



New Solitary Wave Solutions in Nonlinear Physics Using an Extended Rational Exponential Approach

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ABSTRACT: This study introduces an innovative analytical approach to derive exact solitary wave solutions for nonlinear fractional-order equations. By employing the extended rational $\exp\left(-\xi\left(\frac{\phi'}{\phi}\right)\right)$ -expansion method, we obtain novel solitary wave solutions for two prominent nonlinear physical models: the Maccari system and the Higgs equation. The method is demonstrated to effectively solve fractional-order nonlinear partial differential equations, yielding previously unreported solitary wave configurations including kink, singular-kink, and periodic wave solutions. The physical relevance of these results is rigorously analyzed and supported by their graphical illustrations. A comparative study endorses the enhanced reliability and computational efficiency of the proposed method. The study also highlights the adaptability of this method, which has considerable significance for solving a wide range of nonlinear evolution equations that arise in mathematical physics and engineering applications.

Keywords: Extended rational $\exp\left(-\xi\left(\frac{\phi'}{\phi}\right)\right)$ -expansion method, coupled Higgs equation, Maccari system, solitary wave solutions, fractional calculus, Caputo's fractional derivative.

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1. Introduction

Nonlinear systems are widely acknowledged as the foundation for advancements in mathematics, theoretical physics, and engineering, with their relevance expanding substantially in contemporary research. These systems underpin numerous phenomena across assorted disciplines including biology, fluid dynamics, plasma physics, optical fibers, solid-state physics, biophysics, and the Higgs mechanism, where linear frameworks often fail to capture the inherent complexity of their behavior. The intricate nature of nonlinear problems remains a principal concern for researchers, from physicists and engineers to mathematicians, who strive to develop precise analytical frameworks to unravel such dynamics.

In light of this requirement, we present the extended rational $\exp\left(-\xi\left(\frac{\phi'}{\phi}\right)\right)$ -expansion method, an innovative analytical approach to systematically construct exact solitary wave solutions for nonlinear physical models. By deploying this method on established systems such as the Maccari equations and Higgs mechanisms, we categorize previously unexplored wave structures, including kink and periodic solitons, which validate the method's improved accuracy compared to conventional strategies. This contribution not only strengthens theoretical understanding but also refines the practical analysis of

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nonlinear evolution equations, providing rigorous insights for both foundational scientific inquiry and real-world engineering applications.

Many leading methods have been recognized, for example the $\exp\left(-\xi\left(\frac{\phi'}{\phi}\right)\right)$ -expansion method [1,2], the modified simple equation method [3], the homogeneous balancing method [4], the novel (G'/G) -expansion method [5], the (G'/G) -expansion method [6,7], the homotopy perturbation method [8], He's semi-inverse method [9], the exp-function method [10,11,12,13], the generalized exp-function method [14], the ansatz method, the perturbation method [15], the tanh-function method [16,17], the rational $\exp(-\phi(\eta))$ -expansion method [18], the modified exp-function method [19], the F-expansion method [20], Biswas et al. [21], and so on.

Fractional calculus has come to the forefront as a robust mechanism for modeling intricate biological systems characterized by nonlinear dynamics. Fractional-order equations have achieved considerable acclaim by attracting widespread interest from researchers due to their comprehensive ability in managing complex real-world problems in recent years. These equations have proven particularly valuable across diverse disciplines including chemistry, physics, biophysics, hemodynamics, capacitor design, control theory, and electrical circuit analysis, underscoring their broad applicability in both theoretical and applied scientific domains.

Numerous applications of fractional calculus [22,23,24,25,26,38] can be found in mathematical physics, fluid dynamics, engineering, and other fields. A variety of methodologies have been utilized to resolve differential equations with fractional orders, such as the Adomian decomposition approach, the homotopy analysis method [27], the $\exp\left(-\xi\left(\frac{\phi'}{\phi}\right)\right)$ -expansion method [28], the direct algebraic method [29], the local fractional variation iteration method [30], the modification of the truncated expansion method [31], the Kudryashov method [32], the interpolatory HDG method [33], the modified Kudryashov method [34], the breather period limit method [35], the extended exp-function method [36], the sine-Gordon expansion method [37], and so on, including references therein.

2. Fractional form of Caputo's Derivative

In this part, we define key terms and explain fundamental concepts.

Definition I. A real function $h(\kappa)$, $\kappa > 0$ is regarded as being in space C_β , $\beta \in \mathbb{R}$, if there exists a number $\varphi > \beta$, such that

$$h(\kappa) = \kappa^\varphi h_1(\kappa), \quad h_1(\kappa) \in C[0, \infty). \quad (2.1)$$

Definition II. A real function $h(\kappa)$, $\kappa > 0$ is regarded as being in C_β^n (space), $n \in \mathbb{N} \cup \{0\}$, if $h^{(n)} \in C_\beta$.

Definition III. Let $h \in C_\beta$ and $\beta \geq -1$. Then the left-sided Riemann–Liouville integral of order $\gamma > 0$ is defined as

$$I_t^\gamma h(\kappa, t) = \frac{1}{\Gamma(\gamma)} \int_0^t (t-T)^{\gamma-1} h(\kappa, T) dT, \quad t > 0. \quad (2.2)$$

Definition IV. The left-sided Caputo fractional derivative of h with respect to t , where $h \in C_{-1}^n$ and $n \in \mathbb{N} \cup \{0\}$, is given by

$$D_t^\gamma h(\kappa, t) = \begin{cases} \frac{\partial^n}{\partial t^n} h(\kappa, t), & \gamma = n, \\ I_t^{n-\gamma} \frac{\partial^n}{\partial t^n} h(\kappa, t), & n-1 \leq \gamma < n, \quad n \in \mathbb{N}. \end{cases} \quad (2.3)$$

Remark. The following properties hold:

$$I_t^n D_t^\gamma h(\kappa, t) = h(\kappa, t) - \sum_{m=0}^{n-1} \frac{\partial^m h}{\partial t^m}(\kappa, 0) \frac{t^m}{m!}, \quad n-1 < \gamma \leq n, \quad n \in \mathbb{N}, \quad (2.4)$$

$$I_t^\gamma t^\vartheta = \frac{\Gamma(\vartheta+1)}{\Gamma(\gamma+\vartheta+1)} t^{\gamma+\vartheta}. \quad (2.5)$$

3. Portrayal of the Proposed Method

We assume the nonlinear FPDE

$$\Psi\left(v, D_t^\beta v, v_x, v_{xx}, D_t^{2\beta} v, D_t^\beta v_x, \dots\right) = 0, \quad x \in \mathbb{R}, t > 0, 0 \leq \beta \leq 1, \quad (3.1)$$

where $D_t^\beta v$, $D_x^\beta v$, $D_{xx}^\beta v$ denote the fractional derivatives with respect to t, x, xx respectively, and $v(\xi) = v(x, t)$. Here, φ is a specific polynomial of v and its derivatives.

Step 1. Using the transformation

$$v = v(\xi), \quad \xi = x \pm \frac{ut^\beta}{\Gamma(\beta + 1)}, \quad (3.2)$$

where u is a nonzero constant. Substituting Eq. (3.1) into Eq. (3.2), we obtain the corresponding ODE for $v(\xi)$:

$$\Psi(v, \mp Uv', kv', U^2v'', k^2v'', \dots) = 0. \quad (3.3)$$

Step 2. The supposed solution takes the form

$$u(\eta) = \frac{c_0 + \frac{d}{d\eta} \left[\sum_{k=1}^N c_k (e^{Z(\eta)})^k \right]}{\sum_{k=0}^M d_k (e^{Z(\eta)})^k}, \quad (3.4)$$

where c_k and d_k ($k = 0, 1, \dots, N$) are constants with $c_k \neq 0$, $d_k \neq 0$, and $\varphi(\eta)$ satisfies the auxiliary equation

$$\Psi'(\xi) = \mu + \lambda e^{\Psi(\xi)} + e^{-\Psi(\xi)}, \quad (3.5)$$

where the prime denotes derivative of Ψ and the values depend on μ and λ .

Family I: When $\lambda^2 - 4\mu > 0$,

$$\psi(\eta) = \ln \left\{ \left(-\sqrt{\lambda^2 - 4\mu} \tanh \left(\frac{(\eta+c)\sqrt{\lambda^2-4\mu}}{2} \right) - \lambda \right) \frac{1}{2\mu} \right\}.$$

Family II: When $-4\mu + \lambda^2 < 0$,

$$\psi(\eta) = \ln \left\{ \left(\sqrt{-4\mu + \lambda^2} \tan \left(\frac{(\eta+c)\sqrt{\lambda^2-4\mu}}{2} \right) - \lambda \right) \frac{1}{2\mu} \right\}.$$

Family III: When $\lambda \neq 0$, $\mu = 0$ and $\lambda^2 - 4\mu > 0$,

$$\psi(\eta) = -\ln \left\{ \frac{\lambda}{e^{\lambda(\eta+k_1)} - 1} \right\}.$$

Family IV: When $\mu \neq 0$, $\lambda \neq 0$ and $\lambda^2 - 4\mu = 0$,

$$\psi(\eta) = \ln \left\{ \frac{2\lambda(\eta+k_1)+4}{\lambda^2(\eta+k_1)} \right\}.$$

Family V: When $\mu = 0$, $\lambda^2 - 4\mu = 0$ and $\lambda = 0$,

$$\varphi(\eta) = \ln(\eta + k_1).$$

Step 3. The homogeneous balance rule allows us to determine N . We use $e^{N\Psi(\xi)}$ to account for Eqs. (3.3)–(3.5). After equating coefficients of the same powers of $e^{N\Psi(\xi)}$, we obtain several algebraic equations in terms of c_k, U, μ, λ , leading to important conclusions of Eq. (3.1).

4. Solution Procedure

Let us consider the coupled Higgs equation,

$$D_t^{2\beta} \delta - \delta_{xx} + |\delta|^2 \delta = 2\delta\rho, \quad (4.1)$$

$$D_t^{2\alpha} \rho + \rho_{xx} = (|\delta|^2)_{xx}, \quad 0 < \beta \leq 2. \quad (4.2)$$

The present system establishes a framework for characterizing a preserved scalar-nucleon interaction employing a neutral scalar meson. In this context, the symbol δ illustrates a genuine scalar-meson field, while ρ represents a complex nucleon field. Consider,

$$\begin{aligned} \delta(x, t) &= e^{i\theta} v(\xi), \\ \rho(x, t) &= u(\xi), \end{aligned}$$

where,

$$\begin{aligned} \theta &= qx + s \frac{t^\beta}{\Gamma(\beta + 1)}, \\ \xi &= x + a \frac{t^\beta}{\Gamma(\beta + 1)}, \end{aligned}$$

The proposed model undergoes a transformation as follows:

$$(a^2 - 1)v'' + (q^2 - s^2)v + v^3 = 2vu, \quad (4.3)$$

$$(a^3 + 1)u'' = (v^2)''. \quad (4.4)$$

Taking integration of Eq. (4.4) and afterwards including the obtained result into Eq. (4.3):

$$(a^4 - 1)v'' + (a^2 + 1)(q^2 - s^2)v + (a^2 - 1)v^3 = 0, \quad (4.5)$$

The symbol "dash" is used to represent the derivative of the variable ξ . The value of N , i.e., $N = 1$ is determined through the use of the balancing principle. Subsequently, the trial solution that follows is as follows:

$$u(\eta) = \frac{c_0 + \frac{d}{d\eta} \left[\sum_{k=1}^N c_k (e^{Z(\eta)})^k \right]}{\sum_{k=0}^M [d_k (e^{Z(\eta)})^k]} \quad (4.6)$$

From Eq. (4.5) and Eq. (4.6), we have

$$-\frac{1}{(d_0 e^{Z(\eta)} + d_1)^6} (J_0 + J_1 e^{Z(\eta)} + J_2 e^{2Z(\eta)} + J_3 e^{3Z(\eta)} + J_4 e^{4Z(\eta)} + J_5 e^{5Z(\eta)} + J_6 e^{6Z(\eta)}) = 0 \quad (4.7)$$

Comparing the coefficients allows us to arrive at,

$$[J_0 = 0, J_1 = 0, J_2 = 0, J_3 = 0, J_4 = 0, J_5 = 0, J_6 = 0]. \quad (4.8)$$

Eq. (4.6) can be solved to get the following solutions.

Solution 1:

$$a = -1, c_0 = c_0, c_1 = c_1, \mu = \mu, d_0 = d_0, d_1 = d_1, q = -s$$

Family I: When $\lambda \neq 0$ and $\mu^2 - 4\lambda = \varphi > 0$,

$$v_1 = \frac{2c_0\lambda}{\sqrt{\varphi} \tanh \left[\frac{1}{2} \left(\frac{Ut^\beta}{\Gamma(\beta+1)} - x \right) \sqrt{\varphi} \right] d_1 - \mu d_1 + 2d_0\lambda} \quad (4.9)$$

$$u_1 = \frac{4c_0^2\lambda^2}{(a^2 + 1) \left[\sqrt{\varphi} \tanh \left[\frac{1}{2} \left(\frac{Ut^\beta}{\Gamma(\beta+1)} - x \right) \sqrt{\varphi} \right] d_1 - \mu d_1 + 2d_0\lambda \right]^2} \quad (4.10)$$

Family II: Whenever $\lambda \neq 0$ and $\mu^2 - 4\lambda = \varphi < 0$,

$$v_2 = -\frac{2c_0\lambda}{\sqrt{-\mu^2 + 4\lambda} \tan \left[\frac{\sqrt{-\mu^2 + 4\lambda}}{2} \left(\frac{Ut^\beta}{\Gamma(\beta+1)} - x \right) \right]} d_1 + \mu d_1 - 2d_0\lambda \quad (4.11)$$

$$u_2 = \frac{4c_0^2\lambda^2}{(a^2 + 1) \left[\sqrt{-\mu^2 + 4\lambda} \tan \left[\frac{\sqrt{-\mu^2 + 4\lambda}}{2} \left(\frac{Ut^\beta}{\Gamma(\beta+1)} - x \right) \right] d_1 + \mu d_1 - 2d_0\lambda \right]^2} \quad (4.12)$$

Family III: However $\mu^2 - 4\lambda = \varphi > 0$, $\mu = 0$ and $\lambda \neq 0$,

$$v_3 = \frac{c_0\mu}{d_1 e^{-\mu \left(\frac{Ut^\beta}{\Gamma(\beta+1)} - x \right)} + d_0\mu - d_1} \quad (4.13)$$

$$u_3 = \frac{c_0^2\mu^2}{(a^2 + 1) \left(d_1 e^{-\mu \left(\frac{Ut^\beta}{\Gamma(\beta+1)} - x \right)} + d_0\mu - d_1 \right)^2} \quad (4.14)$$

Family IV: If $\mu^2 - 4\lambda = \varphi = 0$, $\lambda \neq 0$, and $\mu \neq 0$,

$$v_4 = \frac{c_0\mu^2 \left(\frac{Ut^\beta}{\Gamma(\beta+1)} - x \right)}{\frac{U\mu^2 t^\beta d_0}{\Gamma(\beta+1)} - \frac{2U\mu t^\beta d_1}{\Gamma(\beta+1)} - \mu^2 x d_0 + 2\mu x d_1 + 2d_1} \quad (4.15)$$

$$u_4 = \frac{c_0^2\mu^4 \left(\frac{Ut^\beta}{\Gamma(\beta+1)} - x \right)^2}{(a^2 + 1) \left(-\frac{2U\mu t^\beta d_1}{\Gamma(\beta+1)} + \frac{U\mu^2 t^\beta d_0}{\Gamma(\beta+1)} - \mu^2 x d_0 + 2\mu x d_1 + 2d_1 \right)^2} \quad (4.16)$$

Family V: While $\mu^2 - 4\lambda = \varphi = 0$, $\mu = 0$ and $\lambda = 0$,

$$v_5 = -\frac{c_0}{\frac{Ut^\beta d_1}{\Gamma(\beta+1)} - x d_1 - d_0} \quad (4.17)$$

$$u_5 = \frac{c_0^2}{(a^2 + 1) \left(\frac{Ut^\beta d_1}{\Gamma(\beta+1)} - x d_1 - d_0 \right)^2} \quad (4.18)$$

Solution 2:

$$a = a, \mu = \mu, d_0 = -\frac{\sqrt{-(q^2 a^2 - s^2 a^2 + q^2 - s^2)(a^2 - 1)} c_0}{q^2 a^2 - s^2 a^2 + q^2 - s^2}, c_0 = c_0, d_1 = 0, q = q$$

Result I: When $\lambda \neq 0$ and $\mu^2 - 4\lambda = \varphi > 0$,

$$v_6 = -\frac{a^2 q^2 - a^2 s^2 + q^2 - s^2}{\sqrt{-(q^2 - s^2)(a^4 - 1)}} \quad (4.19)$$

$$u_6 = -\frac{(a^2 q^2 - a^2 s^2 + q^2 - s^2)^2}{(a^2 + 1)(q^2 - s^2)(a^4 - 1)} \quad (4.20)$$

Result II: Whenever $\lambda \neq 0$ and $\mu^2 - 4\lambda = \varphi < 0$,

$$v_7 = -\frac{a^2 q^2 - a^2 s^2 + q^2 - s^2}{\sqrt{-(q^2 - s^2)(a^4 - 1)}} \quad (4.21)$$

$$u_7 = -\frac{(a^2 q^2 - a^2 s^2 + q^2 - s^2)^2}{(a^2 + 1)(q^2 - s^2)(a^4 - 1)} \quad (4.22)$$

Solution 3:

$$\begin{aligned}
a &= -\frac{1}{2} \frac{\sqrt{2} \sqrt{c_0 \mu (q^2 c_1 - s^2 c_1 + 2\lambda c_0)}}{c_0 \lambda}, \\
\mu &= \frac{2 \sqrt{-(q^2 c_1 - s^2 c_1 + 4\lambda c_0) c_0 c_1 (c_0 + c_1) \lambda}}{\sqrt{-c_0 \lambda (q^2 c_1 - s^2 c_1 + 4\lambda c_0) c_1}}, \\
c_0 &= c_0, \quad c_1 = c_1, \\
d_0 &= -\frac{\sqrt{-(q^2 c_1 - s^2 c_1 + 4\lambda c_0) c_0 c_1 (c_0 + c_1)}}{q^2 a_1 - s^2 a_1 + 4\mu a_0}, \\
d_1 &= -\frac{\sqrt{-c_0 \lambda (q^2 c_1 - s^2 c_1 + 4\lambda c_0) c_1}}{q^2 c_1 - s^2 c_1 + 4\lambda c_0}, \quad q = q.
\end{aligned}$$

Result I: When $\lambda \neq 0$ and $\mu^2 - 4\lambda = \varphi > 0$,

$$v_8 = -\frac{c_0 (q^2 c_1 - s^2 c_1 + 4\lambda c_0) \lambda}{\sqrt{-c_0 \lambda (q^2 c_1 - s^2 c_1 + 4\lambda c_0) c_1} \tan\left(\left(\frac{U t^\beta}{\Gamma(\beta+1)} - x\right) \sqrt{\frac{c_0 \lambda}{c_1}}\right) \sqrt{\frac{c_0 \lambda}{c_1}}} \quad (4.23)$$

$$u_8 = -\frac{q^2 c_1 - s^2 c_1 + 4\lambda c_0}{(a^2 + 1) c_1 \tan^2\left(\left(\frac{U t^\beta}{\Gamma(\beta+1)} - x\right) \sqrt{\frac{c_0 \lambda}{c_1}}\right)} \quad (4.24)$$

Result II: Whenever $\lambda \neq 0$ and $\mu^2 - 4\lambda = \varphi < 0$,

$$v_9 = \frac{c_0 (q^2 c_1 - s^2 c_1 + 4\lambda c_0) \lambda}{\sqrt{-c_0 \lambda (q^2 c_1 - s^2 c_1 + 4\lambda c_0) c_1} \tan\left(\left(\frac{U t^\beta}{\Gamma(\beta+1)} - x\right) \sqrt{-\frac{c_0 \lambda}{c_1}}\right) \sqrt{-\frac{c_0 \lambda}{c_1}}} \quad (4.25)$$

$$u_9 = \frac{q^2 c_1 - s^2 c_1 + 4\lambda c_0}{(a^2 + 1) c_1 \tan^2\left(\left(\frac{U t^\beta}{\Gamma(\beta+1)} - x\right) \sqrt{-\frac{c_0 \lambda}{c_1}}\right)} \quad (4.26)$$

Family III: However $\mu^2 - 4\lambda = \varphi > 0$, $\lambda = 0$ and $\mu \neq 0$,

$$v_{10} = \frac{2 \sqrt{-(q^2 c_1 - s^2 c_1 + 4\lambda c_0) c_0 c_1 (c_0 + c_1)}}{e^{-\frac{2 \sqrt{-(q^2 c_1 - s^2 c_1 + 4\lambda c_0) c_0 c_1 (c_0 + c_1) \lambda} \left(\frac{U t^\beta}{\Gamma(\beta+1)} - x\right)}{\sqrt{-c_0 \lambda (q^2 c_1 - s^2 c_1 + 4\lambda c_0) c_1}} c_1 + 2c_0 + c_1) c_1} \quad (4.27)$$

$$u_{10} = -\frac{4(q^2 c_1 - s^2 c_1 + 4\lambda c_0) c_0 (c_0 + c_1)}{(a^2 + 1) c_1 \left(e^{-\frac{2 \sqrt{-(q^2 c_1 - s^2 c_1 + 4\lambda c_0) c_0 c_1 (c_0 + c_1) \lambda} \left(\frac{U t^\beta}{\Gamma(\beta+1)} - x\right)}{\sqrt{-c_0 \lambda (q^2 c_1 - s^2 c_1 + 4\lambda c_0) c_1}} c_1 + 2c_0 + c_1 \right)^2} \quad (4.28)$$

Family IV: If $\mu^2 - 4\lambda = \varphi = 0$, $\lambda \neq 0$, and $\mu \neq 0$,

$$v_{11} = \frac{-[2c_0(q^2c_1 - s^2c_1 + 4\lambda c_0)(c_0 + c_1)\lambda \left(\frac{Ut^\beta}{\Gamma(\beta+1)} - x\right)]}{2\sqrt{-(q^2c_1 - s^2c_1 + 4\lambda c_0)c_0c_1(c_0 + c_1)}U\lambda t^\beta c_0} \quad (4.29)$$

$$u_{11} = \frac{-2\sqrt{-(q^2c_1 - s^2c_1 + 4\lambda c_0)c_0c_1(c_0 + c_1)}\lambda x c_0 + \sqrt{-c_0\lambda(q^2c_1 - s^2c_1 + 4\lambda c_0)}c_1^2 + 4c_0^2(q^2c_1 - s^2c_1 + 4\lambda c_0)^2(c_0 + c_1)^2\lambda^2 \left(\frac{Ut^\beta}{\Gamma(\beta+1)} - x\right)^2}{(a^2 + 1) \left[\frac{2\sqrt{-(q^2c_1 - s^2c_1 + 4\lambda c_0)c_0c_1(c_0 + c_1)}U\lambda t^\beta c_0}{\Gamma(\beta + 1)} - 2\sqrt{-(q^2c_1 - s^2c_1 + 4\lambda c_0)c_0c_1(c_0 + c_1)}\lambda x c_0 + \sqrt{-c_0\lambda(q^2c_1 - s^2c_1 + 4\lambda c_0)}c_1^2 \right]^2} \quad (4.30)$$

Family V: While $\mu^2 - 4\lambda = \varphi = 0$, $\mu = 0$ and $\lambda = 0$,

$$v_{12} = -\frac{(q^2c_1 - s^2c_1 + 4\lambda c_0)c_0}{\sqrt{-c_0\lambda(q^2c_1 - s^2c_1 + 4\lambda c_0)}Ut^\beta c_1} - \frac{\Gamma(\beta + 1)}{\Gamma(\beta + 1)} \quad (4.31)$$

$$u_{12} = \frac{+ \sqrt{-c_0\lambda(q^2c_1 - s^2c_1 + 4\lambda c_0)}x c_1 + \sqrt{-(q^2c_1 - s^2c_1 + 4\lambda c_0)c_0c_1(c_0 + c_1)}}{(q^2c_1 - s^2c_1 + 4\lambda c_0)^2 c_0^2} \quad (4.32)$$

$$(a^2 + 1) \left(\begin{array}{l} - \frac{\sqrt{-c_0\lambda(q^2c_1 - s^2c_1 + 4\lambda c_0)}Ut^\beta c_1}{\Gamma(\alpha + 1)} \\ + \sqrt{-c_0\mu(q^2c_1 - s^2c_1 + 4\lambda c_0)}x c_1 \\ + \sqrt{-(q^2c_1 - s^2c_1 + 4\lambda c_0)c_0c_1(c_0 + c_1)} \end{array} \right)^2$$

Solution 4:

$$a = a, \mu = \mu, d_0 = \frac{\sqrt{-(q^2c^2 - s^2c^2 + q^2 - s^2)(c^2 - 1)a_0}}{q^2c^2 - s^2c^2 + q^2 - s^2}, d_1 = 0, c_0 = c_0, q = q$$

Solution 5:

$$a = \frac{1}{2} \frac{\sqrt{c_0\lambda(q^2c_1 - s^2c_1 + 4\lambda c_0)}}{c_0\lambda},$$

$$\mu = -\frac{2\sqrt{-(q^2c_1 - s^2c_1 + 8\lambda c_0)c_0c_1(c_0 + 2c_1)}\lambda}{\sqrt{-(2q^2c_1 - 2s^2c_1 + 16\lambda c_0)c_0\lambda c_1}},$$

$$d_0 = \frac{\sqrt{-(q^2c_1 - s^2c_1 + 8\lambda c_0)c_0c_1(c_0 + 2c_1)}\lambda}{q^2c_1 - s^2c_1 + 8\lambda c_0},$$

$$d_1 = -\frac{\sqrt{-(2q^2c_1 - 2s^2c_1 + 16\lambda c_0)c_0\lambda c_1}}{q^2c_1 - s^2c_1 + 8\lambda c_0},$$

$$c_1 = c_1, \quad c_0 = c_0, \quad q = q$$

Solution 6:

$$\begin{aligned}
c &= -\frac{1}{2} \frac{\sqrt{c_0 \lambda (q^2 c_1 - s^2 c_1 + 4 \lambda c_0)}}{c_0 \lambda}, \\
\mu &= -\frac{2 \sqrt{-(q^2 c_1 - s^2 c_1 + 8 \lambda c_0) c_0 c_1 (c_0 + 2 c_1) \lambda}}{\sqrt{-(2 q^2 c_1 - 2 s^2 c_1 + 16 \lambda c_0) c_0 \lambda c_1}}, \\
d_0 &= \frac{\sqrt{-(q^2 c_1 - s^2 c_1 + 8 \lambda c_0) c_0 c_1 (c_0 + 2 c_1) \lambda}}{q^2 c_1 - s^2 c_1 + 8 \lambda c_0}, \\
d_1 &= -\frac{\sqrt{-(2 q^2 c_1 - 2 s^2 c_1 + 16 \lambda c_0) c_0 \lambda c_1}}{q^2 c_1 - s^2 c_1 + 8 \lambda c_0}, \\
c_1 &= c_1, \quad c_0 = c_0, \quad q = q
\end{aligned}$$

Now, consider the Maccari system we have,

$$D_t^\beta V - V_{xx} = -VU, \quad (4.33)$$

$$D_t^\beta U + U_y = -(|V|^2)_x, \quad 0 < \beta \leq 1. \quad (4.34)$$

Let,

$$\begin{aligned}
V(x, y, t) &= e^{i\theta} v(\xi), \\
U(x, y, t) &= u(\xi),
\end{aligned}$$

and assume,

$$\begin{aligned}
\xi &= x + y + a \frac{t^\beta}{\Gamma(\beta + 1)}, \\
\theta &= qx + py + s \frac{t^\beta}{\Gamma(\beta + 1)},
\end{aligned}$$

The intended model is transformed as described above to become

$$v'' + (s + q^2)v = -vu, \quad (4.35)$$

$$(a + 1)u' + (v^2)' = 0. \quad (4.36)$$

By integrating Eq. (4.36) and then using it as a replacement in Eq. (4.35), we obtain

$$(a + 1)u'' - (a + 1)(r + p^2)u - u^3 = 0, \quad (4.37)$$

where the derivative of η is denoted by dash. If we follow the balancing phenomenon, we obtain $M = 1$, and the trial answer is as follows:

$$u(\eta) = \frac{c_0 + \frac{d}{d\eta} \left[\sum_{k=1}^N c_k (e^{Z(\eta)})^k \right]}{\sum_{k=0}^M [d_k (e^{Z(\eta)})^k]} \quad (4.38)$$

From Eqs. (4.35), (4.36) and (4.1), we obtain

$$-\frac{1}{(d_0 e^{Z(\eta)} + d_1)^6} (J_0 + J_1 e^{Z(\eta)} + J_2 e^{2Z(\eta)} + J_3 e^{3Z(\eta)} + J_4 e^{4Z(\eta)} + J_5 e^{5Z(\eta)} + J_6 e^{6Z(\eta)}) = 0 \quad (4.39)$$

We gain from comparing the coefficients,

$$[J_0 = 0, J_1 = 0, J_2 = 0, J_3 = 0, J_4 = 0, J_5 = 0, J_6 = 0], \quad (4.40)$$

By resolving Eq. (4.40), following are some helpful conclusions that we reach.

Solution 1:

$$a = a, \mu = \sqrt{2q^2 - 2s}, d_0 = 0, d_1 = \frac{2c_0}{\sqrt{2a+2}}, \lambda = 0, c_1 = c_0, q = q, c_0 = c_0, s = s$$

Result I: When $\lambda \neq 0$ and $\mu^2 - 4\lambda = \varphi > 0$,

$$\delta_1 = \frac{1}{2} \frac{\sqrt{2q^2 - 2s}\sqrt{2a+2}}{e^{\frac{\sqrt{2q^2 - 2s}Ut^\beta - x}{\Gamma(\beta+1)}} - 1} \quad (4.41)$$

$$\rho_1 = \frac{1}{4} \frac{(2a+2)(2q^2 - 2s)}{(a^2 + 1) \left(e^{\frac{\sqrt{2q^2 - 2s}Ut^\beta - x}{\Gamma(\beta+1)}} - 1 \right)^2} \quad (4.42)$$

Result II: Whenever $\lambda \neq 0$ and $\mu^2 - 4\lambda = \varphi < 0$,

$$\delta_2 = -\frac{1}{2} \frac{\left(\frac{Ut^\beta}{\Gamma(\beta+1)} - x \right) \sqrt{2a+2}(q^2 - s)}{\frac{\sqrt{2q^2 - 2s}Ut^\beta}{\Gamma(\beta+1)} - \sqrt{2q^2 - 2s}x - 1} \quad (4.43)$$

$$\rho_2 = \frac{1}{4} \frac{\left(\frac{Ut^\beta}{\Gamma(\beta+1)} - x \right)^2 (2a+2)(q^2 - s)^2}{(a^2 + 1) \left(-\sqrt{2q^2 - 2s}x - 1 + \frac{\sqrt{2q^2 - 2s}Ut^\beta}{\Gamma(\beta+1)} \right)^2} \quad (4.44)$$

Result III: However $\mu^2 - 4\lambda = \varphi > 0$, $\lambda = 0$ and $\mu \neq 0$,

$$\delta_3 = -\frac{1}{2} \frac{\sqrt{2+2a}}{\frac{Ut^\beta}{\Gamma(\beta+1)} - x} \quad (4.45)$$

$$\rho_3 = \frac{1}{4} \frac{(2+2a)}{(a^2 + 1) \left(\frac{Ut^\beta}{\Gamma(\beta+1)} - x \right)^2} \quad (4.46)$$

Solution 2:

$$s = s, \mu = \mu, \lambda = 0, d_1 = d_1, q = q, c_1 = -\frac{\sqrt{aq^2 - as + q^2 - s}d_1}{\lambda}, d_0 = d, \\ c_0 = -\frac{\sqrt{aq^2 - as + q^2 - s}(\mu d_0 - d_1)}{\mu}$$

Solution 3:

$$s = s, c = -c_0, \mu = 0, d_1 = d, d_0 = d_0, \lambda = 0, q = q, c_0 = c_0$$

Solution 4:

$$s = s, \mu = \mu, d_0 = 0, d_1 = \frac{c_0\mu}{\sqrt{aq^2 - as + q^2 - s}}, c_1 = -c_0, \lambda = 0, q = q, c_0 = c_0$$

Solution 5:

$$s = s, \mu = \mu, d_0 = 0, d_1 = \frac{c_0\mu}{\sqrt{aq^2 - as + q^2 - s}}, c_1 = -c_0, \lambda = 0, q = q, c_0 = c_0$$

Additionally, we may look at the additional outcomes for solutions 2 and 3.

5. Physical Illustrations

This section includes a series of three-dimensional plots that numerically and visually represent the solutions of the proposed nonlinear coupled systems. The graphs are constructed at various time levels and for a continuous range of the parameter α .

Figure 1 illustrates the configuration of soliton solutions for u_5 and v_5 of the Higgs model for $s = 11$, $\beta = 1$, $U = 200$, $q = 12$, $a = 11$, and $\eta = 0.01$. Solitons represent a distinct class of solitary waves that retain their unique characteristics even when interacting with other solitons.

Figure 2 and Figure 3 illustrate the kink wave solutions of v_7 , u_7 , v_{12} , and u_{12} for different values of $s = 21$, $q = 2$, $U = 100$, $\beta = 1$, $a = 5$, and $\eta = 400$. The kink wave is a propagating wave solution characterized by a smooth transition between two distinct asymptotic states, either monotonically increasing or decreasing. As the spatial variable of distance approaches infinity, the wave solution asymptotically converges to a constant equilibrium value. Figure 4 represents the singular kink wave solutions of ρ_1 and δ_1 of the Maccari system for specific parameter values $a = 5$, $\mu = 10$, $q = 4$, $U = 40$, $s = 1$, and $\beta = 0.25$.

Figure 5 also shows the periodic and kink wave solutions of ρ_1 and δ_1 for different parameters $a = 5$, $\mu = 10$, $q = 4$, $U = 2$, $s = 10$, $\beta = 1$, and $\lambda = 1$. For $M > 4$, the solutions exhibit elevated soliton profiles, demonstrating that the system produces several soliton solutions of any order.

Wave solutions that repeat in a predictable manner after fixed intervals of time are known as *periodic solutions*, such as $\cos(\kappa - t)$. The remaining three-dimensional figures are omitted here for simplicity.

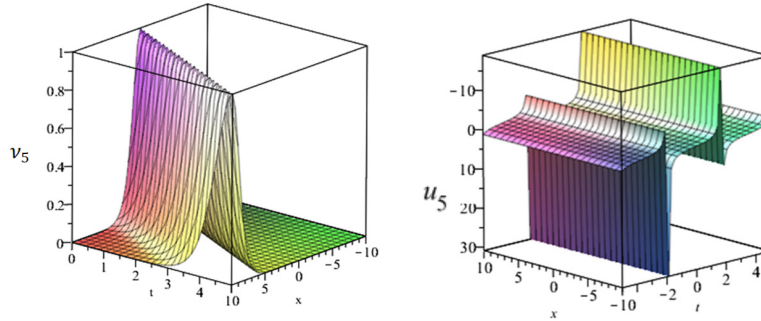


Figure 1: Soliton solution 3-D graphs of v_5 and u_5 for $-4 \leq t \leq 4$ and $-10 \leq x \leq 10$, respectively.

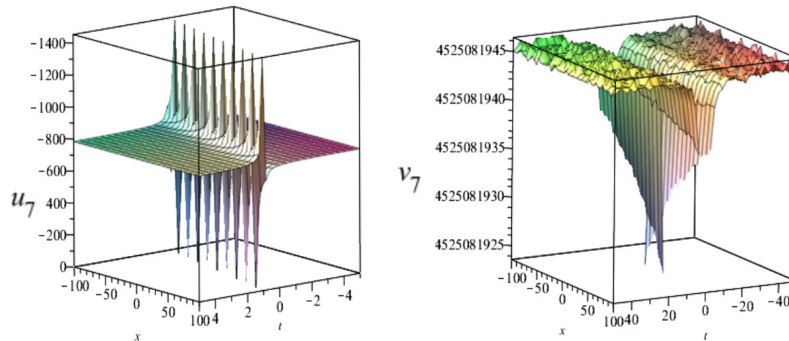


Figure 2: Kink-wave profile three-dimensional graphs of v_7 and u_7 for $-4 \leq t \leq 4$, $-40 \leq t \leq 40$, and $-100 \leq x \leq 100$, respectively.

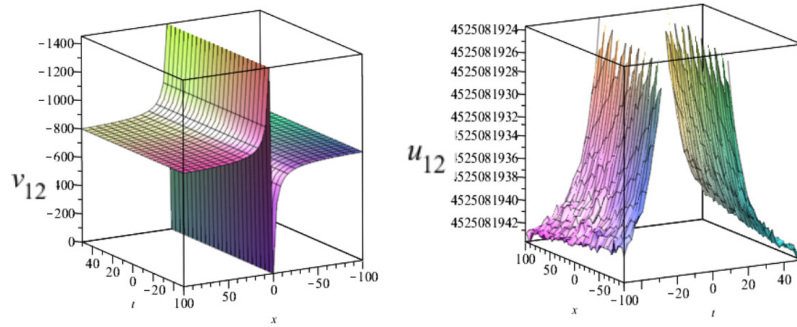


Figure 3: Kink and single-wave outcomes in three-dimensional graphs of v_{12} and u_{12} for $-40 \leq t \leq 40$ and $-100 \leq x \leq 100$, respectively.

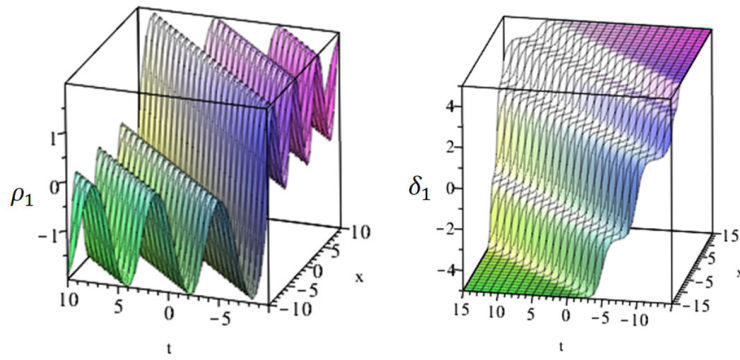


Figure 4: Singular-kink wave solution three-dimensional plots of ρ_1 and δ_1 for $-10 \leq x \leq 10$, $-15 \leq x \leq 15$, and $-10 \leq t \leq 10$, respectively.

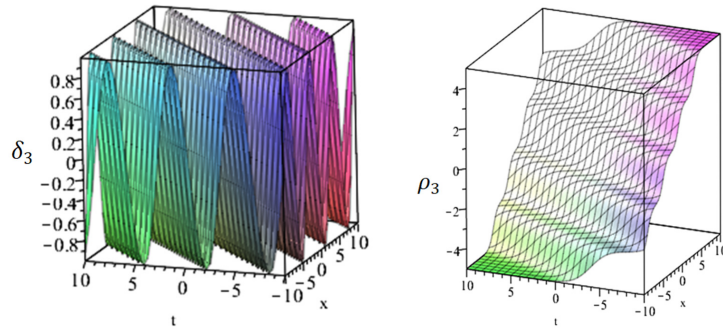


Figure 5: Periodic wave and kink-wave results in three-dimensional plots of δ_3 and ρ_3 for $-10 \leq \kappa \leq 10$, respectively.

6. Results and Discussion

This section presents a comparative analysis of the outcomes obtained through the rational $\exp\left(-\xi\left(\frac{\phi'}{\phi}\right)\right)$ -expansion method for the coupled Higgs and Maccari systems against the results derived from alternative methodologies in existing literature.

The solutions generated in this study demonstrate notable novelty when juxtaposed with the findings of S. T. Mohyud-Din *et al.* [18] and Lee *et al.* [32], as illustrated in Table 1. To broaden the scope of potential applications for this model across diverse scientific fields, the solutions must encompass a comprehensive spectrum of dynamical behaviors.

Notably, within the Higgs system framework:

- Solutions 4 and 5 align with Solution 1 under specific parameter conditions,
- Solutions 2 and 3 correspond to Solution 2.

The proposed method demonstrates enhanced generality, robustness, and efficiency, yielding innovative solitary wave solutions through strategic parameter optimizations. Comparative evaluations against the methodologies of Ali Akbar *et al.* [5], Lee *et al.* [32], and Manafian *et al.* [10,31] reveal the superior performance of our approach.

For instance, in the Higgs equations:

$$(v_4, u_4, v_8, u_8) \equiv (v_2, u_2, v_{10}, u_{10})$$

under adjusted parametric configurations (see Table 2).

Similarly, for the Maccari system:

$$(\delta_3, \rho_3) \equiv (u_{20})$$

when optimal parameter values are designated (see Table 3).

These counterparts underscore the adaptability of the $\exp\left(-\xi\left(\frac{\phi'}{\phi}\right)\right)$ -expansion technique in generating versatile and physically meaningful results across distinct nonlinear systems.

Table 1: A comparative analysis of our solutions and Lee et.al [32] and S.T Mohyud-Din et.al [18] solutions.

For Higgs equation	
Current solutions	Lee et.al [32] and S.T Mohyud-Din et.al [18] solutions
Sol 2: $a = -1, c_1 = c_1, \mu = \mu, d_0 = d_0, c_0 = c_0, q = -s, d_1 = d_1$	Sol 2: $a = -1, q = \pm s, d_0 = d_0, s = s, c_0 = c_0, c_1 = c_1, d_1 = d_1$

Sol 3: $c = -1, \mu = \mu, c_0 = c_0, c_1 = c_1, q = s, d_0 = d_0, d_1 = d_1$	
Sol 4: $a = 1, \mu = \mu, d_0 = d_0, c_1 = c_1, c_0 = c_0, q = -s, d_1 = d_1$	Sol 1: $a = 1, c_1 = c_1, q = \pm s, s = s, c_0 = c_0, d_0 = d_0, d_1 = d_1$
Sol 5: $c = 1, d_0 = d_0, \mu = \mu, c_1 = c_1, c_0 = c_0, q = s, d_1 = d_1$	

Table 2: A comparative analysis of our solutions and Ali Akbar et.al [5], Lee et.al [32] and Manafian et.al [10,31] solutions.

For Higgs equation	
Current solutions	Ali Akbar et.al [5], Lee et.al [32] and Manafian et.al [10, 31] solutions
$v_8 = \frac{\sqrt{-(q^2c_1 - s^2c_1 + 8\lambda c_0)c_0\lambda}}{\tanh\left(\frac{1}{2}\left(\frac{Ut^\beta}{\Gamma(\beta+1)} - \chi\right)\sqrt{2}\sqrt{\frac{c_0\lambda}{c_1}}\right)\sqrt{\frac{c_0\lambda}{c_1}}c_1}$ $u_8 = -\frac{q^2c_1 - s^2c_1 + 8\lambda c_0}{(c^2 + 1)c_1}$ $\times \frac{1}{\tanh\left(\frac{1}{2}\left(\frac{Ut^\beta}{\Gamma(\beta+1)} - \chi\right)\sqrt{2}\sqrt{\frac{c_0\lambda}{c_1}}\right)^2}$ <p>when these values are used for the parameters $\lambda = 1, u = 1, c_1 = 2, c_0 = 1, E = 0, q = 1$</p> $v_4 = \frac{c_0\mu^2\left(\frac{Ut^\beta}{\Gamma(\beta+1)} - \chi\right)}{Ut^\beta d_0/\Gamma(\beta+1) - 2Ut^\beta d_1}$ $\times \frac{1}{\Gamma(\beta+1) - \mu^2\chi d_0 + 2\mu\chi d_1 + 2d_1}$ $u_4 = \frac{c_0^2\mu^4\left(\frac{Ut^\beta}{\Gamma(\beta+1)} - \chi\right)^2}{(c^2 + 1)}$ $\times \frac{1}{\left(\frac{Ut^\beta d_0}{\Gamma(\beta+1)} - \frac{2Ut^\beta d_1}{\Gamma(\beta+1)} - \mu^2\chi d_0 + 2\mu\chi d_1 + 2d_1\right)^2}$	$v_2 = \left\{ -B_1 \left(\frac{2\lambda}{\sqrt{\Omega} \tanh\left(\frac{\sqrt{\Omega}}{2}(x-t+E)\right) + \mu} \right) + B_0 e^{\pm is(x\pm t)} \right.$ $u_2 = \left\{ B_0 - B_1 \left(\frac{2\lambda}{\sqrt{\Omega} \tanh\left(\frac{\sqrt{\Omega}}{2}(x-t+E)\right) + \mu} \right) \right\}^{2 \times \frac{1}{2}}$ <p>when these values are used for the parameters $\lambda = 1, u = 1, c_1 = 2, c_0 = 1, E = 0, q = 1$</p> $v_{10} = \left\{ B_0 + \frac{B_1}{(x-t+E)} \right\} e^{\pm is(x\pm t)}$ $u_{10} = \frac{1}{2} \times \left\{ B_0 + \frac{B_1}{(x-t+E)} \right\}^2$ <p>when these values are used for the parameters $a = 1, d_0 = -1, u = 1, d_1 = -1, s = 0$</p> $v_2 = \left\{ B_0 - B_1 \left(\frac{2\lambda}{\sqrt{\Omega} \tanh\left(\frac{\sqrt{\Omega}}{2}(x-t+E)\right) + \mu} \right) \right\} e^{is(x\pm t)}$ <p>when these values are used for the parameters $d_0 = 0, d_1 = 1, E = 0$</p> $v_{10} = \left\{ B_0 + \frac{B_1}{(x-t+E)} \right\} e^{\pm is(x\pm t)}$ $u_{10} = \frac{1}{2} \times \left\{ B_0 + \frac{B_1}{(x-t+E)} \right\}^2$ <p>when these values are used for the parameters $a = 1, u = 1, d_0 = -1, d_1 = -1, s = 0$</p> $v_2 = \left\{ B_0 - B_1 \left(\frac{2\lambda}{\sqrt{\Omega} \tanh\left(\frac{\sqrt{\Omega}}{2}(x-t+E)\right) + \mu} \right) \right\} e^{is(x\pm t)}$ <p>when these values are used for the parameters $d_0 = 0, d_1 = 1, E = 0$</p>

Table 3: A comparative analysis of our solutions and Ali Akbar et.al [5], Lee et.al [32] and Manafian et.al [10,31] solutions.

For Maccari System	
Current solutions	Ali Akbar et.al [5], Lee et.al [32] and Manafian et.al [10,31] solutions
$\delta_3 = -\frac{\sqrt{(a+1)(q^2-s)}}{\mu \left(\frac{Ut^\beta}{\Gamma(\beta+1)} - \chi \right)}$	$u_{20} = \frac{-B_1^2}{(a+1)} \times \left(\frac{1}{\eta+E} + \frac{\mu}{2} \right)^2$ <p>when these values are used for the parameters $a = 1, s = 1, u = -\frac{1}{2}, \mu = 1$</p>
$\rho_3 = \frac{(a+1)(q^2-s)}{\mu^2(a^2+1)} \left(\frac{Ut^\beta}{\Gamma(\beta+1)} - \chi \right)^2$	$u_{20} = \frac{-B_1^2}{(a+1)} \times \left(\frac{1}{\eta+E} + \frac{\mu}{2} \right)^2$ <p>when these values are used for the parameters $a = 1, E = 0, B_1 = 1, \mu = 0$</p>

7. Conclusion

This study determines the efficacy of the extended rational $\exp\left(-\xi \left(\frac{\phi'}{\phi}\right)\right)$ -expansion method in constructing exact solitary wave solutions for the Higgs equation and the Maccari system. Although retaining its mathematical rigor, the approach proved straightforward, yet powerful in significantly minimizing computational complexity compared to existing techniques. Through the adaptive framework of this method, we derived novel solutions expressed in exponential, hyperbolic, and logarithmic forms, including kink-type waves, periodic structures, and singular solitons that align with the physical behavior of the studied models.

The graphical visualizations of these solutions further validate the precision and reliability of this method in capturing nonlinear wave dynamics. Though applied here to two specific systems, the flexibility of this technique suggests its utility for solving broader classes of coupled nonlinear equations in fields such as optics, fluid dynamics, and plasma physics. While maintaining simplicity with analytical depth, this approach provides significant advantages over conventional methods, particularly in its capacity to derive physically interpretable solutions without imposing restrictive constraints or assumptions. Ultimately, the proposed method demonstrates transparency, computational simplicity, and high efficacy in obtaining precise solutions for numerous nonlinear coupled models with physical significance, in comparison to other techniques.

References

1. K. Khan and M. A. Akbar, The $\exp(-\Phi(\xi))$ -expansion method for finding traveling wave solutions of Vakhnenko-Parkes equation, *Int. J. Dyn. Syst. Differential Equat.*, 5(1), 72–83, 2014a.
2. M. N. Alam, M. A. Akbar and R. Islam, Traveling wave solutions of the simplified MCH equation via $\exp(-\Phi(\xi))$ -expansion method, *British J. Math. Comput. Sci.*, 5(5), 595–605, 2015.
3. A. J. M. Jawad, M. D. Petkovic and A. Biswas, Modified simple equation method for nonlinear evolution equations, *Appl. Math. Comput.*, 217, 869–877, 2010.
4. E. M. E. Zayed, H. A. Zedan and K. A. Gepreel, On the solitary wave solutions for nonlinear Hirota-Sasuma coupled KDV equations, *Chaos Solit. Fract.*, 22, 285–303, 2004.
5. M. N. Alam, M. G. Hafez, F. B. M. Belgacem and M. A. Akbar, Applications of the novel (G'/G) -expansion method to find new exact traveling wave solutions of the nonlinear coupled Higgs field equation, *Nonlinear Studies*, 22(4), 1–21, 2015.
6. G. Ebadi and A. Biswas, The (G'/G) -expansion method and topological soliton solution of the $K(m, n)$ equation, *Commun. Nonlin. Sci. Numer. Simulat.*, 16, 2377–2382, 2011.
7. B. S. Bahrami, H. Abdollahzadeh, I. M. Berijani, D. D. Ganji and M. Abdollahzadeh, Exact travelling wave solutions for some nonlinear physical models by (G'/G) -expansion method, *Pramana J. Phys.*, 77, 263–275, 2011.
8. Y. Yildirim, S. T. Mohyud-Din and S. Sariaydin, Numerical comparison for the solutions of anharmonic vibration of fractionally damped nano-sized oscillator, *J. King Saud Univ. Sci.*, 23, 205–209, 2011.

9. M. Najafi, S. Arbabi and M. Najafi, He's semi-inverse method for Camassa–Holm equation and simplified modified Camassa–Holm equation, *Int. J. Phys. Res.*, 1(1), 1–6, 2013.
10. J. Manafian and I. Zamanpour, Analytical treatment of the coupled Higgs equation and Maccari system via exp-function method, *Apulensis A. U.*, 33, 203–216, 2013.
11. E. Yusufoglu, New solitary solutions for the MBBN equations using exp-function method, *Phys. Lett. A*, 372, 442–446, 2008.
12. J. H. He and M. A. Abdou, New periodic solutions for nonlinear evolution equation using Exp-method, *Chaos Solit. Fract.*, 34, 1421–1429, 2007.
13. S. T. Mohyud-Din, M. A. Noor and A. Waheed, Exp-function method for generalized travelling solutions of good Boussinesq equations, *J. Appl. Math. Comput.*, 30, 439–445, 2009.
14. A. R. Seadawy and D. Lu, Dispersive solitary wave solutions of new coupled Konno-Oono, Higgs field and Maccari equations and their applications, *J. King Saud Univ. Sci.*, 30(3), 417–423, 2018.
15. A. Yildırım and H. Kocak, Homotopy perturbation method for solving the space-time fractional advection-dispersion equation, *Adv. Water Resour.*, 32, 1711–1716, 2009.
16. H. A. Nassar, M. A. Abdel-Razek and A. K. Seddeek, Expanding the tanh function method for solving nonlinear equations, *Appl. Math.*, 2, 1096–1104, 2011.
17. M. A. Abdou, The extended tanh-method and its applications for solving nonlinear physical models, *Appl. Math. Comput.*, 190, 988–996, 2007.
18. A. Ali, M. A. Iqbal and S. T. Mohyud-Din, New analytical solutions for nonlinear physical models of the coupled Higgs equation and the Maccari system via rational $\exp(-\phi(\eta))$ -expansion method, *Pramana*, 87(5), 79, 2016.
19. Y. He, S. Li and Y. Long, Exact solutions of the Klein-Gordon equation by modified Exp-function method, *Int. Math. Forum*, 7(4), 175–182, 2012.
20. Y. L. Ma, B. Q. Li and Y. Y. Fu, A series of the solutions for the Heisenberg ferromagnetic spin chain equation, *Math. Methods Appl. Sci.*, 41(9), 3316–3322, 2018.
21. B. S. Ahmed, A. Biswas, E. V. Krishnan and S. Kumar, Solitons and other solutions to the generalized Maccari system, *Romanian Reports Phys.*, 65(4), 1138–1154, 2013.
22. N. T. Shawagfeh, Analytical approximate solutions for nonlinear fractional differential equations, *Appl. Math. Comput.*, 31(2-3), 517–529, 2002.
23. A. A. Kilbas, H. M. Srivastava and J. J. Trujillo, *Theory and Applications of Fractional Differential Equations*, Comput. Math. Appl., 204, 1079–1087, 2006.
24. X. J. Yang, D. Baleanu, H. M. Srivastava and J. A. Tenreiro Machado, On local fractional continuous wavelet transform, *Abstr. Appl. Anal.*, 5 pages, 2013.
25. X. J. Yang, *Advanced Local Fractional Calculus and Its Applications*, World Science, New York, NY, USA, 2012.
26. X. J. Yang, D. Baleanu and J. A. T. Machado, Mathematical aspects of Heisenberg uncertainty principle within local fractional Fourier analysis, *Boundary Value Problems*, 1, 131–146, 2013.
27. M. Matinfar and M. Saeidy, Application of homotopy analysis method to fourth order parabolic partial differential equations, *Appl. Appl. Math.*, 5, 70–80, 2010.
28. M. G. Hafez, M. N. Alam and M. A. Akbar, Traveling wave solutions for some important coupled nonlinear physical models via the coupled Higgs equation and the Maccari system, *J. King Saud Univ. Sci.*, 27, 105–112, 2015.
29. Sirendaoreji, New exact travelling wave solutions for the Kawahara and modified Kawahara equations, *Chaos Solit. Fract.*, 1, 147–150, 2004.
30. X. J. Yang and D. Baleanu, Fractal heat conduction problem solved by local fractional variation iteration method, *Thermal Science*, 17(2), 625–628, 2013.
31. J. Manafian and J. Zamanpour, Modification of truncated expansion method to some complex nonlinear partial differential equations, *Apulensis A. U.*, 33, 109–216, 2013.
32. J. Lee and J. Sakthivel, Exact travelling wave solutions for some important nonlinear physical models, *Pramana J. Phys.*, 80, 757–769, 2013.
33. B. Cockburn, B. Singler and Z. Zhang, Interpolatory HDG method for parabolic semilinear PDEs, *J. Sci. Comput.*, 79(3), 1777–1800, 2019.
34. A. Gaber, A. Aljohani, A. Ebaid and J. A. Machado, The generalized Kudryashov method for nonlinear space–time fractional partial differential equations of Burgers type, *Nonlinear Dynamics*, 95(1), 361–368, 2019.
35. Y. Jiang, D. Q. Xian and X. R. Kang, Homoclinic breather and rogue wave solutions to Maccari equation, *Comput. Math. Appl.*, 2018.
36. S. Shakeel, S. T. Mohyud-Din and M. A. Iqbal, Closed form solutions for coupled nonlinear Maccari system, *Comput. Math. Appl.*, 76(4), 799–809, 2018.

37. T. Bulut, A. S. Sulaiman and T. Akturk, Complex acoustic gravity wave behaviors to some mathematical models arising in fluid dynamics and nonlinear dispersive media, *Opt. Quantum Electron.*, 50(1), 19, 2018.
38. A. Ahmad, M. Ahmad, Y. B. Yusof, M. Muawwaz, M. Maaz and Ather Qayyum. An Extension of Caputo's k-Fractional Derivative Operator and its Applications, *Jordan Journal of Mathematics and Statistics*, 18(4), 651–666, 2025.

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