



## Perfect Pendant Domination in Graphs

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**ABSTRACT:** Let  $G = (V(G), E(G))$  be a simple connected graph. A set  $S \subseteq V(G)$  is said to be a perfect pendant dominating set of  $G$  if  $S$  is a perfect dominating set and pendant dominating set of  $G$ . The minimum cardinality of a perfect pendant dominating set of  $G$  is called perfect pendant domination number, and is denoted by  $\gamma_{ppe}(G)$ . A perfect pendant dominating set  $S$  with  $|S| = \gamma_{ppe}(G)$  is said to be  $\gamma_{ppe}$ -set. In this paper we give a characterization of perfect pendant dominating set of some graphs and graphs obtained from the join and corona product of two graphs. Moreover, the perfect pendant domination number of the forenamed graphs is determined and also, graphs having no perfect pendant dominating set are examined.

**Keywords:** Dominating set, perfect dominating set, pendant dominating set, perfect pendant dominating set.

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### 1. Introduction

In 1960, the study of domination in graphs began and it became the most interesting topic in graph theory because of its application in networking. In 1990, Livingston and Stout [4], introduced the concept of perfect dominating sets of  $G$ , indicated by  $\gamma_p(G)$ . They studied the existence and construction of perfect dominating sets in families of graphs arising from the interconnection networks of parallel computers. In 2014, Kwon and Lee [3] investigated some results related to perfect domination sets of Cayley graphs.

In 2017, Nayaka S R, Puttaswamy and Purushothama S, studied the idea of pendant domination in graphs. A dominating set  $S$  is called a pendant dominating set if  $\langle S \rangle$  contains at least one pendant node. The least cardinality of a pendant dominating set in  $G$  is called the pendant domination number of  $G$ , denoted by  $\gamma_{pe}(G)$ . For more details about pendant domination parameter refer [6], [7]

In this article, we introduce the idea of perfect pendant domination number. We characterize the perfect pendant dominating set and determine the exact values of perfect pendant domination number of some special graphs and graphs under few binary operations such as join and corona product of two graphs. Not all graphs have perfect pendant dominating set and we called them non- $\gamma_{ppe}(G)$ -graphs.

Let  $G = (V, E)$  be any graph with  $|V(G)| = n$  and  $|E(G)| = m$  edges. Then  $n, m$  are respectively called the order and the size of the graph  $G$ . For each node  $v \in V$ , the open neighborhood of  $v$  is the set  $N(v)$  containing all the vertices  $u$  adjacent to  $v$  and the closed neighborhood of  $v$  is the set  $N[v]$  containing  $v$  and all the vertices  $u$  adjacent to  $v$ . Let  $S$  be any subset of  $V$ , then the open neighborhood of  $S$  is  $N(S) = \bigcup_{v \in S} N(v)$  and the closed neighborhood of  $S$  is  $N[S] = N(S) \cup S$ .

The minimum and maximum of the degree among the vertices of  $G$  is indicated by  $\delta(G)$  and  $\Delta(G)$  respectively. A graph  $G$  is said to be regular if  $\delta(G) = \Delta(G)$ . A node  $v$  of a graph  $G$  is called a *cut node* if its removal increases the number of components. A *bridge* or *cut edge* of a graph is an edge whose removal increases the number of components. A node of degree zero is known as an isolated node and a

node of a degree one is known a pendant node. An edge incident to a pendant node is called a pendant edge. The graph containing no cycle is called a tree. A complete bi-partite graph  $K_{1,3}$  is a tree called as *claw*. Any graph containing no subgraph isomorphic to  $K_{1,3}$  is called a claw-free graph. For more details about graph terminology and domination parameter refer [1], [2], [5].

A set  $S \subseteq V(G)$  is said to be a dominating set if  $N[S] = V(G)$ . A dominating set  $S$  is a minimal dominating set if no proper subset  $S' \subset S$  is a dominating set. The domination number  $\gamma(G)$  of a graph  $G$  is the minimum cardinality of a dominating set of  $G$ . A dominating set  $S$  with  $|S| = \gamma(G)$  is said to be a  $\gamma$ -set.

A set  $S \subseteq V(G)$  is said to be a perfect dominating set if each node  $v \in V(G) - S$  is dominated by exactly one element in  $S$ . The minimum cardinality of a perfect dominating set of  $G$  is called perfect domination number, and is indicated by  $\gamma_p(G)$ . A perfect dominating set  $S$  with  $|S| = \gamma_p(G)$  is said to be a  $\gamma_p$ -set.

A dominating set  $S$  in  $G$  is called a pendant dominating set if  $\langle S \rangle$  contains at least one pendant node. The minimum cardinality of a pendant dominating set is called the pendent domination number and is denoted by  $\gamma_{pe}(G)$ .

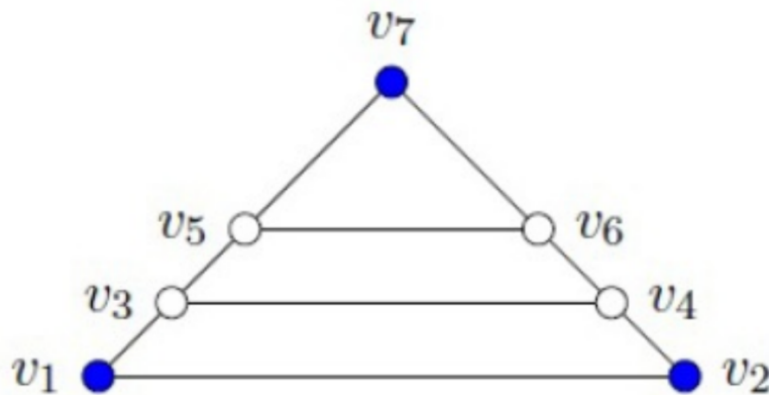
**Corollary 1.1** *Let  $G$  and  $H$  be connected graphs. Then*

$$\gamma(G + H) = \begin{cases} 1, & \gamma(G) = 1 \text{ or } \gamma(H) = 1; \\ 2, & \gamma(G) \neq 1 \text{ or } \gamma(H) \neq 1 \end{cases}$$

## 2. Perfect Pendant Dominating Set

A set  $S \subseteq V(G)$  is said to be a perfect pendant dominating set of  $G$  if  $S$  is a perfect dominating set and the induced subgraph  $\langle S \rangle$  has at least one pendant node. The minimum cardinality of a perfect pendant dominating set of  $G$  is called perfect pendant domination number and is indicated by  $\gamma_{ppe}(G)$ . A perfect pendant dominating set  $S$  with  $|S| = \gamma_{ppe}(G)$  is said to be  $\gamma_{ppe}$ -set. If a graph  $G$  has no perfect pendant dominating set, then we say that the graph  $G$  is a non- $\gamma_{ppe}$ -graph.

**Example 2.1** Let  $G$  be the graph in Figure 1 and  $S = \{v_1, v_2, v_7\}$ . Then  $v_1$  dominates  $v_3, v_2$  dominates  $v_4$ , while  $v_7$  dominates  $v_5, v_6$ . This shows that for all nodes  $v_3, v_4, v_5, v_6$  are elements in  $V(G) - S$  which are dominated by exactly one node in  $S$ . Thus,  $S$  is a perfect dominating set of  $G$ . Observe further that  $v_1, v_2$  is a pendant node of the induced subgraph  $\langle S \rangle$ . Therefore,  $S$  is a perfect pendant dominating set of  $G$  and  $\gamma_{ppe}(G) = 3$



**Proposition 2.1** *Let  $G$  be any graph such that  $G$  has a perfect pendant dominating set. Then  $\gamma_{pe}(G) \leq \gamma_{ppe}(G)$*

**Corollary 2.1** *Let  $G$  be a simple connected graph of order 3 or 4, that is, either  $G \cong P_3$  or  $G \cong C_4$ . Then  $\gamma_{ppe}(G) = 2$*

**Proposition 2.2** *If  $S$  is a dominating set or a perfect dominating set or a pendant dominating set of  $G$  with  $|S| = 2$ , then  $S$  is a perfect pendant dominating set of  $G$ . In particular  $\gamma(G) = \gamma_p(G) = \gamma_{pe}(G) = 2$  if and only if  $\gamma_{ppe}(G) = 2$*

**Theorem 2.1** *Let  $G$  be a connected graph of order  $n$ . Then  $\gamma_{ppe}(G) = n - 2$  if and only if  $G$  is one of the graphs  $P_4, C_4$ .*

**Theorem 2.2** *Let  $G$  be a connected graph of order  $n \geq 2$ . Then  $\gamma_{ppe}(G) = 2$  if and only if  $\Delta(G) = n - 1$ .*

**Theorem 2.3** *Let  $P_n$  be a path graph with  $n \geq 2$  nodes. Then*

$$\gamma_{ppe}(P_n) = \begin{cases} \frac{n}{3} + 1, & \text{if } n \equiv 0 \pmod{3}; \\ \lceil \frac{n}{3} \rceil & \text{if } n \equiv 1 \pmod{3}. \\ \lceil \frac{n}{3} \rceil + 1, & \text{if } n \equiv 2 \pmod{3}; \end{cases}$$

**Proof:** Suppose that  $V(P_n) = v_1, v_2, \dots, v_{n-1}, v_n$  such that  $deg(u_1) = deg(u_n) = 1$  and  $deg(v_i) = 2$  for all  $i = 2, 3, \dots, n - 1$ . Consider the following possible cases:

**Case 1** Suppose that  $n \equiv 0 \pmod{3}$ . If  $n = 3$  then by corollary 3  $\gamma_{ppe}(P_3) = 2 = \lceil \frac{3}{3} \rceil + 1$ . Suppose that  $n > 3$ . Let  $r = \frac{n}{3}$  and  $j = 1, 2, \dots, r - 1, r$ . Group of nodes of  $P_n$  into  $r$  disjoint subsets  $S_j$

$$\begin{aligned} S_1 &= \{v_1, v_2, v_3\} \\ S_2 &= \{v_4, v_5, v_6\} \\ S_3 &= \{v_7, v_8, v_9\} \\ S_4 &= \{v_{10}, v_{11}, v_{12}\} \\ &\vdots \\ S_{r-1} &= \{v_{n-5}, v_{n-4}, v_{n-3}\} \\ S_r &= \{v_{n-2}, v_{n-1}, v_n\} \end{aligned}$$

Clearly the set  $S = \{v_2, v_3, v_6, \dots, v_{n-3}, v_n\}$  is a  $\gamma_{ppe}$ -set of  $P_n$  since closed neighborhood of  $S$  contains all the nodes of  $P_n$ , the  $\langle S \rangle$  has a pendant nodes and  $|S| = \frac{n}{3} + 1$ . Clearly all the nodes of  $v_i \in V(P_n) - S$  are dominated by exactly one node in  $S$ . Therefore  $\gamma_{ppe}(P_n) = \frac{n}{3} + 1$

**Case 2** Suppose that  $n \equiv 1 \pmod{3}$ . If  $n = 4$ , then clearly  $S = \{v_2, v_3\}$  is a  $\gamma_{ppe}$ - set of  $P_4$  since  $|S| = \lceil \frac{4}{3} \rceil = 2$ . Suppose that  $n > 4$ . Let  $r = \frac{n}{3}$  and  $j = 1, 2, \dots, r - 1, r$ . Group of nodes of  $P_n$  into  $r$  disjoint subsets  $S_j$  as in the case 1. Clearly the set  $S = \{v_2, v_3, v_6, \dots, v_{n-3}, v_n\}$  is a  $\gamma_{ppe}$ -set of  $P_n$ , induced subgraph of  $S$  contains a pendant node and all the nodes  $v_i \in V(P_n) - S$  are dominated by exactly one node in  $S$ . Therefore  $\gamma_{ppe}(P_n) = \lceil \frac{n}{3} \rceil$

**Case 3** Suppose that  $n \equiv 2 \pmod{3}$ . If  $n = 5$ , then clearly  $S = \{v_2, v_3, v_5\}$  is a  $\gamma_{ppe}$ - set of  $P_5$  since  $|S| = 3 = \lceil \frac{5}{3} \rceil + 1$ . Suppose that  $n > 5$ . Let  $r = \lceil \frac{n}{3} \rceil$  and  $j = 1, 2, \dots, r - 1, r$ . Group of nodes of  $P_n$  into  $r$  disjoint subsets  $S_j$

$$\begin{aligned} S_1 &= \{v_1, v_2\} \\ S_2 &= \{v_3, v_4, v_5\} \\ S_3 &= \{v_6, v_7, v_8\} \\ S_4 &= \{v_9, v_{10}, v_{11}\} \\ &\vdots \\ S_{r-1} &= \{v_{n-5}, v_{n-4}, v_{n-3}\} \\ S_r &= \{v_{n-2}, v_{n-1}, v_n\} \end{aligned}$$

Clearly the set  $S = \{v_2, v_3, v_4, \dots, v_{n-3}, v_n\}$  is a  $\gamma_{ppe}$ -set of  $P_n$ , induced subgraph of  $S$  contains a pendant node and all the nodes  $v_i \in V(P_n) - S$  are dominated by exactly one node in  $S$ . Therefore  $\gamma_{ppe}(P_n) = \lceil \frac{n}{3} \rceil + 1$

□

**Theorem 2.4** *Let  $C_n$  be a cycle graph with  $n \geq 4$  nodes. Then*

$$\gamma_{ppe}(C_n) = \begin{cases} \frac{n}{3} + 2, & \text{if } n \equiv 0 \pmod{3}; \\ \lceil \frac{n}{3} \rceil, & \text{if } n \equiv 1 \pmod{3}. \\ \lceil \frac{n}{3} \rceil + 1, & \text{if } n \equiv 2 \pmod{3}; \end{cases}$$

**Proof:** Suppose that  $V(C_n) = v_1, v_2, \dots, v_{n-1}, v_n$  such that  $\deg(v_i) = 2$  for all  $i = 2, 3, \dots, n-1$ . Consider the following possible cases:

**Case 1** Suppose that  $n \equiv 0 \pmod{3}$ . If  $n = 6$  then by corollary 3  $\gamma_{ppe}(C_6) = 4 = \lceil \frac{6}{3} \rceil + 2$ . Suppose that  $n > 6$ . Let  $r = \frac{n}{3}$  and  $j = 1, 2, \dots, r-1, r$ . Group of nodes of  $C_n$  into  $r$  disjoint subsets  $T_j$

$$\begin{aligned} T_1 &= \{v_1, v_2, v_3\} \\ T_2 &= \{v_4, v_5, v_6\} \\ T_3 &= \{v_7, v_8, v_9\} \\ T_4 &= \{v_{10}, v_{11}, v_{12}\} \\ &\vdots \\ T_{r-1} &= \{v_{n-5}, v_{n-4}, v_{n-3}\} \\ T_r &= \{v_{n-2}, v_{n-1}, v_n\} \end{aligned}$$

Clearly the set  $S = \{v_1, v_2, v_3, \dots, v_{n-5}, v_{n-2}\}$  is a  $\gamma_{ppe}$ -set of  $C_n$  since closed neighborhood of  $S$  contains all the nodes of  $C_n$ , since  $\langle S \rangle$  has a pendant nodes and  $|S| = \frac{n}{3} + 2$ . Clearly all the nodes of  $v_i \in V(P_n) - S$  are dominated by exactly one node in  $S$ . Therefore  $\gamma_{ppe}(P_n) = \frac{n}{3} + 2$

**Case 2** Suppose that  $n \equiv 1 \pmod{3}$ . If  $n = 4$ , then clearly  $S = \{v_1, v_2\}$  is a  $\gamma_{ppe}$ - set of  $C_4$  since  $S = \lceil \frac{4}{3} \rceil = 2$ . Suppose that  $n > 4$ . Let  $r = \frac{n}{3}$  and  $j = 1, 2, \dots, r-1, r$ . Group of nodes of  $P_n$  into  $r$  disjoint subsets  $T_j$

$$\begin{aligned} T_1 &= \{v_1, v_2\} \\ T_2 &= \{v_3, v_4, v_5\} \\ T_3 &= \{v_6, v_7, v_8\} \\ T_4 &= \{v_9, v_{10}, v_{11}\} \\ &\vdots \\ T_{r-1} &= \{v_{n-5}, v_{n-4}, v_{n-3}\} \\ T_r &= \{v_{n-2}, v_{n-1}, v_n\} \end{aligned}$$

Clearly the set  $S = \{v_1, v_2, v_5, \dots, v_{n-5}, v_{n-2}\}$  is a  $\gamma_{ppe}$ -set of  $C_n$ , induced subgraph of  $S$  contains a pendant node and all the nodes  $v_i \in V(C_n) - S$  are dominated by exactly one node in  $S$ . Therefore  $\gamma_{ppe} = \lceil \frac{n}{3} \rceil$

**Case 3** Suppose that  $n \equiv 2 \pmod{3}$ . If  $n = 5$ , then clearly  $S = \{v_1, v_2, v_3\}$  is a  $\gamma_{ppe}$ - set of  $C_5$  since  $S = 3 = \lceil \frac{5}{3} \rceil + 1$ . Suppose that  $n > 5$ . Let  $r = \lceil \frac{n}{3} \rceil$  and  $j = 1, 2, \dots, r-1, r$ . Group of nodes of  $C_n$  into  $r$  disjoint subsets  $T_j$

$$\begin{aligned} T_1 &= \{v_1, v_2\} \\ T_2 &= \{v_3, v_4, v_5\} \\ T_3 &= \{v_6, v_7, v_8\} \\ T_4 &= \{v_9, v_{10}, v_{11}\} \\ &\vdots \\ T_{r-1} &= \{v_{n-5}, v_{n-4}, v_{n-3}\} \\ T_r &= \{v_{n-2}, v_{n-1}, v_n\} \end{aligned}$$

Clearly the set  $S = \{v_1, v_2, v_3, \dots, v_{n-5}, v_{n-2}\}$  is a  $\gamma_{ppe}$ -set of  $C_n$ , induced subgraph of  $S$  contains a pendant node and all the nodes  $v_i \in V(C_n) - S$  are dominated by exactly one node in  $S$ . Therefore  $\gamma_{ppe}(C_n) = \lceil \frac{n}{3} \rceil + 1$  □

**Theorem 2.5** *The complete graph  $K_n$ ,  $n \geq 4$  is non- $\gamma_{ppe}$ -graph .*

**Proof:** Let  $V(K_n) = \{v_1, v_2, v_3, \dots, v_n\}$  be the node set of  $K_n$ . For the graph  $K_n$ , clearly the sets  $S_1 = \{v_1, v_2\}, S_2 = \{v_2, v_3\}, \dots, S_i = \{v_i, v_j\}$  where  $v_i, v_j$  are any two adjacent nodes of  $K_n$  and  $i \neq j$  are pendant dominating set of  $K_n$  since induced subgraph of  $S_i$  contains a pendant node for all  $i$  but none of them are perfect dominating sets since all the nodes in  $V(K_n) - S_i$  are dominated by more than one nodes of  $K_n$ . Choosing more than three nodes in a pendant dominating set of  $K_n$  is not possible since their induced subgraph has no pendant node. □

**Theorem 2.6** *Let  $G$  and  $H$  be any graphs with  $\gamma(G) \geq 2$  and  $\gamma(H) \geq 2$ . Then the graph  $G + H$  is a non- $\gamma_{ppe}$ - graph.*

**Proof:** By Corollary  $\gamma(G + H) = 2$ . Let  $S$  be a  $\gamma_{ppe}$ - set of  $G + H$ . By proposition  $\gamma_{ppe}(G + H) > 1$ . Thus  $|S| > 1$ . Suppose that  $x, y \in S$ . Consider the following cases:

Case 1:  $x, y \in S \subseteq V(G)$ . Clearly, for all  $z \in V(H)$ ,  $z$  is dominated in  $G + H$  by the two nodes  $x$  and  $y$  in  $S$ . Thus  $S$  is not a perfect dominating set of  $G + H$ . Thus  $S$  is not a  $\gamma_{ppe}$ - set of  $G + H$ . This is contradiction. Similarly, if  $x, y \in S \subseteq V(H)$ , then  $S$  is not a  $\gamma_{ppe}$ - set of  $G + H$ . This is a contradiction. Therefore, having two nodes in  $V(G)$  or in  $V(H)$  is not possible for a set  $S$  to be a  $\gamma_{ppe}$ - set of  $G + H$ . Case 2:  $x, y \in S$  such that  $x \in V(G)$  and  $y \in V(H)$ . Clearly, the induced subgraph  $\langle S \rangle$  has pendant node. Hence,  $S$  is a pendant dominating set of  $G + H$  but each nodes in  $V(G + H) - S$  is dominated by more than one node in  $S$ . Thus  $S$  is not a  $\gamma_{ppe}$ - set of  $G + H$ , a contradiction. Furthermore, adding a node in  $S$  either from  $V(G)$  or  $V(H)$  is not possible by case 1.

Hence, in any case,  $S$  is not a  $\gamma_{ppe}$ -set of  $G + H$ . Therefore, the graph  $G + H$  is a non  $\gamma_{ppe}$ - graph if  $\gamma(G) \geq 2$  and  $\gamma(G) \geq 2$ . □

**Corollary 2.2** *The following graphs are non- $\gamma_{ppe}$ -graphs*

- Complete bipartite graph  $K_{m,n} = \overline{K_m} + \overline{K_n}$ ,  $m \geq 2$  and  $n \geq 2$ .
- Generalized wheel graph  $W_{m,n} = \overline{K_m} + C_n$ ,  $m \geq 3$  and  $n \geq 3$ .
- Generalized fan graph  $F_{n,n} = \overline{K_m} + P_n$ ,  $m \geq 3$  and  $n \geq 4$ .

### 3. The Perfect Pendant Dominating Set of Some Wheel Related Graphs

**Definition 3.1** The closed helm  $CH_n$  is the graph obtained from helm  $H_n$  by joining each pendant node to form a cycle.

**Definition 3.2** The flower graph  $Fl_n$  is the graph obtained from helm  $H_n$  by joining each pendant node to the apex of helm  $H_n$ .

**Theorem 3.1** *If  $G$  is a closed helm graph  $G \cong CH_n$ , for  $n \geq 5$ , then  $\gamma_{ppe}(CH_n) = n - 2$*

**Proof:** The closed helm graph  $CH_n$  contains a wheel graph  $W_n$  and outer cycle  $C_n$ . Let  $v$  in closed helm graph indicate the apex node of wheel graph. Also  $\{u_1, u_2, \dots, u_n\}$  be the rim nodes of wheel graph  $W_n$  of closed helm graph  $CH_n$  and  $\{v_1, v_2, \dots, v_n\}$  be the corresponding adjacent nodes of outer cycle of  $CH_n$ . So  $|V(CH_n)| = 2n + 1$ .

For  $n \geq 5$ ,  $deg(v) = n = \Delta(CH_n)$ . The perfect pendant dominating set of closed helm graph is obtained by taking  $n - 3$  nodes of outer cycle  $C_n$  of closed helm graph  $CH_n$  and select the node  $v$  is not dominated

by  $n-3$  nodes. Therefore the perfect pendant domination number of closed helm graph is  $|(n-3)| + \{v_1\}$ . Hence  $\gamma_{ppe}(CH_n) = |(n-3)| + 1 = n-2$ .

Note that  $S = \{u_1, u_2, \dots, u_{n-3}, v_1\}$  is a perfect pendant dominating set with least cardinality because removal of any one of the nodes from set  $S$  will not dominate all the nodes of  $CH_n$ . Further every node in  $V - S$  is dominated by exactly one node in  $S$  and induced subgraph of  $S$  contains a pendant node. Hence  $\gamma_{ppe}(CH_n) = n-2$   $\square$

**Theorem 3.2** *If  $G$  is a sunflower  $Sf_n$ ,  $n \geq 3$  then,  $\gamma_{ppe}(Sf_n) = n+1$*

**Proof:** Let the node  $v$  be the apex and  $\{v_2, v_4, v_6, \dots, v_{2i}\}$ , for  $i = 1, 2, 3, \dots, n$  be the nodes of degree one in sunflower graph  $Sf_n$  with  $|V(Sf_n)| = 3n+1$ . the apex node  $v$  dominates the  $2n$  nodes and pendant nodes. It is clear to see that any perfect pendant dominating set  $S$  must contain the node  $v$  and pendant nodes. Implies that  $|S| = n+1$ , which is least cardinality and induced subgraph of  $S$  contains a pendant node. Hence  $\gamma_{ppe}(Sf_n) = n+1$ .  $\square$

#### 4. The Perfect Pendant Dominating Set in the Corona of Graphs

The corona of graphs  $G$  and  $H$ ,  $G \circ H$ , is the graph obtained by taking one copy of  $G$  and  $|V(G)|$  copies of  $H$ , and then joining the  $i$ th node of  $G$  to every node of the  $i$ th copy of  $H$ . For every  $v \in V(G)$ , denote by  $H^v$  the copy of  $H$  whose nodes are attached one by one to the node  $v$ . Subsequently, denote by  $v + H^v$  the subgraph of the corona  $G \circ H$  corresponding to the join  $\{v\} + H^v$ ,  $v \in V(G)$ .

**Corollary 4.1** *Let  $G$  be a connected graph of order  $n$  and  $H$  be any graph where  $\gamma(H) = 1$ . Then  $\gamma_{ppe}(G \circ H) = n$ .*

**Corollary 4.2** *Let  $G$  be a connected graph of order  $n$  and  $H$  be any graph where  $\gamma(H) \geq 2$ . Then  $(G \circ H)$  is a non- $\gamma_{ppe}$ -graph.*

#### 5. Conclusion

In recent years, research on domination and related parameters has become one of the most rapidly advancing areas in graph theory, attracting significant attention from scholars and researchers. In this article, we introduce a new invariant called perfect pendant domination. The article is organized as three sections. In the first section presents precise calculations of perfect pendant domination for several standard families of graphs and second section explores the idea of perfect pendant domination in the context of wheel-related graphs. The final section addresses the calculation of perfect pendant domination for corona graphs. This work aims to deepen the understanding of perfect pendant domination and its applications within graph theory.

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