Bol. Soc. Paran. Mat. (3s.) v. 2026 (44) : 1–9.
©SPM – E-ISSN-2175-1188 ISSN-0037-8712
SPM: www.spm.uem.br/bspm doi:10.5269/bspm.79357

On Certain Classes of Univalent Functions Associated with Riemann Fractional Derivative

N. Ravikumar, H. S. Roopa and P. Siva Kota Reddy*

ABSTRACT: In this paper, by making use of the concepts of fractional calculus, we define the subclass $S(r, \lambda, \delta, t)$ of analytic function by using $\Omega^\delta \mathfrak{f}(\tau)$. For function belonging to this class, we obtain co-efficient estimates, inclusions relations, extreme points and some more properties.

Key Words: Univalent functions, Fractional derivatives, Co-efficient inequality, Convex linear function.

Contents

1. Introduction

Let $\mathcal{A}$ denote the class of all analytic functions of the form

$$\mathfrak{f}(\tau) = \tau + \sum_{t=2}^{\infty} a_t \tau^t, \tag{1.1}$$

defined in the unit disc $\mathcal{U} = \{\tau : |\tau| < 1\}$.

Let $\mathcal{T}$ denote the subclass of $\mathcal{A}$ in $\mathcal{U}$, consisting of analytic functions whose non-zero coefficients from the second onwards are negative. That is, an analytic function $\mathfrak{f} \in \mathcal{T}$ if it has a Taylor expansion of the form

$$\mathfrak{f}(\tau) = \tau - \sum_{t=2}^{\infty} a_t \tau^t \qquad (a_t \geq 0),$$

which are analytic in the open disc $\mathcal{U}$.

Definition 1.1 *[5] The fractional derivative operator D_τ^δ of a $\mathfrak{f}(\tau)$ of order $\delta(0 \leq \delta < 1)$ is defined by*

$$D_\tau^\delta \mathfrak{f}(\tau) = \frac{1}{\Gamma(1-\delta)} D \int_0^\tau \frac{\mathfrak{f}(p)}{(\tau - p)^\delta} dp$$

where $0 \leq \delta < 1$, $\mathfrak{f}$ is an analytic function in a simply connected region of the τ-plane containing the origin and the multiplicity of $(\tau - p)^{-\delta}$ is removed by requiring log$(\tau - p)$ to be real when $(\tau - p)$ is greater than 0. Clearly $\mathfrak{f}(\tau) = \lim_{\delta \to 0} D_\tau^\delta \mathfrak{f}(\tau)$ and $\mathfrak{f}'(\tau) = \lim_{\delta \to 1} D_\tau^\delta \mathfrak{f}(\tau)$.

---

* Corresponding author.
2020 *Mathematics Subject Classification:* 30C45, 30C50.
Submitted October 05, 2025. Published February 04, 2026

Definition 1.2 *For an analytic function $\mathfrak{f}(\tau)$ of the form (1.1), we put*

$$\Omega^{\delta}\mathfrak{f}(\tau) = \Gamma(2-\delta)\tau^{\delta}D_{\tau}^{\delta}\mathfrak{f}(\tau) = \tau + \sum_{t=2}^{\infty}\mathcal{K}(t,\delta)a_t\tau^t,$$

where $\mathcal{K}(t,\delta) = \dfrac{\Gamma(t+1)\Gamma(2-\delta)}{\Gamma(t+1-\delta)}$. *The operator Ω^{δ} were defined by Owa and Srivatsava [7].*

Definition 1.3 *A function $\mathfrak{f}(\tau)$ of the form (1.1) is in $S(r,\lambda,\delta,t)$ if it satisfies the condition:*

$$\left|\frac{\dfrac{\tau(\Omega^{\delta}\mathfrak{f}(\tau))'}{(1-\lambda)\Omega^{\delta}\mathfrak{f}(\tau)+\lambda\tau(\Omega^{\delta}\mathfrak{f}(\tau))'}-1}{\dfrac{\tau(\Omega^{\delta}\mathfrak{f}(\tau))'}{(1-\lambda)\Omega^{\delta}\mathfrak{f}(\tau)+\lambda\tau(\Omega^{\delta}\mathfrak{f}(\tau))'}+1}\right| < r,$$

where $0 < r \le 1$, $0 \le \lambda < 1$ and $0 \le \delta < 1$.

It can be seen that, the special cases of the class $S(r,\lambda,\delta,t)$ for different choices of parametrs we get the following results:

(i) The class $S(r,0,0,0) = S(r)$ was studied by Owa [6].

(ii) The class $S(r,0,0,0) = S(r)$ was introduced by Padmanabham [8].

(iii) We note the class $S(r,\lambda,0,0) = S(r,\lambda)$ (See [1,2]).

(iv) Mogra [4] has shown a sufficient condition for a function in the class $S(r)$.

2. Main Results

Theorem 2.1 *A function $\mathfrak{f}(\tau) = \tau - \sum_{t=2}^{\infty}a_t\tau^t$ is in the class $S(r,\lambda,\delta,t)$ if and only if*

$$\sum_{t=2}^{\infty}\mathcal{K}(r,\delta)a_t[(1-\lambda)(t-1)+r(t(1+\lambda)+(1-\lambda))] < 2r. \tag{2.1}$$

Proof: Suppose $\mathfrak{f} \in S(r,\lambda,\delta,t)$. Then

$$\left|\frac{\dfrac{\tau(\Omega^{\delta}\mathfrak{f}(\tau))'}{(1-\lambda)\Omega^{\delta}\mathfrak{f}(\tau)+\lambda\tau(\Omega^{\delta}\mathfrak{f}(\tau))'}-1}{\dfrac{\tau(\Omega^{\delta}\mathfrak{f}(\tau))'}{(1-\lambda)\Omega^{\delta}\mathfrak{f}(\tau)+\lambda\tau(\Omega^{\delta}\mathfrak{f}(\tau))'}+1}\right| < r$$

$$\implies \left|\frac{(1-\lambda)[\tau - \sum_{t=2}^{\infty}\mathcal{K}(t,\delta)a_t t\tau^t - \tau + \sum_{t=2}^{\infty}\mathcal{K}(t,\delta)a_t\tau^t]}{(1+\lambda)(\tau - \sum_{t=2}^{\infty}\mathcal{K}(t,\delta)a_t t\tau^t) + (1-\lambda)(\tau - \sum_{t=2}^{\infty}\mathcal{K}(t,\delta)a_t\tau^t)}\right| < r$$

$$\implies \left|\sum_{t=2}^{\infty}\mathcal{K}(t,\delta)a_t\tau^t((1-\lambda)(t-1)+r[t(1+\lambda)+(1-\lambda)])\right| < 2r|\tau|.$$

Letting $|\tau| \to 1$, we get (2).

Conversley, $\mathfrak{f}(\tau) \in \mathcal{T}$ and satisfies (2).

Since for any $|\tau|$, we have $|Re(\tau)| \leq |\tau|$. So

$$\left| Re \left\{ \frac{\sum_{t=2}^{\infty} \mathcal{K}(t,\delta) a_t (1-\lambda)(t-1)\tau^t}{2t - \sum_{t=2}^{\infty} \mathcal{K}(t,\delta) a_t \tau^t [t(1+\lambda)+(1-\lambda)]} \right\} \right| \leq \left| \frac{\sum_{t=2}^{\infty} \mathcal{K}(t,\delta) a_t (1-\lambda)(t-1)}{2 - \sum_{t=2}^{\infty} \mathcal{K}(t,\delta) a_t [t(1+\lambda)+(1-\lambda)]} \right| < |r| \leq 1.$$

Choose values of τ, so that $\dfrac{\tau(\Omega^\delta \mathfrak{f}(\tau))'}{(1-\lambda)\Omega^\delta \mathfrak{f}(\tau) + \lambda\tau(\Omega^\delta \mathfrak{f}(\tau))'}$ is real.

Therefore,

$$\left| \frac{\dfrac{\tau(\Omega^\delta \mathfrak{f}(\tau))'}{(1-\lambda)\Omega^\delta \mathfrak{f}(\tau) + \lambda\tau(\Omega^\delta \mathfrak{f}(\tau))'} - 1}{\dfrac{\tau(\Omega^\delta \mathfrak{f}(\tau))'}{(1-\lambda)\Omega^\delta \mathfrak{f}(\tau) + \lambda\tau(\Omega^\delta \mathfrak{f}(\tau))'} + 1} \right| < 1,$$

that is

$$\mathfrak{f}(\tau) \in S(r, \lambda, \delta, t).$$

□

Corollary 2.1 *If $\mathfrak{f}(\tau) \in S(r, \lambda, \delta, t)$ then*

$$|a_t| \leq \frac{2r}{\mathcal{K}(t,\delta)[(1-\lambda)(t-1) + r(t(1+\lambda)+(1-\lambda))]}.$$

Theorem 2.2 *Let $0 \leq r < 1, 0 \leq \lambda_1 \leq \lambda_2 < 1$ then $S(r, \lambda_1, \delta, t) \subset S(r, \lambda_2, \delta, t)$.*

Proof: For $\mathfrak{f}(\tau) \in S(r, \lambda_2, \delta, t)$, we have

$$\sum_{t=2}^{\infty} \mathcal{K}(t,\delta) a_t [(1-\lambda_2)(t-1) + r(t(1+\lambda_2)+(1-\lambda_2))]$$

$$\leq \sum_{t=2}^{\infty} \mathcal{K}(t,\delta) a_t [(1-\lambda_1)(t-1) + r(t(1+\lambda_1)+(1-\lambda_1))].$$

Hence $f(z) \in S(r, \lambda_1, \delta, t)$. □

Theorem 2.3 *Let $\mathfrak{f}(\tau) \in S(r, \lambda, \delta, t)$. Define $\mathfrak{f}_1(\tau) = \tau$ and*

$$\mathfrak{f}_t(\tau) = \tau - \frac{2r}{\mathcal{K}(t,\delta)[(1-\lambda)(t-1) + r(t(1+\lambda)+(1-\lambda))]}\tau^t,$$

where $t = 2, 3, \cdots$ for r, $\lambda(0 \leq \lambda < 1)$ and $\tau \in U$.
Then $\mathfrak{f}(\tau) \in S(r, \lambda, \delta, t)$ if and only if $\mathfrak{f}(\tau)$ can be expressed as

$$\mathfrak{f}(\tau) = \sum_{t=1}^{\infty} \mu_t \mathfrak{f}_t(\tau),$$

where $\mu_t \geq 0$ and $\sum_{t=1}^{\infty} \mu_t = 1$.

Proof: If $\mathfrak{f}(\tau) = \sum_{t=1}^{\infty} \mu_t \mathfrak{f}_t(\tau)$ where $\mu_t \geq 0$ and $\sum_{t=1}^{\infty} \mu_t = 1$, then

$$\begin{aligned}\mathfrak{f}(\tau) &= \sum_{t=1}^{\infty} \mu_t \mathfrak{f}_t(\tau) = \mu_1 f_1(z) + \sum_{t=2}^{\infty} \mu_t \mathfrak{f}_t(\tau) \\ &= \tau - \sum_{t=2}^{\infty} \frac{2r\mu_t}{\mathcal{K}(t,\delta)[(1-\lambda)(t-1) + r(t(1+\lambda) + (1-\lambda))]}.\end{aligned}$$

From Theorem 2.1, Consider

$$\begin{aligned}&\sum_{t=2}^{\infty} \frac{2r\mu_t \mathcal{K}(t,\delta)[(1-\lambda)(t-1) + r(t(1+\lambda) + (1-\lambda))]}{\mathcal{K}(t,\delta)[(1-\lambda)(t-1) + r(t(1+\lambda) + (1-\lambda))]} \\ &= \sum_{t=2}^{\infty} \mu_t 2r \\ &= \sum_{t=2}^{\infty} (1-\mu_1) 2r \\ &\leq 2r.\end{aligned}$$

Hence $\mathfrak{f}(\tau) \in S(r, \lambda, \delta, t)$.

Conversley, let $\mathfrak{f}(\tau) = \tau - \sum_{t=2}^{\infty} a_t \tau^t \in S(r, \lambda, \delta, t)$, define

$$\mu_t = \mathcal{K}(t,\delta)[(1-\lambda)(t-1) + r(t(1+\lambda) + (1-\lambda))],$$

for $t = 2, 3, \cdots$, and $\mu_1 = 1 - \sum_{t=2}^{\infty} \mu_t$.

From Theorem 2.1, $\sum_{t=2}^{\infty} \mu_t \leq 1$ and hence $\mu_1 \geq 0$.

Since

$$\begin{aligned}\mu_t \mathfrak{f}_t(\tau) &= \mu_t \left[\tau - \frac{2r\tau^t}{\mathcal{K}(t,\delta)[(1-\lambda)(t-1) + r(t(1+\lambda) + (1-\lambda))]}\right] \\ &= \mu_t \tau - \frac{2r\mu_t \tau^t}{\mathcal{K}(t,\delta)[(1-\lambda)(n-1) + r(t(1+\lambda) + (1-\lambda))]} \\ &= \mu_t \tau - a_t \tau^t,\end{aligned}$$

then

$$\begin{aligned}\sum_{t=1}^{\infty} \mu_t \mathfrak{f}_t(\tau) &= \sum_{t=1}^{\infty} \mu_t \tau - a_t \tau^t \\ &= \sum_{t=1}^{\infty} \mu_t \tau - \sum_{t=1}^{\infty} a_t \tau^t \\ &= \tau - \sum_{t=1}^{\infty} a_t \tau^t = \mathfrak{f}(\tau).\end{aligned}$$

□

Theorem 2.4 *The class $S(r,\lambda,\delta,t)$ is closed under convex linear combination.*

Proof: Let $\mathfrak{f}(\tau),\mathfrak{g}(\tau)\in S(r,\lambda,\delta,t)$ and $\mathfrak{f}(\tau)=\tau-\sum_{t=2}^{\infty}a_t\tau^t$ and $\mathfrak{g}(\tau)=\tau-\sum_{t=2}^{\infty}b_t\tau^t$. For x such that $0\le x<1$, it suffices to show that the function defined by $\mathfrak{h}(\tau)=(1-x)\mathfrak{f}(\tau)+x\mathfrak{g}(\tau),\tau\in(U)$ belongs to $S(r,\lambda,\delta,t)$.
Now

$$\begin{aligned}\mathfrak{h}(\tau)&=(1-x)\mathfrak{f}(\tau)+x\mathfrak{g}(\tau)\\&=(1-x)[\tau-\sum_{t=2}^{\infty}a_t\tau^t]+x[\tau-\sum_{t=2}^{\infty}b_t\tau^t]\\ \mathfrak{h}(\tau)&=\tau-\sum_{t=2}^{\infty}((1-x)a_t+xb_t)\tau^t.\end{aligned}$$

Applying Theorem 2.1 to $\mathfrak{f}(\tau),\mathfrak{g}(\tau)\in S(r,\lambda,\delta,t)$, we have

$$\begin{aligned}&\sum_{t=2}^{\infty}\mathcal{K}(t,\delta)[(1-\lambda)(t-1)+r(t(1+\lambda)+(1-\lambda))][(1-x)a_t+xb_t]\\&=(1-x)\sum_{t=2}^{\infty}\mathcal{K}(t,\delta)[(1-\lambda)(t-1)+r(t(1+\lambda)+(1-\lambda))]a_t\\&+x\sum_{t=2}^{\infty}\mathcal{K}(t,\delta)[(1-\lambda)(t-1)+r(t(1+\lambda)+(1-\lambda))]b_t\\&\le(1-x)2r+x2r\\&=2r.\end{aligned}$$

That is $\mathfrak{h}(\tau)\in S(r,\lambda,\delta,t)$. □

Corollary 2.2 *If $\mathfrak{f}_1(\tau),\mathfrak{f}_2(\tau)$ are in $S(r,\lambda,\delta,t)$ then the function defined by $\mathfrak{g}(\tau)=\frac{1}{2}[\mathfrak{f}_1(\tau)+\mathfrak{f}_2(\tau)]$ is also in $S(r,\lambda,\delta,t)$.*

Theorem 2.5 *Let for $j=1,2,...,n$, $\mathfrak{f}_j(\tau)=\tau-\sum_{t=2}^{\infty}a_{t,j}z^t\in S(r,\lambda,\delta,t)$ and $0<\lambda_j<1$ such that $\sum_{j=1}^{n}\lambda_j=1$, then the function $F(\tau)$ defined by $F(\tau)=\sum_{j=1}^{t}\lambda_j\mathfrak{f}_j(\tau)$ is also in $S(r,\lambda,\delta,t)$.*

Proof: For each $j\in\{1,2,\cdots,n\}$, we obtain $\sum_{t=2}^{\infty}\mathcal{K}(t,\delta)[(1-\lambda)(t-1)+r(t(1+\lambda)+(1-\lambda))]a_t<2r$.

We have

$$\begin{aligned}F(\tau)&=\sum_{j=1}^{t}\lambda_j(\tau-\sum_{t=2}^{\infty}a_{t,j}\tau^t)\\&=\sum_{j=1}^{t}\lambda_j\tau-\sum_{t=2}^{\infty}(\sum_{j=1}^{t}\lambda_ja_{t,j})\tau^t\\&=\tau-\sum_{t=2}^{\infty}(\sum_{j=1}^{t}\lambda_ja_{t,j})\tau^t.\end{aligned}$$

Consider

$$\begin{aligned}
&\sum_{t=2}^{\infty}\mathcal{K}(t,\delta)[(1-\lambda)(t-1)+r(t(1+\lambda)+(1-\lambda))].\sum_{j=1}^{t}\lambda_j a_{t,j}\\
&=\sum_{j=1}^{t}\lambda_j[\sum_{t=2}^{\infty}\mathcal{K}(t,\delta)[(1-\lambda)(t-1)+r(t(1+\lambda)+(1-\lambda))]]a_{t,j}\\
&<\sum_{j=1}^{t}\lambda_j(2r)\\
&<2r.
\end{aligned}$$

Hence $F(\tau)\in S(r,\lambda,\delta,t)$. ◻

Theorem 2.6 *If* $\mathfrak{f}\in S(r,\lambda,\delta,t)$ *then*
$|\tau|-\frac{2r}{\mathcal{K}(2,\delta)[(1-\lambda)+r(3+\lambda)]}|\tau|^2\leq|\mathfrak{f}(\tau)|\leq|\tau|+\frac{2r}{\mathcal{K}(2,\delta)[(1-\lambda)+r(3+\lambda)]}|\tau|^2.$

Proof: Since

$$\xi(t)=\mathcal{K}(t,\delta)[(1-\lambda)(t-1)+r(t(1+\lambda)+(1-\lambda))] \tag{2.2}$$

is an increasing function of $t(t\geq 2)$, from Theorem 2.1,

$$\begin{aligned}
&\mathcal{K}(2,\delta)[(1-\lambda)+r(3+\lambda)]\sum_{t=2}^{\infty}|a_t|\\
&\leq\sum_{t=2}^{\infty}\mathcal{K}(t,\delta)[(1-\lambda)(t-1)+r(t(1+\lambda)+(1-\lambda))]|a_t|\\
&<2r.
\end{aligned}$$

That is,

$$\sum_{t=2}^{\infty}|a_t|\leq\frac{2r}{\mathcal{K}(2,\delta)[(1-\lambda)+r(3+\lambda)]}.$$

Thus,

$$\begin{aligned}
|\mathfrak{f}(\tau)|&=|\tau+\sum_{t=2}^{\infty}a_t\tau^t|\\
&\leq|\tau|+\sum_{t=2}^{\infty}|a_t|\tau|^2\\
&\leq|\tau|+\frac{2r}{\mathcal{K}(2,\delta)[(1-\lambda)+r(3+\lambda)]}.
\end{aligned}$$

Similarly, we get

$$\begin{aligned}
|\mathfrak{f}(\tau)|&\geq|\tau|-\sum_{t=2}^{\infty}|a_t|\tau|^2\\
&\geq|\tau|-\frac{2r}{\mathcal{K}(2,\delta)[(1-\lambda)+r(3+\lambda)]}
\end{aligned}$$

$$|\tau|-\frac{2r}{\mathcal{K}(2,\delta)[(1-\lambda)+r(3+\lambda)]}\leq|\mathfrak{f}(\tau)|\leq|\tau|+\frac{2r}{\mathcal{K}(2,\delta)[(1-\lambda)+r(3+\lambda)]}.$$

◻

Theorem 2.7 *Let* $\mathfrak{f}(\tau) \in S(r, \lambda, \delta, t)$ *then*

$$1 - \frac{4r}{\mathcal{K}(2,\delta)[(1-\lambda)+r(3+\lambda)]}|\tau| \leq |\mathfrak{f}'(\tau)| \leq 1 + \frac{4r}{\mathcal{K}(2,\delta)[(1-\lambda)+r(3+\lambda)]}|\tau|.$$

Proof: Since $\{t(\xi(t))\}$, where $\xi(t)$ given by (2.2), is an increasing function of t, In view of Theorem 2.1, we have

$$\begin{aligned} &\frac{\mathcal{K}(2,\delta)[(1-\lambda)+r(3+\lambda)]}{2}\sum_{t=2}^{\infty} t|a_t| \\ &\leq \mathcal{K}(t,\delta)[(1-\lambda)(n-1)+r(t(1+\lambda))] \\ &< 2r. \end{aligned}$$

That is

$$\sum_{t=2}^{\infty} t|a_t| < \frac{4r}{\mathcal{K}[(1-\lambda)+r(3+\lambda)]}.$$

Thus,

$$\begin{aligned} &|\mathfrak{f}'(\tau)| = |1 + \sum_{t=2}^{\infty} t a_t \tau^{t-1}| \\ &\leq 1 + \sum_{t=2}^{\infty} t|a_t||\tau| \\ &< 1 + \frac{4r}{\mathcal{K}(2,\delta)[(1-\lambda)+r(3+\lambda)]}|\tau|. \end{aligned}$$

Similarly, we get

$$|\mathfrak{f}'(\tau)| \geq 1 - \frac{4r}{\mathcal{K}(2,\delta)[(1-\lambda)+r(3+\lambda)]}|\tau|.$$

□

Theorem 2.8 *Let* $\mathfrak{f}(\tau) \in S(r, \lambda, \delta, t)$ *and Komato operator [3] of* $\mathfrak{f}$ *is defined by*

$$k(\tau) = \int_0^1 \frac{(c+1)^\gamma}{\Gamma(\gamma)} p^c \left(\log \frac{1}{p}\right)^{(\gamma-1)} \frac{\mathfrak{f}(p\tau)}{p} dp,$$

$c > -1$, $\gamma \geq 0$. *Then* $k(\tau) \in S(r, \lambda, \delta, t)$.

Proof: We have $\displaystyle\int_0^1 p^c \left(\log \frac{1}{p}\right)^{\gamma-1} dp = \frac{\Gamma(\gamma)}{(c+1)^\gamma}$.

$$\Longrightarrow \int_0^1 p^{t+c-1} \left(\log \frac{1}{p}\right)^{\gamma-1} dp = \frac{\Gamma(\gamma)}{(c+t)^\gamma},\ t = 2, 3, \ldots$$

$$\begin{aligned} \Longrightarrow k(\tau) &= \frac{(c+1)^\gamma}{\Gamma(\gamma)} \left[\int_0^1 p^c \left(\log \frac{1}{p}\right)^{\gamma-1} \tau dp + \sum_{t=2}^{\infty} \tau^t \int_0^1 a_t p^{t+c-1} \left(\log \frac{1}{p}\right)^{\gamma-1} dp\right] \\ &= \tau + \sum_{t=2}^{\infty} \left(\frac{c+1}{c+t}\right)^\gamma a_t \tau^t. \end{aligned}$$

Since $\mathfrak{f} \in S(r, \lambda, \delta, t)$ and $\left(\frac{c+1}{c+t}\right)^{\gamma} < 1$, we have

$$\sum_{t=2}^{\infty} \mathcal{K}(t, \delta)[(1-\lambda)(t-1) + r(t(1+\lambda) + (1-\lambda))]\left(\frac{c+1}{c+t}\right)^{\gamma} a_t < 2r.$$

□

Theorem 2.9 *Let $\mathfrak{f} \in S(r, \lambda, \delta, t)$ then for every $0 \leq \alpha \leq 1$ the function*

$$\mathcal{H}_{\alpha}(\tau) = (1-\alpha)\mathfrak{f}(\tau) + \alpha \int_0^{\tau} \frac{\mathfrak{f}(p)}{p} dp.$$

Proof: We have

$$\mathcal{H}_{\alpha}(\tau) = \tau + \sum_{t=2}^{\infty}(1 + \frac{\alpha}{t} - \alpha)a_t \tau^t.$$

Since $(1 + \frac{\alpha}{t} - \alpha) < 1, t \geq 2$. So by Theorem (2.1),

$$\begin{aligned}
&\sum_{t=2}^{\infty}(1 + \frac{\alpha}{t} - \alpha)\mathcal{K}(t, \delta)a_t[(1-\lambda)(t-1) + r(t(1+\lambda) + (1-\lambda))] \\
&< \sum_{t=2}^{\infty} \mathcal{K}(t, \delta)a_t[(1-\lambda)(t-1) + r(t(1+\lambda) + (1-\lambda))] \\
&< 2r.
\end{aligned}$$

Therefore,

$$\mathcal{H}_{\alpha}(\tau) \in S(r, \lambda, \delta, t).$$

□

3. Conclusion

In this paper, we have introduced a new subclass $S(r, \lambda, \delta, t)$ of analytic functions by employing the operator $(\Omega^{\delta}\mathfrak{f}(\tau))$ associated with the concept of fractional calculus. For functions belonging to this class, various significant results have been established, including coefficient estimates, inclusion relations, and the determination of extreme points. Furthermore, several geometric and analytical properties of the defined subclass have been discussed in detail. The results obtained in this study provide a foundation for further investigations into more generalized subclasses of analytic functions and their potential applications in geometric function theory.

Acknowledgments

The authors thank the anonymous reviewers for their careful reading of our manuscript and their many insightful comments and suggestions.

References

1. Frasin, B. A., *On certain subclasses of analytic functions associated with Poisson distribution series*, Acta Univ. Sapientiae, Math., 11(1), 78–86, (2019).
2. Frasin, B. A., Al-Hawary, T. and Yousef, F., *Necessary and sufficient conditions for hypergeometric functions to be in a subclass of analytic functions*, Afr. Mat., 30(1-2), 223–230, (2019).
3. Komatu, Y., *On analytic prolongation of a family of operators*, Math. Rev. Anal. Numér. Théor. Approximation, Math., 32(2), 141–145, (1990).
4. Mogra, M. L., *On a class of starlike functions in the unit disc. I*, J. Indian Math. Soc., New Ser., 40, 159–161, (1976).
5. Owa, S, *On the distortion theorems I*, Kyungpook Math. J., 18(1), 53–59, (1978).

6. Owa, S, *On certain classes of univalent functions in the unit disk*, Kyungpook Math. J., 24(2), 127–136, (1984).

7. Owa, S. and Srivastava, H. M., *Univalent and starlike generalized hypergeometric functions*, Can. J. Math., 39(5), 1057–1077, (1987).

8. Padmanabhan, K. S., *On certain classes of starlike functions in the unit disk*, J. Indian Math. Soc., New Ser., 32, 89–103, (1968).

N. Ravikumar
PG Department of Mathematics
JSS College of Arts, Commerce and Science
Mysuru-570 025, India.
E-mail address: ravisn.kumar@gmail.com

and

H. S. Roopa
PG Department of Mathematics
JSS College of Arts, Commerce and Science
Mysuru-570 025, India.
E-mail address: rooparoo1001@gmail.com

and

P. Siva Kota Reddy (Corresponding author)
Department of Mathematics
JSS Science and Technology University
Mysuru-570 006, India.

and

Universidad Bernardo O'Higgins
Facultad de Ingeniería, Ciencia y Tecnología
Departamento de Formación y Desarrollo Científico en Ingeniería
Av. Viel 1497, Santiago, Chile
E-mail address: pskreddy@jssstuniv.in