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Fixed Points of α_s-Interpolative Contractions in S-Metric Spaces

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ABSTRACT: Interpolative contraction is one of the generalization of Banach contraction, recently added in the literature. In this paper, we introduce interpolative contraction of Kannan and Ćirić-Reich-Rus in S-metric spaces via α-admissible mappings. Further, we prove some fixed point theorems for these contractions. We also give an example and discuss various consequences.

Key Words: Fixed point, α-admissible, α-orbital admissible, S-metric space, α_s-interpolative-Reich-Rus type contraction.

Contents

1. Introduction

Kannan [1,2] generalized Banach contraction principle [3] making continuity of the mapping not essential. Karapinar [4] introduced interpolative contraction to generalize Banach and Kannan contractions. He said that in a metric space (X, d), a mapping $T : X \to X$ is an interpolative Kannan type contraction, if there exist constants $\lambda \in [0, 1)$ and $\alpha \in (0, 1)$ such that $d(Tx, Ty) \leq \lambda[d(x, Tx)]^{\alpha}.[d(y, Ty)]^{1-\alpha}$ for all $x, y \in X$ with $x \neq Tx$. He also state the corresponding fixed point theorem as "In a complete metric space (X, d) an interpolative Kannan contraction mapping $T : X \to X$ has a unique fixed point in X".

In [5], Karapinar et al. gave an example ([5], Example 1) showcasing that the fixed point is not necessarily unique and modified the theorem statement as "In a complete metric space (X, d), a mapping $T : X \to X$ possesses a fixed point in X, if there exist constants $\lambda \in [0, 1)$ and $\alpha \in (0, 1)$ such that $d(Tx, Ty) \leq \lambda[d(x, Tx)]^{\alpha}.[d(y, Ty)]^{1-\alpha}$ for all $x, y \in X - Fix(T)$".
Throughout this paper, $Fix(T)$ will denote the collection of all fixed points of T, or points $a \in X$ such that $Ta = a$.

Following theorem was given in [6] stating that was proved independently by Reich, Rus and Ćirić [7,8,9,10,11,12,13,14] to combine and improve fixed point theorems of Banach as well as Kannan.

Theorem 1.1 *[6] "A mapping T on a complete metric space (X, d) satisfying:*

$$d(Tx, Ty) \leq \lambda[d(x, y) + d(x, Tx) + d(y, Ty)],$$

for all $x, y \in X$, where $\lambda \in [0, \frac{1}{3})$, has a distinct fixed point".

Following variation of Reich was also stated in [5];

$$d(Tx, Ty) \leq ad(x, y) + bd(x, Tx) + cd(y, Ty),$$

where $a, b, c \in (0, \infty)$ such that $0 \leq a + b + c < 1$.

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Let Ψ be the collection of all nondecreasing mappings ψ constructed on the interval $[0, +\infty)$ with:

$$\sum_{n=1}^{\infty} \psi^n(t) < \infty$$

for each $t > 0$.

Observe that given $\psi \in \Psi$, it maintains that $\psi(0) = 0$ and $\psi(t) < t$ across every $t > 0$.

Many papers used and generalized above concept in order to prove variant (common) fixed point results (see, for instance, [6]).

Samet et al. [15] gave the following definition of α-admissible.

Definition 1.1 *[15] Let $T : X \to X$ be a mapping and $\alpha : X \times X \to [0, +\infty)$ be a function. We identify T as α-admissible if for $x, y \in X$, the condition $\alpha(x, y) \geq 1$ necessitates that $\alpha(Tx, Ty) \geq 1$.*

Popescu [16] gave the definition of α-orbital admissible.

Definition 1.2 *[16] Let $T : X \to X$ be a mapping and $\alpha : X \times X \to [0, +\infty)$ be a function. We identify T as α-orbital admissible if for $x \in X$, the condition $\alpha(x, Tx) \geq 1$ necessitates that $\alpha(Tx, T^2x) \geq 1$.*

Sedghi et al. [17] define S-metric space as follows

Definition 1.3 *[17] A mapping $S : X^3 \to [0, +\infty)$, with X as a nonempty set, is classified as an S-metric space if it fulfills the subsequent circumstances for every $x_1, x_2, x_3, t \in X$:*

(1) $S(x_1, x_2, x_3) \geq 0$

(2) $S(x_1, x_2, x_3) = 0$ *iff* $x_1 = x_2 = x_3 = 0$

(3) $S(x_1, x_2, x_3) \leq S(x_1, x_1, t) + S(x_2, x_2, t) + S(x_3, x_3, t)$

The duo (X, S) is termed as S-metric space.

For detail discussion about S-metric space we refer the reader to [17].

α-admissible and its various form are extended to S-metric spaces by Priyobarta et al. [18],Khomdram et al. [19] and Poddar & Rohen [20]. Here, for requirement we pick up the following definition of α-admissible in S- metric space.

Definition 1.4 *[18] Let (X, S) be an S-metric space, $T : X \to X$, and $\alpha_s : X^3 \to [0, +\infty)$. Then T is termed α_s-admissible if for $x, y, z \in X$, the condition $\alpha_s(x, y, z) \geq 1$ necessitates that $\alpha_s(Tx, Ty, Tz) \geq 1$.*

In the same line, we gave the following definition of α-orbital admissible in S-metric spaces, by extending Definition 1.2.

Definition 1.5 *Let (X, S) be an S-metric space, $T : X \to X$, and $\alpha_s : X^3 \to [0, +\infty)$. Then T is called α_s-orbital admissible if for $x \in X$, the condition $\alpha_s(x, x, Tx) \geq 1$ necessitates that $\alpha_s(Tx, Tx, T^2x) \geq 1$.*

2. Main Results

We commence with the principal outcome of our research by presenting the following definition.

Definition 2.1 *The mapping T on the S-metric space (X, S) is designated as an α_s-interpolative Kannan type contraction if there occurs a function $\psi \in \Psi$ and a mapping $\alpha_s : X \times X \times X \to [0, +\infty)$, with $p, q \in (0, 1)$ such that*

$$\alpha_s(x, y, z)S(Tx, Ty, Tz) \leq \psi([S(x, x, Tx)]^p [S(y, y, Ty)]^q [[S(z, z, Tz)]^{1-p-q}) \tag{2.1}$$

for each $x, y, z \in X - Fix(T)$.

The principal outcome is presented as follows.

Theorem 2.1 *Let $T : X \to X$ be a mapping defined on the complete S-metric space (X, S) that follows the subsequent circumstances:*

1. *T is continuous.*
2. *T is α_s-orbital admissible.*
3. *There occurs an element $x_0 \in X$ that gives $\alpha_s(x_0, x_0, Tx_0) \geq 1$.*
4. *T is α_s-interpolative Kannan type contraction.*

Then a fixed point of T exists in X.

Proof:
Let $x_0 \in X$ be a point that corresponds to $\alpha_s(x_0, x_0, Tx_0) \geq 1$. Let $\{x_n\}$ be the sequence defined by $x_n = T^n(x_0)$ for $n \geq 1$. If for a certain n_0, $x_{n_0} = x_{n_0+1}$, then x_{n_0} becomes a fixed point of T, otherwise, $x_n \neq x_{n+1}$ for all $n \geq 1$. We have $\alpha_s(x_0, x_0, x_1) \geq 1$. Since T is α_s-orbital admissible,

$$\alpha_s(x_1, x_1, x_2) = \alpha_s(Tx_0, Tx_0, Tx_1) \geq 1.$$

Continuing as above, we obtain that

$$\alpha_s(x_n, x_n, x_{n+1}) \geq 1 \quad for \quad all \quad n \geq 0 \tag{2.2}$$

Taking $x = y = x_{n-1}$ and $z = x_n$ in (2.1), we find that

$$\begin{aligned} S(x_n, x_n, x_{n+1}) &\leq \alpha_s(x_{n-1}, x_{n-1}, x_n) S(Tx_{n-1}, Tx_{n-1}, Tx_n) \\ &\leq \psi([S(x_{n-1}, x_{n-1}, Tx_{n-1})]^p [S(x_{n-1}, x_{n-1}, Tx_{n-1})]^q [S(x_n, x_n, Tx_n)]^{1-p-q}) \\ &= \psi([S(x_{n-1}, x_{n-1}, x_n)]^{p+q} [S(x_n, x_n, x_{n+1})]^{1-p-q}) \end{aligned} \tag{2.3}$$

Specifically, since $\psi(t) < t$ for any $t > 0$,

$$S(x_n, x_n, x_{n+1}) \leq [S(x_{n-1}, x_{n-1}, x_n)]^{p+q} [S(x_n, x_n, x_{n+1})]^{1-p-q} \tag{2.4}$$

We derive

$$[S(x_n, x_n, x_{n+1})]^{p+q} < [S(x_{n-1}, x_{n-1}, x_n)]^{p+q}$$

Therefore,

$$S(x_n, x_n, x_{n+1}) < S(x_{n-1}, x_{n-1}, x_n) \quad for \quad all \quad n \geq 1. \tag{2.5}$$

Therefore, the positive sequence $\{S(x_{n-1}, x_{n-1}, x_n)\}$ is monotonically decreasing. Ultimately, we have a real number $l \geq 0$ that gives $\lim_{n \to +\infty} S(x_{n-1}, x_{n-1}, x_n) = l$. Taking into account (2.5), so (2.3) in conjunction coupled with the nondecreasing nature of ψ results in:

$$\begin{aligned} S(x_n, x_n, x_{n+1}) &\leq \psi([S(x_{n-1}, x_{n-1}, x_n)]^{1-p-q} [S(x_n, x_n, x_{n+1})]^{p+q}) \\ &\leq \psi[S(x_{n-1}, x_{n-1}, x_n)] \end{aligned}$$

By reiterating this contention, we get

$$\begin{aligned} S(x_n, x_n, x_{n+1}) &\leq \psi(S(x_{n-1}, x_{n-1}, x_n)) \leq \psi^2(S(x_{n-2}, x_{n-2}, x_{n-1})) \\ &\leq ... \leq \psi^n(S(x_0, x_0, x_1)) \end{aligned} \tag{2.6}$$

By letting $n \to +\infty$ in (2.6) and employing the fact that $\lim_{n\to+\infty} \psi^n(t) = 0$ for any $t > 0$, we conclude that $l = 0$, precisely,

$$\lim_{n\to+\infty} S(x_n, x_n, x_{n+1}) = 0$$

We claim that $\{x_n\}$ constitutes a Cauchy sequence, specifically that $\lim_{n\to+\infty} S(x_n, x_n, x_{n+p}) = 0$ for all $p \in \mathbb{N}$. By virtue of the triangle inequality in conjunction with (2.6), we ascertain:

$$\begin{aligned} S(x_n, x_n, x_{n+p}) &\leq 2\psi^n(S(x_0, x_0, x_1)) + ... + 2\psi^{n+p-2}(S(x_0, x_0, x_1)) + \psi^{n+p-1}(S(x_0, x_0, x_1)) \\ &\leq 2\sum_{i=n}^{n+p-1} \psi^i(S(x_0, x_0, x_1)) \end{aligned}$$

Taking the limit as n approaches infinity in the aforementioned inequality, we can deduce that the right-hand side converges to zero. Consequently, the series $\{x_n\}$ is a Cauchy sequence. Concerning the completeness of the S-metric space (X, S), we conclude that there exists an element $x \in X$ such that

$$\lim_{n\to+\infty} S(x_n, x_n, x) = 0 \tag{2.7}$$

Given that T is continuous, we obtain

$$x = \lim_{n\to+\infty} x_{n+1} = \lim_{n\to+\infty} Tx_n = T \lim_{n\to+\infty}(x_n) = Tx$$

□

Additionally, the condition labelled as (H) has frequently been examined to bypass the continuity of the relevant contractive mappings.

(H) If $\{x_n\}$ is a sequence in X such that $\alpha_s(x_n, x_n, x_{n+1}) \geq 1$ for each n and $x_n \to x \in X$ as $n \to +\infty$ then there exists $\{x_{n(k)}\}$ from $\{x_n\}$ such that $\alpha_s(x_{n(k)}, x_{n(k)}, x) \geq 1$ for each k.

In the meantime, we substitute the continuity criteria with the weakened condition (H).

Theorem 2.2 *Let $T : X \to X$ be a self-mapping defined on a complete S-metric space (X, S) that satisfies condition (H) together with the subsequent conditions:*

1. *T is α_s-orbital admissible.*
2. *There occurs an element $x_0 \in X$ that gives $\alpha_s(x_0, x_0, Tx_0) \geq 1$.*
3. *T is α_s-interpolative Kannan type contraction.*

Then a fixed point of T exists in X.

Proof:

According to the proof of Theorem 2.1, we deduce that the sequence $\{x_n\}$ is Cauchy and that (2.7) is satisfied. Assume that condition (H) is satisfied. We employ proof by contradiction by presuming that $x \neq Tx$. Note that $x_{n(k)} \neq Tx_{n(k)}$ for every $k \geq 0$. As a result of (H), there exists a partial subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha_s(x_{n(k)}, x_{n(k)}, x) \geq 1$ for all k. Since $S(x_{n(k)}, x_{n(k)}, x) \to 0$, $S(x_{n(k)}, x_{n(k)}, Tx_{n(k)}) \to 0$ and $S(x, x, Tx) > 0$, There occurs $N \in \mathbb{N}$ such that, for every $k \geq N$,

$$S(x_{n(k)}, x_{n(k)}, x) \leq S(x, x, Tx)$$

and

$$S(x_{n(k)}, x_{n(k)}, Tx_{n(k)}) \leq S(x, x, Tx).$$

Taking $x = y = x_{n(k)}$ and $z = x$ in (2.1), we get

$$\begin{aligned} S(x_{n(k)+1}, x_{n(k)+1}, Tx) &\leq \alpha_s(S(x_{n(k)}, x_{n(k)}, x))S(Tx_{n(k)}, Tx_{n(k)}, Tx) \\ &\leq \psi([S(x_{n(k)}, x_{n(k)}, Tx_{n(k)})]^p[S(x_{n(k)}, x_{n(k)}, Tx_{n(k)})]^q \\ &\quad [S(x, x, Tx)]^{1-p-q}) \end{aligned} \tag{2.8}$$

Since ψ is nondecreasing, it emerges from (2.8) that

$$\begin{aligned} S(x_{n(k)+1}, x_{n(k)+1}, Tx) &\leq \psi([S(x, x, Tx)]^p[S(x, x, Tx)]^q[S(x, x, Tx)]^{1-p-q}) \\ &= \psi(S(x, x, Tx)) \end{aligned}$$

Letting $k \to +\infty$, we find that

$$0 \leq S(x, x, Tx) \leq \psi(S(x, x, Tx)) < S(x, x, Tx)$$

which is a contradiction. Thus $x = Tx$. □

To start with the second main result of our study, we state the following definition

Definition 2.2 *Let (X, S) be an S-metric space. The mapping $T : X \to X$ is said to be an α_s-interpolative Ćirić-Reich-Rus-type contraction if there occurs $\psi \in \Psi$, $\alpha_s : X \times X \times X \to [0, +\infty)$ and positive reals $p, q, r > 0$, verifying $p + q + r < 1$, such that*

$$\alpha_s(x, y, z)S(Tx, Ty, Tz) \leq \psi([S(x, y, z)]^p[S(x, x, Tx)]^q[S(y, y, Ty)]^r[S(z, z, Tz)]^{1-p-q-r}) \tag{2.9}$$

for all $x, y, z \in X - Fix(T)$.

The following one is our second main result.

Theorem 2.3 *Let $T : X \to X$ be a self mapping defined on the complete S-metric space (X, S) satisfying the following conditions:*

1. *T is continuous.*
2. *T is α_s-orbital admissible.*
3. *There occurs an element $x_0 \in X$ that gives $\alpha_s(x_0, x_0, Tx_0) \geq 1$.*
4. *T is α_s-interpolative Ćirić-Reich-Rus type contraction.*

Then a fixed point of T exists in X.

Proof:
Let $x_0 \in X$ be a point such that $\alpha_s(x_0, x_0, Tx_0) \geq 1$. Let $\{x_n\}$ be the sequence defined by $x_n = T^n(x_0)$, $n \geq 1$. If for some n_0 , we have $x_{n_0} = x_{n_0+1}$, then x_{n_0} is a fixed point of T otherwise, $x_n \neq x_{n+1}$ for each $n \geq 1$. We have $\alpha_s(x_0, x_0, x_1) \geq 1$. Since T is α_s-orbital admissible,

$$\alpha_s(x_1, x_1, x_2) = \alpha_s(Tx_0, Tx_0, Tx_1) \geq 1.$$

Continuing as above, we obtain that

$$\alpha_s(x_n, x_n, x_{n+1}) \geq 1 \quad for \quad all \quad n \geq 0 \tag{2.10}$$

Taking $x = y = x_{n-1}$ and $z = x_n$ in (2.9), we find that

$$\begin{aligned} S(x_n,x_n,x_{n+1}) &\leq \alpha_s(x_{n-1},x_{n-1},x_n)S(Tx_{n-1},Tx_{n-1},Tx_n) \\ &\leq \psi([S(x_{n-1},x_{n-1},x_n)]^p[S(x_{n-1},x_{n-1},Tx_{n-1})]^q[S(x_{n-1},x_{n-1},Tx_{n-1})]^r[S(x_n,x_n,Tx_n)]^{1-p-q-r}) \\ &= \psi([S(x_{n-1},x_{n-1},x_n)]^{p+q+r}[S(x_n,x_n,x_{n+1})]^{1-p-q-r}) \end{aligned} \tag{2.11}$$

In particular, as $\psi(t) < t$ for each $t > 0$,

$$S(x_n,x_n,x_{n+1}) < [S(x_{n-1},x_{n-1},x_n)]^{p+q+r}[S(x_n,x_n,x_{n+1})]^{1-p-q-r} \tag{2.12}$$

We derive

$$[S(x_n,x_n,x_{n+1})]^{p+q+r} < [S(x_{n-1},x_{n-1},x_n)]^{p+q+r}$$

Therefore,

$$S(x_n,x_n,x_{n+1}) < S(x_{n-1},x_{n-1},x_n) \quad for \quad all \quad n \geq 1. \tag{2.13}$$

Hence, the positive sequence $\{S(x_{n-1},x_{n-1},x_n)\}$ is decreasing. Eventually, there is a real $l \geq 0$ in order that $\lim_{n\to+\infty} S(x_{n-1},x_{n-1},x_n) = l$. Taking into account (2.13),

$$\begin{aligned} [S(x_{n-1},x_{n-1},x_n)]^{p+q+r}[S(x_n,x_n,x_{n+1})]^{1-p-q-r} &\leq [S(x_{n-1},x_{n-1},x_n)]^{p+q+r}[S(x_{n-1},x_{n-1},x_n)]^{1-p-q-r} \\ &= S(x_{n-1},x_{n-1},x_n) \end{aligned}$$

so (2.11) along with the nondecreasing nature of ψ, this results in:

$$\begin{aligned} S(x_n,x_n,x_{n+1}) &\leq \psi([S(x_{n-1},x_{n-1},x_n)]^{p+q+r}[S(x_n,x_n,x_{n+1})]^{1-p-q-r}) \\ &\leq \psi[S(x_{n-1},x_{n-1},x_n)] \end{aligned}$$

By reiterating this argument, we obtain

$$\begin{aligned} S(x_n,x_n,x_{n+1}) &\leq \psi(S(x_{n-1},x_{n-1},x_n)) \leq \psi^2(S(x_{n-2},x_{n-2},x_{n-1})) \\ &\leq ... \leq \psi^n(S(x_0,x_0,x_1)) \end{aligned} \tag{2.14}$$

By letting $n \to +\infty$ in (2.14) and utilising the fact that $\lim_{n\to+\infty} \psi^n(t) = 0$ for any $t > 0$, we conclude that $l = 0$, that is,

$$\lim_{n\to+\infty} S(x_n,x_n,x_{n+1}) = 0$$

We claim that $\{x_n\}$ constitutes a Cauchy sequence, specifically that $\lim_{n\to+\infty} S(x_n,x_n,x_{n+p}) = 0$ for all $p \in \mathbb{N}$. By virtue of the triangle inequality in conjunction with (2.14), we ascertain:

$$\begin{aligned} S(x_n,x_n,x_{n+p}) &\leq 2\psi^n(S(x_0,x_0,x_1)) + ... + 2\psi^{n+p-1}(S(x_0,x_0,x_1)) \\ &\leq 2\sum_{i=n}^{+\infty} \psi^i(S(x_0,x_0,x_1)). \end{aligned}$$

Taking the limit as n approaches infinity in the aforementioned inequality, we may deduce that the right-hand side converges to zero. Consequently, the series $\{x_n\}$ is a Cauchy sequence. Concerning the completeness of the S-metric space (X,S), we conclude that there exists an element $x \in X$ such that

$$\lim_{n\to\infty} S(x_n,x_n,x) = 0. \tag{2.15}$$

Since T is continuous, we have

$$x = \lim_{n\to+\infty} x_{n+1} = \lim_{n\to+\infty} Tx_n = T \lim_{n\to+\infty}(x_n) = Tx.$$

Subsequently, we substitute the continuity criteria by a weakened condition (H).

□

Theorem 2.4 *Let $T : X \to X$ be a mapping defined on the complete S-metric space (X, S) satisfying condition (H) along with the following conditions:*

1. *T is α_s-orbital admissible.*
2. *There occurs an element $x_0 \in X$ that gives $\alpha_s(x_0, x_0, Tx_0) \geq 1$.*
3. *T is α_s-interpolative Ćirić–Reich-Rus type contraction.*

Then a fixed point of T exists in X.

Proof:

By the direct application of Theorem 2.3, we deduce that the sequence $\{x_n\}$ is Cauchy and that (2.15) is satisfied. Assume that condition (H) is satisfied. We employ proof by contradiction by presuming that $x \neq Tx$. Note that $x_{n(k)} \neq Tx_{n(k)}$ for every $k \geq 0$. As a result of (H), there exists a partial subsequence $\{x_{n(k)}\}$ of $\{x_n\}$ such that $\alpha_s(x_{n(k)}, x_{n(k)}, x) \geq 1$ for every k. Since $S(x_{n(k)}, x_{n(k)}, x) \to 0$, $S(x_{n(k)}, x_{n(k)}, Tx_{n(k)}) \to 0$ and $S(x, x, Tx) > 0$, there is $N \in \mathbb{N}$ such that, for each $k \geq N$,

$$S(x_{n(k)}, x_{n(k)}, x) \leq S(x, x, Tx)$$

and

$$S(x_{n(k)}, x_{n(k)}, Tx_{n(k)}) \leq S(x, x, Tx).$$

Taking $x = y = x_{n(k)}$ and $z = x$ in (2.9), we get

$$\begin{aligned} S(x_{n(k)+1}, x_{n(k)+1}, Tx) &\leq \alpha_s(x_{n(k)}, x_{n(k)}, x) S(Tx_{n(k)}, Tx_{n(k)}, Tx) \\ &\leq \psi([S(x_{n(k)}, x_{n(k)}, x)]^p [S(x_{n(k)}, x_{n(k)}, Tx_{n(k)})]^q \\ &\quad [S(x_{n(k)}, x_{n(k)}, Tx_{n(k)})]^r [S(x, x, Tx)]^{1-p-q-r}) \end{aligned} \tag{2.16}$$

so (2.16) with the nondecreasing nature of ψ, this results in

$$\begin{aligned} S(x_{n(k)+1}, x_{n(k)+1}, Tx) &\leq \psi([S(x, x, Tx)]^p [S(x, x, Tx)]^q [S(x, x, Tx)]^r [S(x, x, Tx)]^{1-p-q-r}) \\ &= \psi(S(x, x, Tx)) \end{aligned}$$

Letting $k \to +\infty$, we find that

$$0 \leq S(x, x, Tx) \leq \psi(S(x, x, Tx)) < S(x, x, Tx)$$

which is a contradiction. Thus $x = Tx$.

□

By considering $\alpha_s(x, y, z) = 1$ in Theorem 2.3, we state the following.

Corollary 2.1 *Let T be a continuous self-mapping on a complete S-metric space (X, S) such that*

$$\begin{aligned} S(Tx, Ty, Tz) &\leq \psi([S(x, x, Tx)]^p [S(y, y, Ty)]^q \\ &\quad [S(z, z, Tz)]^{1-p-q}) \end{aligned} \tag{2.17}$$

for all $x, y, z \in X - Fix(T)$, where $0 < p, q < 1$. Then, T possesses a fixed point in X.

Corollary 2.2 *Let T be a continuous self-mapping on a complete S-metric space (X,S) such that*

$$\begin{aligned} S(Tx,Ty,Tz) \leq\ & \psi([S(x,y,z)]^p[S(x,x,Tx)]^q \\ & [S(y,y,Ty)]^r[S(z,z,Tz)]^{1-p-q-r}) \end{aligned} \tag{2.18}$$

for all $x,y,z \in X - Fix(T)$, where $p,q,r > 0$ are positive reals satisfying $p+q+r < 1$. Then, T possesses a fixed point in X.

Taking $\psi(t) = \lambda t$ (where $\lambda \in [0,1)$) in Corollary 2.2, we state

Corollary 2.3 *Let T be a continuous self-mapping on a complete S-metric space (X,S) such that*

$$\begin{aligned} S(Tx,Ty,Tz) \leq\ & \lambda([S(x,y,z)]^p[S(x,x,Tx)]^q \\ & [S(y,y,Ty)]^r[S(z,z,Tz)]^{1-p-q-r}) \end{aligned} \tag{2.19}$$

for all $x,y,z \in X - Fix(T)$, where p,q,r are positive reals verifying $p+q+r < 1$ and $\lambda \in [0,1)$. Then, T possesses a fixed point in X.

Taking $p = 1$ and $q = r = 0$ in Corollary 2.3, we get Banach contraction principle in S-metric space. Which may be stated as

Let T be a continuous self-mapping on a complete S-metric space (X,S) such that

$$S(Tx,Ty,Tz) \leq \lambda([S(x,y,z)])$$

for all $x,y,z \in X - Fix(T)$. Then, T possesses a fixed point in X.

Taking $\psi(t) = \lambda t$ (where $\lambda \in [0,1)$) in Corollary 2.1, we state

Corollary 2.4 *Let T be a continuous self-mapping on a complete S-metric space (X,S) such that*

$$\begin{aligned} S(Tx,Ty,Tz) \leq\ & \lambda([S(x,x,Tx)]^p[S(y,y,Ty)]^q \\ & [S(z,z,Tz)]^{1-p-q}) \end{aligned} \tag{2.20}$$

for all $x,y,z \in X - Fix(T)$, where $0 < p,q < 1$ and $\lambda \in [0,1)$. Then, T possesses a fixed point of T.

Example 2.1 *Let us consider the set $X = [a,b]$ with the S-metric defined as follows $S(x,y,z) =\mid x-y \mid + \mid y-z \mid + \mid z-x \mid$. Let T be a self mapping on X defined by:*

$$Tx = \begin{cases} \frac{a+b}{2}, & if \quad x \in [c,b], \quad a < c < \frac{a+b}{2} \\ a, & otherwise. \end{cases}$$

Take

$$\alpha_s(x,y,z) = \begin{cases} 1, & if \quad x,y,z \in [c,b], \\ 0, & otherwise. \end{cases}$$

Let $x,y,z \in X$ be such that $x \neq Tx$, $y \neq Ty$, $z \neq Tz$ and $\alpha_s(x,y,z) \geq 1$. Then $x,y,z \in [c,b]$ and $x,y,z \notin \{\frac{a+b}{2}\}$. We have $Tx = Ty = Tz = \frac{a+b}{2}$. Hence for $x_0 = b$, we have:

$$\alpha_s(x_0,x_0,Tx_0) = \alpha_s(b,b,\frac{a+b}{2}) = 1$$

Now, let $x,y,z \in X$ be such that $\alpha_s(x,y,z) \geq 1$. It yields that $x,y,z \in [c,b]$ so that $Tx = Ty = Tz = \frac{a+b}{2} \in [c,b]$ Hence,$\alpha_s(Tx,Ty,Tz) = \alpha_s(\frac{a+b}{2},\frac{a+b}{2},\frac{a+b}{2}) = 1$.
That is T is α_s-orbital admissible.
Since T is not continuous. We will demonstrate that (H) holds. Let $\{x_n\}$ be a sequence in X such that $\alpha_s(x_n,x_n,x_{n+1}) \geq 1$ for each $n \in \mathbb{N}$. Then, $\{x_n\} \subset [c,b]$.
If $\{x_n\} \to u$ as $n \to \infty$, we have $\mid x_n - x_n \mid + \mid x_n - u \mid + \mid x_n - u \mid \to 0$ as $n \to \infty$. Hence $u \in [c,b]$, and so $\alpha_s(x_n,x_n,u) = 1$. All prerequisites of Theorem 2.3 are satisfied. In this situation a and $\frac{a+b}{2}$ are two fixed points of T.

3. Conclusion

In conclusion, utilizing the notion of α_S-admissibility, interpolation, and the simulation function within the framework of S-metric space, we present the concepts of α_S-interpolative Kannan type contraction and α_S-interpolative Ćirić-Reich-Rus type contractions to establish several fixed point theorems. A comparable outcome using the Banach Contraction principle is derived in the context of S-metric spaces as a corollary of our findings. Moreover, our findings can be extrapolated to additional generalized metric spaces.

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