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A Computational Study of Time Fractional Phytoplankton Nutrient Model Arising in the Biological System

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ABSTRACT: Microorganisms from the ocean are known as phytoplankton. We discuss an example of the nitrogen-limited plankton in the oceans. There are two types of plankton: zooplankton, or animal plankton, which feeds on phytoplankton, and phytoplankton, or plant plankton, which photosynthesises and requires essential nutrients to thrive. Here, we used the Shehu Adomian decomposition method to find the approximate solutions of the time fractional phytoplankton nutrient model (TFPNM). Fractional-order derivatives are considered in the Caputo, Atangana-Baleanu-Caputo (ABC), and Caputo-Fabrizio (CF) sense. The biological system emphasises the dynamics of phytoplankton-nutrient interaction in the recycling of nutrients. Existence and uniqueness of TFPNM are provided. Rapidly convergent series solutions are provided by the proposed technique. Various graphical representations are used to present the findings of the study.

Keywords: Phytoplankton nutrient model, Caputo, Caputo-Fabrizio, Atangana-Baleanu-Caputo, Shehu Adomian decomposition method.

Contents

1. Introduction

Microorganisms from the ocean are known as phytoplankton. By utilising external solar energy, they transform mineral nutrients into basic biotic material. Assuming that the system is nitrogen-closed, discuss an example of the nitrogen-limited plankton in the oceans. There are two types of plankton: zooplankton, or animal plankton, which feeds on phytoplankton, and phytoplankton, or plant plankton, which photosynthesises and requires essential nutrients to thrive. The model further assumes that the activity takes place on a surface of the ocean that is well-mixed. Phytoplankton consumes nitrogen that has been absorbed from the ocean as a dissolved gas or as an element of compounds. Through the consumption of phytoplankton, zooplankton absorbs nitrogen. It is transferred to the ocean by plankton mortality and excretion.The mathematical model for these processes is given by Gonzalez-Parra et al. [1],

$$\begin{aligned} D_t\Phi(t) &= a\omega(t) + bO(t) - c\Phi(t)\omega(t), \\ D_t\omega(t) &= c\Phi(t)P(t) - d\omega(t)O(t) - a\omega(t), \\ D_tO(t) &= d\omega(t)O(t) - bO(t). \end{aligned} \tag{1.1}$$

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Where $\Phi(t)$ indicates the concentration of nitrogen at the time t, $\omega(t)$ indicates the concentration of phytoplankton at the time t, and $O(t)$ indicates the concentration of zooplankton at the time t, they are all given in terms of the same unit of measurement, which is the nitrogen mass assimilated into the plankton per unit ocean surface area [2]. Britton investigated the dynamic of a phytoplankton group and nutrients that are one of the key ideas in mathematical biology [2]. Ruan investigated the dynamics of the interactions between phytoplankton and zooplankton, as well as the conditions under which they cohabit [3]. As a result of positive periodic bifurcation, zooplankton and phytoplankton may coexist. They are specifically included in plankton models because of how nutrients affect plankton growth investigated by Pada Das et al. [4]. Consider the time fractional phytoplankton nutrient model [5] of the form:

$$\begin{aligned} D_t^{\beta}\Phi(t) &= a\omega(t) + bO(t) - c\Phi(t)\omega(t), \\ D_t^{\beta}\omega(t) &= c\Phi(t)P(t) - d\omega(t)O(t) - a\omega(t), \\ D_t^{\beta}O(t) &= d\omega(t)O(t) - bO(t). \end{aligned} \tag{1.2}$$

with initial conditions $\Phi(0) = h_1, \omega(0) = h_2$ and $O(0) = h_3$.

Jena et al. [5] studied the approximate solutions of TFPNM using homotopy perturbation Elzaki transform method (HPETM) by considering the fractional deriative Atangana-Baleanu-Caputo (ABC) sense. The fractional calculus (FC) [6,7,8,9,10,11,12,13] provides an extensive use of inherited features, memory and dissipative effects to solve problems in the real world. Through the use of power-law distribution, several mathematicians have applied the concepts of fractional differential and integral operators to the fields of science and engineering [14,15]. Nevertheless, this power-law distribution is not statistically significant when the fractional order is smaller than 1. Certain fundamental requirements, such as classical mechanical law, index law, and singular kernels, are satisfied by the class of fractional differential operators based on the power-law kernel. It implies that the power-law-based operators are weak physically and are unable to handle more complicated phenomena.

Another issue is singularity, which prevents objects from being mathematically characterised; as a result, it is difficult to characterise many natural phenomena with singularity. Due to the recent advancement of the fractional calculus, two significant fractional derivatives, namely Caputo-Fabrizio and Atangana-Baleanu, have developed. Although they feature non-singular kernels, their current derivatives do not operate under a power distribution. When modelling complicated problems, choosing a kernel is quite important. If we look at the experimental results from the real-world model, we can see that many physical issues don't always follow the power law based on the function $\chi^{-\beta}$, which is right now the most popular kernel in the literature. Therefore, it will be appropriate to depict the advanced version of fractional derivatives with Mittag-Leffler and exponential kernels, namely Atangana-Baleanu and Caputo-Fabrizio derivates, to formulate a complex model such as the phytoplankton nutrient model. In certain instances, it may be difficult to determine analytic solutions to fractional differential equations. Consequently, acquiring numerical solutions to these equations is essential. Numerous approaches and methodologies for solving FDEs have been reported in the literature, including the Homotopy Perturbation Elzaki Transform Method [5], the natural transform decomposition method [10,11], variationl iteration transform method [14], and Laplace Adomian decomposition method [16], among others.

This study's primary objective is to study a TFPNM with Caputo (C), Caputo-Fabrizio (CF), and ABC derivatives; we use SADM to approximate the solitons in this case. ST and ADM are combined in SADM. It is simpler to estimate the series terms because the proposed approach, in contrast to the traditional Adomian process, excludes the computation of the derivatives of fractions or integrals with fractions into the recurrent process. With SADM, round-off errors are avoided, and it is not necessary to linearize, prescribe assumptions, perturbation, or discretize. SADM is used to solve various FDEs, including the Zakharov-Kuznetsov equation [17], the KdV equations [18], the system of nonlinear equations for unsteady flow of a polytropic gas [19], Korteweg–de Vries Equations [20], Kundu–Eckhaus equation and massive thirring system [21]and so on.

The article has been organised as follows: Section 2 provides a brief overview of the basic definitions of fractional calculus, as well as the Shehu transform (ST) of Caputo, CF, and ABC derivatives. We provided the procedure of SADM to solve TFPNM in Section 3. We discussed the uniqueness and convergence of SADM in Section 4. TFPNM solutions are provided in Section 5 to examine the current technique. The results and discussions are presented in Section 6. Conclusions are mentioned in Section 7.

2. Basic Definitions

This section will highlight the fundamental fractional calculus definitions.

Definition 1 *[22] The fractional differential operator Caputo of order μ is given as*

$$D^{\beta}f(t) = I^{m-\beta}D^{m}f(t) = \frac{1}{\Gamma(m-\beta)}\int_0^t (t-\rho)^{m-\beta-1}f^{m}(\rho)d\rho,\ \ t>0,$$

where $m-1<\beta\leq m,\ \ m\in\mathbb{N}$.

Definition 2 *[23] Let $f\in\mathbb{H}^1(c,d),\ \ d>c$, then the CF of order $\beta\in(0,1)$ is given as*

$${}_{c}^{CF}D_t^{\beta}(f(t)) = \frac{B(\beta)}{1-\beta}\int_c^t exp\Big(-\beta\frac{(t-\rho)}{1-\beta}\Big)f'(\rho)d\rho.$$

Definition 3 *[24] Let $f\in\mathbb{H}^1(c,d),\ \ d>c$, then the ABC of order $\beta\in(0,1)$ is given as*

$${}_{c}^{ABC}D_t^{\beta}(f(t)) = \frac{B(\beta)}{1-\beta}\int_c^t E_{\beta}\Big(-\beta\frac{(t-\rho)^{\beta}}{1-\beta}\Big)f'(\rho)d\rho.$$

Where E_{β} indicates Mittag-Leffler function [25] and $B(\beta)$ represents a normalization function.

Definition 4 *[26] The ST of $\omega(t)$ for all $t\geq 0$ is regarded as*

$$\mathbb{S}[\omega(t)] = \int_0^{\infty} e^{-\frac{st}{\vartheta}}\omega(t)\ \ dt \qquad \vartheta>0,\ \ s>0.$$

Where, $\mathbb{S}$ represents ST operator.

Definition 5 *[27] The ST of C is expressed as*

$$\mathbb{S}[D_t^{\beta}\omega(t)] = \frac{s^{\beta}}{\vartheta^{\beta}}\Big(\mathbb{S}[\omega(t)] - \frac{\vartheta}{s}\omega(t)\Big). \tag{2.1}$$

Definition 6 *[15] The ST of CF is expressed as*

$$\mathbb{S}[D_t^{\beta}\omega(t)] = \frac{1}{1-\beta+\beta\frac{\vartheta}{s}}\Big(\mathbb{S}[\omega(t)] - \frac{\vartheta}{s}\omega(t)\Big). \tag{2.2}$$

Definition 7 *[28] The ST of ABC is expressed as*

$$\mathbb{S}[D_t^{\beta}\omega(t)] = \frac{B(\beta)}{1-\beta+\beta\frac{\vartheta^{\beta}}{s^{\beta}}}\Big(\mathbb{S}[\omega(t)] - \frac{\vartheta}{s}\omega(t)\Big). \tag{2.3}$$

3. Existence and Uniqueness of Solution

Theorem 1 *[16] Let*

$$\begin{aligned}
z_1' &= f_1(z_1,z_2,z_3,\dots z_m,t), z_1(t_0)=z_{10}\\
z_2' &= f_2(z_1,z_2,z_3,\dots z_m,t), z_2(t_0)=z_{20}\\
z_3' &= f_3(z_1,z_2,z_3,\dots z_m,t), z_3(t_0)=z_{30}\\
&\ \vdots\\
z_m' &= f_m(z_1,z_2,z_3,\dots z_m,t), z_m(t_0)=z_{m0}.
\end{aligned} \tag{3.1}$$

If the partial derivatives $\frac{\partial f}{\partial z_j}$, are continuous in $\mathbb{R}$ where $j=1,2,\dots m$ and $\mathbb{R}=\{(z,t):|t-t_0|\leq a, |z-z_0|\leq b\}$ then their constant $\mathit{\Lambda}\geq 0$ such that there exist a unique continuous vector solution $z=[z_1(t),z_2(t),\dots,z_m(t)]$ in the interval $|t-t_0|\leq\mathit{\Lambda}$.

4. The Procedure of SADM

The SADM technique is provided in this section and is then employed to the following nonlinear fractional differential equation (NFDE):

$$\begin{aligned} D_t^{\beta}\Phi(t) &= R_1(\Phi,\omega,O)+N_1(\Phi,\omega,O)+\Psi_1(t),\\ D_t^{\beta}\omega(t) &= R_2(\Phi,\omega,O)+N_2(\Phi,\omega,O)+\Psi_2(t),\\ D_t^{\beta}O(t) &= R_3(\Phi,\omega,O)+N_3(\Phi,\omega,O)+\Psi_3(t), \end{aligned} \tag{4.1}$$

with the initial conditions (I.Cs)

$$\Phi(0)=h_1,\ \omega(0)=h_2,\ O(0)=h_3.$$

Here, R_1, R_2 and R_3 are the linear differential operators and N_1, N_2 and N_3 are the nonlinear differential operators. Whereas Ψ_1,Ψ_2 and Ψ_3 are the non-homogeneous terms Φ,ω and O are unknown functions. Now, we employ ST on Eq. (4.1) by taking C, CF, and ABC operators.

SADM$_C$: Taking ST to Eq. (4.1) and using (2.1), we get

$$\begin{aligned} \mathbb{S}(\Phi(t)) =&h_1\frac{\vartheta}{s}+\frac{\vartheta^{\beta}}{s^{\beta}}\mathbb{S}[R_1(\Phi,\omega,O)+N_1(\Phi,\omega,O)+\Psi_1(t)],\\ \mathbb{S}(\omega(t)) =&h_2\frac{\vartheta}{s}+\frac{\vartheta^{\beta}}{s^{\beta}}\mathbb{S}[R_2(\Phi,\omega,O)+N_2(\Phi,\omega,O)+\Psi_2(t)],\\ \mathbb{S}(O(t)) =&h_3\frac{\vartheta}{s}+\frac{\vartheta^{\beta}}{s^{\beta}}\mathbb{S}[R_3(\Phi,\omega,O)+N_3(\Phi,\omega,O)+\Psi_3(t)]. \end{aligned} \tag{4.2}$$

By taking inverse ST on Eq. (4.2), we obtain

$$\begin{aligned} \Phi(t) =&H_1(t)+\mathbb{S}^{-1}\Big[\frac{\vartheta^{\beta}}{s^{\beta}}\mathbb{S}[R_1(\Phi,\omega,O)+N_1(\Phi,\omega,O)]\Big],\\ \omega(t) =&H_2(t)+\mathbb{S}^{-1}\Big[\frac{\vartheta^{\beta}}{s^{\beta}}\mathbb{S}[R_2(\Phi,\omega,O)+N_2(\Phi,\omega,O)]\Big],\\ O(t) =&H_3(t)+\mathbb{S}^{-1}\Big[\frac{\vartheta^{\beta}}{s^{\beta}}\mathbb{S}[R_3(\Phi,\omega,O)+N_3(\Phi,\omega,O)]\Big]. \end{aligned} \tag{4.3}$$

Here H_1,H_2,H_3 indicate the terms acquired from the known functions Ψ_1,Ψ_2,Ψ_3 and I.Cs. through the use of inverse ST. The nonlinear terms N_1,N_2,N_3 can be written as

$$N_1(\Phi,\omega,O)=\sum_{\eta=0}^{\infty}\mathbb{B}_{\eta},\ N_2(\Phi,\omega,O)=\sum_{\eta=0}^{\infty}\mathbb{C}_{\eta},\ N_3(\Phi,\omega,O)=\sum_{\eta=0}^{\infty}\mathbb{E}_{\eta}, \tag{4.4}$$

where $\mathbb{B}_{\eta},\mathbb{C}_{\eta},\mathbb{E}_{\eta}$ indicate the Adomian polynomials [29].

$$\Phi(t)=\sum_{\eta=0}^{\infty}\Phi_{\eta}(t),\ \omega(t)=\sum_{\eta=0}^{\infty}\omega_{\eta}(t),\ O(t)=\sum_{\eta=0}^{\infty}O_{\eta}(t). \tag{4.5}$$

Now, put Eq. (4.4) and (4.5) into Eq.(4.3) to get

$$\begin{aligned} \sum_{\eta=0}^{\infty}\Phi_{\eta}(t) =&H_1(t)+\mathbb{S}^{-1}\Big[\frac{\vartheta^{\beta}}{s^{\beta}}\mathbb{S}\Big[R_1\Big(\sum_{\eta=0}^{\infty}(\Phi_{\eta},\omega_{\eta},O_{\eta})\Big)+\sum_{\eta=0}^{\infty}\mathbb{B}_{\eta}\Big]\Big]\\ \sum_{\eta=0}^{\infty}\omega_{\eta}(t) =&H_2(t)+\mathbb{S}^{-1}\Big[\frac{\vartheta^{\beta}}{s^{\beta}}\mathbb{S}\Big[R_2\Big(\sum_{\eta=0}^{\infty}(\Phi_{\eta},\omega_{\eta},O_{\eta})\Big)+\sum_{\eta=0}^{\infty}\mathbb{C}_{\eta}\Big]\Big]\\ \sum_{\eta=0}^{\infty}O_{\eta}(t) =&H_3(t)+\mathbb{S}^{-1}\Big[\frac{\vartheta^{\beta}}{s^{\beta}}\mathbb{S}\Big[R_3\Big(\sum_{\eta=0}^{\infty}(\Phi_{\eta},\omega_{\eta},O_{\eta})\Big)+\sum_{\eta=0}^{\infty}\mathbb{E}_{\eta}\Big]\Big]. \end{aligned} \tag{4.6}$$

From Eq. (4.6), we obtain

$$\begin{aligned}
\Phi_0^C(t) =& H_1(t),\ \omega_0^C(t) = H_2(t),\ O_0^C(t) = H_3(t),\\
\Phi_1^C(t) =& \mathbb{S}^{-1}\Big[\frac{\vartheta^\beta}{s^\beta}\mathbb{S}\Big[R_1(\Phi_0,\omega_0,O_0)+\mathbb{B}_0\Big]\Big],\\
\omega_1^C(t) =& \mathbb{S}^{-1}\Big[\frac{\vartheta^\beta}{s^\beta}\mathbb{S}\Big[R_2(\Phi_0,\omega_0,O_0)+\mathbb{C}_0\Big]\Big],\\
O_1^C(t) =& \mathbb{S}^{-1}\Big[\frac{\vartheta^\beta}{s^\beta}\mathbb{S}\Big[R_3(\Phi_0,\omega_0,O_0)+\mathbb{E}_0\Big]\Big],\\
&\vdots\\
\Phi_{\eta+1}^C(t) =& \mathbb{S}^{-1}\Big[\frac{\vartheta^\beta}{s^\beta}\mathbb{S}\Big[R_1(\Phi_\eta,\omega_\eta,O_\eta)+\mathbb{B}_\eta\Big]\Big], \eta \geq 0,\\
\omega_{\eta+1}^C(t) =& \mathbb{S}^{-1}\Big[\frac{\vartheta^\beta}{s^\beta}\mathbb{S}\Big[R_2(\Phi_\eta,\omega_\eta,O_\eta)+\mathbb{C}_\eta\Big]\Big], \eta \geq 0,\\
O_{\eta+1}^C(t) =& \mathbb{S}^{-1}\Big[\frac{\vartheta^\beta}{s^\beta}\mathbb{S}\Big[R_3(\Phi_\eta,\omega_\eta,O_\eta)+\mathbb{E}_\eta\Big]\Big], \eta \geq 0.
\end{aligned} \tag{4.7}$$

By putting Eq.(4.7) into Eq. (4.5), we get the solution of NFDE *i.e*

$$\begin{aligned}
\Phi^C(t) =& \Phi_0^C + \Phi_1^C + \Phi_2^C + \dots\ ,\\
\omega^C(t) =& \omega_0^C + \omega_1^C + \omega_2^C + \dots\ ,\\
O^C(t) =& O_0^C + O_1^C + O_2^C + \dots\ .
\end{aligned} \tag{4.8}$$

SADM$_{CF}$: Consider $\zeta(\beta,t,s) = (1-\beta+\beta\frac{\vartheta}{s})$.
Taking ST on Eq. (4.1) and using Eq. (2.2), we get

$$\begin{aligned}
\mathbb{S}(\Phi(t)) =& \frac{\vartheta}{s}h_1 + \zeta(\beta,t,s)\mathbb{S}[R_1(\Phi,\omega,O)+N_1(\Phi,\omega,O)+\Psi_1(t)],\\
\mathbb{S}(\omega(t)) =& \frac{\vartheta}{s}h_2 + \zeta(\beta,t,s)\mathbb{S}[R_2(\Phi,\omega,O)+N_2(\Phi,\omega,O)+\Psi_2(t)],\\
\mathbb{S}(O(t)) =& \frac{\vartheta}{s}h_3 + \zeta(\beta,t,s)\mathbb{S}[R_3(\Phi,\omega,O)+N_3(\Phi,\omega,O)+\Psi_3(t)].
\end{aligned} \tag{4.9}$$

By taking inverse ST on Eq. (4.9), we get

$$\begin{aligned}
\Phi(t) =& H_1(t) + \mathbb{S}^{-1}\Big[\zeta(\beta,t,s)\mathbb{S}[R_1(\Phi,\omega,O)+N_1(\Phi,\omega,O)]\Big],\\
\omega(t) =& H_2(t) + \mathbb{S}^{-1}\Big[\zeta(\beta,t,s)\mathbb{S}[R_2(\Phi,\omega,O)+N_2(\Phi,\omega,O)]\Big]\\
O(t) =& H_3(t) + \mathbb{S}^{-1}\Big[\zeta(\beta,t,s)\mathbb{S}[R_3(\Phi,\omega,O)+N_3(\Phi,\omega,O)]\Big].
\end{aligned} \tag{4.10}$$

Here H_1, H_2, H_3 indicate the terms acquired from the known functions Ψ_1, Ψ_2, Ψ_3 and I.Cs. through the use of inverse ST.
Now, put Eq.(4.4) and (4.5) into (4.10) to get

$$\begin{aligned}
\sum_{\eta=0}^{\infty}\Phi_\eta(t) =& H_1(t) + \mathbb{S}^{-1}\Big[\zeta(\beta,t,s)\mathbb{S}\Big[R_1\Big(\sum_{\eta=0}^{\infty}(\Phi_\eta,\omega_\eta,O_\eta)\Big)+\sum_{\eta=0}^{\infty}\mathbb{B}_\eta\Big]\Big],\\
\sum_{\eta=0}^{\infty}\omega_\eta(t) =& H_2(t) + \mathbb{S}^{-1}\Big[\zeta(\beta,t,s)\mathbb{S}\Big[R_2\Big(\sum_{\eta=0}^{\infty}(\Phi_\eta,\omega_\eta,O_\eta)\Big)+\sum_{\eta=0}^{\infty}\mathbb{C}_\eta\Big]\Big],\\
\sum_{\eta=0}^{\infty}O_\eta(t) =& H_3(t) + \mathbb{S}^{-1}\Big[\zeta(\beta,t,s)\mathbb{S}\Big[R_3\Big(\sum_{\eta=0}^{\infty}(\Phi_\eta,\omega_\eta,O_\eta)\Big)+\sum_{\eta=0}^{\infty}\mathbb{E}_\eta\Big]\Big],
\end{aligned} \tag{4.11}$$

From (4.11) we get

$$\begin{aligned}
\Phi_0^{CF}(t) =& H_1(t),\ \omega_0^{CF}(t) = H_2(t),\ O_0^{CF}(t) = H_3(t),\\
\Phi_1^{CF}(t) =& \mathbb{S}^{-1}\Big[\zeta(\beta,t,s)\mathbb{S}\Big[R_1(\Phi_0,\omega_0,O_0)+\mathbb{B}_0\Big]\Big],\\
\omega_1^{CF}(t) =& \mathbb{S}^{-1}\Big[\zeta(\beta,t,s)\mathbb{S}\Big[R_2(\Phi_0,\omega_0,O_0)+\mathbb{C}_0\Big]\Big],\\
O_1^{CF}(t) =& \mathbb{S}^{-1}\Big[\zeta(\beta,t,s)\mathbb{S}\Big[R_3(\Phi_0,\omega_0,O_0)+\mathbb{E}_0\Big]\Big],\\
&\vdots\\
\Phi_{\eta+1}^{CF}(t) =& \mathbb{S}^{-1}\Big[\zeta(\beta,t,s)\mathbb{S}\Big[R_1(\Phi_\eta,\omega_\eta,O_\eta)+\mathbb{B}_\eta\Big]\Big], \eta\geq 0,\\
\omega_{\eta+1}^{CF}(t) =& \mathbb{S}^{-1}\Big[\zeta(\beta,t,s)\mathbb{S}\Big[R_2(\Phi_\eta,\omega_\eta,O_\eta)+\mathbb{C}_\eta\Big]\Big], \eta\geq 0,\\
O_{\eta+1}^{CF}(t) =& \mathbb{S}^{-1}\Big[\zeta(\beta,t,s)\mathbb{S}\Big[R_3(\Phi_\eta,\omega_\eta,O_\eta)+\mathbb{E}_\eta\Big]\Big], \eta\geq 0.
\end{aligned} \tag{4.12}$$

By putting Eq. (4.12) into (4.5), we get the solution of of NFDE *i.e*

$$\begin{aligned}
\Phi^{CF}(t) =& \Phi_0^{CF}+\Phi_1^{CF}+\Phi_2^{CF}+\dots\ .\\
\omega^{CF}(t) =& \omega_0^{CF}+\omega_1^{CF}+\omega_2^{CF}+\dots\ .\\
O^{CF}(t) =& O_0^{CF}+O_1^{CF}+O_2^{CF}+\dots\ .
\end{aligned} \tag{4.13}$$

SADM$_{ABC}$: Consider $\sigma(\beta,t,s)=\frac{(1-\beta+\beta\frac{\vartheta^\beta}{s^\beta})}{B(\beta)}$.
Taking ST to Eq. (4.1) and utilising (2.3), we obtain

$$\begin{aligned}
\mathbb{S}(\Phi(t)) =& \frac{\vartheta}{s}h_1+\sigma(\beta,t,s)\mathbb{S}[R_1(\Phi,\omega,O)+N_1(\Phi,\omega,O)+\Psi_1(t)],\\
\mathbb{S}(\omega(t)) =& \frac{\vartheta}{s}h_2+\sigma(\beta,t,s)\mathbb{S}[R_2(\Phi,\omega,O)+N_2(\Phi,\omega,O)+\Psi_2(t)],\\
\mathbb{S}(O(t)) =& \frac{\vartheta}{s}h_3+\sigma(\beta,t,s)\mathbb{S}[R_3(\Phi,\omega,O)+N_3(\Phi,\omega,O)+\Psi_3(t)],
\end{aligned} \tag{4.14}$$

By applying inverse ST on Eq. (4.14), we obtain

$$\begin{aligned}
\Phi(t) =& H_1(t)+\mathbb{S}^{-1}\Big[\sigma(\beta,t,s)\mathbb{S}[R_1(\Phi,\omega,O)+N_1(\Phi,\omega,O]\Big],\\
\omega(t) =& H_2(t)+\mathbb{S}^{-1}\Big[\sigma(\beta,t,s)\mathbb{S}[R_2(\Phi,\omega,O)+N_2(\Phi,\omega,O]\Big],\\
O(t) =& H_3(t)+\mathbb{S}^{-1}\Big[\sigma(\beta,t,s)\mathbb{S}[R_3(\Phi,\omega,O)+N_3(\Phi,\omega,O)]\Big],
\end{aligned} \tag{4.15}$$

Here H_1, H_2, H_3 indicate the term acquired from the known functions Ψ_1, Ψ_2, Ψ_3 and I.Cs. through the use of inverse ST.
Now, put Eq.(4.4) and (4.5) into (4.15) to get

$$\begin{aligned}
\sum_{\eta=0}^{\infty}\Phi_\eta(t) =& H_1(t)+\mathbb{S}^{-1}\Big[\frac{1}{\sigma(\beta,t,s)}\mathbb{S}\Big[R_1\Big(\sum_{\eta=0}^{\infty}(\Phi_\eta,\omega_\eta,O_\eta)\Big)+\sum_{\eta=0}^{\infty}\mathbb{B}_\eta\Big]\Big],\\
\sum_{\eta=0}^{\infty}\omega_\eta(t) =& H_2(t)+\mathbb{S}^{-1}\Big[\frac{1}{\sigma(\beta,t,s)}\mathbb{S}\Big[R_2\Big(\sum_{\eta=0}^{\infty}(\Phi_\eta,\omega_\eta,O_\eta)\Big)+\sum_{\eta=0}^{\infty}\mathbb{C}_\eta\Big]\Big],\\
\sum_{\eta=0}^{\infty}O_\eta(t) =& H_3(t)+\mathbb{S}^{-1}\Big[\frac{1}{\sigma(\beta,t,s)}\mathbb{S}\Big[R_3\Big(\sum_{\eta=0}^{\infty}(\Phi_\eta,\omega_\eta,O_\eta)+\sum_{\eta=0}^{\infty}\mathbb{E}_\eta\Big]\Big],
\end{aligned} \tag{4.16}$$

From Eq. (4.16) we obtain

$$\begin{aligned}
\Phi_0^{ABC}(t) =& H_1(t),\ \omega_0^{ABC}(t) = H_2(t),\ O_0^{ABC}(t) = H_3(t),\\
\Phi_1^{ABC}(t) =& \mathbb{S}^{-1}\Big[\frac{1}{\sigma(\beta,t,s)}\mathbb{S}\Big[R_1(\Phi_0,\omega_0,O_0)+\mathbb{B}_0\Big]\Big],\\
\omega_1^{ABC}(t) =& \mathbb{S}^{-1}\Big[\frac{1}{\sigma(\beta,t,s)}\mathbb{S}\Big[R_2(\Phi_0,\omega_0,O_0)+\mathbb{C}_0\Big]\Big],\\
O_1^{ABC}(t) =& \mathbb{S}^{-1}\Big[\frac{1}{\sigma(\beta,t,s)}\mathbb{S}\Big[R_3(\Phi_0,\omega_0,O_0)+\mathbb{E}_0\Big]\Big],\\
&\vdots\\
\Phi_{\eta+1}^{ABC}(t) =& \mathbb{S}^{-1}\Big[\frac{1}{\sigma(\beta,t,s)}\mathbb{S}\Big[R_1(\Phi_\eta,\omega_\eta,O_\eta)+\mathbb{B}_\eta\Big]\Big], \eta\geq 0.\\
\omega_{\eta+1}^{ABC}(t) =& \mathbb{S}^{-1}\Big[\frac{1}{\sigma(\beta,t,s)}\mathbb{S}\Big[R_2(\Phi_\eta,\omega_\eta,O_\eta)+\mathbb{C}_\eta\Big]\Big], \eta\geq 0.\\
O_{\eta+1}^{ABC}(t) =& \mathbb{S}^{-1}\Big[\frac{1}{\sigma(\beta,t,s)}\mathbb{S}\Big[R_3(\Phi_\eta,\omega_\eta,O_\eta)+\mathbb{E}_\eta\Big]\Big], \eta\geq 0.
\end{aligned} \tag{4.17}$$

By putting Eq. (4.17) into (4.5), we get the solution of of NFDE *i.e*

$$\begin{aligned}
\Phi^{ABC}(t) =& \Phi_0^{ABC}+\Phi_1^{ABC}+\Phi_2^{ABC}+\dots\ .\\
\omega^{ABC}(t) =& \omega_0^{ABC}+\omega_1^{ABC}+\omega_2^{ABC}+\dots\ .\\
O^{ABC}(t) =& O_0^{ABC}+O_1^{ABC}+O_2^{ABC}+\dots\ .
\end{aligned} \tag{4.18}$$

5. Convergence Analysis

Theorem 2 [30] *The series solution* $\sum_{\eta=0}^{\infty}\varphi_\eta(t)$ *converges* $\forall\ \epsilon>0,\ \exists$ *a number* $n\in\mathbb{N}$ *such that* $|\varphi_{m+p}|<\epsilon,\ \forall\ m\geq n$ *and* $p=1,2,3,\dots.$

Proof: Let us take the sequence of function $\{J_m\}_{m=0}^{\infty}$is defined as follows:

$$\begin{aligned}
J_0 &= \varphi_0,\\
J_1 &= \varphi_0+\varphi_1,\\
J_2 &= \varphi_0+\varphi_1+\varphi_2,\\
&\vdots\\
J_m &= \varphi_0+\varphi_1+\varphi_2+\cdots+\varphi_m.
\end{aligned}$$

It is noted that $\{J_m\}_{m=0}^{\infty}$ converges uniformly on $\Re$. From Theorem, we obtain $|J_{m+p}-J_m|=|\varphi_{m+p}|<\epsilon, \forall\ m\geq n$ and $p=1,2,3,\dots.$

Hence, by Cauchy criteria, $\{J_m\}_{m=0}^{\infty}$ converges uniformly on $\Re$, and thus the series solution $\sum_{\eta=0}^{\infty}\varphi_\eta(t)$ converges. □

6. Time Fractional Phytoplankton Nutrient Model

Here, application of the SADM is tested on TFPNM with the fractional differential operators C, ABC, and CF.Consider

$$\begin{aligned}
D_t^{\beta}N(t) &= aP(t)+bZ(t)-cN(t)P(t),\\
D_t^{\beta}P(t) &= cN(t)P(t)-dP(t)Z(t)-aP(t),\\
D_t^{\beta}Z(t) &= dP(t)Z(t)-bZ(t).
\end{aligned} \tag{6.1}$$

with the I.Cs $N(0) = \epsilon_1,\ P(0) = \epsilon_2$ and $Z(0) = \epsilon_3$.
SADM$_C$: By employing the $SADM_C$, we obtain the following successive solutions,

$$
\begin{aligned}
&N_0^C(t) = \epsilon_1,\ P_0^C(t) = \epsilon_2,\ Z_0^C(t) = \epsilon_3,\\
&N_1^C(t) = (a\epsilon_2 + b\epsilon_3 - c\epsilon_1\epsilon_2)\frac{t^\beta}{\Gamma(1+\beta)},\\
&P_1^C(t) = (c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2)\frac{t^\beta}{\Gamma(1+\beta)},\\
&Z_1^C(t) = (d\epsilon_2\epsilon_3 - b\epsilon_3)\frac{t^\beta}{\Gamma(1+\beta)},\\
&N_2^C(t) = \Big(a[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2] + b[d\epsilon_2\epsilon_3 - b\epsilon_3] - c(\epsilon_1[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2]\\
&\qquad + \epsilon_2[a\epsilon_2 + b\epsilon_3 - c\epsilon_1\epsilon_2])\Big)\frac{t^{2\beta}}{\Gamma(1+2\beta)},\\
&P_2^C(t) = \Big(c\big(\epsilon_1\big[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2\big] + \epsilon_2[a\epsilon_2 + b\epsilon_3 - c\epsilon_1\epsilon_2]\big) - d\big(\epsilon_2[d\epsilon_2\epsilon_3 - b\epsilon_3]\\
&\qquad + \epsilon_3[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2]\big) - a[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2]\Big)\frac{t^{2\beta}}{\Gamma(1+2\beta)},\\
&Z_2^C(t) = \Big(d\big(\epsilon_2[d\epsilon_2\epsilon_3 - b\epsilon_3] + \epsilon_3[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2]\big) - b[d\epsilon_2\epsilon_3 - b\epsilon_3]\Big)\frac{t^{2\beta}}{\Gamma(1+2\beta)},\\
&\qquad\vdots
\end{aligned}
\tag{6.2}
$$

by substituting (6.2) in (4.8), we get

$$
\begin{aligned}
N^C(t) &= \epsilon_1 + (a\epsilon_2 + b\epsilon_3 - c\epsilon_1\epsilon_2)\frac{t^\beta}{\Gamma(\beta+1)} + \Big(a[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2] + b[d\epsilon_2\epsilon_3 - b\epsilon_3]\\
&\quad - c(\epsilon_1[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2] + \epsilon_2[a\epsilon_2 + b\epsilon_3 - c\epsilon_1\epsilon_2])\Big)\frac{t^{2\beta}}{\Gamma(2\beta+1)} + \dots,\\
P^C(t) &= \epsilon_2(c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2)\frac{t^\beta}{\Gamma(\beta+1)} + \Big(c\big(\epsilon_1\big[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2\big] + \epsilon_2[a\epsilon_2 + b\epsilon_3 - c\epsilon_1\epsilon_2]\big)\\
&\quad - d\big(\epsilon_2[d\epsilon_2\epsilon_3 - b\epsilon_3] + \epsilon_3[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2]\big) - a[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2]\Big)\frac{t^{2\beta}}{\Gamma(2\beta+1)} + \dots,\\
Z^C(t) &= \epsilon_3 + (d\epsilon_2\epsilon_3 - b\epsilon_3)\frac{t^\beta}{\Gamma(\beta+1)} + \Big(d\big(\epsilon_2[d\epsilon_2\epsilon_3 - b\epsilon_3] + \epsilon_3[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2]\big) - b[d\epsilon_2\epsilon_3 - b\epsilon_3]\Big)\\
&\quad \frac{t^{2\beta}}{\Gamma(2\beta+1)} + \dots.
\end{aligned}
$$

SADM$_{CF}$: By employing the $SADM_{CF}$, we obtain the following successive solutions,

$$
\begin{aligned}
&N_0^{CF}(t) = \epsilon_1,\ P_0^{CF}(t) = \epsilon_2,\ Z_0^{CF}(t) = \epsilon_3,\\
&N_1^{CF}(t) = (a\epsilon_2 + b\epsilon_3 - c\epsilon_1\epsilon_2)(1 - \beta + \beta t),\\
&P_1^{CF}(t) = (c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2)(1 - \beta + \beta t),\\
&Z_1^{CF}(t) = (d\epsilon_2\epsilon_3 - b\epsilon_3)(1 - \beta + \beta t),
\end{aligned}
$$

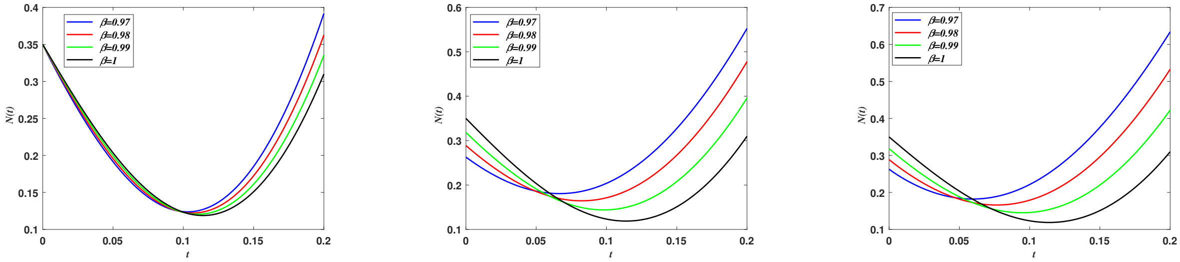
$$
\begin{aligned}
N_2^{CF}(t) &= \Big(a[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2] + b[d\epsilon_2\epsilon_3 - b\epsilon_3] - c(\epsilon_1[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2] + \epsilon_2[a\epsilon_2 + b\epsilon_3 - c\epsilon_1\epsilon_2])\Big) \\
&\quad \Big[\beta^2 \frac{t^2}{\Gamma(3)} - (2\beta^2 - 2\beta)t + (\beta - 1)^2\Big], \\
P_2^{CF}(t) &= \Big(c\big(\epsilon_1\big[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2\big] + \epsilon_2[a\epsilon_2 + b\epsilon_3 - c\epsilon_1\epsilon_2]\big) - d\big(\epsilon_2[d\epsilon_2\epsilon_3 - b\epsilon_3] + \epsilon_3[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2]\big) \\
&\quad - a[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2]\Big[\beta^2 \frac{t^2}{\Gamma(3)} - (2\beta^2 - 2\beta)t + (\beta - 1)^2\Big], \\
Z_2^{CF}(t) &= \Big(d\big(\epsilon_2[d\epsilon_2\epsilon_3 - b\epsilon_3] + \epsilon_3[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2]\big) - b[d\epsilon_2\epsilon_3 - b\epsilon_3]\Big)\Big[\beta^2 \frac{t^2}{\Gamma(3)} - (2\beta^2 - 2\beta)t + (\beta - 1)^2\Big], \\
&\vdots
\end{aligned}
\tag{6.3}
$$

by substituting (6.3) in (4.13), we get

$$
\begin{aligned}
N^{CF}(t) &= \epsilon_1 + (a\epsilon_2 + b\epsilon_3 - c\epsilon_1\epsilon_2)(1 - \beta + \beta t) + \Big(a[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2] + b[d\epsilon_2\epsilon_3 - b\epsilon_3] \\
&\quad - c(\epsilon_1[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2] + \epsilon_2[a\epsilon_2 + b\epsilon_3 - c\epsilon_1\epsilon_2])\Big) \\
&\quad \Big[\beta^2 \frac{t^2}{\Gamma(3)} - (2\beta^2 - 2\beta)t + (\beta - 1)^2\Big] + \dots. \\
P^{CF}(t) &= \epsilon_2 + (c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2)(1 - \beta + \beta t) + \Big(c\big(\epsilon_1\big[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2\big] + \epsilon_2[a\epsilon_2 + b\epsilon_3 - c\epsilon_1\epsilon_2]\big) \\
&\quad - d\big(\epsilon_2[d\epsilon_2\epsilon_3 - b\epsilon_3] + \epsilon_3[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2]\big) - a[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2] \\
&\quad \Big[\beta^2 \frac{t^2}{\Gamma(3)} - (2\beta^2 - 2\beta)t + (\beta - 1)^2\Big] + \dots. \\
Z^{CF}(t) &= \epsilon_3 + (d\epsilon_2\epsilon_3 - b\epsilon_3)(1 - \beta + \beta t) + \Big(d\big(\epsilon_2[d\epsilon_2\epsilon_3 - b\epsilon_3] + \epsilon_3[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2]\big) - b[d\epsilon_2\epsilon_3 - b\epsilon_3]\Big) \\
&\quad \Big[\beta^2 \frac{t^2}{\Gamma(3)} - (2\beta^2 - 2\beta)t + (\beta - 1)^2\Big] + \dots.
\end{aligned}
$$

SADM$_{ABC}$: By employing the $SADM_{ABC}$, we obtain the following successive solutions,

$$
\begin{aligned}
&N_0^{ABC}(t) = \epsilon_1,\ P_0^{ABC}(t) = \epsilon_2,\ Z_0^{ABC}(t) = \epsilon_3, \\
&N_1^{ABC}(t) = (a\epsilon_2 + b\epsilon_3 - c\epsilon_1\epsilon_2)\Big[\beta \frac{t^\beta}{\Gamma(1+\beta)} - \beta + 1\Big], \\
&P_1^{ABC}(t) = (c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2)\Big[\beta \frac{t^\beta}{\Gamma(1+\beta)} - \beta + 1\Big], \\
&Z_1^{ABC}(t) = (d\epsilon_2\epsilon_3 - b\epsilon_3)\Big[\beta \frac{t^\beta}{\Gamma(1+\beta)} - \beta + 1\Big],
\end{aligned}
$$



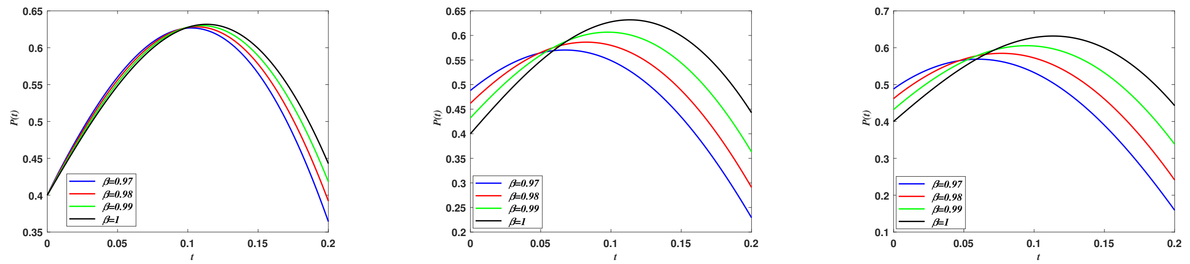
(a) $SADM_C$ (b) $SADM_{CF}$ (c) SADM$_{ABC}$

Figure 1: Approximate solution of $N(t)$.

$$
\begin{aligned}
N_2^{ABC}(t) =&\Big(a[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2] + b[d\epsilon_2\epsilon_3 - b\epsilon_3] - c(\epsilon_1[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2] \\
&+ \epsilon_2[a\epsilon_2 + b\epsilon_3 - c\epsilon_1\epsilon_2])\Big)\Big[\beta^2\frac{t^{2\beta}}{\Gamma(1+2\beta)} - (2\beta^2 - 2\beta)\frac{t^{\beta}}{\Gamma(1+\beta)} + (\beta-1)^2\Big], \\
P_2^{ABC}(t) =& \Big(c\big(\epsilon_1\big[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2] + \epsilon_2[a\epsilon_2 + b\epsilon_3 - c\epsilon_1\epsilon_2]\big) - d\big(\epsilon_2[d\epsilon_2\epsilon_3 - b\epsilon_3] \\
&+ \epsilon_3[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2]\big) - a[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2] \\
&\Big[\beta^2\frac{t^{2\beta}}{\Gamma(1+2\beta)} - (2\beta^2 - 2\beta)\frac{t^{\beta}}{\Gamma(1+\beta)} + (\beta-1)^2\Big], \\
Z_2^{ABC}(t) =& \Big(d\big(\epsilon_2[d\epsilon_2\epsilon_3 - b\epsilon_3] + \epsilon_3[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2]\big) - b[d\epsilon_2\epsilon_3 - b\epsilon_3]\Big) \\
&\Big[\beta^2\frac{t^{2\beta}}{\Gamma(1+2\beta)} - (2\beta^2 - 2\beta)\frac{t^{\beta}}{\Gamma(1+\beta)} + (\beta-1)^2\Big], \\
&\vdots
\end{aligned} \tag{6.4}
$$

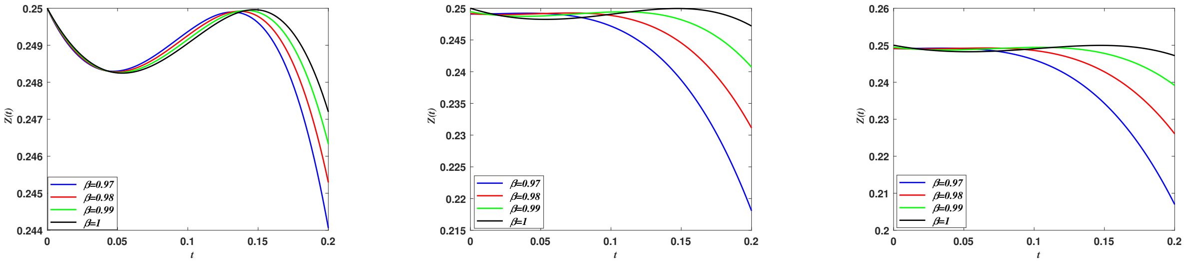
by substituting (6.4) in (4.18), we get

$$
\begin{aligned}
N^{ABC}(t) =&\epsilon_1 + (a\epsilon_2 + b\epsilon_3 - c\epsilon_1\epsilon_2)\Big[\beta\frac{t^{\beta}}{\Gamma(1+\beta)} - \beta + 1\Big] + \Big(a[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2] \\
&+ b[d\epsilon_2\epsilon_3 - b\epsilon_3] - c(\epsilon_1[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2] + \epsilon_2[a\epsilon_2 + b\epsilon_3 - c\epsilon_1\epsilon_2])\Big) \\
&\Big[\beta^2\frac{t^{2\beta}}{\Gamma(1+2\beta)} - (2\beta^2 - 2\beta)\frac{t^{\beta}}{\Gamma(1+\beta)} + (\beta-1)^2\Big] + \ldots. \\
P^{ABC}(t) =&\epsilon_2 + (c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2)\Big[\beta\frac{t^{\beta}}{\Gamma(1+\beta)} - \beta + 1\Big] + \Big(c\big(\epsilon_1\big[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2] \\
&+ \epsilon_2[a\epsilon_2 + b\epsilon_3 - c\epsilon_1\epsilon_2]\big) - d\big(\epsilon_2[d\epsilon_2\epsilon_3 - b\epsilon_3] + \epsilon_3[c\epsilon_1\epsilon_2 - d\epsilon_2\epsilon_3 - a\epsilon_2]\big) - a[c\epsilon_1\epsilon_2 \\
&- d\epsilon_2\epsilon_3 - a\epsilon_2]\Big)\Big[\beta^2\frac{t^{2\beta}}{\Gamma(1+2\beta)} - (2\beta^2 - 2\beta)\frac{t^{\beta}}{\Gamma(1+\beta)} + (\beta-1)^2\Big] + \ldots. \\
Z^{ABC}(t) =& \epsilon_3 + (d\epsilon_2\epsilon_3 - b\epsilon_3)\Big[\beta\frac{t^{\beta}}{\Gamma(1+\beta)} - \beta + 1\Big] + \Big(d\big(\epsilon_2[d\epsilon_2\epsilon_3 - b\epsilon_3] + \epsilon_3[c\epsilon_1\epsilon_2 \\
&- d\epsilon_2\epsilon_3 - a\epsilon_2]\big) - b[d\epsilon_2\epsilon_3 - b\epsilon_3]\Big)\Big[\beta^2\frac{t^{2\beta}}{\Gamma(1+2\beta)} - (2\beta^2 - 2\beta)\frac{t^{\beta}}{\Gamma(1+\beta)} \\
&+ (\beta-1)^2\Big] + \ldots.
\end{aligned}
$$



(a) $SADM_C$ (b) $SADM_{CF}$ (c) SADM_{ABC}

Figure 2: Approximate solution of $P(t)$.



(a) $SADM_C$ (b) $SADM_{CF}$ (c) SADM_{ABC}

Figure 3: Approximate solution of $Z(t)$.

Table 1: Approximate solution of TFPNM model with different values of β.

	t	$\beta = 0.98$			$\beta = 0.99$			$\beta = 1$		
		C	CF	ABC	C	CF	ABC	C	CF	ABC
N	0	0.35000	0.28884	0.28884	0.35000	0.31831	0.31831	0.35000	0.35000	0.35000
N	0.05	0.195505	0.185494	0.181811	0.199485	0.191208	0.188250	0.203459	0.203459	0.203459
N	0.1	0.123224	0.171276	0.179029	0.123009	0.144040	0.145770	0.123301	0.123301	0.123301
N	0.15	0.172421	0.267191	0.296382	0.161053	0.207707	0.220059	0.150892	0.150892	0.150892
N	0.2	0.362669	0.477933	0.532784	0.335413	0.396117	0.422682	0.309914	0.309914	0.309914
P	0	0.400000	0.462002	0.462002	0.400000	0.432240	0.432240	0.400000	0.400000	0.400000
P	0.05	0.556215	0.565362	0.569008	0.552251	0.560016	0.562948	0.548288	0.548288	0.548288
P	0.1	0.627513	0.579857	0.572381	0.627834	0.606558	0.604810	0.627649	0.627649	0.627649
P	0.15	0.577800	0.488270	0.460792	0.589055	0.544105	0.532142	0.599152	0.599152	0.599152
P	0.2	0.392040	0.290927	0.241150	0.418257	0.363165	0.338173	0.442886	0.442886	0.442886
Z	0	0.250000	0.249159	0.249159	0.250000	0.249446	0.249446	0.250000	0.250000	0.250000
Z	0.05	0.248280	0.249145	0.249181	0.248264	0.248776	0.248802	0.248252	0.248252	0.248252
Z	0.1	0.249263	0.248868	0.248591	0.249157	0.249402	0.249419	0.249050	0.249050	0.249050
Z	0.15	0.249779	0.244539	0.242826	0.249892	0.248187	0.247799	0.249956	0.249956	0.249956
Z	0.2	0.245291	0.231140	0.226066	0.246330	0.240717	0.239145	0.247200	0.247200	0.247200

7. Results and Discussion

Figures 1, 2 and 3 show the solutions of PNM based on the operators C, CF, and ABC taken in various fractional order β respectively. From Figure 1 we observe that the concentration of nitrogen $N(t)$ decreases while increasing the time from 0 to 0.1, then after it starts increasing gradually over the time from 0.1 to 0.2, and also found that the increment rate becomes higher as β decreases from 1 to 0.25 within the time interval 0.1 to 0.2. From Figure 2 we observe that the concentration of Phytoplankton $P(t)$ increases while increasing the time from 0 to 0.1, then after it starts decreasing gradually over the time from 0.1 to 0.2, and also found that as β falls from 1 to 0.25 throughout the region of 0.1 to 0.2, the decrement rate reduces. From Figure 3(a) we observe that the concentration of Zootoplankton $Z(t)$ increses while increasing the time from 0.05 to 0.1, then after it starts decreasing gradually over the time from 0.15 to 0.2, and also found that as β falls from 1 to 0.25 throughout the region of 0.15 to 0.2, the decrement rate reduces. Similarly Figures 3(b) and (c) we see that the concentration of Zootoplankton $Z(t)$ decreases while increasing the time from 0.1 to 0.2, and also found that as β falls from 1 to 0.25 throughout the region of 0.1 to 0.2, the decrement rate reduces. In terms of plots, a comparison has also been demonstrated. In these plots, we compared the solutions obtained by considering three distinct fractional differential operators C, CF, and ABC. Table 1 contains the PNM model solutions for numerous fractional values $0.97, 0.98, 0.99, 1$ using three fractional order derivatives.

8. Conclusion

In the present study, to find the approximate analytical solutions for the TFPNM, we used SADM by considering the fractional derivative in the Caputo, CF and ABC manner. The obtained solutions are given in a series form, which converges rapidly. Graphical solutions are similar to graphical results obtained by existing methods in the literature, which shows the efficacy and accuracy of the SADM. We can see that the fractional order and the system parameters have a significant impact on the model. Specifically, they influence the overall behaviour and performance of the system. These parameters can also affect the model's stability, potentially leading to variations in the system's responses and behaviours under different conditions. We draw the conclusion that the SADM is highly methodical and more efficient and that it may be used to study nonlinear fractional mathematical models describing biological processes. Additionally, the application of fractional calculus opens up novel perspectives in the field of mathematical modelling, such as improving the accuracy of predictions in complex biological systems and enhancing the understanding of dynamic processes over time. Furthermore, this method can be extended to analyze nonlinear fractional-order mathematical models arising in oceanography and in infectious disease dynamics, such as SIR, Ebola, tuberculosis, hepatitis, and related models.

References

1. G. Gonzalez-Parra, A. J. Arenas, and B. M. Chen-Charpentier, *Combination of nonstandard schemes and Richardson's extrapolation to improve the numerical solution of population models*, Mathematical and Computer Modelling. 52 (7-8), 1030-1036, (2010).
2. N. F. Britton and N. F. Britton,*Essential mathematical biology*, London: Springer. (Vol. 453), (2003).
3. S. Ruan, *Persistence and coexistence in zooplankton-phytoplankton-nutrient models with instantaneous nutrient recycling*, Journal of Mathematical Biology. 31, 633-654, (1993).
4. K. Pada Das, S. Chatterjee and J. Chattopadhyay, *Dynamics of nutrient-phytoplankton interaction in the presence of viral infection and periodic nutrient input*, Mathematical Modelling of Natural Phenomena. 3(3), 149-169, (2008).
5. R. M. Jena, S. Chakraverty and S. K. Jena, *Analysis of the dynamics of phytoplankton nutrient and whooping cough models with nonsingular kernel arising in the biological system*, Chaos, Solitons and Fractals. 141, 110373, (2020).
6. H. Sun, Y. Zhang, D. Baleanu, W. Chen, and Y. Chen,*A new collection of real world applications of fractional calculus in science and engineering*, Communications in Nonlinear Science and Numerical Simulation. 64, 213-231, (2018).
7. C. Ionescu, A. Lopes, D. Copot, J. T. Machado and J. H. Bates, *The role of fractional calculus in modeling biological phenomena*: A review, Communications in Nonlinear Science and Numerical Simulation. 51, 141-159, (2017).
8. V. M. Martinez, A. N. Barbosa, P. F. Mancera, A. Rodrigues, D. S. and R. F. Camargo, *A fractional calculus model for HIV dynamics: real data, parameter estimation and computational strategies*, Chaos, Solitons and Fractals. 152, 111398, (2021).
9. R. M. Jena, S. Chakraverty and K. S. Nisar, *Dynamical behavior of rotavirus epidemic model with non-probabilistic uncertainty under Caputo–Fabrizio derivative*, Mathematical Methods in the Applied Sciences. 46(9), 10672-10697, (2023).
10. A. S. V. Ravi Kanth, K. Aruna, and K. Raghavendar, *Natural transform decomposition method for the numerical treatment of the time fractional Burgers–Huxley equation*, Numer Methods Partial Differ Equ. 39(3), 2690-2718, (2023).
11. K. Pavani and K. Raghavendar, *Approximate Solutions of Time-Fractional Swift–Hohenberg Equation via Natural Transform Decomposition Method*, International Journal of Applied and Computational Mathematics, 9 (3), 29,(2023).
12. K. Manikandan, D. Aravinthan, J. B. Sudharsan and S. R. R. Reddy,*Soliton and rogue wave solutions of the space–time fractional nonlinear Schrödinger equation with PT-symmetric and time-dependent potentials*, Optik. 266, 169594, (2022).

13. R. M. Jena, S. Chakraverty, M. Yavuz and T. Abdeljawad, *A new modeling and existence–uniqueness analysis for Babesiosis disease of fractional order*, Modern Physics Letters B. 35(30), 2150443, (2021).
14. Y. M. Chu, E. H. Bani Hani, E. R. El-Zahar, A. Ebaid, and N. A. Shah,*Combination of Shehu decomposition and variational iteration transform methods for solving fractional third order dispersive partial differential equations*,Numerical Methods for Partial Differential Equations, 40(2), e22755, (2024).
15. M. L. Rupa, K. Aruna and K. Raghavendar,*Insights into the time Fractional Belousov-Zhabotinsky System Arises in Thermodynamics*, International Journal of Theoretical Physics. 63(9), 222, (2024).
16. A. O. Yunus, M. O. Olayiwola, K. A. Adedokun, J. A. Adedeji, and I. A. Alaje, *Mathematical analysis of fractional-order Caputo's derivative of coronavirus disease model via Laplace Adomian decomposition method*, Beni-Suef University Journal of Basic and Applied Sciences. 11(1), 144, (2022).
17. M. Al-Qurashi, S. Rashid, F. Jarad, M. Tahir, and A. M. Alsharif,*New computations for the two-mode version of the fractional Zakharov–Kuznetsov model in plasma fluid by means of the Shehu decomposition method*,AIMS Math. 7(2), 2044–2060,(2022).
18. S. Rashid, A. Khalid, S. Sultana, Z. Hammouch, R. Shah, and A. M. Alsharif,*A novel analytical view of time-fractional Korteweg–De Vries equations via a new integral transform*,Symmetry 13(7), 1254,(2021).
19. H. Khan, S. Islam, and M. Arif, *The fractional view analysis of polytropic gas, unsteady flow system*, Mathematical Problems in Engineering,2021, 5130136,(2021).
20. M. L. Rupa, and K. Aruna, *Soliton Solutions of Time Fractional Seventh Order Korteweg–de Vries Equations Arises in Applied Sciences.* International Journal of Applied and Computational Mathematics. 11(4), 119, (2025).
21. M. L. Rupa and K. Aruna,*Optical solitons of time fractional Kundu–Eckhaus equation and massive Thirring system arises in quantum field theory*, Optical and Quantum Electronics. 56(3), 460, (2024).
22. I. Podlubny, *Fractional differential equations: an introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications.* Elsevier. (1998)
23. M. Caputo, , and M. Fabrizio, *A new definition of fractional derivative without singular kernel*, Progress in Fractional Differentiation and Applications. 1(2), 1-13, (2015).
24. A. Atangana and D. Baleanu, *New fractional derivatives with nonlocal and non-singular kernel: theory and application to heat transfer model*, Thermal Science. 20(2), 763-769, (2016).
25. H. J. Haubold, A. M. Mathai and R. K. Saxena, *Mittag-Leffler functions and their applications*, Journal of applied mathematics. 2011, (2011).
26. S. Maitama and W. Zhao, *New integral transform: Shehu transform a generalization of Sumudu and Laplace transform for solving differential equations*, International Journal of Analysis and Applications. 17(2), 167-190, (2019).
27. L. Akinyemi and O. S. Iyiola, *Exact and approximate solutions of time-fractional models arising from physics via Shehu transform*, Mathematical Methods in the Applied Sciences. 43(12), 7442-7464, (2020).
28. A. Bokhari,*Application of Shehu transform to Atangana-Baleanu derivatives*, Journal of Mathematics and Computer Science. 20, 101-107, (2019).
29. G. Adomian,*A new approach to nonlinear partial differential equations*, Journal of Mathematical Analysis and Applications. 102(2), 420-434, (1984).
30. R. M. Jena, S. Chakraverty, and D. Baleanu, *SIR epidemic model of childhood diseases through fractional operators with Mittag-Leffler and exponential kernels*, Mathematics and Computers in Simulation. 182, 514-534, (2021).

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