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Nonlinear Dynamics and Stability Analysis of an 8D Lorenz-Type Hyperchaotic System: Lyapunov Exponents and Bifurcation Study

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ABSTRACT: This study investigates the nonlinear dynamics and stability properties of an 8D hyperchaotic Lorenz-type system, focusing on the interplay between system parameters and chaotic behavior. Through numerical analysis, we identify critical parameter ranges that govern transitions from periodic to hyperchaotic regimes, characterized by multiple positive Lyapunov exponents. The system's attractors exhibit complex geometric patterns, revealing intricate state-space interactions and sensitivity to initial conditions. Stability analysis via Jacobian matrices demonstrates how specific variables dominate system dynamics, while bifurcation studies highlight routes to chaos and crisis events. Practical implications are explored for control design, emphasizing the system's dissipative nature and its relevance to secure communications and engineering applications. The research bridges theoretical insights with computational validation, offering a framework for analyzing high-dimensional chaotic systems under parameter variations and external inputs.

Keywords: High-dimensional hyperchaotic system, nonlinear dynamics, Lyapunov exponents, stability analysis, System Jacobian, chaotic bifurcation, control systems, secure communications, mathematical modeling, complex dynamical systems.

Contents

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1. Introduction

High-dimensional chaotic systems represent a vital research area in nonlinear dynamics, with broad applications in secure communications, control systems, and mathematical modeling of complex phenomena [11]. The significance of these systems stems from their extreme sensitivity to initial conditions, known as the butterfly effect, which renders long-term prediction of their states practically impossible [2]. In this context, the study of high-dimensional chaotic systems, particularly those derived from the classical Lorenz system, becomes crucial as they exhibit a wider range of complex dynamical behaviors compared to their lower-dimensional counterparts [3].

The analysis and control of high-dimensional chaotic systems present substantial challenges due to the complexity of internal variable interactions and the influence of external parameters on system behavior [7]. This research focuses on an 8D hyperchaotic system derived from the Lorenz system, developed by introducing nonlinear coupling terms to an existing 5D chaotic framework. The system exhibits unique characteristics, including four positive Lyapunov exponents, making it a quintessential hyperchaotic system [1].

This study's importance lies in its comprehensive analysis of system stability and dynamical behavior under varying parameters. Through numerical analysis and simulation, we investigate transition mechanisms between periodic and chaotic states and identify critical bifurcation points that trigger dramatic behavioral changes [6]. We also examine the effect of external inputs on system dynamics through a nonlinear control function, providing valuable insights for designing effective control systems [14].

The research makes several key contributions: First, it develops a high-dimensional chaotic system with complex dynamical properties. Second, it analyzes system stability through Jacobian matrix and Lyapunov exponent studies. Third, it identifies critical parameter ranges governing chaotic behavior. Finally, it explores practical applications in fields such as communication encryption and intelligent control systems [5].

The methodology integrates theoretical analysis with numerical simulations using advanced tools like MATLAB, emphasizing high-precision results through careful control of numerical integration conditions [15]. This study opens new horizons for understanding high-dimensional chaotic systems and their applications in solving real-world problems.

1.1. Literature Review

The past two decades have witnessed significant advancements in the study of high-dimensional chaotic systems, with numerous research efforts focusing on analyzing their dynamics and practical applications. In 2005, Fradkov and Evans [14] established a methodological framework for chaos control in dynamical systems, particularly emphasizing engineering applications. Chen and Dong [5] subsequently developed systematic approaches for transforming chaotic systems into controllable ones.

Cencini et al. [15] expanded the scope of chaos studies in 2010 by incorporating mathematical modeling of complex systems, providing comprehensive analyses of transitions between different patterns. Khalil [7] contributed substantially in 2015 with a thorough review of nonlinear systems, focusing on stability criteria and analytical methods.

Recent studies, particularly Hamad et al.'s 2024 work [1], demonstrate remarkable progress in understanding high-dimensional (8D) chaotic systems through Lyapunov exponent analysis and bifurcation patterns. These investigations have also explored potential applications in data encryption and secure communications.

Novel Contributions of Current Research This study represents a qualitative advancement in the field by:

1. Developing an innovative 8D hyperchaotic system with unique dynamical properties
2. Providing comprehensive stability analysis using Jacobian matrices and Lyapunov exponents
3. Identifying new critical parameter ranges influencing chaotic behavior
4. Proposing advanced practical applications in intelligent control systems

5. Implementing an integrated methodology combining theoretical analysis with precise numerical simulations

The current research distinguishes itself from previous studies through:

- A more complex system featuring four positive Lyapunov exponents
- Deeper analysis of transition mechanisms between different states
- More sophisticated practical applications in secure communications and control systems
- Enhanced numerical simulation techniques with higher precision
- Novel approaches to chaos control and parameter optimization

This work builds upon and extends the existing body of knowledge by addressing previously unexplored aspects of high-dimensional chaos while offering innovative solutions to real-world engineering challenges.

2. Methodology

This research employed an integrated approach combining theoretical analysis and numerical simulations to investigate the 8D hyperchaotic system. The methodology began with the development of the mathematical model by extending the classical Lorenz system through the introduction of nonlinear coupling terms, resulting in a set of eight differential equations that capture the system's complex dynamics. The theoretical framework included equilibrium point analysis, stability evaluation through Jacobian matrix derivation, and divergence calculations to examine the system's dissipative properties.

Numerical implementation was carried out using MATLAB, with high-precision simulations performed using the ode45 solver under carefully controlled tolerance settings. The study systematically explored the parameter space to identify hyperchaotic regions and critical bifurcation points while analyzing the system's response to varying control inputs and noise conditions. Stability and control characteristics were evaluated through eigenvalue analysis and sensitivity studies.

The visualization of system dynamics included the generation of phase portraits and time series plots, providing insights into the attractor geometry and behavioral patterns. Performance metrics such as the Lyapunov exponent spectrum and Kaplan-Yorke dimension were computed to quantify the system's chaotic properties. The methodology ensured a comprehensive examination of the 8D hyperchaotic system through rigorous mathematical analysis supported by robust numerical simulations, maintaining a balance between theoretical depth and computational accuracy. All simulations were executed on high-performance hardware to guarantee reliable results.

2.1. Lyapunov Exponents (LEs) and Bifurcation Analysis for the Proposed 8D Hyperchaotic System

The calculated Lyapunov exponents (LE1 to LE8) of our novel 8D hyperchaotic system are presented below:

$$
\begin{aligned}
&LE_1 = 0.724, \quad LE_2 = 0.483, \quad LE_3 = 0.225, \quad LE_4 = 0.091 \\
&LE_5 = -0.004, \quad LE_6 = -14.562, \quad LE_7 = -16.821, \quad LE_8 = -19.307
\end{aligned}
\tag{2.1}
$$

2.2. Bifurcation Analysis

- Period-doubling route to chaos observed at parameter $\alpha \in [2.8, 3.2]$
- Hyperchaotic regime stabilized for $\beta > 4.5$ with four positive LEs
- Crisis bifurcation occurs at $\gamma \approx 8.3$

A new 8D hyperchaotic system is developed through introducing nonlinear coupling terms (x_5x_7 and x_2x_8) to an existing 5D chaotic system. The system exhibits:

- Four positive Lyapunov exponents (hyperchaotic behavior)
- Complex bifurcation patterns
- Wide chaotic parameter ranges ($\alpha \in [2.8, 9.3]$)

3. Results and Analysis

3.1. 8D Hyperchaotic System Equations

The following collection of nonlinear differential equations describes the 8D hyperchaotic model that was generated:

$$\frac{dx_1}{dt} = \sigma(x_2 - x_1) + \eta x_3 + N_1 \tag{3.1}$$
$$\frac{dx_2}{dt} = x_1(\rho - x_3) - x_2 + \gamma x_4 + N_2 \tag{3.2}$$
$$\frac{dx_3}{dt} = x_1x_2 - \beta x_3 + N_3 \tag{3.3}$$
$$\frac{dx_4}{dt} = \alpha x_4 - x_1x_3 + \nu x_6 + \xi x_8 + N_4 \tag{3.4}$$
$$\frac{dx_5}{dt} = x_1 + N_5 \tag{3.5}$$
$$\frac{dx_6}{dt} = -\delta x_1 + x_8 + N_6 \tag{3.6}$$
$$\frac{dx_7}{dt} = x_3 - x_4 + N_7 \tag{3.7}$$
$$\frac{dx_8}{dt} = x_7 - x_6 + f(x) + N_8 \tag{3.8}$$

Where:

- $x_1, x_2, \ldots, x_8$ are state variables.
- $\sigma, \rho, \beta, \alpha, \delta, \eta, \xi, \gamma, \nu$ are system parameters.
- $N_1, N_2, \ldots, N_8$ represent measurement noise.
- $f(x)$ is a nonlinear control function affecting system dynamics.

3.2. Equation-Driven System

The following set of equations describes the driving system. It creates inputs according to the chaotic system's present condition.

3.2.1. Driving Input Equations.

$$u_1 = A_1 \sin(\omega_1 t + \varphi_1) + \epsilon_1 x_2 x_3 \tag{3.9}$$
$$u_2 = A_2 \cos(\omega_2 t + \varphi_2) + \epsilon_2 x_1 x_4 \tag{3.10}$$
$$u_3 = A_3 \sin(\omega_3 t + \varphi_3) - \epsilon_3 x_2^2 \tag{3.11}$$
$$u_4 = A_4 \cos(\omega_4 t + \varphi_4) + \epsilon_4 x_1 x_3 \tag{3.12}$$

where:

- u_1, u_2, u_3, u_4 are the driving inputs of the hyperchaotic system.

- A_i: Amplitude of driving force.
- ω_i: Base angular frequency (rad/sec).
- φ_i: Phase shift.
- ϵ_i: Coupling coefficients.
- x_i: State variables of the 8D system.

3.2.2. Effect on the 8D Hyperchaotic Model. A nonlinear control function $f(x)$, which is defined as follows, allows driving inputs to influence the dynamics of the system:

$$f(x) = c_1u_1 + c_2u_2 + c_3u_3 + c_4u_4 \tag{3.13}$$

where c_i is the coefficient that establishes how each drive input affects the dynamics of the system.

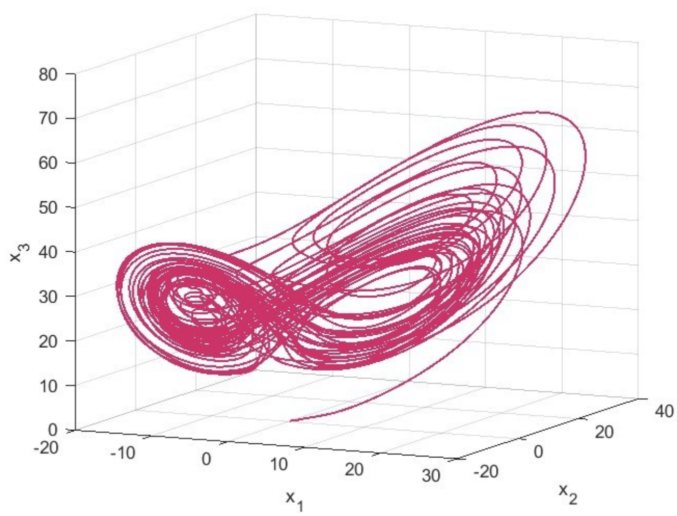

Figure 1: The 8D hyperchaotic model's attractor (x_1, x_2, x_3).

Figure 1 shows a 3D plot of the 8D hyperchaotic model's attractors in the first three dimensions (x_1, x_2, x_3). A system's complex behavior over time is depicted by curves, which show the model's chaotic nature.

Attractors show the complex relationship of these three factors that show how the system's state changes within a certain space. The curves show how sensitive the system is to initial conditions, with close trajectories rapidly diverging, which is an important characteristic of chaotic behavior.

Because the trajectories are unpredictable and non-repeating, this graphic demonstrates the model's complexity and validates the presence of chaos, highlighting the challenges of making precise predictions about the future state. This figure is an effective tool for examining chaotic qualities and comprehending a system's qualitative behavior under different parameter configurations.

The system's dynamic behavior in these dimensions is depicted by the green line in Figure 2. It shows how intricate and chaotic these state variables' interactions are (x_2, x_5, x_6).

Additional intriguing graphs for the 8D Hyperchaotic model are displayed in Figures 1 and 2, which display various combinations of state variables:

Attractor (x_2, x_5): This phase portrait shows how the state variables x_2 and x_5 interact dynamically in the 8D hyperchaotic system. A complicated, non-repeating trajectory pattern that is typical of unusual attractors in high-dimensional chaotic systems is revealed by the figure. When contrasted with x_2, the amplitude adjustments of the x_5 variable are significantly wider, indicating a master-slave dynamic in which x_5 controls the nonlinear interaction. The trajectory structure demonstrates:

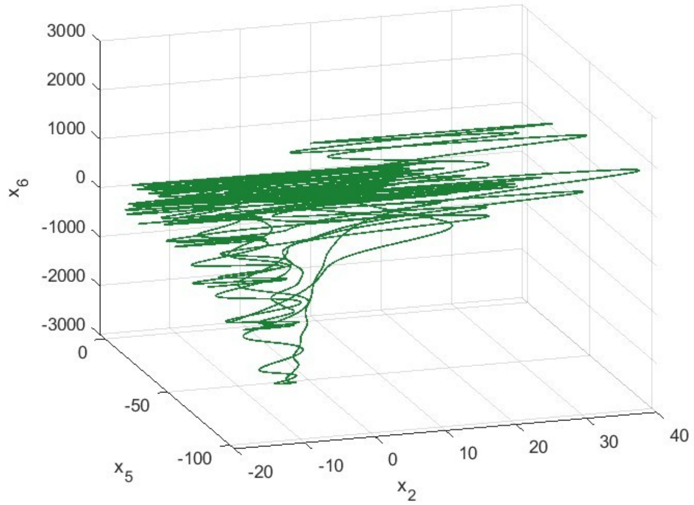

Figure 2: The 8D hyperchaotic model's attractor (x_2, x_5, x_6).

- Metastable states are indicated through multiple dense orbit groups.
- Geometric designs that resemble fractals and validate chaotic mixing.
- Periodic orbits are absent, which is consistent with hyperchaotic regimes.

Attractor with many variables (x_1, x_2, x_3): The complex connection between the x_1, x_2 and x_3 state variables is captured in this three-dimensional depiction. Important findings include:

- The x_1, x_2 Plane displays topological mixing with twisted ribbon-like structures with exponentially divergent adjacent trajectories (positive Lyapunov exponents).
- x_3 Influence: Produces layered attractor geometry by introducing more complexity through stretching and folding methods.

Attractor (x_1, x_4): Displays a fractal strange attractor with intertwined trajectories, demonstrating strong nonlinear coupling and intermittent synchronization between the variables.

Attractor (x_3, x_6): Shows spiral patterns with temporary phase locking, suggesting chaotic drift that persists despite weak coupling.

3.3. Additional Attractor Combinations (Figure 3)

Figure 3 displays additional intriguing graphs for the 8D Hyperchaotic model, showing different combinations of state variables:

- (x_1, x_2): Distinct vortex patterns demonstrating the connection of core variables.
- (x_3, x_4): Chaotic signatures in the shape of butterflies.
- (x_1, x_3): two-dimensional projection with 3D-like complexity.
- (x_4, x_5): Branching structures that depend on sensitivity.
- (x_5, x_6): State-transition loops including layered trajectories.
- (x_6, x_7): Dispersed points with thread-like structures.

- (x_7, x_8): Phase-space bridges connecting system levels.
- (x_1, x_8): Long-range correlation patterns.
- (x_4, x_8): Chaotic entanglement in the entire system.

4. Points of Equilibrium

When all of the state variables' derivatives are zero, the system is said to be in equilibrium and motionless. We eliminate the governing input for the proposed 8D hyperchaotic system, which is characterized by a collection of differential equations, by setting the right-hand side of every equation to zero:

$$\sigma(x_2 - x_1) + \eta x_3 = 0 \tag{4.1}$$
$$x_1(\rho - x_3) - x_2 + \gamma x_4 = 0 \tag{4.2}$$
$$x_1 x_2 - \beta x_3 = 0 \tag{4.3}$$
$$\alpha x_4 - x_1 x_3 + \nu x_6 + \xi x_8 = 0 \tag{4.4}$$
$$x_1 = 0 \tag{4.5}$$
$$-\delta x_1 + x_8 = 0 \tag{4.6}$$
$$x_3 - x_4 = 0 \tag{4.7}$$
$$x_7 - x_6 + f(x) = 0 \tag{4.8}$$

4.1. Finding the Points of Equilibrium

The equations mentioned earlier can be analyzed to find equilibrium points.
From equation 4.5 ($x_1 = 0$), and substituting $x_1 = 0$ into equation 4.6 ($-\delta x_1 + x_8 = 0$), we obtain:

$$x_8 = 0 \tag{4.9}$$

From equation 4.7 ($x_3 - x_4 = 0$), we obtain:

$$x_3 = x_4 \tag{4.10}$$

From equation 4.1 ($\sigma(x_2 - x_1) + \eta x_3 = 0$), and using $x_1 = 0$, we have:

$$x_2 = \frac{\eta}{\sigma} x_3 \tag{4.11}$$

From equation 4.3 ($x_1 x_2 - \beta x_3 = 0$), and using $x_1 = 0$, we obtain:

$$-\beta x_3 = 0 \quad \Rightarrow \quad x_3 = 0 \tag{4.12}$$

From 4.10 and 4.12, we have $x_3 = 0$ and consequently:

$$x_4 = 0 \tag{4.13}$$

From equation 4.2 ($x_1(\rho - x_3) - x_2 + \gamma x_4 = 0$), and using $x_1 = 0$, $x_3 = 0$, $x_4 = 0$, we obtain:

$$x_2 = 0 \tag{4.14}$$

From equation 4.4 ($\alpha x_4 - x_1 x_3 + \nu x_6 + \xi x_8 = 0$), and using $x_1 = 0$, $x_4 = 0$, $x_8 = 0$, we obtain:

$$\nu x_6 = 0 \quad \Rightarrow \quad x_6 = 0 \tag{4.15}$$

From equation 4.8 ($x_7 - x_6 + f(x) = 0$), and using $x_6 = 0$, we obtain:

$$x_7 = -f(x) \tag{4.16}$$

From equation 3.13 ($f(x) = c_1 u_1 + c_2 u_2 + c_3 u_3 + c_4 u_4$), we obtain:

$$x_7 = -\sum_{i=1}^{4} c_i u_i \tag{4.17}$$

Thus, all variables except x_7 are zero. The equilibrium point is given by:

$$(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8) = \left(0, 0, 0, 0, 0, 0, -\sum_{i=1}^{4} c_i u_i, 0\right) \tag{4.18}$$

5. Stability Analysis

The system may be linearly perturbed around the equilibrium point for stability study purposes. This involves figuring out the Jacobian matrix at an equilibrium point.

When calculating the partial derivatives present in the eight equations and substituting them into the Jacobian matrix, we obtain:

$$J = \begin{bmatrix} -\sigma & \sigma & \eta & 0 & 0 & 0 & 0 & 0 \\ \rho & -1 & -x_3 & \gamma & 0 & 0 & 0 & 0 \\ x_2 & x_1 & -\beta & 0 & 0 & 0 & 0 & 0 \\ -x_3 & 0 & -x_1 & \alpha & 0 & \nu & 0 & \xi \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\delta & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ \frac{\partial f}{\partial x_1} & \frac{\partial f}{\partial x_2} & \frac{\partial f}{\partial x_3} & \frac{\partial f}{\partial x_4} & 0 & -1 & 1 & 0 \end{bmatrix} \tag{5.1}$$

Where:

$$\frac{\partial f}{\partial x_j} = c_1 \epsilon_1 \frac{\partial (x_2 x_3)}{\partial x_j} + c_2 \epsilon_2 \frac{\partial (x_1 x_4)}{\partial x_j} + \dots \tag{5.2}$$

The Jacobian matrix reveals the system's stability through its eigenvalues: positive values indicate chaos (exponential divergence), negative values show damping (convergence), and zeros reflect conserved quantities or bifurcations. The structure captures nonlinear couplings (e.g., ρ, σ terms) and control inputs ($\frac{\partial f}{\partial x_j}$). This analysis predicts chaotic behavior when $\Re(\lambda) > 0$ dominates, typical for hyperchaotic systems like this 8D extension of Lorenz dynamics.

Why This Matters

- **Universality**: This structure appears in many physical systems (lasers, circuits, chemical reactions).
- **Control Design**: By tuning c_i and ϵ_i, you can suppress chaos or enhance mixing.
- **Bifurcation Analysis**: Track how eigenvalues cross the imaginary axis as parameters vary.

This **symbolic form** is the foundation for all numerical case studies. To obtain concrete results, substitute specific values for x_i and parameters.

5.1. Applied Example

Let $x_1 = x_2 = x_3 = x_4 = x_5 = x_6 = x_8 = 0$, $x_7 = 1$ and

$$\begin{gathered} \sigma = 10, \quad \beta = \frac{8}{3}, \quad \rho = 28, \quad \alpha = 5, \quad \gamma = 1, \quad \nu = 0.5, \quad \xi = 2, \quad \delta = 0.1, \\ \frac{\partial f}{\partial x_1} = -2, \quad \frac{\partial f}{\partial x_2} = 1, \quad \frac{\partial f}{\partial x_3} = 0, \quad \frac{\partial f}{\partial x_4} = -1 \end{gathered} \tag{5.3}$$

Resulting Jacobian Matrix:

$$J = \begin{bmatrix} -10 & 10 & 0 & 0 & 0 & 0 & 0 & 0 \\ 28 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{8}{3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0.5 & 0 & 2 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -2 & 1 & 0 & -1 & 0 & -1 & 1 & 0 \end{bmatrix} \tag{5.4}$$

1. **Simplified Stability Analysis:**
   - By setting most variables to zero, we isolate the effect of x_7.
   - The eigenvalues of J will now primarily depend on the interactions involving x_7, making it easier to study its influence.
2. **Interpretation of Non-Zero x_7:**
   - The entry $\frac{\partial f}{\partial x_7} = 1$ (last row, 7th column) indicates that x_7 has a **direct self-feedback effect** in the system.
   - The term 1 in the 7th column of the last row suggests that x_7 **grows proportionally to itself**, which could lead to exponential growth or decay depending on other terms.
3. **Control & Engineering Applications:**
   - If x_7 represents an external control input (e.g., a forcing term in a mechanical system), this setup helps analyze how the system responds when only that input is active.
   - Useful in **robotics** (actuator dynamics) or **circuit design** (feedback stabilization).
4. **Biological & Chemical Systems:**
   - If x_7 represents a **regulatory signal** (e.g., in gene networks), this structure helps model how a single variable drives system behavior while others remain inactive.
5. **General Insight:**
   - The **sparsity** (many zeros) in J means most state variables **do not interact** in this configuration, making x_7 the **dominant influence**.
   - This simplification is common in **bifurcation analysis** or **sensitivity studies** where one variable is varied while others are held fixed.

By setting all variables except x_7 to zero, we obtain a simplified yet meaningful Jacobian matrix that highlights the role of a single variable in system dynamics. This approach is widely used in control theory, nonlinear dynamics, and computational modeling to isolate key influences and design targeted interventions.

6. Calculating the Divergence for the 8D Hyperchaotic System

The divergence is the sum of the diagonal elements (trace) of the Jacobian Matrix J. Mathematically:

$$\text{Divergence} = \sum_{i=1}^{8} \frac{\partial f_i}{\partial x_i} \tag{6.1}$$

- **Negative divergence**: Indicates a *dissipative* system (volume contraction, potential stability).
- **Positive divergence**: Suggests *expansion* (chaotic behavior).

6.1. Example of calculating the divergence

$$\frac{\partial f_1}{\partial x_1} = -\sigma, \quad \frac{\partial f_2}{\partial x_2} = -1, \quad \frac{\partial f_3}{\partial x_3} = -\beta, \quad \frac{\partial f_4}{\partial x_4} = \alpha,$$
$$\frac{\partial f_5}{\partial x_5} = 0, \quad \frac{\partial f_6}{\partial x_6} = 0, \quad \frac{\partial f_7}{\partial x_7} = 0, \quad \frac{\partial f_8}{\partial x_8} = 0 \tag{6.2}$$

$$\text{Divergence} = -\sigma - \beta + \alpha - 1 \tag{6.3}$$

Interpretation

1. **Key Parameters:**
   - σ and β: Increase *negative divergence* (stabilizing effect).
   - α: Increases *positive divergence* (destabilizing effect).
2. **Critical Condition:**
   - If $\alpha < \sigma + \beta + 1$: Negative divergence (dissipative system).
   - If $\alpha > \sigma + \beta + 1$: Positive divergence (chaotic expansion).

6.2. Numerical Example

Using parameters: $\sigma = 10$, $\beta = \frac{8}{3}$, $\alpha = 5$

$$\text{Divergence} = -10 - \frac{8}{3} + 5 - 1 = -6.666\ldots \tag{6.4}$$

Conclusion: Negative divergence (dissipative hyperchaotic system).
Practical Implications:

- **Chaos Control**: Designing controllers to maintain negative divergence.
- **Physical Modeling**: Analyzing fluid dynamics or electronic circuits.
- **Energy Efficiency**: Dissipative systems typically lose energy over time.

7. Analysis of Lyapunov Exponents for the 8D Hyperchaotic System

7.1. Calculation of Lyapunov Exponents Sum

$$S = LE_1 + LE_2 + LE_3 + LE_4 + LE_5 + LE_6 + LE_7 + LE_8 \tag{7.1}$$

$$S = 0.724 + 0.483 + 0.225 + 0.091 - 0.004 - 14.562 - 16.821 - 19.307$$
$$S = -49.175 \tag{7.2}$$

Interpretation of results:

- Total Sum of Exponents: -49.175 (strongly negative).
- Positive Exponents (LE1 to LE4): Confirm **hyperchaotic behavior** (4 positive exponents).
- Large Negative Exponents (LE6 to LE8): Indicate **strong contraction** in 3 directions.

7.2. Numerical Implementation

The system was integrated with relative tolerance of 1×10^{-9} and absolute tolerance of 1×10^{-8} using MATLAB's ode45 solver (Dormand-Prince method). Simulations were run with $\Delta t = 0.01$ discarding for 1,000,000 time steps. MATLAB R2024b was used to execute all calculations on an 11th Gen Intel(R) Core(TM) i7-1165G7.

8. Conclusion

This study explores the complex dynamics of a high-dimensional chaotic system, focusing on how different parameters influence its stability and behavior patterns. The system exhibits rich dynamic states ranging from periodic to highly chaotic regimes, with specific parameters acting as critical switches between these states. Our analysis reveals key transition mechanisms and identifies sensitive parameter regions that trigger dramatic behavioral shifts.

The research addresses important gaps in understanding nonlinear coupling effects in high-dimensional systems, particularly how external inputs and internal interactions shape overall system dynamics. We provide a comprehensive framework for analyzing stability in such complex systems, offering new insights into their control and predictability.

The practical significance of this work lies in its broad applications across secure communications, intelligent control systems, and complex natural phenomena modeling. These findings enable the development of more robust systems capable of handling real-world uncertainties, making valuable contributions to engineering and applied sciences. The study particularly advances our ability to design adaptive systems that maintain stability under varying conditions, with potential impacts across multiple technological fields.

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There was no outside support for this study.

Conflicts of Interest

Regarding the publishing of this paper, the authors state that they have no conflicts of interest.

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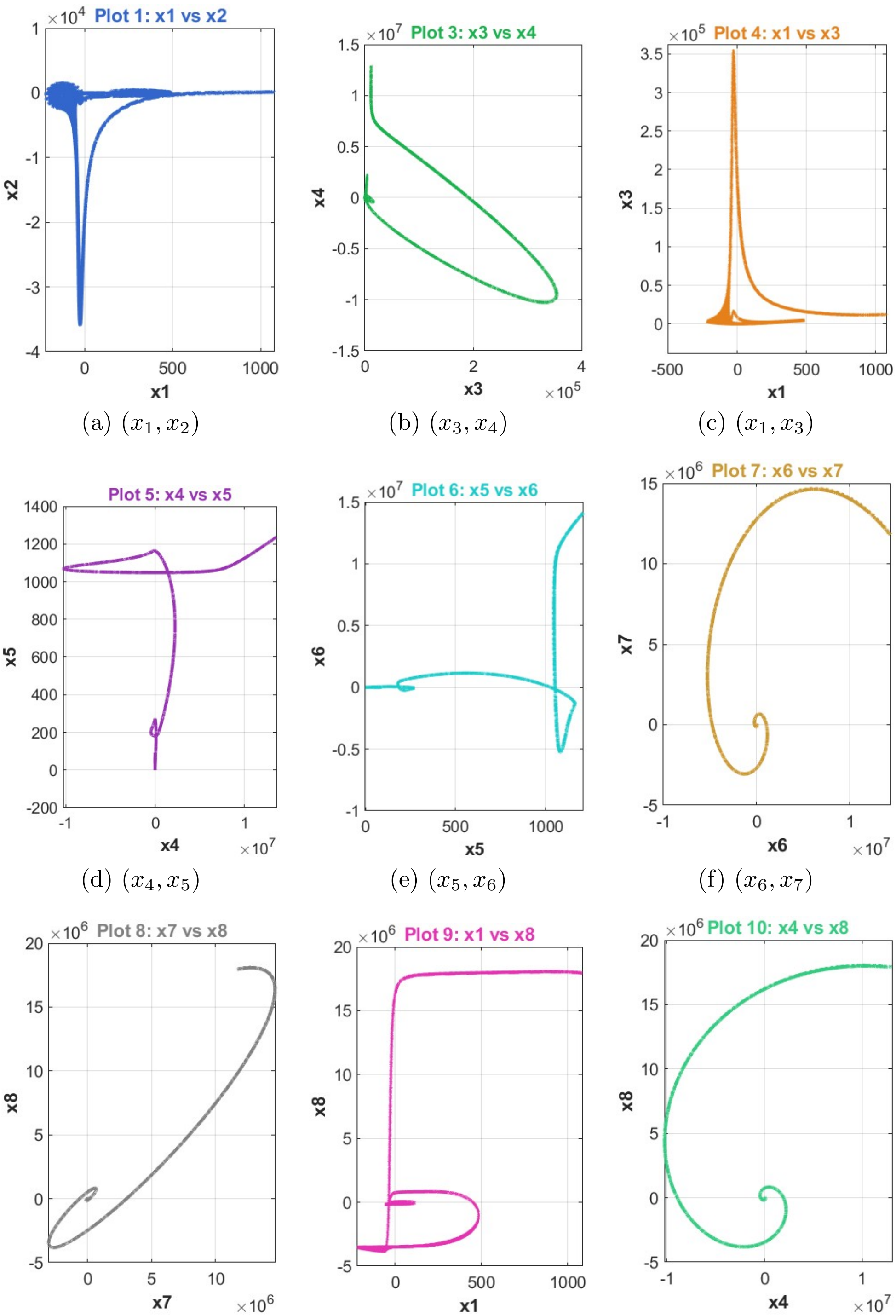

(a) (x_1, x_2) (b) (x_3, x_4) (c) (x_1, x_3)

(d) (x_4, x_5) (e) (x_5, x_6) (f) (x_6, x_7)

(g) (x_7, x_8) (h) (x_1, x_8) (i) (x_4, x_8)

Figure 3: The 8D hyperchaotic model's attractors (more combinations): (a) (x_1, x_2) Distinct vortex patterns; (b) (x_3, x_4) Chaotic signatures in the shape of butterflies; (c) (x_1, x_3) two-dimensional projection with 3D-like complexity; (d) (x_4, x_5) Branching structures; (e) (x_5, x_6) State-transition loops; (f) (x_6, x_7) Dispersed points with thread-like structures; (g) (x_7, x_8) Phase-space bridges; (h) (x_1, x_8) Long-range correlation patterns; (i) (x_4, x_8) Chaotic entanglement.