theta\text{-Cl} \min\{\widehat{Q}, \widetilde{K}\}}, \text{ thus } \min\{\mu_{\widehat{Q}}(x), \mu_{\widetilde{K}}(x)\} \text{ is } \text{fn}\theta\text{-e}^*\text{o}.$$

Theorem 2.23

Let \widehat{Q} be a fns of fnts $(\mathfrak{X}, \mathcal{T}_{\mathbb{R}}(\mathfrak{X}))$, then the following are equivalent:

- i. \widehat{Q} is $\text{fn}\theta\text{-e}^*\text{o}$.
- ii. \widehat{Q} is the intersection of $\text{fn}\theta\text{-so}$ and $\text{fn}\theta\text{-dense}$.
- iii. \widehat{Q} is the intersection of $\text{fn}\theta\text{-so}$ and $\text{fn}\theta\text{-po}$.

Proof

(i \Rightarrow ii) Let \widehat{Q} is $\text{fn}\theta\text{-e}^*\text{o}$, then $\mu_{\text{nano } \theta\text{-Cl}(\widehat{Q})(x)}$ is fnRc and so $\text{fn}\theta\text{-so}$.

Take $\mu_{\widetilde{M}}(x) = \max\{\mu_{\widehat{Q}}(x), 1 - \mu_{\text{nano } \theta\text{-Cl}(\widehat{Q})(x)}\}$. This implies that $\mu_{\widetilde{M}}(x)$ is $\text{fn}\theta\text{-dense}$ and $\mu_A(x) = \min\{\mu_{\text{nano } \theta\text{-Cl}(\widehat{Q})(x)}, \mu_{\widetilde{M}}(x)\}$.

(ii \Rightarrow iii) Obvious.

(iii \Rightarrow i) It follows from Theorem 2.22.

4. Medical Application by Using Fuzzy Nano Topology

In this section we take medical case “Diabetics” and we apply the fuzzy nano topology and fuzzy nano bases to find the key factors of “Diabetics”.

Example 3.1

In the following table we gave information about patients that have Overweight, Advanced age, Family history, High blood pressure, Lethargy, and defined its membership function between zero and one according to the intensity of infection of patients which is defined as follows.

patients	Advance dage	Family history	Overweight	High blood pressure	Lethargy and lack of movement	Result
u_1^*	$\sqrt{0.8}$	$\sqrt{0.7}$	Nil 0.0	$\sqrt{0.7}$	Nil 0.0	Yes
u_2^*	$\sqrt{0.9}$	Nil 0.0	$\sqrt{0.8}$	Nil 0.0	$\sqrt{0.8}$	Yes
u_3^*	Nil 0.0	$\sqrt{0.4}$	$\sqrt{0.9}$	$\sqrt{0.9}$	Nil 0.0	Yes
u_4^*	$\sqrt{0.8}$	$\sqrt{0.7}$	Nil 0.0	$\sqrt{0.6}$	Nil 0.0	No
u_5^*	Nil 0.0	Nil 0.0	$\sqrt{0.9}$	$\sqrt{0.9}$	Nil 0.0	Yes
u_6^*	Nil 0.0	$\sqrt{0.5}$	$\sqrt{0.8}$	$\sqrt{0.9}$	Nil 0.0	No
u_7^*	Nil 0.0	$\sqrt{0.6}$	Nil 0.0	$\sqrt{0.7}$	Nil 0.0	No

Table 1: Patient Data

The set of patients is $\widetilde{U} = \{u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\}$, And $\widetilde{A} = \{\text{Advanced age, Family history, Overweight, High blood pressure, Lethargy and lack of movement}\}$ and its divided into two fuzzy classes $\widetilde{C} = \{\text{AA, FH, OW, HBP, LLM}\}$ and $\widetilde{C} = \{\text{Diabetics}\}$. The family of fuzzy equivalent classes U/\widetilde{C} with respect to \widetilde{C} is defined by

$$U/\widehat{\mathcal{R}}(\widetilde{C}) = \{\{u_1^*, u_4^*\}, \{u_2^*\}, \{u_3^*, u_6^*\}, \{u_5^*\}, \{u_7^*\}\}.$$

Case 1 patients have Diabetics

Let $\hat{X} = \{u_1^*, u_2^*, u_3^*, u_5^*\}$, the fuzzy set of patients have Diabetics.

$$\mathcal{Q}_{\hat{\mathcal{R}}_{\tilde{C}}}(\hat{X}) = \{u_1^*, u_2^*, u_4^*, u_5^*, u_7^*\}, \quad U_{\hat{\mathcal{R}}_{\tilde{C}}}(\hat{X}) = \{u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*\}, \quad \mathcal{B}_{\hat{\mathcal{R}}}(\hat{X}) = \{u_3^*, u_6^*\}.$$

Therefore the fnt is $\mathcal{T}_{\hat{\mathcal{R}}_{\tilde{C}}}(\hat{X}) = \{\tilde{U}, \emptyset, \{u_1^*, u_2^*, u_4^*, u_5^*, u_7^*\}, \{u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*\}, \{u_3^*, u_6^*\}\}$,

the fuzzy nano base of $\tilde{T}_{\tilde{C}}(\hat{X})$ is $\tilde{\beta}_{\tilde{C}} = \{\tilde{U}, \{u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\}\}$.

Step 1 ((Advanced age is removed))

$$U/\hat{\mathcal{R}}(\tilde{C})/AA = \{\{u_1^*, u_4^*, \hat{\theta}_7\}, \{u_2^*\}, \{u_3^*, u_6^*\}, \{u_5^*\}\},$$

$$\mathcal{Q}_{\hat{\mathcal{R}}/AA}(\hat{X}) = \{u_1^*, u_2^*, u_4^*, u_5^*, \hat{\theta}_7\}$$

$$U_{\hat{\mathcal{R}}/AA}(\hat{X}) = \{u_2^*, u_3^*, u_4^*, u_5^*, u_6^*\}, \quad \mathcal{B}_{\hat{\mathcal{R}}/AA}(\hat{X}) = \{u_3^*, u_6^*\}, \text{ therefore the fnt is}$$

$$\mathcal{T}_{\hat{\mathcal{R}}_{\tilde{C}}/AA}(\hat{X}) = \{\tilde{U}, \emptyset, \{u_1^*, u_2^*, u_4^*, u_5^*, u_7^*\}, \{u_1^*, u_2^*, u_4^*, u_5^*, u_6^*\}, \{u_3^*, u_6^*\}\},$$

$$\tilde{\beta}_{CAA} = \{\tilde{U}, \{u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*\}\} = \tilde{\beta}_{\tilde{C}}.$$

Step 2 ((Family history is removed))

$$U/\hat{\mathcal{R}}(\tilde{C})/FH = \{\{u_1^*, u_4^*\}, \{u_2^*\}, \{u_3^*, u_6^*\}, \{u_5^*\}, \{u_7^*\}\},$$

$$\mathcal{Q}_{\hat{\mathcal{R}}/FH}(\hat{X}) = \{u_1^*, u_2^*, u_4^*, u_7^*\}, \quad U_{\hat{\mathcal{R}}/FH}(\hat{X}) = \{u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*\},$$

$$\mathcal{B}_{\hat{\mathcal{R}}/FH}(\hat{X}) = \{u_3^*, u_5^*, u_6^*\}, \text{ therefore the fnt is}$$

$$\mathcal{T}_{\hat{\mathcal{R}}_{\tilde{C}}/FH}(\hat{X}) = \{\tilde{U}, \emptyset, \{u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*\}, \{u_1^*, u_2^*, u_4^*, u_5^*, u_6^*\}, \{u_1^*, u_2^*, u_4^*, u_5^*\}\},$$

$$\tilde{\beta}_{CFH} = \{\tilde{U}, \{u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*\}\} \neq \tilde{\beta}_{\tilde{C}}.$$

Step 3 ((Overweight is removed))

$$U/\hat{\mathcal{R}}(\tilde{C})/OW = \{\{u_1^*, u_4^*\}, \{u_2^*\}, \{u_3^*, u_6^*, u_7^*\}, \{u_5^*\}\},$$

$$\mathcal{Q}_{\hat{\mathcal{R}}/OW}(\hat{X}) = \{u_1^*, u_2^*, u_4^*, u_5^*\}, \quad U_{\hat{\mathcal{R}}/OW}(\hat{X}) = \{u_1^*, u_2^*, u_4^*, u_5^*, u_7^*\},$$

$$\mathcal{B}_{\hat{\mathcal{R}}/OW}(\hat{X}) = \emptyset, \text{ therefore the fnt is}$$

$$\mathcal{T}_{\hat{\mathcal{R}}_{\tilde{C}}/OW}(\hat{X}) = \{\tilde{U}, \emptyset, \{u_1^*, u_2^*, u_4^*, u_5^*, u_7^*\}, \{u_1^*, u_2^*, u_4^*, u_5^*, u_6^*\}\},$$

$$\tilde{\beta}_{COW} = \{\tilde{U}, \{u_1^*, u_2^*, u_4^*, u_5^*, u_6^*\}\} \neq \tilde{\beta}_{\tilde{C}}.$$

Step 4 ((High blood pressure is removed))

$$U/\hat{\mathcal{R}}(\tilde{C})/HBP = \{\{u_1^*, u_4^*\}, \{u_2^*\}, \{u_3^*, u_6^*\}, \{u_5^*\}, \{u_7^*\}\},$$

$$\mathcal{Q}_{\hat{\mathcal{R}}/HBP}(\hat{X}) = \{u_1^*, u_2^*, u_4^*, u_5^*, u_7^*\}, \quad U_{\hat{\mathcal{R}}/HBP}(\hat{X}) = \{u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*\},$$

$$\mathcal{B}_{\hat{\mathcal{R}}/HBP}(\hat{X}) = \{u_3^*, u_6^*\}, \text{ therefore the fnt is}$$

$$\mathcal{T}_{\hat{\mathcal{R}}_{\tilde{C}}/HBP}(\hat{X}) = \{\tilde{U}, \emptyset, \{u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*\}, \{u_1^*, u_2^*, u_4^*, u_5^*, u_6^*\}, \{u_3^*, u_6^*\}\},$$

$$\tilde{\beta}_{CHBP} = \{\tilde{U}, \{u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*\}\} = \tilde{\beta}_{\tilde{C}}.$$

Step 5 ((Lethargy and lack of movement is removed))

$$U/\hat{\mathcal{R}}(\tilde{C})/LLM = \{\{u_1^*, u_4^*\}, \{u_2^*\}, \{u_3^*, u_6^*\}, \{u_5^*\}, \{u_7^*\}\},$$

$$\mathcal{Q}_{\hat{\mathcal{R}}/LLM}(\hat{X}) = \{u_1^*, u_2^*, u_4^*, u_5^*, u_7^*\}, \quad U_{\hat{\mathcal{R}}/LLM}(\hat{X}) = \{u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*\},$$

$$\mathcal{B}_{\hat{\mathcal{R}}/LLM}(\hat{X}) = \{u_3^*, u_6^*\}, \text{ therefore the fnt is}$$

$$\mathcal{T}_{\hat{\mathcal{R}}_{\tilde{C}}/LLM}(\hat{X}) = \{\tilde{U}, \emptyset, \{u_1^*, u_2^*, u_4^*, u_5^*, u_7^*\}, \{u_1^*, u_2^*, u_4^*, u_5^*, u_6^*\}, \{u_3^*, u_6^*\}\},$$

$$\tilde{\beta}_{CLLM} = \{\tilde{U}, \{u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*\}\} = \tilde{\beta}_{\tilde{C}}.$$

Case 2 patients not have Diabetics

Let $\hat{X} = \{u_4^*, u_6^*, u_7^*\}$ the fuzzy set of patients have Diabetics

$$\mathcal{Q}_{\widehat{\mathcal{R}}_{\tilde{C}}}(\hat{X}) = \{u_7^*\}, \quad U_{\widehat{\mathcal{R}}_{\tilde{C}}}(\hat{X}) = \{u_1^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\}, \quad \mathcal{B}_{\widehat{\mathcal{R}}_{\tilde{C}}}(\hat{X}) = \{u_1^*, u_3^*, u_4^*, u_5^*, u_6^*\}.$$

Therefore the fnt is $\mathcal{T}_{\widehat{\mathcal{R}}_{\tilde{C}}}(\hat{X}) = \{\tilde{U}, \emptyset, \{u_1^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\}, \{u_1^*\}, \{u_1^*, u_3^*, u_4^*, u_5^*, u_6^*\}, \{u_3^*, u_6^*\}\}$, The fuzzy nano base of $\tilde{T}_{\tilde{C}}(\hat{X})$ is $\tilde{\beta}_{\tilde{C}} = \{\tilde{U}, \{u_1^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\}\}$.

Step 1 ((Advanced age is removed))

$$U/\widehat{\mathcal{R}}(\tilde{C})/AA = \{\{u_1^*, u_4^*, u_7^*\}, \{u_2^*\}, \{u_3^*, u_6^*\}, \{u_5^*\}\},$$

$$\mathcal{Q}_{\widehat{\mathcal{R}}/AA}(\hat{X}) = \{\emptyset\}, \quad U_{\widehat{\mathcal{R}}/AA}(\hat{X}) = \{u_1^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\},$$

$$\mathcal{B}_{\widehat{\mathcal{R}}/AA}(\hat{X}) = \{u_1^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\}.$$

$$\text{Therefore the fnt is } \mathcal{T}_{\widehat{\mathcal{R}}_{\tilde{C}}/AA}(\hat{X}) = \{\tilde{U}, \emptyset, \{u_1^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\}\},$$

$$\tilde{\beta}_{CAA} = \{\tilde{U}, \{u_1^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\}\} = \tilde{\beta}_{\tilde{C}}.$$

Step 2 ((Family history is removed))

$$U/\widehat{\mathcal{R}}(\tilde{C})/FH = \{\{u_1^*, u_4^*\}, \{u_2^*\}, \{u_3^*, u_6^*, u_7^*\}, \{u_5^*\}\},$$

$$\mathcal{Q}_{\widehat{\mathcal{R}}/FH}(\hat{X}) = \{\hat{\theta}_7\}, \quad U_{\widehat{\mathcal{R}}/FH}(\hat{X}) = \{u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*\},$$

$$\mathcal{B}_{\widehat{\mathcal{R}}/FH}(\hat{X}) = \{u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*\}.$$

$$\text{Therefore the fnt is } \mathcal{T}_{\widehat{\mathcal{R}}_{\tilde{C}}/FH}(\hat{X}) = \{\tilde{U}, \emptyset, \{u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*\}, \{u_1^*, u_2^*, u_4^*, u_5^*\}\},$$

$$\tilde{\beta}_{CFH} = \{\tilde{U}, \{u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*\}\} \neq \tilde{\beta}_{\tilde{C}}.$$

Step 3 ((Overweight is removed))

$$U/\widehat{\mathcal{R}}(\tilde{C})/OW = \{\{u_1^*, u_4^*\}, \{u_2^*\}, \{u_3^*, u_6^*, u_7^*\}, \{u_5^*\}\},$$

$$\mathcal{Q}_{\widehat{\mathcal{R}}/OW}(\hat{X}) = \{u_2^*\}, \quad U_{\widehat{\mathcal{R}}/OW}(\hat{X}) = \{u_1^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\},$$

$$\mathcal{B}_{\widehat{\mathcal{R}}/OW}(\hat{X}) = \{u_1^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\}.$$

$$\text{Therefore the fnt is } \mathcal{T}_{\widehat{\mathcal{R}}_{\tilde{C}}/OW}(\hat{X}) = \{\tilde{U}, \emptyset, \{u_1^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\}, \{\hat{\theta}_2\}\},$$

$$\tilde{\beta}_{C/OW} = \{\tilde{U}, \{u_1^*, u_2^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\}\} \neq \tilde{\beta}_{\tilde{C}}.$$

Step 4 ((High blood pressure is removed))

$$U/\widehat{\mathcal{R}}(\tilde{C})/HBP = \{\{u_1^*, u_4^*\}, \{u_2^*\}, \{u_3^*, u_6^*, \hat{\theta}_7\}, \{u_5^*\}\},$$

$$\mathcal{Q}_{\widehat{\mathcal{R}}/HBP}(\hat{X}) = \{u_7^*\}, \quad U_{\widehat{\mathcal{R}}/HBP}(\hat{X}) = \{u_1^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\},$$

$$\mathcal{B}_{\widehat{\mathcal{R}}/HBP}(\hat{X}) = \{u_1^*, u_3^*, u_4^*, u_5^*, u_6^*\}.$$

$$\text{Therefore the fnt is } \mathcal{T}_{\widehat{\mathcal{R}}_{\tilde{C}}/HBP}(\hat{X}) = \{\tilde{U}, \emptyset, \{u_1^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\}, \{\hat{\theta}_7\}, \{\hat{\theta}_1, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\}\},$$

$$\tilde{\beta}_{C/HBP} = \{\tilde{U}, \{u_1^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\}\} = \tilde{\beta}_{\tilde{C}}.$$

Step 5 ((Lethargy and lack of movement is removed))

$$U/\widehat{\mathcal{R}}(\tilde{C})/LI = \{\{u_1^*, u_4^*\}, \{u_2^*\}, \{u_3^*, u_5^*, u_6^*\}, \{\hat{\theta}_7\}\},$$

$$\mathcal{Q}_{\widehat{\mathcal{R}}/LI}(\hat{X}) = \{u_7^*\}, \quad U_{\widehat{\mathcal{R}}/LI}(\hat{X}) = \{u_1^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\},$$

$$\mathcal{B}_{\widehat{\mathcal{R}}/LI}(\hat{X}) = \{u_1^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\}.$$

$$\text{Therefore the fnt is } \mathcal{T}_{\widehat{\mathcal{R}}_{\tilde{C}}/LI}(\hat{X}) = \{\tilde{U}, \emptyset, \{u_1^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\}, \{\hat{\theta}_7\}, \{\hat{\theta}_1, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\}\},$$

$$\tilde{\beta}_{C/LI} = \{\tilde{U}, \{u_1^*, u_3^*, u_4^*, u_5^*, u_6^*, u_7^*\}\} = \tilde{\beta}_{\tilde{C}}.$$

5. Conclusion

This study presents a groundbreaking exploration of fuzzy nano open sets, introducing new classifications and examining their intricate relationships within fuzzy nano topological spaces. The theoretical insights gained, including key characterizations and counterexamples, highlight the robustness and versatility of fuzzy nano topology as a mathematical framework. Moreover, the application of this framework in the medical field demonstrates its practical significance. By addressing the elimination of prodromes and identifying critical factors contributing to diabetes, this research bridges the gap between abstract

mathematical theories and real-world medical challenges. These findings pave the way for future studies to leverage fuzzy nano topology in tackling complex problems across diverse scientific and technological domains, making it a promising tool for innovation and discovery. Also, These study indicates that "Family History" and "Overweight" are key factors contributing to the development of Diabetes. However, proper medical care and positive changes in lifestyle and behavioral patterns can help reduce the risk

References

1. C. L. Chang, Fuzzy topological spaces, *J. Math. Anal. Appl.* 24 (1968), 182–190.
2. J. M. Jamil, W. A. Hussein, On Nano ic-open set and some of its applications, In: *AIP Conference Proceedings*, 2834,080083, 2023.
3. K.Kavithamani, R.Kasthuri, C.Gayathri, and A.Kavitha" Introduction of g^*c - Closed Sets in Nano Topological Spaces and Elimination of Prodromes in Rheumatoid Arthritis Disease Through Nano Topology" *Advances in Nonlinear Variational Inequalities*, Vol 28 No. 4s, 2025.
4. Al-Khafaji,M.A.K, Abdulhusein,G.H, Study about fuzzy ω -paracompact space in fuzzy Topological space. *journal of physics conference series*, 2020, 159(1), 012067.
5. Khailik Al-Khafaji, M.A, Abdulhusein, G.H, Some results on fuzzy ω -covering dimension function in fuzzy topological space, *Iraqi journal of science*, 2021, 62(3), pp.961-971
6. Ming, P. P. and Ming, L. Y., "Fuzzy Topology I. Neighborhood Structure of a Fuzzy point and Moor-smith Convergence", *J. Math. Anal. Appl.*, Vol.76, PP. 571-599, 1980.
7. M.Lellis Thivagar, Carmel Richard, on Nano forms of weakly open sets. *International Journal of Mathematics and Statistics Invention*, volume 1, Issue 1, pp-31-37, August 2013.
8. Thivagar, M. Lellis, and Carmel Richard. "On nano continuity." *Mathematical theory and modeling* 3.7, 32-37, 2013.
9. MohammadS.and Mostafa H."Fuzzy equivalence classes and fuzzy partitions" *international journal of Applied mathematics* Vol.21, PP. 607-613, 2008.
10. M. parimala,C.indirani,and S.jafari "on nano b – open set in nano topological space" *Jordan Journal of Mathematics and Statistics (JJMS)* 9(3), pp 173- 184, 2016.
11. Mary, D. A., and I. Arockiarani. "On characterizations of nano rbg-closed sets in nano topological spaces." *Int. J. Mod. Eng. Res* 5.1, 68-76, 2015
12. Mary, D. A., and I. Arockiarani. "On b-open sets and b-continuous functions in nano topological spaces." *Int. J. Innov. Res. Stud* 3.11, 98-116, 2014.
13. Z. Pawlak, Rough sets, *Inter. J. Comput. Inform. Sci.* 11, 341-356, 1982.
14. Z. Pawlok , *Rough Sets: Theoretical Aspects of Reasoning about Data*, Kluwer Academic Publishers, Boston, 1991.
15. Pazar Varol, B.; Malkoç, H. Bipolar Fuzzy Supra Topology via (Q-) Neighborhood and Its Application in Data Mining *Process. Symmetry* 2024, 16, 216
16. Rachna Navalakhe and Purva Rajwade" Nano Fuzzy b-open Sets in Nano fuzzy Topological Spaces" *Annals of Pure and Applied Mathematics*, Vol. 25, No. 1, pp55-61, 2022.
17. R. Thangammal, M. Saraswathi, A. Vadivel, C. John Sundard " Fuzzy nano Z-open sets in fuzzy nano topological spaces" *Journal of Linear and Topological Algebra*, Vol. 11, No. 01, pp27- 38, 2022.
18. X. Arul Selvaraj, U. Balakrishna, Z-open sets in nano topological spaces, *AIP Conf Proc.* 2364 , 020037, 2022
19. Zadeh, L.A., "Fuzzy Sets", *Inform. Control*, Vol.8, PP. 338-353, 1965.
20. P. Rajwade, R. Navalakhe " Nano Fuzzy Z - Open Sets and its Application in Nano Fuzzy Topological Space " *J. Sci. Res.* 16 (1), 201-211 (2024).
21. Dynamical Analysis Within-Host and Between-Host for HIV AIDS with the Application of Optimal Control Strategy *Dynamical analysis within-host and between-host for an HIV AIDS.* (2020). *Iraqi Journal of Science*, 61(5), 1173-1189.
22. Minimal wave speed and traveling wave in nonlocal dispersion SIS epidemic model with delay. *Bound Value Probl* 2025, 67 (2025).
23. Stability analysis and Bifurcation for an Bacterial Meningitis Spreading with Stage Structure : *Mathematical Modeling.* (2024). *Iraqi Journal of Science*, 65(5), 2630-2648.

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