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# Second-Order Differential Subordination Applications for a Class of Analytic Function Defined by Differential Operator

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ABSTRACT: In this work, we use the differential operator to establish a new class of analytic mappings and derive certain subordination results.

Keywords: Symmetric, differential subordination, convex, analytic.

**Contents**



## 1. Introduction

Let $\mathfrak{C}$ be complex plane and let $\mathfrak{U} = \{\zeta : \zeta \in \mathfrak{C}$ and $|\zeta| < 1\} = \mathfrak{U} \setminus \{0\}$ be an open unit disc in $\mathfrak{C}$. Also let $H(\mathfrak{U})$ be a class of functions analytic in $\mathfrak{U}$. For $n \in \mathbb{N} = \{1, 2, 3, \cdots, \}$ and $a \in \mathfrak{C}$, let $H[a, n]$ be a subclass of $H(\mathfrak{U})$ formed by functions of the form

$$f(\zeta) = \zeta + a_n\zeta^n + a_{n+1}\zeta^{n+1} + \cdots$$

with $H_0 \equiv H[0, 1]$ and $H \equiv H[1, 1]$. Suppose that $A_n$ is a class of all analytic functions of the form

$$f(\zeta) = \zeta + \sum_{k=n+1}^{\infty} a_n\zeta^n \tag{1.1}$$

within the unit disk that is open $\mathfrak{U}$ with $A_1 = A$. A mapping $f \in H(\mathfrak{U})$ is a one-to-one mapping called as univalent in $\mathfrak{U}$. Use $S$ to demonstrate the subcategory of $A$ generated by one-to-one mappings in $\mathfrak{U}$. Suppose the mapping $f \in A$ to $\mathfrak{U}$ onto a domain, which is convex as well as $f$ is one-to-one, after that $f$ is referred to as a mapping which is convex. Taking

$$K = \left\{ f \in A : \Re\left\{1 + \frac{\zeta f''(\zeta)}{f'(\zeta)}\right\} > 0, \quad \zeta \in \mathfrak{U} \right\}$$

with the normalized conditions $f(0) = 0$ and $f'(0) = 1$ ,we represent the category or class of all convex mappings demonstrated in $\mathfrak{U}$. Suppose that the elements $f$ and $F$ are in $H(\mathfrak{U})$. A mapping $f$ is known as subordinate to $F$whether a Schwartz function exists $w$ which is analytic in $\mathfrak{U}$ and satisfying

$$|w(\zeta)| < 1 \quad \text{and} \quad w(0) = 0, \quad \zeta \in \mathfrak{U},$$

so that

$$F(w(\zeta)) = f(\zeta).$$

In this case, we write

$$f(\zeta) \prec F(\zeta) \quad \text{or} \quad f \prec F.$$

Furthermore, if the function $F$ is univalent in $\mathfrak{U}$, then we get the following equivalence [2,6]:

$$f(\zeta) \prec F(\zeta) \Leftrightarrow f(0) = F(0) \quad \text{and} \quad f(\mathfrak{U}) \prec F(\mathfrak{U}).$$

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The method of differential subordinations (also known as the method of admissible functions ) was first introduced and the development of the theory was originated by Miller and Mocanu [7,8]. All details can be found in the book by Miller and Mocanu [6]. In recent years, numerous authors studied the properties of differential subordinations (see [1,3,10,12], etc.).

Let $\Psi : \mathfrak{C}^3 \times \mathfrak{U} \to \mathfrak{C}$ and let $g$ be univalent in $\mathfrak{U}$. If $u$ is analytic in $\mathfrak{U}$ and satisfies the second-order differential subordination

$$\Psi\left(u(\zeta), \zeta u'(\zeta), \zeta u''(\zeta); \zeta\right) \prec g(\zeta), \tag{1.2}$$

then $u$ referred to as the differential subordination solution. The univalent mapping $v$ is known as a dominant of the primitive of the differential subordination or, simply, a dominant if $u \prec v$ for all $u$ fulfilling (1.2). The dominant $v_1$ fulfilling $v_1 \prec v$ for every dominants $v$ of (1.2) referred to as the best dominant of (1.2).

The linear multiplier differential operator $D^s_\mu f$ was defined in [4] as follows:

$$\begin{aligned}
&\mathscr{D}^0_\mu f(\zeta) = f(\zeta)\\
&\mathscr{D}^1_\mu f(\zeta) = \mathscr{D}_\mu f(\zeta) = \mu z^3 (f(\zeta))''' + (2\mu+1)\zeta^2 (f(\zeta))'' + \zeta f'(\zeta)\\
&\mathscr{D}^2_\mu f(\zeta) = \mathscr{D}_\mu\left(\mathscr{D}^1_\mu f(\zeta)\right)\\
&\qquad\qquad \vdots\\
&\mathscr{D}^s_\mu f(\zeta) = \mathscr{D}_\mu\left(\mathscr{D}^{s-1}_\lambda f(\zeta)\right)
\end{aligned}$$

where $\mu \geq 0$ and $s \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

If $f$ is given by (1.1) then from the definition of the operator $\mathscr{D}^s_\mu f$ it is easy to see that

$$\mathscr{D}^s_\mu f(\zeta) = \zeta + \sum_{k\geq 2} \mathscr{L}(k,\mu,s) a_k \zeta^k \tag{1.3}$$

where

$$\mathscr{L}(k,\mu,s)) = k^{2s}[\mu(k-1)+1]^s. \tag{1.4}$$

We adopt the methods of [5] and introduce a new class by using the differential operator $\mathscr{D}^s_\mu$

**Definition 1.1** Let $\mathfrak{L}_{\mu,s}(\alpha)$ is a class or category of mapping $f \in A$ fulfilling inequality as

$$\Re\left(\mathscr{D}^s_\mu f(\zeta)\right) \geq \alpha,$$

where $\zeta \in \mathfrak{U}$, $0 \leq \alpha < 1$ and $\mathscr{D}^s_\mu f(\zeta)$ is the operator for differential.

**Lemma 1.1** *[5] Let $g$ be a convex mapping holding $g(0) = a$ and consider $\gamma \in \mathfrak{C}^* := \mathfrak{C} \setminus \{0\}$ is a number which is complex, having $\Re\{\gamma\} \geq 0$. If $u \in H[a,n]$ as well as*

$$u(\zeta) + \frac{1}{\gamma}\zeta u'(\zeta) \prec g(\zeta), \tag{1.5}$$

*then*

$$u(\zeta) \prec v(\zeta) \prec g(\zeta),$$

*where*

$$v(\zeta) = \frac{\gamma}{n\zeta^{\frac{\gamma}{n}}} \int_0^\zeta t^{\frac{\gamma}{n-1}} g(t)\,dt, \quad \zeta \in \mathfrak{U}.$$

*The function $v$ is convex and is the best dominant for subordination (1.5).*

**Lemma 1.2** *[9] Let $\Re\{\mu\} > 0, \quad n \in \mathbb{N}$ and*

$$w = \frac{n^2 + |\mu|^2 - |n^2 - \mu^2|}{4n\Re\{\mu\}}.$$

*Also let $g$ be an analytic function in $\mathfrak{U}$ with $g(0) = 1$. Suppose that*

$$\Re\left\{1 + \frac{\zeta g''(\zeta)}{g'(\zeta)}\right\} > -w.$$

*If*

$$u(\zeta) = 1 + u_n\zeta^n + u_{n+1}\zeta^{n+1} + \cdots$$

*is analytic in $\mathfrak{U}$ and*

$$u(\zeta) + \frac{1}{\mu}\zeta u'(\zeta) \prec g(\zeta), \tag{1.6}$$

*then*

$$u(\zeta) \prec v(\zeta),$$

*where $v$ is a solution of the differential equation*

$$v(\zeta) + \frac{n}{\mu}\zeta v'(\zeta) = g(\zeta), \quad v(0) = 1,$$

*given by*

$$v(\zeta) = \frac{\mu}{n\zeta^{\frac{\mu}{n}}}\int_0^\zeta t^{\frac{\mu}{n-1}} g(t)dt, \quad \zeta \in \mathfrak{U}.$$

*Moreover, in terms of differential subordination (1.6), $v$ is the most desirable dominant.*

**Lemma 1.3** *[11] Assume that $r$ is a convex mapping within $\mathfrak{U}$ also consider*

$$g(\zeta) = r(\zeta) + n\alpha\zeta r'(\zeta), \quad \zeta \in \mathfrak{U},$$

*where $\alpha > 0$ and $n \in \mathbb{N}$. If*

$$u(\zeta) = r(0) + u_n\zeta^n + u_{n+1}\zeta^{n+1} + \cdots, \quad \zeta \in \mathfrak{U},$$

*is holomorphic in $\mathfrak{U}$ and*

$$u(\zeta) + \alpha\zeta u'(\zeta) \prec g(\zeta), \quad \zeta \in \mathfrak{U},$$

*then*

$$u(\zeta) \prec r(\zeta)$$

*This outcome is notable.*

In the present paper, we use the subordination results from [5] and [9] to prove our main results.

## 2. Main Results

**Theorem 2.1** *The set $\mathfrak{L}_{\mu,s}(\alpha)$ is convex.*

**Proof:** Let

$$f_j(\zeta) = \zeta + \sum_{k\geq 2} a_{k,j}\zeta^k, \quad \zeta \in \mathfrak{U}, \quad j = 1, \cdots, m$$

be in the category $\mathfrak{L}_{\mu,s}(\alpha)$. Afterthat, using the definition 1.1, we obtain

$$\Re\left\{(\mathscr{D}_\mu^s f(\zeta))'\right\} = \Re\left\{1 + \sum_{k\geq 2}\mathscr{L}(k,\mu,s)a_{k,j}k\zeta^{k-1}\right\} > \alpha. \tag{2.1}$$

For any positive numbers $\varsigma_1, \varsigma_2, \varsigma_3, \cdots, \varsigma_m$ in such a way

$$\sum_{j=1}^{m} \varsigma_j = 1,$$

it is essential to demonstrate the mapping

$$g(\zeta) = \sum_{j=1}^{m} \varsigma_j f_j(\zeta)$$

represents a component of $\mathfrak{L}_{\mu,s}(\alpha)$, that means

$$\Re\left\{(\mathscr{D}_{\mu}^{s} g(\zeta))'\right\} > \alpha. \tag{2.2}$$

Thus, we have

$$\mathscr{D}_{\mu}^{s} g(\zeta) = \zeta + \sum_{k=2}^{\infty} \mathscr{L}(k, \mu, s) \left\{ \sum_{j=1}^{m} \varsigma_j a_{k,j} \right\} \zeta^k. \tag{2.3}$$

If we differentiate (2.3) with respect to $\zeta$, then we obtain

$$(\mathscr{D}_{\mu}^{s} g(\zeta))' = 1 + \sum_{k=2}^{\infty} k\mathscr{L}(k, \mu, s) \left\{ \sum_{j=1}^{m} \varsigma_j a_{k,j} \right\} \zeta^{k-1}.$$

Thus, we have

$$\begin{aligned} \Re\left\{(\mathscr{D}_{\mu}^{s} g(\zeta))'\right\} =& 1 + \sum_{j=1}^{m} \varsigma_j \Re\left\{ \sum_{k=2}^{\infty} k\mathscr{L}(k, \mu, s) a_{k,j} \zeta^{k-1} \right\} \\ >& 1 + \sum_{j=1}^{m} \varsigma_j (\alpha - 1) \quad [ \ \textit{by} \ \ (2.2) \ ] \\ =& \alpha. \end{aligned}$$

Hence, inequality (2.1) is true and we arrive at the desired result.
Hence the theorem is proved. □

**Theorem 2.2** *Let $v$ be convex function in $\mathfrak{U}$ with $v(0) = 1$ and*

$$g(\zeta) = v(\zeta) + \frac{1}{\gamma + 1} \zeta v'(\zeta), \quad \zeta \in \mathfrak{U},$$

*while $\gamma$ gives a number which is complex, holding $\Re\{\gamma\} > -1$. If $f \in \mathfrak{L}_{\mu,s}(\alpha)$ as well as $\aleph = \Upsilon_\gamma f$, also*

$$\aleph(\zeta) = \Upsilon_\gamma f(\zeta) = \frac{\gamma + 1}{\zeta^\gamma} \int_0^\zeta t^{\gamma - 1} f(t) dt, \tag{2.4}$$

*then*

$$(\mathscr{D}_{\mu}^{s} f(\zeta))' \prec g(\zeta) \tag{2.5}$$

*implies that*

$$(\mathscr{D}_{\mu}^{s} \aleph(\zeta))' \prec v(\zeta)$$

*This outcome is notable.*

**Proof:** Given equality (2.4), we are able to write as

$$\zeta^{\gamma}\aleph(\zeta) = (\gamma+1)\int_{0}^{\zeta} t^{\gamma-1} f(t)dt. \tag{2.6}$$

With respect to $\zeta$, differentiate (2.6) we acquire

$$(\gamma)\aleph(\zeta) + \zeta\aleph'(\zeta) = (\gamma+1)f(\zeta).$$

Additionally, when we apply operator $\mathscr{D}_{\mu}^{s}$ to the above equation, we acquire

$$(\gamma)\mathscr{D}_{\mu}^{s}\aleph(\zeta) + \zeta(\mathscr{D}_{\mu}^{s}\aleph(\zeta))' = (\gamma+1)\mathscr{D}_{\mu}^{s}f(\zeta). \tag{2.7}$$

With respect to $\zeta$ differentiate (2.7), further obtained as

$$(\mathscr{D}_{\mu}^{s}\aleph(\zeta))' + \frac{1}{\gamma+1}\zeta(\mathscr{D}_{\mu}^{s}f(\zeta))'' = (\mathscr{D}_{\mu}^{s}f(\zeta))'. \tag{2.8}$$

using equality (2.8)'s differential subordination provided by (2.5) yields

$$(\mathscr{D}_{\mu}^{s}\aleph(\zeta))' + \frac{1}{\gamma+1}\zeta(\mathscr{D}_{\mu}^{s}f(\zeta))'' \prec g(\zeta). \tag{2.9}$$

We define

$$u(\zeta) = (\mathscr{D}_{\mu}^{s}\aleph(\zeta))'. \tag{2.10}$$

Hence, as a result of simple computations, we get

$$\begin{aligned} u(\zeta) &= \left[\zeta + \sum_{k=2}^{\infty}\mathscr{L}(k,\mu,s)\frac{\gamma+1}{\gamma+k}a_k\zeta^k\right]' \\ &= 1 + u_1\zeta + u_2\zeta^2 + \cdots, \quad u \in H[1,1]. \end{aligned}$$

Utilizing (2.10) in subordination (2.9) yields

$$u(\zeta) + \frac{1}{\gamma+1}\zeta u'(\zeta) \prec g(\zeta) = v(\zeta) + \frac{1}{\gamma+1}\zeta v'(\zeta), \quad \zeta \in \mathfrak{U}.$$

When applying Lemma 1.2, we write

$$u(\zeta) \prec v(\zeta).$$

Thus, we obtained the desired result and $v$ is the best dominant.
Hence the theorem is proved. □

**Example 2.1** If we choose in Theorem 2.2,

$$\gamma = i+1 \;\; and \;\; v(\zeta) = \frac{1+\zeta}{1-\zeta},$$

then we get

$$g(\zeta) = \frac{(i+2) - ((i+2)\zeta + 2)\zeta}{(i+2)(1-\zeta)^2}.$$

If $f \in \mathfrak{L}_{\mu,s}(\alpha)$ and $\aleph$ is given by

$$\aleph(\zeta) = \Upsilon_i f(\zeta) = \frac{i+2}{\zeta^{i+1}}\int_{0}^{\zeta} t^{i} f(t)dt,$$

Consequently, using Theorem 2.2, we discover

$$(\mathscr{D}_\mu^s f(\zeta))' \prec g(\zeta) = \frac{(i+2) - ((i+2)\zeta + 2)\zeta}{(i+2)(1-\zeta)^2}$$
$$\implies (\mathscr{D}_\mu^s f(\zeta))' \prec \frac{1+\zeta}{1-\zeta}.$$

**Theorem 2.3** *Suppose $\Re\{\gamma\} > -1$ and let*

$$w = \frac{1 + |\gamma+1|^2 - |\gamma^2 + 2\gamma|}{4\Re\{\gamma+1\}}.$$

*Assume $g$ is holomorphic mapping in $\mathfrak{U}$ holding $g(0) = 1$ as well as*

$$\Re\left\{1 + \frac{\zeta g''(\zeta)}{g'(\zeta)}\right\} > -w.$$

*Consequently $f \in \mathfrak{L}_{\mu,s}(\alpha)$ as well as $\aleph = \Upsilon_\mu^s f$, which $\aleph$ demonstrated as (2.4), afterwards*

$$(\mathscr{D}_\mu^s f(\zeta))' \prec g(\zeta) \tag{2.11}$$

*suggested as*

$$(\mathscr{D}_\mu^s \aleph(\zeta))' \prec v(\zeta),$$

*In this case,v represents the differential equation's solution.*

$$g(\zeta) = v(\zeta) + \frac{1}{\gamma+1}\zeta v'(\zeta), \quad v(0) = 1,$$

*provided as*

$$v(\zeta) = \frac{\gamma+1}{\zeta^{\gamma+1}} \int_0^\zeta t^\gamma f(t) dt.$$

*Furthermore, for subordination (2.11), $v$ is the most effective dominant.*

Proof. The proof of the theorem is acquired by using the result of Theorem 2.2 and if we substitute $n = 1$ and $\mu = \gamma + 1$ in Lemma 1.1.

**Theorem 2.4** *Suppose*

$$g(\zeta) = \frac{1 + (2\alpha - 1)\zeta}{1+\zeta}, \quad 0 \le \alpha < 1 \tag{2.12}$$

*holding condition $g(0) = 1$ and is convex in $\mathfrak{U}$. If $f \in A$ and satisfies the differential subordination*

$$(\mathscr{D}_\mu^s f(\zeta))' \prec g(\zeta),$$

*then*

$$(\mathscr{D}_\mu^s \aleph(\zeta))' \prec v(\zeta) = (2\alpha - 1) + \frac{2(1-\alpha)(\gamma+1)\tau(\gamma)}{\zeta^{\gamma+1}},$$

*while $\tau$ is provided with the expression*

$$\tau(\gamma) = \int_0^\zeta \frac{t^\gamma}{t+1} dt \tag{2.13}$$

*as well as $\aleph$ can be found using equation (2.4). Also the mapping $v$ which is best dominant as well as convex.*

**Proof:** If

$$g(\zeta) = \frac{1+(2\alpha-1)\zeta}{1+\zeta}, \quad 0 \leq \alpha < 1,$$

Consequently, $g$ is convex, and we may write using Theorem 2.3

$$(\mathscr{D}_{\mu}^{s}\aleph(\zeta))' \prec v(\zeta).$$

We obtained by utilising lemma 1.1

$$\begin{aligned} v(\zeta) &= \frac{\gamma+1}{\zeta^{\gamma+1}} \int_0^{\zeta} t^{\gamma} g(t)dt \\ &= \frac{\gamma+1}{\zeta^{\gamma+1}} \int_0^{\zeta} t^{\gamma} \left\{ \frac{1+(2\alpha-1)t}{1+t} \right\} dt \\ &= (2\alpha-1) + \frac{2(1-\alpha)(\gamma+1)}{\zeta^{\gamma+1}} \tau(\gamma), \end{aligned}$$

while $\tau$ is provided by (2.13). Thus, we acquire

$$(\mathscr{D}_{\mu}^{s}\aleph(\zeta))' \prec v(\zeta) = (2\alpha-1) + \frac{2(1-\alpha)(\gamma+1)\tau(\gamma)}{\zeta^{\gamma+1}}.$$

$v$ is the most dominant and the mapping is convex.
Thus, the theorem is demonstrated. □

**Theorem 2.5** *If* $0 \leq \mu < 1, 0 \leq \alpha < 1, \Re\{\gamma\} > -1, \delta \geq 0,$ *as well as* $\aleph = \Upsilon_{\gamma} f$ *has been described with* (2.4)*, after that*

$$\Upsilon_{\gamma}(\mathfrak{L}_{\mu,s}(\alpha)) \subset \mathfrak{L}_{\mu,s}(\rho),$$

*however*

$$\rho = \min_{|z|=1} \Re\{v(\zeta)\} = \rho(\gamma,\alpha) = (2\alpha-1) + 2(1-\alpha)(\gamma+1)\tau(\gamma) \tag{2.14}$$

*While* $\tau$ *be provided with* (2.13)*.*

**Proof:** Take the assumption with equation (2.12) provides $g$ , $f \in \mathfrak{L}_{\mu,s}(\alpha)$, as well as $\aleph = \Upsilon_{\gamma} f$ is described as (2.4). Afterwards $g$ be convex and utilising Theorem 2.3, obtained us

$$(\mathscr{D}_{\mu}^{s}\aleph(\zeta))' \prec v(\zeta) = (2\alpha-1) + \frac{2(1-\alpha)(\gamma+1)\tau(\gamma)}{\zeta^{\gamma+1}}, \tag{2.15}$$

$\tau$ is provided by (2.13). Considering that $v$ is convex, $v(\mathfrak{U})$ which is symmetric with respect to the real axis, as well as $\Re\{\gamma\} > -1$, we discover

$$\begin{aligned} \Re\left\{(\mathscr{D}_{\mu}^{s}\aleph(\zeta))'\right\} &\geq \min_{|\zeta|=1} \Re\{v(\zeta)\} = \Re\{v(1)\} = \rho(\gamma,\alpha) \\ &= (2\alpha-1) + 2(1-\alpha)(\gamma+1)(1-\alpha)\tau(\gamma). \end{aligned}$$

Based on inequality (2.15), it can be observed as

$$\Upsilon_{\gamma}(\mathfrak{L}_{\mu,s}(\alpha)) \subset \mathfrak{L}_{\mu,s}(\rho),$$

however (2.14) provides $\rho$ .
Consequently, proved the outcome of theorem. □

**Theorem 2.6** *Suppose $v$ considered as a convex mapping holding $v(0) = 1$ as well as $g$ is a mapping in such a way*

$$g(\zeta) = v(\zeta) + \zeta v'(\zeta), \quad \zeta \in \mathfrak{U}.$$

*If $f \in A$, then the subordination*

$$(\mathscr{D}^s_\mu f(\zeta))' \prec g(\zeta) \tag{2.16}$$

*implies that*

$$\frac{\mathscr{D}^s_\mu f(\zeta)}{\zeta} \prec v(\zeta),$$

*and the result is sharp.*

**Proof:** Let

$$u(\zeta) = \frac{\mathscr{D}^s_\mu f(\zeta)}{\zeta}. \tag{2.17}$$

Differentiating (2.17), we find

$$(\mathscr{D}^s_\mu f(\zeta))' = u(\zeta) + \zeta u'(\zeta).$$

We now compute $u(\zeta)$. This gives

$$\begin{aligned} u(\zeta) = \frac{\mathscr{D}^s_\mu f(\zeta)}{\zeta} &= \frac{\zeta + \sum_{k=2}^{\infty} \mathscr{L}(k, \mu, s) a_k \zeta^k}{\zeta} \\ &= 1 + u_1\zeta + u_2\zeta^2 + \cdots, \quad u \in H[1, 1]. \end{aligned} \tag{2.18}$$

Equation (2.18) is used in subordination (2.16) to determine

$$u(\zeta) + \zeta u'(\zeta) \prec g(\zeta) = v(\zeta) + \zeta v'(\zeta).$$

Therefore, using Lemma 1.3, we deduce that

$$u(\zeta) \prec v(\zeta)$$

That means

$$\frac{\mathscr{D}^s_\mu f(\zeta)}{\zeta} \prec v(\zeta).$$

$v$ is the most effective dominant, and this outcome is notable.
Thus, the theorem is demonstrated. □

**Example 2.2** If we put $\mu = 0$ as well as $s = 1$ in equality (1.3) also $v(\zeta) = \frac{1}{1-\zeta}$ in the previous Theorem 2.5, afterwards

$$g(\zeta) = \frac{1}{(1-\zeta)^2}$$

and

$$\mathscr{D}^1_0 f(\zeta) = \zeta + \sum_{k\geq 2} k^2 a_k \zeta^k. \tag{2.19}$$

With respect to $\zeta$, we can take the differentiation of (2.19) and acquired

$$\begin{aligned} (\mathscr{D}^1_0 f(\zeta))' &= 1 + \sum_{k\geq 2} 2k a_k \zeta^{k-1}. \\ &= 1 + u_1\zeta + u_2\zeta^2 + \cdots, \quad u \in H[1, 1]. \end{aligned}$$

By using Theorem 2.5, we find

$$(\mathscr{D}^1_0 f(\zeta))' \prec g(\zeta) = \frac{1}{(1-\zeta)^2}.$$

This yields

$$\frac{\mathscr{D}^1_0 f(\zeta)}{\zeta} \prec v(\zeta) = \frac{1}{1-\zeta}.$$

**Theorem 2.7** *Suppose*

$$g(\zeta) = \frac{1+(2\alpha-1)\zeta}{1+\zeta}, \quad \zeta \in \mathbb{U}$$

*with* $g(0) = 1$ *also* $0 \leq \alpha < 1$, *which is convex in* $\mathfrak{U}$ *If* $f \in A$ *meets the requirements for the differential subordination*

$$(\mathscr{D}_\mu^s f(\zeta))' \prec g(\zeta), \tag{2.20}$$

*afterward,*

$$\frac{\mathscr{D}_\mu^s f(\zeta)}{\zeta} \prec v(\zeta) = (2\alpha-1) + \frac{2(1-\alpha)ln(1+\zeta)}{\zeta}.$$

*Also the mapping* $v$ *is the best dominant, which is convex.*

**Proof:** Let

$$u(z) = \frac{\mathscr{D}_\mu^s f(\zeta)}{\zeta} = 1 + u_1\zeta + u_2\zeta^2 + \cdots, \quad u \in H[1,1]. \tag{2.21}$$

Differentiating (2.21), we find

$$(\mathscr{D}_\mu^s f(\zeta))' = u(\zeta) + \zeta u'(z). \tag{2.22}$$

The differential subordination (2.20) obtained by utilizing the equation (2.22),

$$(\mathscr{D}_\mu^s f(\zeta))' \prec g(\zeta) = \frac{1+(2\alpha-1)\zeta}{1+\zeta}.$$

By using Lemma1.1, we conclude

$$u(\zeta) \prec v(\zeta) = \frac{1}{\zeta}\int g(t)dt = (2\alpha-1) + \frac{2(1-\alpha)ln(1+\zeta)}{\zeta}.$$

We get the required outcome by using the equality (2.21).
Hence the theorem is proved. □

**Corollary 2.1** *Suppose* $f \in \mathfrak{L}_{\mu,s}(\alpha)$, *after prove that*

$$\Re\left(\frac{\mathscr{D}_\mu^s f(\zeta)}{\zeta}\right) > (2\alpha-1) + 2(1-\alpha)ln(2).$$

**Proof:** If $f$ from Definition 1.1 and $f \in \mathfrak{L}_{\mu,s}(\alpha)$, then it gives that

$$\Re\left\{(\mathscr{D}_\mu^s f(\zeta))'\right\} > \alpha, \quad \zeta \in \mathfrak{U}.$$

That is comparable to

$$(\mathscr{D}_\mu^s f(\zeta))' \prec g(\zeta) = \frac{1+(2\alpha-1)\zeta}{1+\zeta}.$$

By using Theorem 2.7, we obtain

$$\frac{\mathscr{D}_\mu^s f(\zeta)}{\zeta} \prec v(\zeta) = (2\alpha-1) + \frac{2(1-\alpha)ln(1+\zeta)}{\zeta}.$$

We deduce that since $v(\mathfrak{U})$ as symmetric along with the real axis as well as $v$ is convex,

$$\Re\left(\frac{\mathscr{D}_\mu^s f(\zeta)}{\zeta}\right) > \Re(v(1)) = (2\alpha-1) + 2(1-\alpha)ln(2).$$

□

**Theorem 2.8** *Suppose $v$ considering as convex mapping with $v(0) = 1$ as well as $g$ is the mapping provided by the formula*

$$g(\zeta) = v(\zeta) + \zeta v'(\zeta), \quad \zeta \in \mathfrak{U}.$$

*By taking $f \in A$ along with the differential subordination*

$$\left\{ \frac{\zeta \mathscr{D}_\mu^s f(\zeta)}{\mathscr{D}_\mu^s \aleph(\zeta)} \right\}' \prec g(\zeta), \quad \zeta \in \mathfrak{U}, \tag{2.23}$$

*then*

$$\frac{\mathscr{D}_\mu^s f(\zeta)}{\mathscr{D}_\mu^s \aleph(\zeta)} \prec v(\zeta), \quad \zeta \in \mathfrak{U},$$

*as well as, this outcome is pronounced.*

**Proof:** suppose the mapping $f \in A$, reffered as of the form (1.1), considering

$$\mathscr{D}_\mu^s \aleph(\zeta) = \zeta + \sum_{k \geq 2} \mathscr{L}(k, \mu, s) \frac{\gamma + 1}{k + \gamma} a_k b_k \zeta^k, \quad \zeta \in \mathfrak{U}.$$

We now consider the function

$$u(\zeta) = \frac{\mathscr{D}_\mu^s f(\zeta)}{\mathscr{D}_\mu^s \aleph(\zeta)} = \frac{\zeta + \sum_{k \geq 2} \mathscr{L}(k, \mu, s) a_k b_k \zeta^k}{\zeta + \sum_{k \geq 2} \mathscr{L}(k, \mu, s) \frac{\gamma+1}{k+\gamma} a_k b_k \zeta^k}$$
$$= \frac{1 + \sum_{k \geq 2} \mathscr{L}(k, \mu, s) a_k b_k \zeta^{k-1}}{1 + \sum_{k \geq 2} \mathscr{L}(k, \mu, s) \frac{\gamma+1}{k+\gamma} a_k b_k \zeta^{k-1}}.$$

In this case, we get

$$(u(\zeta))' = \frac{(\mathscr{D}_\mu^s f(\zeta))'}{\mathscr{D}_\mu^s \aleph(\zeta)} - u(\zeta) \frac{(\mathscr{D}_\mu^s \aleph(\zeta))'}{\mathscr{D}_\mu^s \aleph(\zeta)}.$$

Then

$$u(\zeta) + \zeta u'(\zeta) = \left\{ \frac{\zeta \mathscr{D}_\mu^s f(\zeta)}{\mathscr{D}_\mu^s \aleph(\zeta)} \right\}', \quad \zeta \in \mathfrak{U}. \tag{2.24}$$

We acquire by utilisig equality (2.24) in inequality (2.23)

$$u(\zeta) + \zeta u'(\zeta) \prec g(\zeta) = v(\zeta) + \zeta v'(\zeta)$$

also, because of Lemma 1.3,

$$u(\zeta) \prec v(\zeta).$$

That means

$$\frac{\mathscr{D}_\mu^s f(\zeta)}{\mathscr{D}_\mu^s \aleph(\zeta)} \prec v(\zeta).$$

The outcome of theorem obtained. □

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