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Extended Type-II Topp-Leone Half-Logistic Distribution: Properties, Estimation and Applications

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ABSTRACT: This study advances the Type II Topp-Leone half-logistic distribution by introducing a scale parameter, enhancing its flexibility and applicability to real-world datasets. We introduce this refined model as the extended Type II Topp-Leone half-logistic distribution and thoroughly derive its statistical properties, including the hazard function, reliability function, odds function, quantile function, order statistics, and entropy, providing a comprehensive theoretical framework. Parameter estimation was conducted using 16 diverse strategies, validated through extensive simulations across varying sample sizes and metrics, affirming the consistency of all estimates. Among these, the Kolmogorov method emerged as superior, demonstrating exceptional estimation accuracy and robustness. Applying the extended model to real-life datasets, specifically focusing on the failure times of repairable items in reliability engineering, revealed that it substantially outperformed existing models, as demonstrated by various performance metrics. This study offers a powerful and versatile tool for modelling complex phenomena in reliability engineering, enhancing maintenance scheduling and reliability assessments. The extended Type II Topp-Leone half-logistic distribution sets a new standard in reliability analysis, providing deeper insights and more reliable predictions for engineering applications.

Key Words: Type II Topp Leone half logistic distribution, scale parameter, hazard function, survival function, Kolmogorov estimation.

Contents

1. Introduction

Probability distributions are fundamental to statistical analysis, providing essential frameworks for modeling and interpreting diverse phenomena across various scientific and practical domains (for more on the importance of probability distributions, see [1,2]). Examples such as the Gamma, Exponential, Weibull, Gompertz, and Rayleigh distributions have long been instrumental in modeling real-world phenomena. However, the rapid increase in the volume and complexity of emerging datasets has exposed the limitations of these traditional distributions, often rendering them inadequate for fitting such diverse and complex datasets. This has prompted researchers to explore innovative methods for extending and adapting these distributions to better accommodate the nuances of contemporary data.

One promising approach involves extending classical distributions by introducing additional parameters, such as scale, location, shape, and transmuted parameters, enhancing flexibility and adaptability.

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Studies have demonstrated that these extensions significantly improve the ability of existing distributions to model real-world complexities; see references [3], [4], [5], [6], [7], [8], [9], [10].

This study explores an enhancement to the Type II Topp-Leone Half-Logistic ($T_{II}TLHL$) distribution, initially proposed by [11]. Although the $T_{II}TLHL$ distribution is promising as a versatile model, its sole reliance on a shape parameter limits its ability to accommodate datasets with complex characteristics. To address this limitation, we introduce the Extended Type II Topp-Leone Half-Logistic ($E_{T_{II}TLHL}$) distribution, which introduces a scale parameter to the original $T_{II}TLHL$ distribution. This enhancement aims to fill a critical gap in the literature by providing a more adaptable and implementable statistical model, potentially improving accuracy in today's data-rich environments.

The motivation for this study is twofold. First, extending the Type II Topp Leone half-logistic distribution by introducing a scale parameter significantly enhances its flexibility, making it more adaptable to datasets with diverse characteristics. To validate this enhancement, we applied 16 different parameter estimation methods and conducted a comprehensive simulation study to evaluate their performance under various conditions. This rigorous testing ensures the robustness and reliability of our proposed model. Second, we applied the extended model to two real-life datasets involving the failure times of repairable items in reliability engineering. Accurate modeling of failure times is crucial for designing effective maintenance schedules and improving product reliability. The distribution can also be applied to a wide range of practical data across various fields, including biomedical sciences, environmental studies, and physics. Our study aims to contribute to the statistical literature by offering a robust and versatile tool for tackling complex data challenges across various domains.

The following are the objectives of this paper:

- To propose an extension of the $T_{II}TLHL$ model by incorporating a scale parameter to improve its flexibility and adaptability.
- To compute statistical properties of the model, such as the quantile function, moments, moment-generating function, survival function, hazard function, Rényi entropy, and order statistics.
- To employ sixteen different estimation methods to estimate the parameters of the distribution. These methods include maximum likelihood estimation, Anderson-Darling, Cramér-von Mises, maximum product of spacings, least squares, percentile, right-tail Anderson-Darling, weighted least squares, left-tail Anderson-Darling, minimum spacing absolute distance, minimum spacing absolute-log distance, Anderson-Darling left-tail second order, Kolmogorov, minimum spacing square distance, minimum spacing square-log distance, and minimum spacing Linex distance.
- To conduct a simulation study to investigate the consistency of the estimates obtained from the various estimation techniques.
- To demonstrate that the proposed model provides a better fit compared to several established models, such as the Type II Topp-Leone half-logistic (HL), the exponentiated generalized standard HL, the exponentiated HL, the Topp-Leone Fréchet, the Type II half-logistic Weibull, and the Poisson generalized HL models.

The article is structured as follows: Section 2 outlines the formulation of the $ET_{II}TLHL$ distribution. Section 3 explores the primary statistical properties of the model. A description of the sixteen estimation methods is provided in Section 4. Section 5 details simulation studies that validate the consistency of the estimates. The application of the new distribution is illustrated through the analysis of two real datasets in Section 6. Lastly, Section 7 presents the concluding insights derived from the study.

2. Model Formulation

In 2023, Adubisi and Adubisi [11] derived a novel distribution called type II Topp Leone half logistic ($T_{II}TLHL$) distribution. Its cumulative distribution function (CDF) is defined as follows

$$R(x) = 1 - \left(1 - \left(\frac{1-e^{-x}}{e^{-x}+1}\right)^2\right)^{\delta},\ x,\ \delta > 0, \tag{2.1}$$

and its probability density function (PDF) is given by

$$r(x) = 4^{\delta} \delta \left(e^{x} - 1\right) e^{\delta x} \left(e^{x} + 1\right)^{-2\delta - 1}. \tag{2.2}$$

Now, we will add a new scale parameter to the CDF (2.1), to have a new statistical model called extended $T_{II}TLHL$ ($ET_{II}TLHL$), its CDF is given by

$$F(x) = 1 - \left(1 - \left(\frac{1 - e^{-\gamma x}}{e^{-\gamma x} + 1}\right)^{2}\right)^{\delta}, \; x, \; \delta, \; \gamma > 0, \tag{2.3}$$

and its PDF is defined as follows

$$f(x) = 4^{\delta} \delta \gamma \left(e^{\gamma x} - 1\right) e^{\delta \gamma x} \left(e^{\gamma x} + 1\right)^{-2\delta - 1}. \tag{2.4}$$

Both of PDF and CDF Plots of the E$T_{II}TLHL$ distribution are presented in Figure 1.

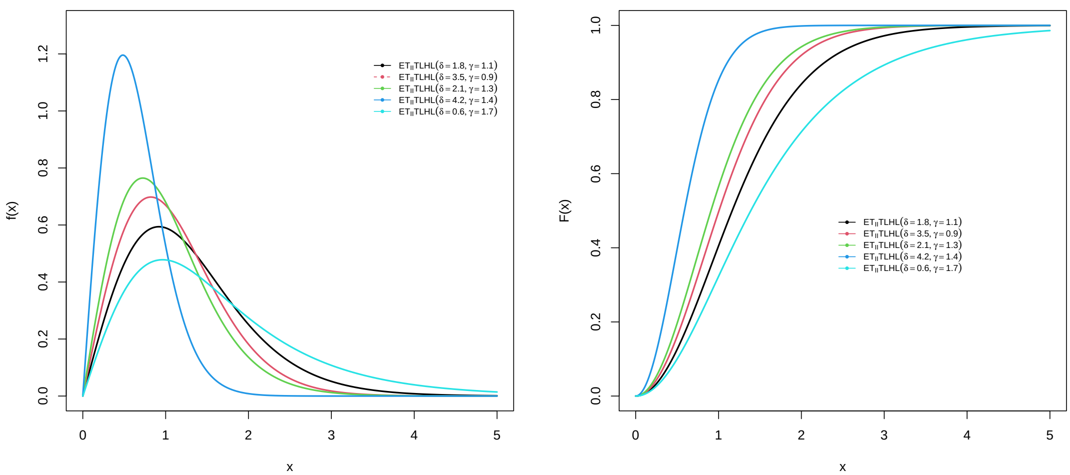

Figure 1: PDF and CDF Plots of the E$T_{II}TLHL$ distribution

3. Statistical Properties

This section explores various properties of the $ET_{II}TLHL$ distribution, including its significant representations, moments, moment-generating function, order statistics, entropy, survival function, hazard function, quantile function, and odds function.

Quantile function

The quantile function indeed plays a pivotal role in statistics. It allows us to identify specific points within a distribution corresponding to given probabilities. This function helps determine various important values such as medians, quartiles, and percentiles. These values are crucial in describing the spread and central tendency of data. Moreover, the quantile function is essential for various statistical techniques, including accurate simulation methods like Monte Carlo simulations. More discussion on quantile function can be found in the works of [12,13,14]. The quantile function of E$T_{II}TLHL$ distribution is calculated as:

$$Q\left(u\right)=-\frac{1}{\gamma}\log\left[\frac{1-\left(1-\left(1-u\right)^{1/\delta}\right)^{1/2}}{\left(1-\left(1-u\right)^{1/\delta}\right)^{1/2}+1}\right]. \tag{3.1}$$

The numerical representation for the proposed model quantile function is presented in Table 1.

Table 1: Quantiles for different values of δ and γ

u	$\delta=0.5,\gamma=0.3$	$\delta=2.5,\gamma=3.4$	$\delta=0.2,\gamma=2.1$	$\delta=4.5,\gamma=3.0$
0.1	3.3994	0.2391	0.6328	0.2596
0.2	4.1758	0.2590	0.8952	0.2750
0.3	5.0122	0.2781	1.1980	0.2892
0.4	5.9725	0.2979	1.5553	0.3037
0.5	7.1194	0.3200	1.9840	0.3194
0.6	8.5442	0.3459	2.5129	0.3374
0.7	10.4094	0.3783	3.1969	0.3594
0.8	13.0729	0.4231	4.1621	0.3891
0.9	17.6693	0.4992	5.8124	0.4382

The median for the E$T_{II}TLHL$ distribution is obtained by substituting u = 0.5 in (3.1) as:

$$Median=-\frac{1}{\gamma}\log\left[\frac{1-\left(1-\left(1-0.5\right)^{1/\delta}\right)^{1/2}}{\left(1-\left(1-0.5\right)^{1/\delta}\right)^{1/2}+1}\right]. \tag{3.2}$$

Reliability, hazard, and odd functions

The reliability function quantifies the likelihood that an item will continue to function beyond a designated time threshold. The reliability function of the E$T_{II}TLHL$ distribution is derived as:

$$s(x)=\left(1-\left(\frac{1-e^{-\gamma x}}{e^{-\gamma x}+1}\right)^{2}\right)^{\delta}. \tag{3.3}$$

The hazard function represents the conditional probability density that an event will occur at a specific time, given that the event has not occurred up to that time. The hazard function of the E$T_{II}TLHL$ distribution is computed as:

$$h(x)=\frac{4^{\delta}\delta\gamma(\exp(\gamma x)-1)\exp(\delta\gamma x)(\exp(\gamma x)+1)^{-2\delta-1}}{\left(1-\left(\frac{1-\exp(-\gamma x)}{\exp(-\gamma x)+1}\right)^{2}\right)^{\delta}}. \tag{3.4}$$

Both of survival and hazard plots of the E$T_{II}TLHL$ distribution are presented in Figure 2.

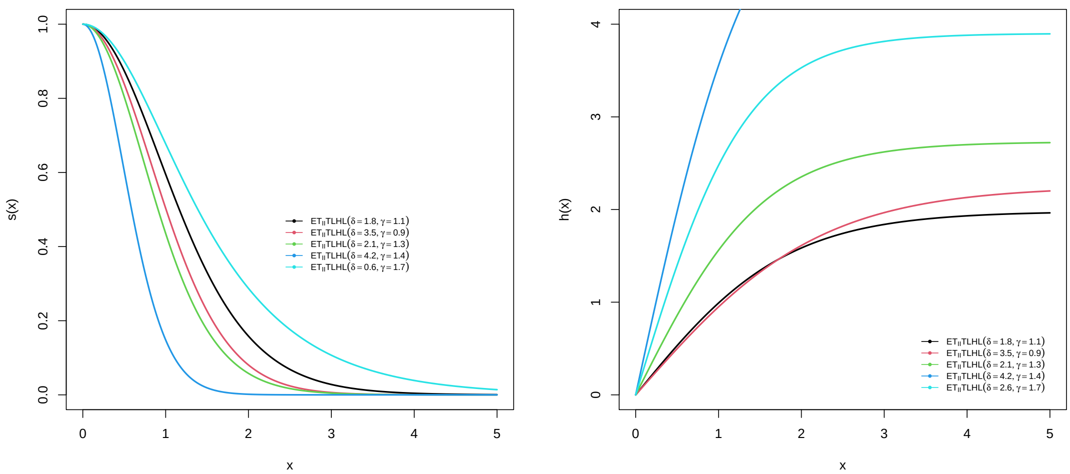

Figure 2: Survival and Hazard Plots of the E$T_{II}TLHL$ distribution

The odd function of E$T_{II}TLHL$ distribution is calculated as:

$$O\left(x\right)=\frac{1-\left(1-\left(\frac{1-e^{-\gamma x}}{e^{-\gamma x}+1}\right)^{2}\right)^{\delta}}{\left(1-\left(\frac{1-e^{-\gamma x}}{e^{-\gamma x}+1}\right)^{2}\right)^{\delta}}. \tag{3.5}$$

Linear representation

The CDF of our proposed model is written by

$$F(x)=1-\left(1-\left(\frac{1-e^{-\gamma x}}{e^{-\gamma x}+1}\right)^{2}\right)^{\delta},\ x,\ \delta,\ \gamma>0,$$

Then, by using a suitable series to simplify our model statistical properties calculations, we have the $ET_{II}TLHL$ CDF as follows

$$\begin{aligned}
F(x) &= 1-\sum_{v=0}^{\infty}\left[(-1)^{v}\binom{\lambda}{v}\left(1-v^{-\mu x}\right)^{2v}\left(v^{-\mu x}+1\right)^{-2v}\right],\\
&= 1-\sum_{v=0}^{\infty}\sum_{h=0}^{2v}\sum_{w=0}^{\infty}\binom{2v}{h}\binom{\lambda}{v}\binom{2v+w-1}{w}(-1)^{v+h+w}e^{-\mu(h+w)x},\\
&= 1-\sum_{v,w=0}^{\infty}\sum_{h=0}^{2v}\phi_{v,h,w}e^{-\mu(h+w)x}. \qquad (3.6)
\end{aligned}$$

where $\phi_{v,h,w}=(-1)^{v+h+w}\left(\begin{array}{c}2v\\h\end{array}\right)\left(\begin{array}{c}\lambda\\v\end{array}\right)\left(\begin{array}{c}2v+w-1\\w\end{array}\right)$

Due to the convergence of series in Equation (3.6), we can have a simple version of our proposed model PDF as follows

$$f(x) = \sum_{v=0}^{\infty}\sum_{h=0}^{2v}\sum_{w=0}^{\infty}\binom{2v}{h}\binom{\lambda}{v}\binom{2v+w-1}{w}(-1)^{v+h+w}[\mu(h+w)]e^{-\mu(h+w)x},$$
$$= \sum_{v,w=0}^{\infty}\sum_{h=0}^{2v}\Psi_{v,h,w}[\mu(h+w)]e^{-\mu(h+w)x}. \tag{3.7}$$

where $\Psi_{v,h,w} = \left(\begin{array}{c} 2v \\ h \end{array}\right)\left(\begin{array}{c} \lambda \\ v \end{array}\right)\left(\begin{array}{c} 2v+w-1 \\ w \end{array}\right)(-1)^{v+h+w}$

Moment

Moments play a pivotal role in capturing the essence of a distribution's characteristics. For a random variable X, moments are precisely defined as its expected values. These moments serve as fundamental descriptors that encapsulate the statistical properties and shape of the distribution, providing crucial insights into its behavior and variability.

For the $ET_{II}TLHL$ distribution, the r-th model is derived as follows:

$$\mu_r = \sum_{v,w=0}^{\infty}\sum_{h=0}^{2v}\Psi_{v,h,w}\int_0^{\infty} x^r[\mu(h+w)]e^{-\mu(h+w)x}dx,$$

$$\mu_r = \sum_{v,w=0}^{\infty}\sum_{h=0}^{2v}\Psi_{v,h,w}\frac{\Gamma(r+1)}{[\mu(h+w)]^{r+1}}. \tag{3.8}$$

Tables 2 and 3 showcase the statistical computation of the first four moments, variance(Var), skewness (S_K), and kurtosis(K_U) for the $ET_{II}TLHL$ distribution at various parameter values of γ and δ . Results of the skewness and kurtosis values indicates that the distribution is positively skewed and platykurtic

Table 2: Outcome of statistical measures μ_1', μ_2', μ_3', μ_4', Var, S_K , K_U for the $ET_{II}TLHL$ distribution

γ	δ	μ_1'	μ_2'	μ_3'	μ_4'	Var	S_K	K_U
2.5	1.5	0.6283	0.5324	0.5638	0.7174	0.1376	1.1032	4.9556
	1.7	0.5846	0.4579	0.4452	0.5165	0.1161	1.0540	4.7509
	1.9	0.5489	0.4015	0.3625	0.3885	0.1001	1.0135	4.5859
	2.1	0.5189	0.3573	0.3023	0.3022	0.0879	0.9798	4.4510
	2.3	0.4933	0.3218	0.2569	0.2415	0.0783	0.9513	4.3390
	2.5	0.4712	0.2926	0.2218	0.1973	0.0705	0.9269	4.245
	2.7	0.4518	0.2683	0.1939	0.1640	0.0641	0.9059	4.1650
	2.9	0.4345	0.2476	0.1714	0.1385	0.0588	0.8875	4.0962
2.3	1.5	0.6829	0.6290	0.7241	1.0014	0.1626	1.1032	4.9556
	1.7	0.6355	0.5410	0.5717	0.7211	0.1371	1.0540	4.7509
	1.9	0.5966	0.4743	0.4655	0.5423	0.1183	1.0135	4.5859
	2.1	0.5641	0.4221	0.3882	0.4219	0.1039	0.9798	4.4510
	2.3	0.5363	0.3802	0.3300	0.3371	0.0925	0.9513	4.3390
	2.5	0.5122	0.3457	0.2848	0.2754	0.0834	0.9269	4.2450
	2.7	0.4910	0.3170	0.2490	0.2290	0.0758	0.9059	4.1650
	2.9	0.4723	0.2926	0.2201	0.1933	0.0695	0.8875	4.0962
2.0	1.5	0.7853	0.8319	1.1012	1.7514	0.2150	1.1032	4.9556
	1.7	0.7308	0.7155	0.8696	1.2612	0.1814	1.0540	4.7509
	1.9	0.6861	0.6273	0.7081	0.9486	0.1565	1.0135	4.5859
.	2.1	0.6487	0.5583	0.5905	0.7379	0.1374	0.9798	4.4510
	2.3	0.6167	0.5028	0.5019	0.5897	0.1224	0.9513	4.3390
	2.5	0.5890	0.4572	0.4332	0.4816	0.1100	0.9269	4.245
	2.7	0.5647	0.4192	0.3788	0.4005	0.1003	0.9059	4.1650
	2.9	0.5432	0.3870	0.3348	0.3382	0.0919	0.8875	4.0962
1.5	1.5	1.0471	1.4789	2.6104	5.5355	0.3823	1.1032	4.9556
	1.7	0.9744	1.2721	2.0612	3.9860	0.3225	1.0540	4.7509
	1.9	0.9148	1.1153	1.6784	2.9980	0.2783	1.0135	4.5859
	2.1	0.8649	0.9925	1.3997	2.3324	0.2444	0.9798	4.4510
	2.3	0.8223	0.8939	1.1897	1.8639	0.2176	0.9513	4.3390
	2.5	0.7853	0.8129	1.0270	1.5223	0.1960	0.9269	4.245
	2.7	0.7530	0.7453	0.8980	1.2660	0.1783	0.9059	4.1650
	2.9	0.7242	0.6880	0.7937	1.0690	0.1634	0.8875	4.0962
1.3	1.5	1.2083	1.9690	4.0101	9.8119	0.5090	1.1032	4.9556
	1.7	1.1243	1.6936	3.1665	7.0654	0.4293	1.0540	4.7509
	1.9	1.0556	1.4849	2.5784	5.3141	0.3705	1.0135	4.5859
	2.1	0.9980	1.3214	2.1502	4.1342	0.3254	0.9798	4.4510
	2.3	0.9488	1.1901	1.8276	3.3038	0.2898	0.9513	4.3390
	2.5	0.9062	1.0823	1.5776	2.6984	0.2610	0.9269	4.245
	2.7	0.8688	0.9923	1.3795	2.2441	0.2374	0.9059	4.1650
	2.9	0.8357	0.9160	1.2193	1.8948	0.2176	0.8875	4.0962

Table 3: Outcome of statistical measures μ_1', μ_2', μ_3', μ_4', Var, S_K , K_U for the $ET_{II}TLHL$ distribution

γ	δ	μ_1'	μ_2'	μ_3'	μ_4'	Var	S_K	K_U
3.2	0.4	1.1497	2.0162	4.9011	15.4908	0.6943	1.7045	7.6740
	0.6	0.8670	1.0935	1.8472	3.9730	0.3418	1.5328	6.8770
	0.8	0.7185	0.7295	0.9681	1.6058	0.2133	1.3939	6.2377
	1.0	0.625	0.5415	0.6023	0.8253	0.1508	1.2850	5.7458
	1.2	0.5597	0.4284	0.4156	0.4908	0.1151	1.1996	5.3688
	1.4	0.5109	0.3535	0.3069	0.3215	0.0924	1.1318	5.0770
	1.6	0.4729	0.3005	0.2378	0.2253	0.0769	1.0773	4.8475
	1.8	0.4421	0.2611	0.1909	0.1661	0.0656	1.0328	4.6641
3.6	0.4	1.0219	1.5930	3.4422	9.6708	0.5486	1.7045	7.6740
	0.6	0.7706	0.8640	1.2974	2.4803	0.2700	1.5328	6.8770
	0.8	0.6386	0.5764	0.6799	1.0025	0.1685	1.3939	6.2377
	1.0	0.5555	0.4278	0.4230	0.5152	0.1192	1.2850	5.7458
	1.2	0.4975	0.3385	0.2919	0.3064	0.0910	1.1996	5.3688
	1.4	0.4542	0.2793	0.2156	0.2007	0.0730	1.1318	5.0770
	1.6	0.4203	0.2374	0.1670	0.1407	0.0607	1.0773	4.8475
	1.8	0.3930	0.2063	0.1341	0.1037	0.0518	1.0328	4.6641
4.0	0.4	0.9197	1.2903	2.5093	6.3450	0.4444	1.7045	7.6740
	0.6	0.6936	0.6998	0.9458	1.6273	0.2187	1.5328	6.8770
	0.8	0.5748	0.4669	0.4956	0.6577	0.1365	1.3939	6.2377
.	1.0	0.5000	0.3465	0.3084	0.3380	0.0965	1.2850	5.7458
	1.2	0.4477	0.2742	0.2128	0.2010	0.0737	1.1996	5.3688
	1.4	0.4087	0.2262	0.1571	0.1316	0.0591	1.1318	5.0770
	1.6	0.3783	0.1923	0.1217	0.0923	0.0492	1.0773	4.8475
	1.8	0.3537	0.1671	0.0977	0.0680	0.0420	1.0328	4.6641
4.4	0.4	0.8361	1.0664	1.8853	4.3337	0.3672	1.7045	7.6740
	0.6	0.6305	0.5784	0.7106	1.1115	0.1808	1.5328	6.8770
	0.8	0.5225	0.3859	0.3724	0.4492	0.1128	1.3939	6.2377
	1.0	0.4545	0.2864	0.2317	0.2309	0.0798	1.2850	5.7458
	1.2	0.4070	0.2266	0.1598	0.1373	0.0609	1.1996	5.3688
	1.4	0.3716	0.1870	0.1180	0.0899	0.0489	1.1318	5.0770
	1.6	0.3439	0.1589	0.0915	0.0630	0.0406	1.0773	4.8475
	1.8	0.3215	0.1381	0.0734	0.0464	0.0347	1.0328	4.6641
4.8	0.4	0.7664	0.8961	1.4521	3.0599	0.3086	1.7045	7.6740
	0.6	0.5780	0.4860	0.5473	0.7847	0.1519	1.5328	6.8770
	0.8	0.4790	0.3242	0.2868	0.3172	0.0948	1.3939	6.2377
	1.0	0.4166	0.2406	0.1784	0.16303	0.06706	1.2850	5.7458
	1.2	0.3731	0.1904	0.1231	0.0969	0.0511	1.1996	5.3688
	1.4	0.3406	0.1571	0.0909	0.0635	0.0410	1.1318	5.0770
	1.6	0.3152	0.1335	0.0704	0.0445	0.0341	1.0773	4.8475
	1.8	0.2947	0.1160	0.0565	0.0328	0.0291	1.0328	4.6641

Moment generating function

The moment generating function (MGF) of a continuous random variable x is defined as:

$$M_x\left(t\right) = \int_{-\infty}^{\infty} e^{tx} f(x) dx,$$

using the written form in (3.7), the MGF of the $ET_{II}TLHL$ distribution is obtained as follows:

$$M_x(t)=\sum_{v,w=0}^{\infty}\sum_{h=0}^{2v}\Psi_{v,h,w}\int_0^{\infty}e^{tx}[\mu(h+w)]e^{-\mu(h+w)x}dx,$$

$$M_x(t)=\sum_{v,w=0}^{\infty}\sum_{h=0}^{2v}\Psi_{v,h,w}\left[\frac{\mu(h+w)}{\mu(h+w)-t}\right]. \tag{3.9}$$

Order statistic

The PDF of the Kth order statistic for a random sample of size n from a distribution F(X) and an associated pdf f(x) is given by:

$$f_{k:n}=\frac{n!}{(k-1)!\,(n-k)!}f(x)\,[F(x)]^{k-1}[1-F(x)]^{n-k},$$

The kth order statistic of the $\mathrm{ET}_{II}TLHL$ distribution is given as:

$$f_{k:n}=\frac{n!}{(k-1)!\,(n-k)!}\left[1-\sum_{v,w=0}^{\infty}\sum_{h=0}^{2v}\phi_{v,h,w}e^{-\mu(h+w)x}\right]^{k-1}\left[\sum_{v,w=0}^{\infty}\sum_{h=0}^{2v}\phi_{v,h,w}e^{-\mu(h+w)x}\right]^{n-k}$$
$$\sum_{v,w=0}^{\infty}\sum_{h=0}^{2v}\Psi_{v,h,w}[\mu(h+w)]e^{-\mu(h+w)x}.$$

Entropy

Entropy serves as a profound metric quantifying the degree of uncertainty or disorder within a distribution: heightened entropy signifies amplified unpredictability or indeterminacy in the potential outcomes of a random variable. This pivotal concept lies at the heart of information theory, wielding significant influence across diverse disciplines including statistics, physics, and computer science. The Renyi entropy, a variant of entropy, is generally expressed as:

$$I_y(x)=\frac{1}{1-\lambda}\log\int_{-\infty}^{\infty}f(x)^{\lambda}dx,$$

The Renyi entropy of the $\mathrm{ET}_{II}TLHL$ distribution can be computed as follows:

$$f(x)^{\lambda}=\left[\sum_{v,w=0}^{\infty}\sum_{h=0}^{2v}\Psi_{v,h,w}\right]^{\lambda}\left[[\mu(h+w)]e^{-\mu(h+w)x}\right]^{\lambda},$$

let, $\beta=\left[[\mu(h+w)]e^{-\mu(h+w)x}\right]$,

$$f(x)^{\lambda}=\left[\sum_{v,w=0}^{\infty}\sum_{h=0}^{2v}\Psi_{v,h,w}\right]^{\lambda}\beta^{\lambda},$$

$$I_y(x)=\frac{1}{1-\lambda}\left[\left(\sum_{v,w=0}^{\infty}\sum_{h=0}^{2v}\Psi_{v,h,w}\right)^{\lambda}\log\int_0^{\infty}\beta^{\lambda}dx\right]. \tag{3.10}$$

4. Estimation Methods

In this section, sixteen different estimation techniques are used to estimate the parameters δ and γ of the proposed model $ET_{II}TLHL$. These methods, some of which have been discussed in earlier studies (see, [15]; [16]; [17]), are presented as follows.

Method of maximum likelihood (MLE)

The MLE technique estimates this subsection's $ET_{II}TLHLD$ parameters. Let $x_1, \ldots, x_n$ are n observed sample from PDF (2.4). The log-likelihood density function of $ET_{II}TLHLD$ can be expressed as below:

$$L(x;\delta,\gamma) = n\delta\log(4) + n\log(\delta\gamma) + \sum_{i=1}^{n}\log(e^{\gamma x_i}-1) + \delta\gamma\sum_{i=1}^{n}x_i - (2\delta+1)\sum_{i=1}^{n}\log(e^{\gamma x_i}+1).$$

The log-likelihood function can be maximized to determine the MLEs $\hat{\delta}$ and $\hat{\gamma}$ of the parameters δ and γ using some optimization package in R, Python, Matlab, etc. software.

Method of Anderson-Darling (ADE)

The ADE is selected for its sensitivity to deviations from the assumed distribution. For the $ET_{II}TLHLD$, the ADE can be obtained by optimizing the following function with respect to the parameters of the model.

$$A(x_i) = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\left[\log F(x_{i:n}) + \log S(x_{n-i-1:n})\right]$$

Method of Cramer-von-Mises (CVME)

In parameter estimation using the CVME method, the goal is to find the parameter values of a theoretical distribution that minimizes the Cramér-von Mises statistic. It is chosen for its sensitivity to differences in the CDF and its ability to detect departure from the assumed distribution. This involves optimizing the parameters to achieve the best possible fit between the observed data and the theoretical distribution.

$$C(x_i) = \frac{1}{12n} + \sum_{i=1}^{n}\left[F(x_{i:n}) - \frac{2i-1}{2n}\right]^2$$

Method of maximum product of spacings (MPSE)

The MPS is a nonparametric estimation method used in statistics to estimate the parameters of the probability model and is suitable when the underlying distribution is not known. The Estimation Method of MPS involves maximizing the product of the observed spacings between ordered data points. This method is particularly employed when estimating parameters for continuous probability distributions.

$$\delta\left(x_i\right) = \frac{1}{n+1}\sum_{i=1}^{n+1}\log I_i(x_i),$$

where $I_i(x_i) = F(x_{i:n}) - F(x_{i-1:n})$, $F(x_{0:n}) = 0$ and $F(x_{n+1:n}) = 1$.

Method of least squares

The method of LSE is a statistical technique used to estimate the parameters of a mathematical model by minimizing the sum of the squares of the differences between observed and predicted values. It is selected for its simplicity and ease of interpretation.

$$V(x_i) = \sum_{i=1}^{n}\left[F(x_{i:n}) - \frac{i}{n+1}\right]^2.$$

Method of percentile (PEM)

The PEM is an approach for estimating model parameters by equating population percentiles (as functions of the parameters) to sample percentiles.

$$PCE = \sum_{i=1}^{n}[x_{i:n} - Q(p_i)]^2, \;\; p_i = \frac{i}{n+1}.$$

Method of right-tail Anderson Darling (RTAD)

The Estimation Method RTAD involves determining the parameters of a statistical distribution. This method is chosen to specifically assess the fit of the model in the right tail of the distribution, typically a CDF, by optimizing the right-tail version of the ADE statistic.

$$R(x_i) = \frac{n}{2} - 2\sum_{i=1}^{n} F(x_{i:n}) - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\log S(x_{i:n})$$

Method of weighted least squares (WLSE)

WLSE is a statistical method used to estimate the parameters of a probability model, where varying weights are assigned to each observation. WLSE is used when certain data points are considered more important or reliable than others, allowing for a more nuanced estimation.

$$W(x_i) = \sum_{i=1}^{n} \frac{(n+1)^2(n+2)}{i(n-i+1)} \left[F(x_{i:n}) - \frac{i}{n+1}\right]^2.$$

Method of left tail Anderson Darling (LTADE)

The LTAD estimation method refers to approaches used to estimate the parameters of a distribution, particularly focusing on the left tail, based on the ADE statistic. The ADE statistic is a measure of how well a given distribution fits a set of data. By minimizing the CDF of $ET_{II}TLHLD$ for ordered random variables, we can estimate the parameters of the suggested model. It complements the right-tail method by examining deviations in the opposite tail, providing a more comprehensive evaluation of model fit.

$$L(x_i) = -\frac{3}{2}n + 2\sum_{i=1}^{n} F(x_{i:n}) - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\log F(x_{i:n})$$

Minimum spacing absolute distance (MSADE)

Because of its simplicity and ability to capture the spacing between data points, it can be useful for parameter estimation. If we have an ordered sample denoted as $X_{1:n}, \ldots, X_{n:n}$ from $ET_{II}TLHLD$, the minimum spacing absolute distance estimates can be derived by minimizing the following function

$$\zeta\left(x_i\right) = \sum_{i=1}^{n+1} |I_i - \frac{1}{n+1}|.$$

Minimum spacing absolute-log distance (MSALDE)

This method extends the minimum spacing approach by considering the logarithm of the distances, which can be useful for handling data with varying scales. Using this estimation method, we can estimate the model parameters by minimizing the following expression

$$\Upsilon\left(x_i\right) = \sum_{i=1}^{n+1} |\log I_i - \log\frac{1}{n+1}|.$$

Anderson Darling left tail second order (ADLTSOE

This method is chosen to explore second-order effects in the left tail of the distribution, providing a more detailed analysis. The $ET_{II}TLHLD$ distribution parameters can be derived for Left-Tail Second-Order estimation using the ADLTSOE, achieved by minimizing the following function.

$$LTS = 2\sum_{i=1}^{n} \log F(x_i) + \frac{1}{n}\sum_{i=1}^{n} \frac{(2i-1)}{F(x_i)}$$

Kolmogorov Method (KME)

For its simplicity and effectiveness in assessing the maximum deviations between empirical and theoretical distributions. The Kolmogorov estimators for the parameters of the $ET_{II}TLHLD$ distribution can be obtained by optimizing the following expression.

$$KM = \underset{1\leq i\leq n}{MAX}\left[\frac{i}{n} - F(x_i), F(x_i) - \frac{i-1}{n}\right]$$

Minimum spacing square distance

The minimum spacing square distance estimation (MSSDE) is employed to obtain our suggested model estimators by optimizing the following equation.

$$\phi_{(x_i)} = \sum_{i=1}^{n+1}(I_i - \frac{1}{n+1})^2.$$

Minimum spacing square-log distance

The minimum spacing square distance estimation (MSSLDE) is employed to obtain our suggested model estimators by minimizing the following equation.

$$\Psi_{(x_i)} = \sum_{i=1}^{n+1}\left(\log I_i - \log\frac{1}{n+1}\right)^2.$$

Minimum spacing linex distance

The minimum spacing linex distance estimation (MSLNDE) is employed to obtain our suggested model estimators by minimizing the following equation.

$$\varDelta_{(x_i)} = \sum_{i=1}^{n+1}\left[e^{I_i - \frac{1}{n+1}} - \left(I_i - \frac{1}{n+1}\right) - 1\right].$$

5. Simulation

This section compares the performance of several estimation approaches for estimating the parameters of the proposed model using a vast quantity of simulated data. In our simulation, we used the suggested model quantity function to create random data sets for various sample sizes ($n = 25, 100, 200, 350, 450$ and 600). This section will investigate our model estimators' performance and behavior. Furthermore, we will assess the performance of various estimating strategies using a variety of metrics such as such as average of bias ($|Bias(\widehat{\boldsymbol{B}})| = \frac{1}{L}\sum_{i=1}^{L}|\widehat{\boldsymbol{B}} - \boldsymbol{B}|$), mean squared errors ($MSE = \frac{1}{L}\sum_{i=1}^{L}(\widehat{\boldsymbol{B}} - \boldsymbol{B})^2$), mean relative errors ($MRE = \frac{1}{L}\sum_{i=1}^{L}|\widehat{\boldsymbol{B}} - \boldsymbol{B}|/\boldsymbol{B}$), average absolute difference ($D_{abs} = \frac{1}{n\ L}\sum_{i=1}^{L}\sum_{j=1}^{n}|F(x_{ij}|\boldsymbol{B}) - F(x_{ij}|\widehat{\boldsymbol{B}})|$), maximum absolute difference ($D_{max} = \frac{1}{L}\sum_{i=1}^{L}\max_{j}|F(x_{ij}|\boldsymbol{B}) - F(x_{ij}|\widehat{\boldsymbol{B}})|$), and average squared absolute error ($ASAE = \frac{1}{n}\sum_{i=1}^{n}\frac{|x_i - \hat{x_i}|}{x_n - x_1}$), where x_i are the ascending ordered observations and $\boldsymbol{B} = (\delta, \gamma)$.

Tables 4 through 8 present the results of simulating the specified model parameter with 16 estimation approaches. Figures 3 through 8 visually show the data from Table 5. It is critical to stress that all parameter estimates for the proposed distribution are accurate and near to their real values. The estimation methodologies provided are exact. As n increases, all of the forecasted metrics for each scenario are expected to fall. All estimating approaches do exceptionally well in predicting the appropriate model estimators. With a total score of 31.0, Table 9 shows that KE is the best for the factors we analyzed followed by MLE with a score of 98.5. Table 9 displays the total rankings for all estimating procedures.

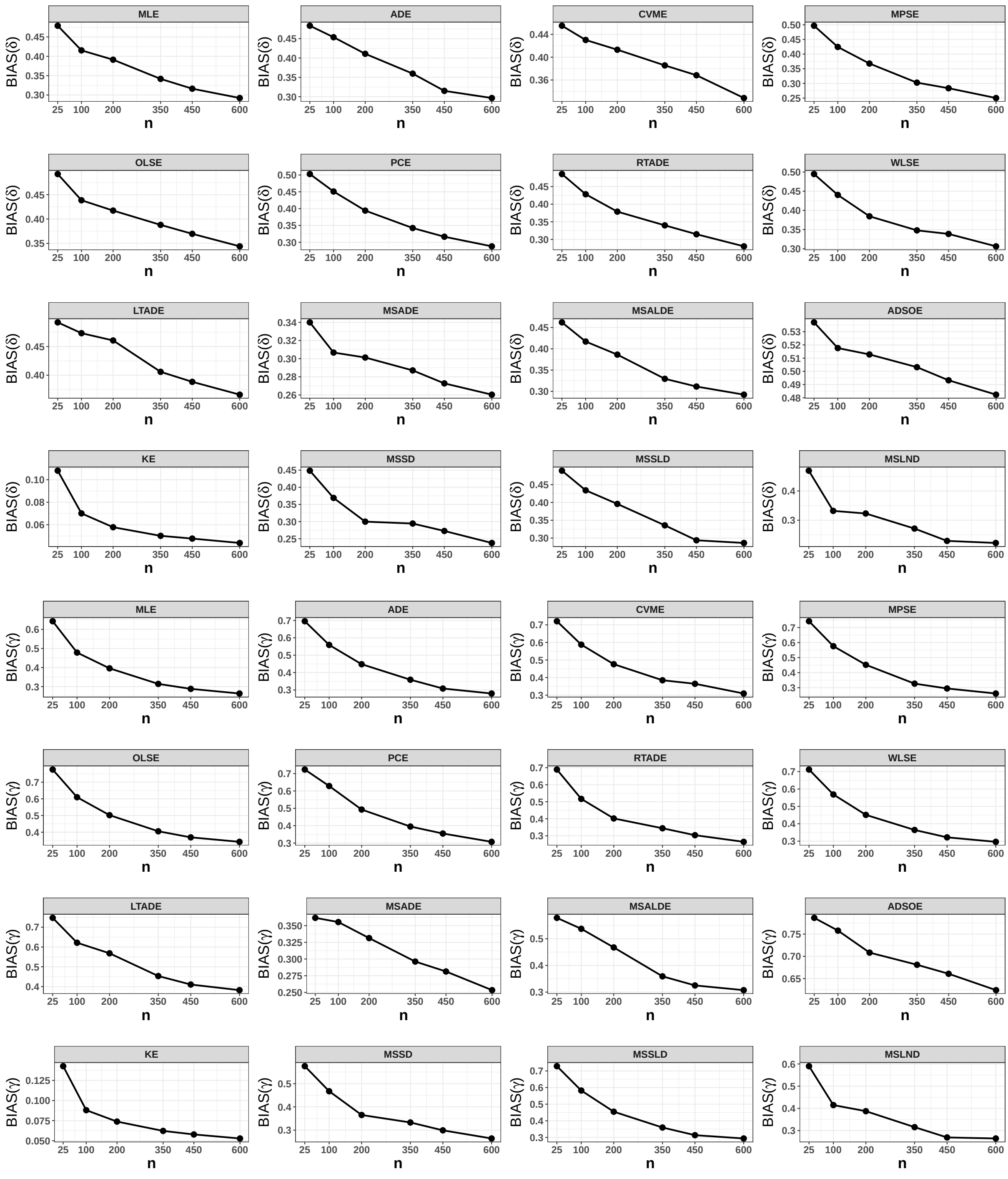

Figure 3: Graphical representation for BIAS values presented in Table 5.

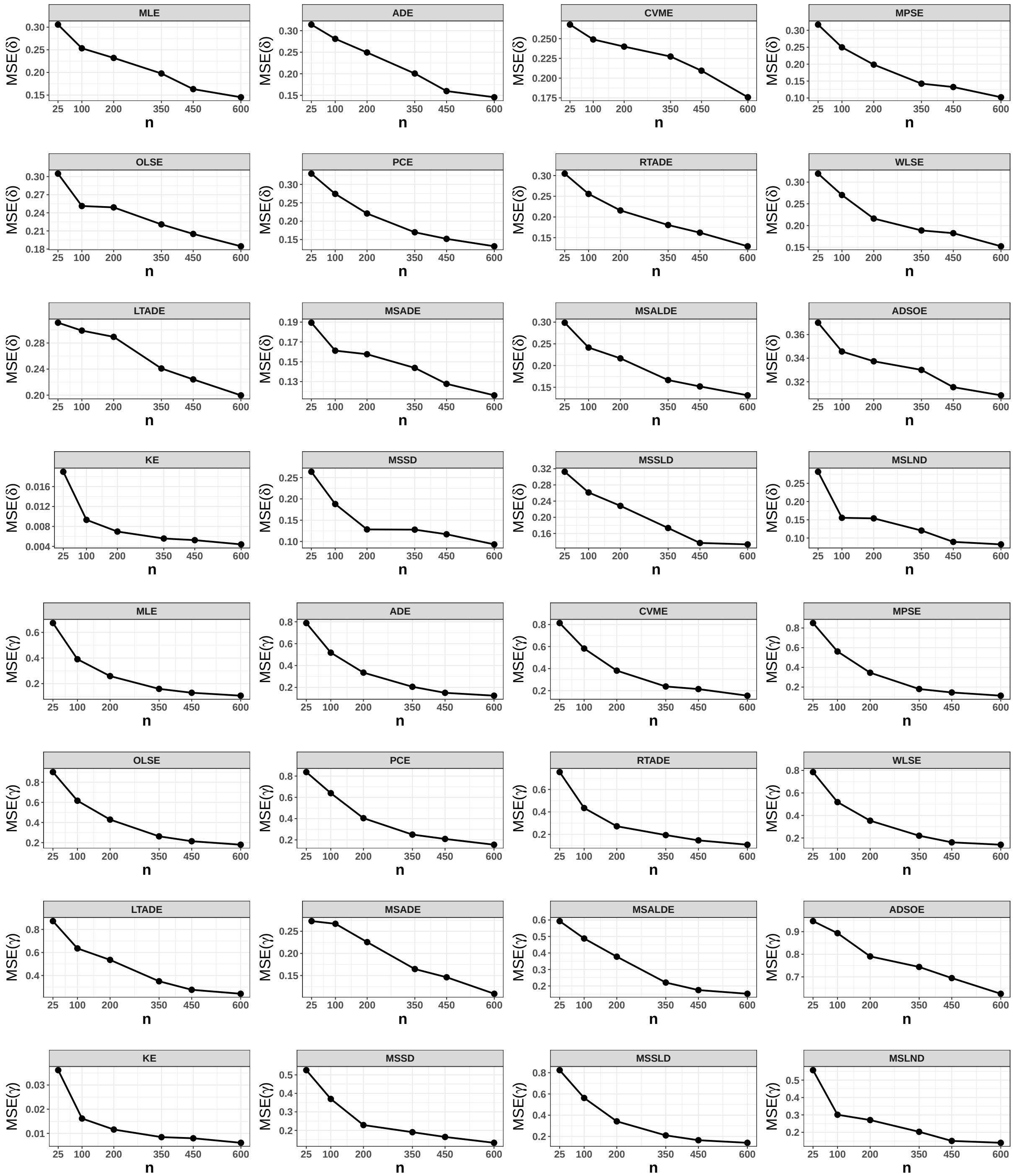

Figure 4: Graphical representation for MSE values presented in Table 5.

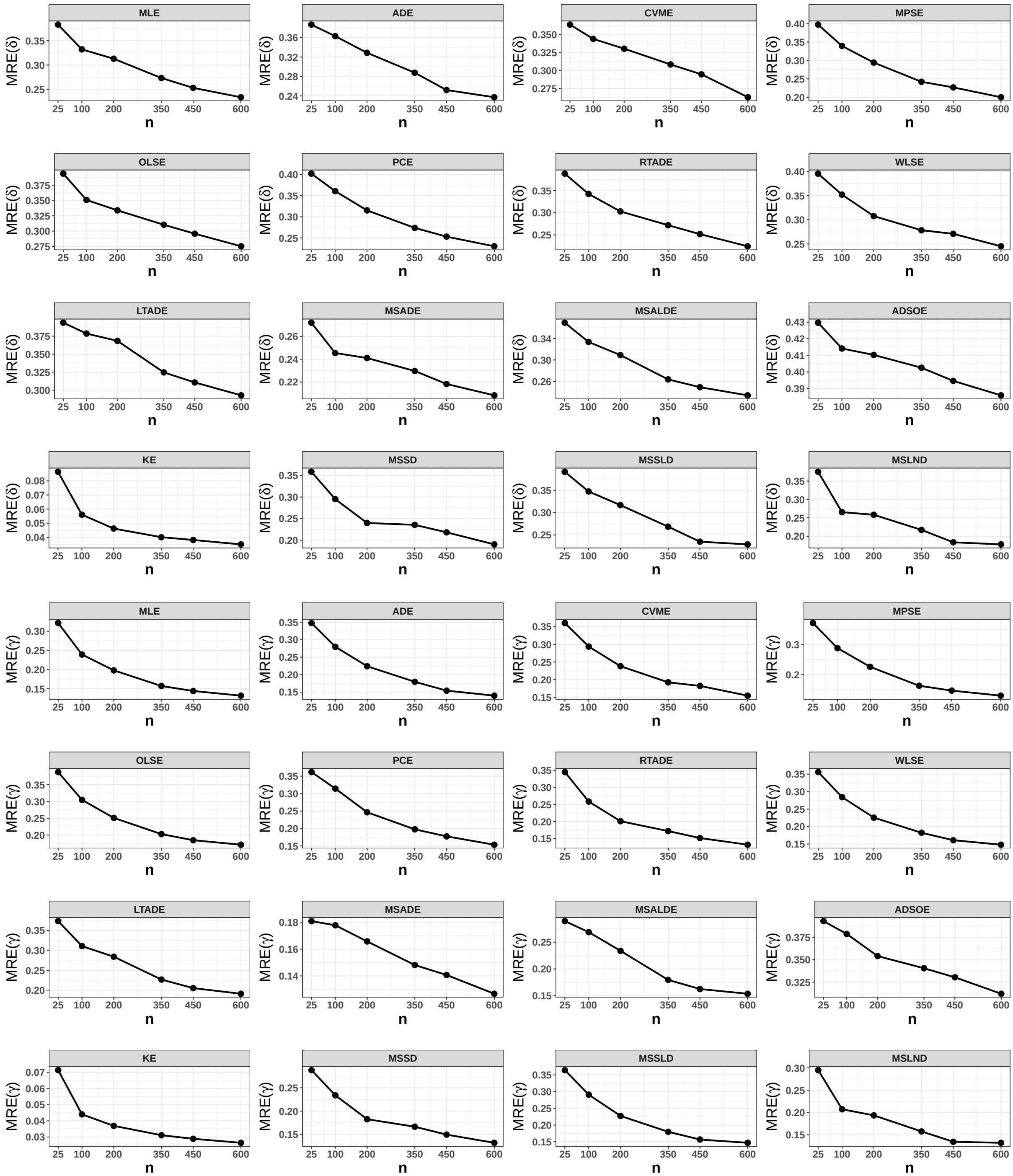

Figure 5: Graphical representation for MRE values presented in Table 5.

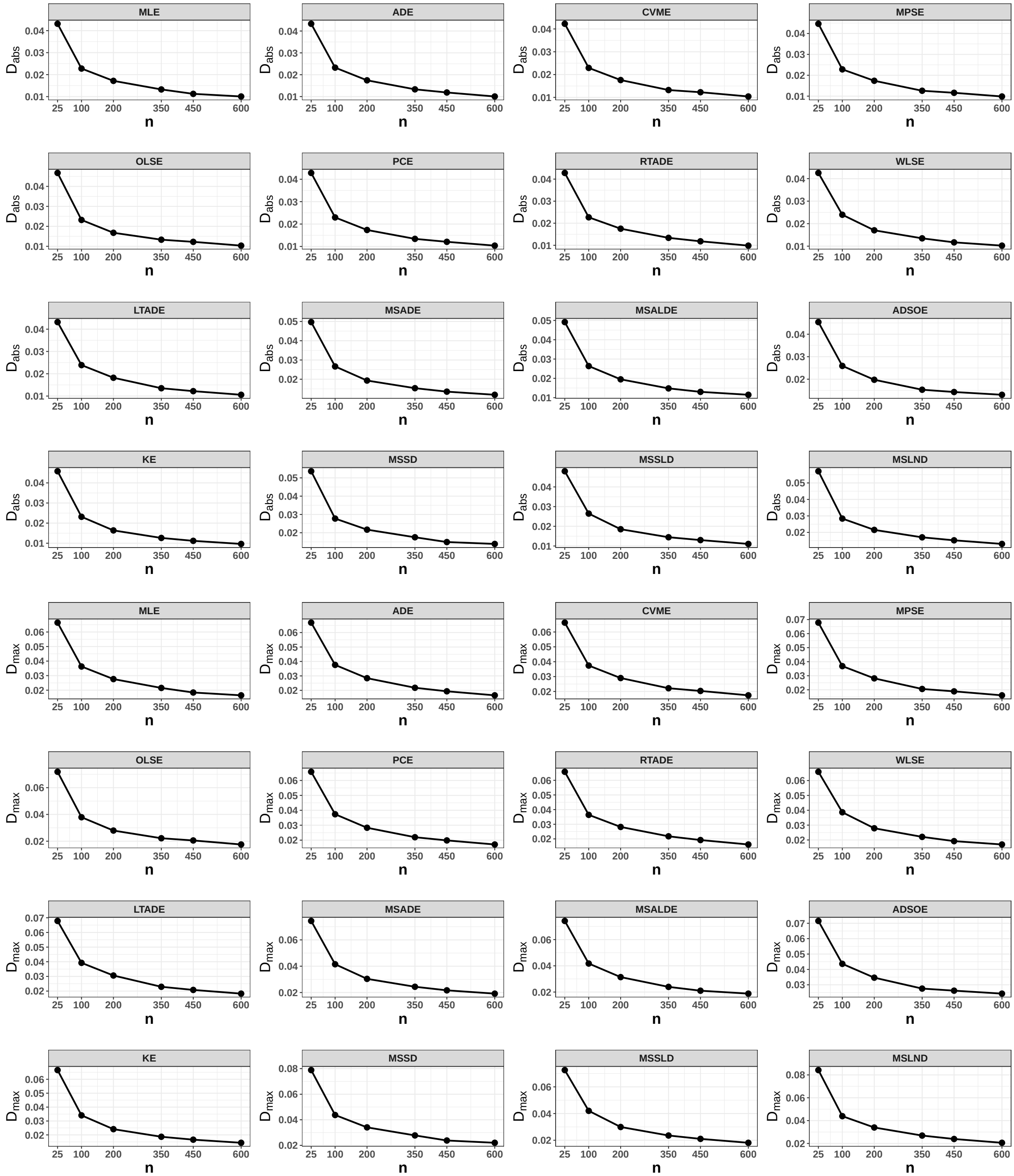

Figure 6: Graphical representation for D_{abs} and D_{max} values presented in Table 5.

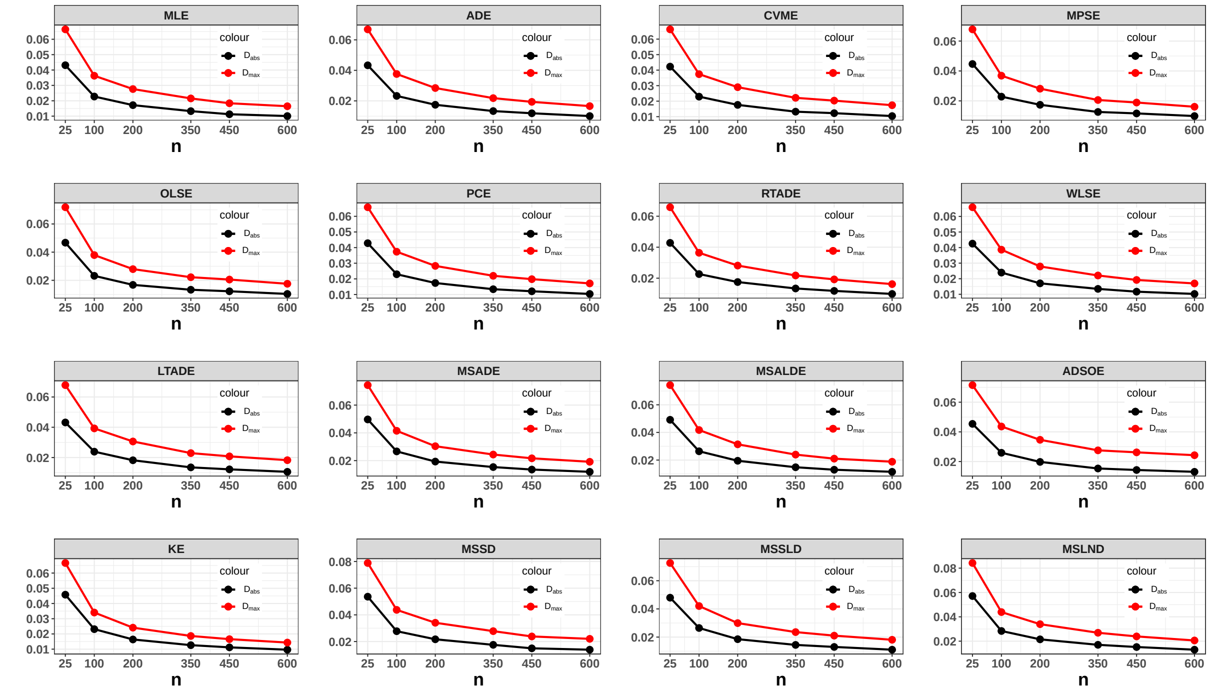

Figure 7: Comparison between D_{abs} and D_{max} values presented in Table 5.

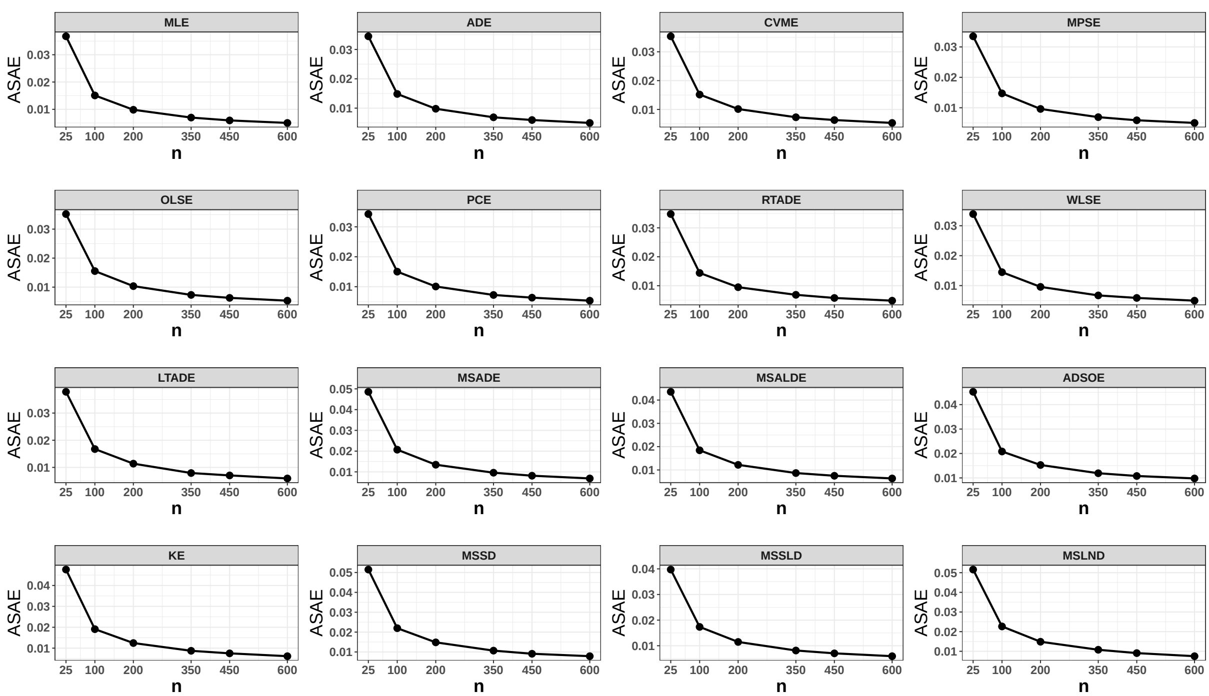

Figure 8: Graphical representation for ASAE values presented in Table 5.

Table 4: Numerical values of simulation measures for $\delta = 0.25,\ \gamma = 0.75$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	PCE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
25	$\mathrm{BIAS}(\hat{\delta})$	$0.0925^{\{11\}}$	$0.08499^{\{4\}}$	$0.08988^{\{7\}}$	$0.09283^{\{13\}}$	$0.08948^{\{6\}}$	$0.09061^{\{9\}}$	$0.08891^{\{5\}}$	$0.09263^{\{12\}}$	$0.09011^{\{8\}}$	$0.06827^{\{2\}}$	$0.07966^{\{3\}}$	$0.09325^{\{14\}}$	$0.0508^{\{1\}}$	$0.10494^{\{16\}}$	$0.09126^{\{10\}}$	$0.10074^{\{15\}}$
	$\mathrm{BIAS}(\hat{\gamma})$	$0.25189^{\{10\}}$	$0.22365^{\{4\}}$	$0.23091^{\{5\}}$	$0.26809^{\{14\}}$	$0.2587^{\{11\}}$	$0.24556^{\{8\}}$	$0.23929^{\{6\}}$	$0.26561^{\{13\}}$	$0.24438^{\{7\}}$	$0.15051^{\{2\}}$	$0.2133^{\{3\}}$	$0.25064^{\{9\}}$	$0.07649^{\{1\}}$	$0.32376^{\{16\}}$	$0.25963^{\{12\}}$	$0.31127^{\{15\}}$
	$\mathrm{MSE}(\hat{\delta})$	$0.01164^{\{11\}}$	$0.0106^{\{4\}}$	$0.0118^{\{12.5\}}$	$0.01133^{\{7\}}$	$0.01125^{\{6\}}$	$0.01158^{\{9\}}$	$0.01103^{\{5\}}$	$0.0118^{\{12.5\}}$	$0.01161^{\{10\}}$	$0.00735^{\{2\}}$	$0.00945^{\{3\}}$	$0.01217^{\{14\}}$	$0.00506^{\{1\}}$	$0.01401^{\{16\}}$	$0.01154^{\{8\}}$	$0.01321^{\{15\}}$
	$\mathrm{MSE}(\hat{\gamma})$	$0.09825^{\{10\}}$	$0.08018^{\{4\}}$	$0.08722^{\{5\}}$	$0.11145^{\{14\}}$	$0.10693^{\{12\}}$	$0.09342^{\{8\}}$	$0.0898^{\{6\}}$	$0.10989^{\{13\}}$	$0.0934^{\{7\}}$	$0.05089^{\{2\}}$	$0.07812^{\{3\}}$	$0.09814^{\{9\}}$	$0.01331^{\{1\}}$	$0.15016^{\{16\}}$	$0.10279^{\{11\}}$	$0.14225^{\{15\}}$
	$\mathrm{MRE}(\hat{\delta})$	$0.36998^{\{11\}}$	$0.33995^{\{4\}}$	$0.35952^{\{7\}}$	$0.37134^{\{13\}}$	$0.35792^{\{6\}}$	$0.36242^{\{9\}}$	$0.35562^{\{5\}}$	$0.37051^{\{12\}}$	$0.36045^{\{8\}}$	$0.27307^{\{2\}}$	$0.31865^{\{3\}}$	$0.37299^{\{14\}}$	$0.20321^{\{1\}}$	$0.41978^{\{16\}}$	$0.36505^{\{10\}}$	$0.40294^{\{15\}}$
	$\mathrm{MRE}(\hat{\gamma})$	$0.33586^{\{10\}}$	$0.29821^{\{4\}}$	$0.30788^{\{5\}}$	$0.35745^{\{14\}}$	$0.34494^{\{11\}}$	$0.32742^{\{8\}}$	$0.31905^{\{6\}}$	$0.35415^{\{13\}}$	$0.32584^{\{7\}}$	$0.20068^{\{2\}}$	$0.2844^{\{3\}}$	$0.33418^{\{9\}}$	$0.10198^{\{1\}}$	$0.43168^{\{16\}}$	$0.34618^{\{12\}}$	$0.41503^{\{15\}}$
	D_{abs}	$0.04418^{\{2\}}$	$0.0461^{\{8\}}$	$0.04588^{\{6\}}$	$0.04549^{\{4\}}$	$0.04584^{\{5\}}$	$0.04782^{\{11\}}$	$0.04395^{\{1\}}$	$0.04447^{\{3\}}$	$0.04657^{\{9\}}$	$0.05251^{\{14\}}$	$0.05018^{\{13\}}$	$0.04972^{\{12\}}$	$0.04676^{\{10\}}$	$0.05869^{\{16\}}$	$0.04595^{\{7\}}$	$0.05793^{\{15\}}$
	D_{max}	$0.06989^{\{4\}}$	$0.07093^{\{7\}}$	$0.0711^{\{8\}}$	$0.07074^{\{6\}}$	$0.07052^{\{5\}}$	$0.07294^{\{11\}}$	$0.0686^{\{1\}}$	$0.06935^{\{3\}}$	$0.07241^{\{10\}}$	$0.07907^{\{14\}}$	$0.07649^{\{12\}}$	$0.07703^{\{13\}}$	$0.06876^{\{2\}}$	$0.0921^{\{16\}}$	$0.07141^{\{9\}}$	$0.09073^{\{15\}}$
	ASAE	$0.03899^{\{4\}}$	$0.03632^{\{7\}}$	$0.03679^{\{8\}}$	$0.03632^{\{6\}}$	$0.03605^{\{5\}}$	$0.03347^{\{11\}}$	$0.03497^{\{1\}}$	$0.03482^{\{3\}}$	$0.04033^{\{10\}}$	$0.04741^{\{14\}}$	$0.04282^{\{12\}}$	$0.04751^{\{13\}}$	$0.0469^{\{2\}}$	$0.05011^{\{16\}}$	$0.04101^{\{9\}}$	$0.04977^{\{15\}}$
	$\sum Ranks$	$77^{\{10\}}$	$45^{\{3\}}$	$62.5^{\{6\}}$	$90^{\{13\}}$	$66^{\{7\}}$	$74^{\{8\}}$	$38^{\{2\}}$	$83.5^{\{11\}}$	$75^{\{9\}}$	$53^{\{4\}}$	$54^{\{5\}}$	$108^{\{14\}}$	$30^{\{1\}}$	$144^{\{16\}}$	$89^{\{12\}}$	$135^{\{15\}}$
100	$\mathrm{BIAS}(\hat{\delta})$	$0.07204^{\{5\}}$	$0.07257^{\{6\}}$	$0.08202^{\{14\}}$	$0.07061^{\{4\}}$	$0.07769^{\{9\}}$	$0.08198^{\{13\}}$	$0.07894^{\{11\}}$	$0.07451^{\{8\}}$	$0.07803^{\{10\}}$	$0.06254^{\{2\}}$	$0.06835^{\{3\}}$	$0.08853^{\{16\}}$	$0.03316^{\{1\}}$	$0.08251^{\{15\}}$	$0.07345^{\{7\}}$	$0.07956^{\{12\}}$
	$\mathrm{BIAS}(\hat{\gamma})$	$0.16284^{\{4\}}$	$0.16194^{\{3\}}$	$0.19905^{\{11\}}$	$0.18547^{\{9\}}$	$0.20342^{\{12\}}$	$0.22501^{\{15\}}$	$0.17666^{\{7\}}$	$0.1896^{\{10\}}$	$0.18233^{\{8\}}$	$0.14163^{\{2\}}$	$0.16743^{\{5\}}$	$0.22057^{\{14\}}$	$0.05473^{\{1\}}$	$0.23639^{\{16\}}$	$0.1758^{\{6\}}$	$0.21555^{\{13\}}$
	$\mathrm{MSE}(\hat{\delta})$	$0.00817^{\{5\}}$	$0.00838^{\{7.5\}}$	$0.00992^{\{15\}}$	$0.00749^{\{4\}}$	$0.00894^{\{9\}}$	$0.00961^{\{13\}}$	$0.00965^{\{14\}}$	$0.00838^{\{7.5\}}$	$0.00921^{\{12\}}$	$0.00654^{\{2\}}$	$0.00727^{\{3\}}$	$0.01094^{\{16\}}$	$0.00273^{\{1\}}$	$0.0092^{\{11\}}$	$0.0082^{\{6\}}$	$0.00908^{\{10\}}$
	$\mathrm{MSE}(\hat{\gamma})$	$0.04109^{\{3\}}$	$0.043^{\{4\}}$	$0.06374^{\{11\}}$	$0.05509^{\{8\}}$	$0.07132^{\{12\}}$	$0.08237^{\{15\}}$	$0.0481^{\{6\}}$	$0.06216^{\{10\}}$	$0.05552^{\{9\}}$	$0.04054^{\{2\}}$	$0.04708^{\{5\}}$	$0.07945^{\{14\}}$	$0.0073^{\{1\}}$	$0.08673^{\{16\}}$	$0.04866^{\{7\}}$	$0.07472^{\{13\}}$
	$\mathrm{MRE}(\hat{\delta})$	$0.28816^{\{5\}}$	$0.29028^{\{6\}}$	$0.32809^{\{14\}}$	$0.28243^{\{4\}}$	$0.31078^{\{9\}}$	$0.32792^{\{13\}}$	$0.31575^{\{11\}}$	$0.29804^{\{8\}}$	$0.31214^{\{10\}}$	$0.25017^{\{2\}}$	$0.2734^{\{3\}}$	$0.35414^{\{16\}}$	$0.13265^{\{1\}}$	$0.33006^{\{15\}}$	$0.29379^{\{7\}}$	$0.31823^{\{12\}}$
	$\mathrm{MRE}(\hat{\gamma})$	$0.21712^{\{4\}}$	$0.21592^{\{3\}}$	$0.26541^{\{11\}}$	$0.24729^{\{9\}}$	$0.27122^{\{12\}}$	$0.30001^{\{15\}}$	$0.23555^{\{7\}}$	$0.2528^{\{10\}}$	$0.2431^{\{8\}}$	$0.18883^{\{2\}}$	$0.22325^{\{5\}}$	$0.2941^{\{14\}}$	$0.07297^{\{1\}}$	$0.31518^{\{16\}}$	$0.2344^{\{6\}}$	$0.2874^{\{13\}}$
	D_{abs}	$0.02419^{\{2\}}$	$0.02434^{\{3\}}$	$0.02558^{\{8\}}$	$0.02415^{\{1\}}$	$0.02608^{\{11\}}$	$0.02473^{\{5\}}$	$0.02482^{\{7\}}$	$0.02444^{\{4\}}$	$0.02597^{\{10\}}$	$0.02752^{\{14\}}$	$0.02615^{\{12\}}$	$0.02704^{\{13\}}$	$0.02481^{\{6\}}$	$0.03192^{\{16\}}$	$0.02569^{\{9\}}$	$0.03144^{\{15\}}$
	D_{max}	$0.03938^{\{3\}}$	$0.03958^{\{4\}}$	$0.04232^{\{10\}}$	$0.0393^{\{2\}}$	$0.04261^{\{12\}}$	$0.04117^{\{7\}}$	$0.04062^{\{6\}}$	$0.0401^{\{5\}}$	$0.04258^{\{11\}}$	$0.04364^{\{13\}}$	$0.04215^{\{9\}}$	$0.04512^{\{14\}}$	$0.03738^{\{1\}}$	$0.05179^{\{16\}}$	$0.04192^{\{8\}}$	$0.05062^{\{15\}}$
	ASAE	$0.01523^{\{3\}}$	$0.01497^{\{4\}}$	$0.01548^{\{10\}}$	$0.01473^{\{2\}}$	$0.01534^{\{12\}}$	$0.01467^{\{7\}}$	$0.01445^{\{6\}}$	$0.01479^{\{5\}}$	$0.01657^{\{11\}}$	$0.02001^{\{13\}}$	$0.0183^{\{9\}}$	$0.02038^{\{14\}}$	$0.0185^{\{1\}}$	$0.02162^{\{16\}}$	$0.01755^{\{8\}}$	$0.02148^{\{15\}}$
	$\sum Ranks$	$37^{\{2\}}$	$41.5^{\{3\}}$	$102^{\{13\}}$	$44^{\{4\}}$	$93^{\{11\}}$	$98^{\{12\}}$	$70^{\{9\}}$	$66.5^{\{8\}}$	$87^{\{10\}}$	$52^{\{5\}}$	$56^{\{6\}}$	$131^{\{15\}}$	$25^{\{1\}}$	$137^{\{16\}}$	$66^{\{7\}}$	$118^{\{14\}}$
200	$\mathrm{BIAS}(\hat{\delta})$	$0.05762^{\{5\}}$	$0.06162^{\{9\}}$	$0.06721^{\{11\}}$	$0.05529^{\{3\}}$	$0.06345^{\{10\}}$	$0.07486^{\{15\}}$	$0.0605^{\{7\}}$	$0.06126^{\{8\}}$	$0.06824^{\{13\}}$	$0.05492^{\{2\}}$	$0.05732^{\{4\}}$	$0.08531^{\{16\}}$	$0.02446^{\{1\}}$	$0.06822^{\{12\}}$	$0.06043^{\{6\}}$	$0.06911^{\{14\}}$
	$\mathrm{BIAS}(\hat{\gamma})$	$0.12377^{\{3\}}$	$0.13061^{\{6\}}$	$0.15072^{\{10\}}$	$0.12934^{\{5\}}$	$0.1534^{\{11\}}$	$0.21515^{\{15\}}$	$0.12932^{\{4\}}$	$0.14614^{\{9\}}$	$0.15708^{\{12\}}$	$0.11733^{\{2\}}$	$0.13468^{\{7\}}$	$0.21713^{\{16\}}$	$0.04588^{\{1\}}$	$0.17319^{\{14\}}$	$0.14239^{\{8\}}$	$0.17088^{\{13\}}$
	$\mathrm{MSE}(\hat{\delta})$	$0.0056^{\{5\}}$	$0.00632^{\{9\}}$	$0.00723^{\{13\}}$	$0.00483^{\{2\}}$	$0.0065^{\{10\}}$	$0.00803^{\{15\}}$	$0.00631^{\{8\}}$	$0.006^{\{7\}}$	$0.00738^{\{14\}}$	$0.00557^{\{4\}}$	$0.00525^{\{3\}}$	$0.01052^{\{16\}}$	$0.00174^{\{1\}}$	$0.007^{\{11\}}$	$0.00594^{\{6\}}$	$0.00704^{\{12\}}$
	$\mathrm{MSE}(\hat{\gamma})$	$0.02379^{\{2\}}$	$0.0266^{\{6\}}$	$0.03724^{\{10\}}$	$0.02591^{\{4\}}$	$0.03954^{\{11\}}$	$0.0756^{\{15\}}$	$0.02612^{\{5\}}$	$0.03668^{\{9\}}$	$0.04156^{\{12\}}$	$0.02493^{\{3\}}$	$0.02967^{\{7\}}$	$0.07768^{\{16\}}$	$0.00601^{\{1\}}$	$0.04563^{\{14\}}$	$0.03245^{\{8\}}$	$0.04285^{\{13\}}$
	$\mathrm{MRE}(\hat{\delta})$	$0.23047^{\{5\}}$	$0.24648^{\{9\}}$	$0.26882^{\{11\}}$	$0.22116^{\{3\}}$	$0.2538^{\{10\}}$	$0.29945^{\{15\}}$	$0.242^{\{7\}}$	$0.24504^{\{8\}}$	$0.27296^{\{13\}}$	$0.21969^{\{2\}}$	$0.22929^{\{4\}}$	$0.34122^{\{16\}}$	$0.09785^{\{1\}}$	$0.27286^{\{12\}}$	$0.24172^{\{6\}}$	$0.27642^{\{14\}}$
	$\mathrm{MRE}(\hat{\gamma})$	$0.16502^{\{3\}}$	$0.17414^{\{6\}}$	$0.20096^{\{10\}}$	$0.17246^{\{5\}}$	$0.20454^{\{11\}}$	$0.28687^{\{15\}}$	$0.17243^{\{4\}}$	$0.19486^{\{9\}}$	$0.20944^{\{12\}}$	$0.15644^{\{2\}}$	$0.17957^{\{7\}}$	$0.2895^{\{16\}}$	$0.06117^{\{1\}}$	$0.23093^{\{14\}}$	$0.18986^{\{8\}}$	$0.22784^{\{13\}}$
	D_{abs}	$0.01737^{\{2\}}$	$0.01769^{\{4\}}$	$0.01856^{\{7\}}$	$0.01747^{\{3\}}$	$0.01909^{\{9\}}$	$0.01985^{\{12\}}$	$0.01781^{\{5\}}$	$0.01841^{\{6\}}$	$0.01867^{\{8\}}$	$0.02008^{\{13\}}$	$0.01927^{\{11\}}$	$0.02103^{\{14\}}$	$0.01694^{\{1\}}$	$0.02303^{\{15\}}$	$0.01915^{\{10\}}$	$0.02317^{\{16\}}$
	D_{max}	$0.02871^{\{2\}}$	$0.02937^{\{5\}}$	$0.03121^{\{7\}}$	$0.02875^{\{3\}}$	$0.03165^{\{11\}}$	$0.0335^{\{13\}}$	$0.02924^{\{4\}}$	$0.03058^{\{6\}}$	$0.03148^{\{8\}}$	$0.03227^{\{12\}}$	$0.03152^{\{9.5\}}$	$0.03652^{\{14\}}$	$0.02563^{\{1\}}$	$0.03768^{\{15\}}$	$0.03152^{\{9.5\}}$	$0.03808^{\{16\}}$
	ASAE	$0.00959^{\{2\}}$	$0.0095^{\{5\}}$	$0.01003^{\{7\}}$	$0.0096^{\{3\}}$	$0.00998^{\{11\}}$	$0.00998^{\{13\}}$	$0.00931^{\{4\}}$	$0.00923^{\{6\}}$	$0.01071^{\{8\}}$	$0.01254^{\{12\}}$	$0.01181^{\{9.5\}}$	$0.01505^{\{14\}}$	$0.01152^{\{1\}}$	$0.0136^{\{15\}}$	$0.01126^{\{9.5\}}$	$0.01361^{\{16\}}$
	$\sum Ranks$	$31^{\{2\}}$	$57^{\{6\}}$	$87^{\{10\}}$	$33^{\{3\}}$	$89^{\{11\}}$	$122^{\{14\}}$	$46^{\{4\}}$	$63^{\{7\}}$	$101^{\{12\}}$	$53^{\{5\}}$	$64.5^{\{8\}}$	$140^{\{16\}}$	$19^{\{1\}}$	$121^{\{13\}}$	$71.5^{\{9\}}$	$126^{\{15\}}$
350	$\mathrm{BIAS}(\hat{\delta})$	$0.04222^{\{2\}}$	$0.04792^{\{6\}}$	$0.05232^{\{11\}}$	$0.04266^{\{3\}}$	$0.05177^{\{10\}}$	$0.06632^{\{15\}}$	$0.04898^{\{8\}}$	$0.05044^{\{9\}}$	$0.05635^{\{12\}}$	$0.04369^{\{5\}}$	$0.04305^{\{4\}}$	$0.07569^{\{16\}}$	$0.018^{\{1\}}$	$0.0589^{\{14\}}$	$0.04885^{\{7\}}$	$0.05702^{\{13\}}$
	$\mathrm{BIAS}(\hat{\gamma})$	$0.09042^{\{2\}}$	$0.10124^{\{6\}}$	$0.11813^{\{10\}}$	$0.09762^{\{4\}}$	$0.11987^{\{11\}}$	$0.18567^{\{15\}}$	$0.10295^{\{7\}}$	$0.11344^{\{9\}}$	$0.12807^{\{12\}}$	$0.09482^{\{3\}}$	$0.09777^{\{5\}}$	$0.18589^{\{16\}}$	$0.03554^{\{1\}}$	$0.1367^{\{14\}}$	$0.10742^{\{8\}}$	$0.13366^{\{13\}}$
	$\mathrm{MSE}(\hat{\delta})$	$0.00315^{\{3\}}$	$0.00388^{\{6\}}$	$0.00445^{\{11\}}$	$0.003^{\{2\}}$	$0.00439^{\{10\}}$	$0.00653^{\{15\}}$	$0.00424^{\{9\}}$	$0.0041^{\{8\}}$	$0.00506^{\{12\}}$	$0.00345^{\{5\}}$	$0.00322^{\{4\}}$	$0.00847^{\{16\}}$	$0.00083^{\{1\}}$	$0.00553^{\{14\}}$	$0.00402^{\{7\}}$	$0.00515^{\{13\}}$
	$\mathrm{MSE}(\hat{\gamma})$	$0.01282^{\{2\}}$	$0.01581^{\{5\}}$	$0.02305^{\{10\}}$	$0.01494^{\{3\}}$	$0.02423^{\{11\}}$	$0.06008^{\{16\}}$	$0.01653^{\{7\}}$	$0.02037^{\{9\}}$	$0.02677^{\{13\}}$	$0.01628^{\{6\}}$	$0.01557^{\{4\}}$	$0.05938^{\{15\}}$	$0.00374^{\{1\}}$	$0.02723^{\{14\}}$	$0.01842^{\{8\}}$	$0.02653^{\{12\}}$
	$\mathrm{MRE}(\hat{\delta})$	$0.16887^{\{2\}}$	$0.19168^{\{6\}}$	$0.20929^{\{11\}}$	$0.17066^{\{3\}}$	$0.20708^{\{10\}}$	$0.26529^{\{15\}}$	$0.19593^{\{8\}}$	$0.20177^{\{9\}}$	$0.2254^{\{12\}}$	$0.17474^{\{5\}}$	$0.17219^{\{4\}}$	$0.30278^{\{16\}}$	$0.07201^{\{1\}}$	$0.23561^{\{14\}}$	$0.1954^{\{7\}}$	$0.22807^{\{13\}}$
	$\mathrm{MRE}(\hat{\gamma})$	$0.12057^{\{2\}}$	$0.13498^{\{6\}}$	$0.1575^{\{10\}}$	$0.13015^{\{4\}}$	$0.15983^{\{11\}}$	$0.24756^{\{15\}}$	$0.13727^{\{7\}}$	$0.15126^{\{9\}}$	$0.17077^{\{12\}}$	$0.12642^{\{3\}}$	$0.13036^{\{5\}}$	$0.24786^{\{16\}}$	$0.04739^{\{1\}}$	$0.18227^{\{14\}}$	$0.14323^{\{8\}}$	$0.17821^{\{13\}}$
	D_{abs}	$0.01297^{\{2\}}$	$0.01365^{\{4\}}$	$0.01374^{\{5\}}$	$0.01324^{\{3\}}$	$0.01425^{\{8\}}$	$0.01582^{\{12\}}$	$0.01378^{\{6\}}$	$0.01379^{\{7\}}$	$0.0148^{\{9\}}$	$0.0164^{\{14\}}$	$0.01507^{\{11\}}$	$0.01637^{\{13\}}$	$0.01277^{\{1\}}$	$0.01852^{\{16\}}$	$0.01494^{\{10\}}$	$0.01787^{\{15\}}$
	D_{max}	$0.02134^{\{2\}}$	$0.02268^{\{4\}}$	$0.02315^{\{6.5\}}$	$0.02183^{\{3\}}$	$0.02397^{\{8\}}$	$0.02736^{\{13\}}$	$0.02293^{\{5\}}$	$0.02315^{\{6.5\}}$	$0.02516^{\{11\}}$	$0.02637^{\{12\}}$	$0.02432^{\{9\}}$	$0.02891^{\{14\}}$	$0.01932^{\{1\}}$	$0.03055^{\{16\}}$	$0.02457^{\{10\}}$	$0.02963^{\{15\}}$
	ASAE	$0.00686^{\{2\}}$	$0.00677^{\{4\}}$	$0.00694^{\{6.5\}}$	$0.00674^{\{3\}}$	$0.00696^{\{8\}}$	$0.00724^{\{13\}}$	$0.0064^{\{5\}}$	$0.00656^{\{6.5\}}$	$0.0077^{\{11\}}$	$0.00908^{\{12\}}$	$0.00836^{\{9\}}$	$0.0111^{\{14\}}$	$0.00816^{\{1\}}$	$0.01011^{\{16\}}$	$0.00773^{\{10\}}$	$0.01002^{\{15\}}$
	$\sum Ranks$	$22^{\{2\}}$	$47^{\{4\}}$	$80.5^{\{10\}}$	$28^{\{3\}}$	$86^{\{11\}}$	$124^{\{14\}}$	$58^{\{5.5\}}$	$68.5^{\{8\}}$	$102^{\{12\}}$	$66^{\{7\}}$	$58^{\{5.5\}}$	$138^{\{16\}}$	$19^{\{1\}}$	$131^{\{15\}}$	$75^{\{9\}}$	$121^{\{13\}}$
450	$\mathrm{BIAS}(\hat{\delta})$	$0.03807^{\{3\}}$	$0.04205^{\{5\}}$	$0.0492^{\{12\}}$	$0.03755^{\{2\}}$	$0.04645^{\{10\}}$	$0.05979^{\{15\}}$	$0.04429^{\{8\}}$	$0.04517^{\{9\}}$	$0.05054^{\{13\}}$	$0.04278^{\{6\}}$	$0.04187^{\{4\}}$	$0.07406^{\{16\}}$	$0.01663^{\{1\}}$	$0.05237^{\{14\}}$	$0.0429^{\{7\}}$	$0.04763^{\{11\}}$
	$\mathrm{BIAS}(\hat{\gamma})$	$0.07922^{\{2\}}$	$0.09076^{\{6\}}$	$0.10814^{\{11\}}$	$0.08438^{\{3\}}$	$0.10682^{\{10\}}$	$0.16981^{\{15\}}$	$0.09394^{\{8\}}$	$0.10191^{\{9\}}$	$0.11114^{\{13\}}$	$0.08946^{\{4\}}$	$0.09015^{\{5\}}$	$0.17966^{\{16\}}$	$0.03351^{\{1\}}$	$0.1207^{\{14\}}$	$0.09198^{\{7\}}$	$0.11109^{\{12\}}$
	$\mathrm{MSE}(\hat{\delta})$	$0.00271^{\{3\}}$	$0.00307^{\{4\}}$	$0.00404^{\{12\}}$	$0.00237^{\{2\}}$	$0.00359^{\{11\}}$	$0.00545^{\{15\}}$	$0.00336^{\{9\}}$	$0.00335^{\{8\}}$	$0.00436^{\{13\}}$	$0.00333^{\{7\}}$	$0.0032^{\{6\}}$	$0.00813^{\{16\}}$	$0.00072^{\{1\}}$	$0.00451^{\{14\}}$	$0.00315^{\{5\}}$	$0.00355^{\{10\}}$
	$\mathrm{MSE}(\hat{\gamma})$	$0.01039^{\{2\}}$	$0.01301^{\{5\}}$	$0.01909^{\{11\}}$	$0.01114^{\{3\}}$	$0.01937^{\{12\}}$	$0.05194^{\{15\}}$	$0.01362^{\{7\}}$	$0.01649^{\{9\}}$	$0.02012^{\{13\}}$	$0.01408^{\{8\}}$	$0.01335^{\{6\}}$	$0.05606^{\{16\}}$	$0.00311^{\{1\}}$	$0.02175^{\{14\}}$	$0.01299^{\{4\}}$	$0.01887^{\{10\}}$
	$\mathrm{MRE}(\hat{\delta})$	$0.15229^{\{3\}}$	$0.16821^{\{5\}}$	$0.19681^{\{12\}}$	$0.15019^{\{2\}}$	$0.1858^{\{10\}}$	$0.23915^{\{15\}}$	$0.17718^{\{8\}}$	$0.18067^{\{9\}}$	$0.20214^{\{13\}}$	$0.17114^{\{6\}}$	$0.16747^{\{4\}}$	$0.29622^{\{16\}}$	$0.06654^{\{1\}}$	$0.20948^{\{14\}}$	$0.17159^{\{7\}}$	$0.19054^{\{11\}}$
	$\mathrm{MRE}(\hat{\gamma})$	$0.10562^{\{2\}}$	$0.12102^{\{6\}}$	$0.14419^{\{11\}}$	$0.11251^{\{3\}}$	$0.14242^{\{10\}}$	$0.22642^{\{15\}}$	$0.12525^{\{8\}}$	$0.13588^{\{9\}}$	$0.14819^{\{13\}}$	$0.11928^{\{4\}}$	$0.1202^{\{5\}}$	$0.23955^{\{16\}}$	$0.04468^{\{1\}}$	$0.16093^{\{14\}}$	$0.12264^{\{7\}}$	$0.14812^{\{12\}}$
	D_{abs}	$0.01165^{\{3\}}$	$0.01215^{\{5\}}$	$0.01266^{\{8\}}$	$0.0116^{\{2\}}$	$0.01298^{\{10\}}$	$0.01418^{\{13\}}$	$0.012^{\{4\}}$	$0.01236^{\{6\}}$	$0.01275^{\{9\}}$	$0.0141^{\{12\}}$	$0.01315^{\{11\}}$	$0.0154^{\{15\}}$	$0.01145^{\{1\}}$	$0.01631^{\{16\}}$	$0.01254^{\{7\}}$	$0.01527^{\{14\}}$
	D_{max}	$0.01912^{\{2\}}$	$0.0202^{\{5\}}$	$0.02145^{\{8\}}$	$0.01913^{\{3\}}$	$0.02173^{\{11\}}$	$0.02458^{\{13\}}$	$0.02011^{\{4\}}$	$0.0207^{\{6\}}$	$0.02164^{\{10\}}$	$0.02301^{\{12\}}$	$0.02156^{\{9\}}$	$0.02746^{\{16\}}$	$0.0174^{\{1\}}$	$0.02691^{\{15\}}$	$0.02086^{\{7\}}$	$0.02511^{\{14\}}$
	ASAE	$0.0057^{\{2\}}$	$0.00582^{\{5\}}$	$0.00598^{\{8\}}$	$0.00573^{\{3\}}$	$0.00598^{\{11\}}$	$0.00617^{\{13\}}$	$0.00554^{\{4\}}$	$0.00568^{\{6\}}$	$0.00653^{\{10\}}$	$0.0077^{\{12\}}$	$0.00725^{\{9\}}$	$0.00986^{\{16\}}$	$0.00695^{\{1\}}$	$0.00849^{\{15\}}$	$0.00668^{\{7\}}$	$0.00836^{\{14\}}$
	$\sum Ranks$	$23^{\{2\}}$	$46^{\{4\}}$	$92^{\{11\}}$	$24^{\{3\}}$	$90^{\{10\}}$	$124^{\{14\}}$	$57^{\{5\}}$	$67^{\{8\}}$	$106^{\{12\}}$	$72^{\{9\}}$	$62^{\{7\}}$	$143^{\{16\}}$	$19^{\{1\}}$	$130^{\{15\}}$	$61^{\{6\}}$	$108^{\{13\}}$
600	$\mathrm{BIAS}(\hat{\delta})$	$0.03447^{\{3\}}$	$0.0363^{\{7\}}$	$0.03886^{\{9\}}$	$0.0321^{\{2\}}$	$0.04221^{\{11\}}$	$0.05106^{\{15\}}$	$0.03549^{\{4\}}$	$0.03752^{\{8\}}$	$0.04536^{\{13\}}$	$0.0394^{\{10\}}$	$0.03622^{\{6\}}$	$0.06892^{\{16\}}$	$0.01504^{\{1\}}$	$0.04602^{\{14\}}$	$0.03573^{\{5\}}$	$0.04384^{\{12\}}$
	$\mathrm{BIAS}(\hat{\gamma})$	$0.07263^{\{3\}}$	$0.07839^{\{6\}}$	$0.08694^{\{10\}}$	$0.0708^{\{2\}}$	$0.09306^{\{11\}}$	$0.14288^{\{15\}}$	$0.07626^{\{4\}}$	$0.08593^{\{9\}}$	$0.09555^{\{12\}}$	$0.08395^{\{8\}}$	$0.07859^{\{7\}}$	$0.15763^{\{16\}}$	$0.0278^{\{1\}}$	$0.10544^{\{14\}}$	$0.07771^{\{5\}}$	$0.10345^{\{13\}}$
	$\mathrm{MSE}(\hat{\delta})$	$0.00214^{\{4\}}$	$0.00227^{\{7\}}$	$0.00255^{\{9\}}$	$0.00178^{\{2\}}$	$0.00295^{\{11\}}$	$0.00405^{\{15\}}$	$0.00211^{\{3\}}$	$0.00224^{\{5\}}$	$0.00348^{\{14\}}$	$0.00286^{\{10\}}$	$0.00232^{\{8\}}$	$0.00739^{\{16\}}$	$0.00062^{\{1\}}$	$0.00339^{\{13\}}$	$0.00225^{\{6\}}$	$0.00301^{\{12\}}$
	$\mathrm{MSE}(\hat{\gamma})$	$0.00839^{\{3\}}$	$0.00961^{\{5\}}$	$0.01215^{\{9\}}$	$0.00821^{\{2\}}$	$0.0141^{\{11\}}$	$0.0374^{\{15\}}$	$0.00898^{\{4\}}$	$0.01169^{\{8\}}$	$0.0148^{\{12\}}$	$0.01242^{\{10\}}$	$0.01005^{\{7\}}$	$0.0429^{\{16\}}$	$0.00217^{\{1\}}$	$0.01652^{\{14\}}$	$0.00993^{\{6\}}$	$0.01584^{\{13\}}$
	$\mathrm{MRE}(\hat{\delta})$	$0.13787^{\{3\}}$	$0.14518^{\{7\}}$	$0.15543^{\{9\}}$	$0.1284^{\{2\}}$	$0.16882^{\{11\}}$	$0.20422^{\{15\}}$	$0.14197^{\{4\}}$	$0.15007^{\{8\}}$	$0.18142^{\{13\}}$	$0.1576^{\{10\}}$	$0.14488^{\{6\}}$	$0.27568^{\{16\}}$	$0.06017^{\{1\}}$	$0.18408^{\{14\}}$	$0.14292^{\{5\}}$	$0.17535^{\{12\}}$
	$\mathrm{MRE}(\hat{\gamma})$	$0.09685^{\{3\}}$	$0.10453^{\{6\}}$	$0.11593^{\{10\}}$	$0.0944^{\{2\}}$	$0.12408^{\{11\}}$	$0.19051^{\{15\}}$	$0.10168^{\{4\}}$	$0.11458^{\{9\}}$	$0.1274^{\{12\}}$	$0.11193^{\{8\}}$	$0.10478^{\{7\}}$	$0.21017^{\{16\}}$	$0.03706^{\{1\}}$	$0.14059^{\{14\}}$	$0.10361^{\{5\}}$	$0.13793^{\{13\}}$
	D_{abs}	$0.00979^{\{2\}}$	$0.01066^{\{6\}}$	$0.01067^{\{7\}}$	$0.00998^{\{3\}}$	$0.01108^{\{9\}}$	$0.01229^{\{12\}}$	$0.01002^{\{4\}}$	$0.01063^{\{5\}}$	$0.0113^{\{10\}}$	$0.01264^{\{13\}}$	$0.01154^{\{11\}}$	$0.01341^{\{14\}}$	$0.00974^{\{1\}}$	$0.01375^{\{16\}}$	$0.01085^{\{8\}}$	$0.01354^{\{15\}}$
	D_{max}	$0.01624^{\{2\}}$	$0.01766^{\{5\}}$	$0.01784^{\{7\}}$	$0.01644^{\{3\}}$	$0.01874^{\{9\}}$	$0.02132^{\{13\}}$	$0.01671^{\{4\}}$	$0.01779^{\{6\}}$	$0.01926^{\{11\}}$	$0.02073^{\{12\}}$	$0.01897^{\{10\}}$	$0.02397^{\{16\}}$	$0.01487^{\{1\}}$	$0.02298^{\{15\}}$	$0.01795^{\{8\}}$	$0.02247^{\{14\}}$
	ASAE	$0.00484^{\{2\}}$	$0.00481^{\{5\}}$	$0.00502^{\{7\}}$	$0.00484^{\{3\}}$	$0.00503^{\{9\}}$	$0.00525^{\{13\}}$	$0.00462^{\{4\}}$	$0.00476^{\{6\}}$	$0.00558^{\{11\}}$	$0.00647^{\{12\}}$	$0.0061^{\{10\}}$	$0.00835^{\{16\}}$	$0.00577^{\{1\}}$	$0.00702^{\{15\}}$	$0.00563^{\{8\}}$	$0.00699^{\{14\}}$
	$\sum Ranks$	$27^{\{3\}}$	$52^{\{5\}}$	$76^{\{9\}}$	$23^{\{2\}}$	$91^{\{10\}}$	$123^{\{14\}}$	$32^{\{4\}}$	$60^{\{7\}}$	$106^{\{12\}}$	$94^{\{11\}}$	$74^{\{8\}}$	$142^{\{16\}}$	$19^{\{1\}}$	$129^{\{15\}}$	$58^{\{6\}}$	$118^{\{13\}}$

Table 5: Numerical values of simulation measures for $\delta = 1.25,\ \gamma = 2.0$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	PCE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
25	BIAS($\hat{\delta}$)	$0.47892^{\{7\}}$	$0.48332^{\{8\}}$	$0.45512^{\{4\}}$	$0.49673^{\{14\}}$	$0.49244^{\{12\}}$	$0.50286^{\{15\}}$	$0.48553^{\{9\}}$	$0.49475^{\{13\}}$	$0.49226^{\{11\}}$	$0.34001^{\{2\}}$	$0.46211^{\{5\}}$	$0.53706^{\{16\}}$	$0.10804^{\{1\}}$	$0.44844^{\{3\}}$	$0.48915^{\{10\}}$	$0.46934^{\{6\}}$
	BIAS($\hat{\gamma}$)	$0.64275^{\{6\}}$	$0.69573^{\{8\}}$	$0.72085^{\{10\}}$	$0.74207^{\{13\}}$	$0.77556^{\{15\}}$	$0.72345^{\{11\}}$	$0.68922^{\{7\}}$	$0.71192^{\{9\}}$	$0.74709^{\{14\}}$	$0.36165^{\{2\}}$	$0.57842^{\{4\}}$	$0.7865^{\{16\}}$	$0.14273^{\{1\}}$	$0.57544^{\{3\}}$	$0.72863^{\{12\}}$	$0.5901^{\{5\}}$
	MSE($\hat{\delta}$)	$0.30566^{\{9\}}$	$0.31405^{\{12\}}$	$0.26779^{\{4\}}$	$0.31691^{\{13\}}$	$0.3049^{\{8\}}$	$0.32933^{\{15\}}$	$0.30467^{\{7\}}$	$0.31936^{\{14\}}$	$0.31114^{\{10\}}$	$0.18939^{\{2\}}$	$0.29869^{\{6\}}$	$0.37009^{\{16\}}$	$0.01897^{\{1\}}$	$0.26389^{\{3\}}$	$0.31265^{\{11\}}$	$0.28174^{\{5\}}$
	MSE($\hat{\gamma}$)	$0.67319^{\{6\}}$	$0.78885^{\{9\}}$	$0.81381^{\{10\}}$	$0.85062^{\{13\}}$	$0.90095^{\{15\}}$	$0.83775^{\{12\}}$	$0.75482^{\{7\}}$	$0.7855^{\{8\}}$	$0.8721^{\{14\}}$	$0.27297^{\{2\}}$	$0.59322^{\{5\}}$	$0.9467^{\{16\}}$	$0.03615^{\{1\}}$	$0.52554^{\{3\}}$	$0.82402^{\{11\}}$	$0.55711^{\{4\}}$
	MRE($\hat{\delta}$)	$0.38314^{\{7\}}$	$0.38666^{\{8\}}$	$0.36409^{\{4\}}$	$0.39738^{\{14\}}$	$0.39396^{\{12\}}$	$0.40229^{\{15\}}$	$0.38842^{\{9\}}$	$0.3958^{\{13\}}$	$0.39381^{\{11\}}$	$0.27201^{\{2\}}$	$0.36969^{\{5\}}$	$0.42965^{\{16\}}$	$0.08643^{\{1\}}$	$0.35875^{\{3\}}$	$0.39132^{\{10\}}$	$0.37548^{\{6\}}$
	MRE($\hat{\gamma}$)	$0.32137^{\{6\}}$	$0.34786^{\{8\}}$	$0.36043^{\{10\}}$	$0.37103^{\{13\}}$	$0.38778^{\{15\}}$	$0.36173^{\{11\}}$	$0.34461^{\{7\}}$	$0.35596^{\{9\}}$	$0.37355^{\{14\}}$	$0.18082^{\{2\}}$	$0.28921^{\{4\}}$	$0.39325^{\{16\}}$	$0.07136^{\{1\}}$	$0.28772^{\{3\}}$	$0.36431^{\{12\}}$	$0.29505^{\{5\}}$
	D_{abs}	$0.04313^{\{5\}}$	$0.04332^{\{7\}}$	$0.04227^{\{1\}}$	$0.04466^{\{8\}}$	$0.04674^{\{11\}}$	$0.04283^{\{4\}}$	$0.04281^{\{3\}}$	$0.04254^{\{2\}}$	$0.04318^{\{6\}}$	$0.04969^{\{14\}}$	$0.04915^{\{13\}}$	$0.0454^{\{9\}}$	$0.0458^{\{10\}}$	$0.05363^{\{15\}}$	$0.04797^{\{12\}}$	$0.0571^{\{16\}}$
	D_{max}	$0.06645^{\{5\}}$	$0.06688^{\{7\}}$	$0.0663^{\{4\}}$	$0.06783^{\{8\}}$	$0.07188^{\{11\}}$	$0.06582^{\{1.5\}}$	$0.06582^{\{1.5\}}$	$0.06591^{\{3\}}$	$0.06785^{\{9\}}$	$0.07441^{\{14\}}$	$0.07424^{\{13\}}$	$0.07153^{\{10\}}$	$0.06662^{\{6\}}$	$0.07888^{\{15\}}$	$0.07258^{\{12\}}$	$0.08424^{\{16\}}$
	ASAE	$0.03676^{\{5\}}$	$0.03449^{\{7\}}$	$0.03538^{\{4\}}$	$0.0335^{\{8\}}$	$0.0352^{\{11\}}$	$0.03428^{\{1.5\}}$	$0.03478^{\{1.5\}}$	$0.03391^{\{3\}}$	$0.03783^{\{9\}}$	$0.04859^{\{14\}}$	$0.04355^{\{13\}}$	$0.04525^{\{10\}}$	$0.04764^{\{6\}}$	$0.05149^{\{15\}}$	$0.03974^{\{12\}}$	$0.0516^{\{16\}}$
	$\sum Ranks$	$59^{\{5\}}$	$71^{\{8\}}$	$54^{\{2.5\}}$	$97^{\{12\}}$	$105^{\{15\}}$	$87.5^{\{11\}}$	$55.5^{\{4\}}$	$73^{\{9\}}$	$98^{\{13\}}$	$54^{\{2.5\}}$	$66^{\{7\}}$	$127^{\{16\}}$	$35^{\{1\}}$	$63^{\{6\}}$	$100^{\{14\}}$	$79^{\{10\}}$
100	BIAS($\hat{\delta}$)	$0.41526^{\{5\}}$	$0.45363^{\{14\}}$	$0.43015^{\{9\}}$	$0.42431^{\{7\}}$	$0.43859^{\{11\}}$	$0.45101^{\{13\}}$	$0.42822^{\{8\}}$	$0.44036^{\{12\}}$	$0.47335^{\{15\}}$	$0.30675^{\{2\}}$	$0.41704^{\{6\}}$	$0.51762^{\{16\}}$	$0.07009^{\{1\}}$	$0.36901^{\{4\}}$	$0.43401^{\{10\}}$	$0.33193^{\{3\}}$
	BIAS($\hat{\gamma}$)	$0.47852^{\{5\}}$	$0.55935^{\{8\}}$	$0.58747^{\{12\}}$	$0.57603^{\{10\}}$	$0.60958^{\{13\}}$	$0.62856^{\{15\}}$	$0.51691^{\{6\}}$	$0.56853^{\{9\}}$	$0.6215^{\{14\}}$	$0.35538^{\{2\}}$	$0.53736^{\{7\}}$	$0.75781^{\{16\}}$	$0.08802^{\{1\}}$	$0.46755^{\{4\}}$	$0.58202^{\{11\}}$	$0.41486^{\{3\}}$
	MSE($\hat{\delta}$)	$0.25339^{\{9\}}$	$0.28106^{\{14\}}$	$0.24898^{\{6\}}$	$0.24975^{\{7\}}$	$0.25128^{\{8\}}$	$0.27421^{\{13\}}$	$0.25569^{\{10\}}$	$0.27017^{\{12\}}$	$0.299^{\{15\}}$	$0.16121^{\{3\}}$	$0.24143^{\{5\}}$	$0.34566^{\{16\}}$	$0.00932^{\{1\}}$	$0.18799^{\{4\}}$	$0.26137^{\{11\}}$	$0.15546^{\{2\}}$
	MSE($\hat{\gamma}$)	$0.39055^{\{5\}}$	$0.51792^{\{8\}}$	$0.58234^{\{12\}}$	$0.56155^{\{10\}}$	$0.6164^{\{13\}}$	$0.63939^{\{15\}}$	$0.43452^{\{6\}}$	$0.51874^{\{9\}}$	$0.63517^{\{14\}}$	$0.26695^{\{2\}}$	$0.48778^{\{7\}}$	$0.89336^{\{16\}}$	$0.01614^{\{1\}}$	$0.36997^{\{4\}}$	$0.56227^{\{11\}}$	$0.30099^{\{3\}}$
	MRE($\hat{\delta}$)	$0.33221^{\{5\}}$	$0.3629^{\{14\}}$	$0.34412^{\{9\}}$	$0.33945^{\{7\}}$	$0.35087^{\{11\}}$	$0.36081^{\{13\}}$	$0.34258^{\{8\}}$	$0.35229^{\{12\}}$	$0.37868^{\{15\}}$	$0.2454^{\{2\}}$	$0.33363^{\{6\}}$	$0.4141^{\{16\}}$	$0.05607^{\{1\}}$	$0.29521^{\{4\}}$	$0.34721^{\{10\}}$	$0.26555^{\{3\}}$
	MRE($\hat{\gamma}$)	$0.23926^{\{5\}}$	$0.27967^{\{8\}}$	$0.29373^{\{12\}}$	$0.28801^{\{10\}}$	$0.30479^{\{13\}}$	$0.31428^{\{15\}}$	$0.25845^{\{6\}}$	$0.28426^{\{9\}}$	$0.31075^{\{14\}}$	$0.17769^{\{2\}}$	$0.26868^{\{7\}}$	$0.37891^{\{16\}}$	$0.04401^{\{1\}}$	$0.23378^{\{4\}}$	$0.29101^{\{11\}}$	$0.20743^{\{3\}}$
	D_{abs}	$0.02277^{\{2\}}$	$0.02325^{\{8\}}$	$0.02292^{\{4\}}$	$0.02283^{\{3\}}$	$0.0232^{\{7\}}$	$0.02296^{\{5\}}$	$0.02268^{\{1\}}$	$0.02396^{\{10\}}$	$0.02388^{\{9\}}$	$0.02665^{\{14\}}$	$0.02637^{\{12\}}$	$0.02591^{\{11\}}$	$0.02315^{\{6\}}$	$0.02775^{\{15\}}$	$0.02646^{\{13\}}$	$0.02838^{\{16\}}$
	D_{max}	$0.03628^{\{2\}}$	$0.03762^{\{7\}}$	$0.03742^{\{6\}}$	$0.03686^{\{4\}}$	$0.03792^{\{8\}}$	$0.03737^{\{5\}}$	$0.03641^{\{3\}}$	$0.03865^{\{9\}}$	$0.03926^{\{10\}}$	$0.04149^{\{11\}}$	$0.04175^{\{12\}}$	$0.04363^{\{14\}}$	$0.03407^{\{1\}}$	$0.04374^{\{15\}}$	$0.04204^{\{13\}}$	$0.04391^{\{16\}}$
	ASAE	$0.01509^{\{2\}}$	$0.01484^{\{7\}}$	$0.01516^{\{6\}}$	$0.01473^{\{4\}}$	$0.01554^{\{8\}}$	$0.01503^{\{5\}}$	$0.01442^{\{3\}}$	$0.01449^{\{9\}}$	$0.01675^{\{10\}}$	$0.02069^{\{11\}}$	$0.01844^{\{12\}}$	$0.02082^{\{14\}}$	$0.01909^{\{1\}}$	$0.02197^{\{15\}}$	$0.01735^{\{13\}}$	$0.02265^{\{16\}}$
	$\sum Ranks$	$44^{\{2\}}$	$85^{\{11\}}$	$77^{\{9\}}$	$61^{\{5\}}$	$92^{\{12\}}$	$99^{\{13\}}$	$49^{\{3\}}$	$84^{\{10\}}$	$115^{\{15\}}$	$51^{\{4\}}$	$73^{\{8\}}$	$135^{\{16\}}$	$25^{\{1\}}$	$69^{\{7\}}$	$100^{\{14\}}$	$65^{\{6\}}$
200	BIAS($\hat{\delta}$)	$0.39119^{\{9\}}$	$0.41084^{\{12\}}$	$0.41306^{\{13\}}$	$0.36779^{\{5\}}$	$0.41738^{\{14\}}$	$0.39414^{\{10\}}$	$0.37887^{\{6\}}$	$0.3847^{\{7\}}$	$0.46065^{\{15\}}$	$0.30128^{\{3\}}$	$0.38635^{\{8\}}$	$0.51279^{\{16\}}$	$0.0578^{\{1\}}$	$0.29981^{\{2\}}$	$0.39585^{\{11\}}$	$0.32297^{\{4\}}$
	BIAS($\hat{\gamma}$)	$0.39593^{\{5\}}$	$0.44818^{\{7\}}$	$0.47622^{\{12\}}$	$0.45219^{\{9\}}$	$0.50208^{\{14\}}$	$0.493^{\{13\}}$	$0.4021^{\{6\}}$	$0.45121^{\{8\}}$	$0.56814^{\{15\}}$	$0.33137^{\{2\}}$	$0.46749^{\{11\}}$	$0.70833^{\{16\}}$	$0.07383^{\{1\}}$	$0.36521^{\{3\}}$	$0.45481^{\{10\}}$	$0.38735^{\{4\}}$
	MSE($\hat{\delta}$)	$0.23206^{\{11\}}$	$0.24938^{\{14\}}$	$0.23991^{\{12\}}$	$0.19858^{\{5\}}$	$0.24902^{\{13\}}$	$0.2208^{\{9\}}$	$0.21578^{\{6\}}$	$0.21605^{\{7\}}$	$0.28939^{\{15\}}$	$0.15761^{\{4\}}$	$0.21659^{\{8\}}$	$0.33737^{\{16\}}$	$0.00697^{\{1\}}$	$0.12832^{\{2\}}$	$0.22826^{\{10\}}$	$0.15392^{\{3\}}$
	MSE($\hat{\gamma}$)	$0.25898^{\{4\}}$	$0.33532^{\{7\}}$	$0.3817^{\{12\}}$	$0.3456^{\{9\}}$	$0.42887^{\{14\}}$	$0.40431^{\{13\}}$	$0.27218^{\{6\}}$	$0.35352^{\{10\}}$	$0.53561^{\{15\}}$	$0.22551^{\{2\}}$	$0.37752^{\{11\}}$	$0.79054^{\{16\}}$	$0.01158^{\{1\}}$	$0.22881^{\{3\}}$	$0.34202^{\{8\}}$	$0.27047^{\{5\}}$
	MRE($\hat{\delta}$)	$0.31296^{\{9\}}$	$0.32868^{\{12\}}$	$0.33045^{\{13\}}$	$0.29423^{\{5\}}$	$0.33391^{\{14\}}$	$0.31531^{\{10\}}$	$0.3031^{\{6\}}$	$0.30776^{\{7\}}$	$0.36852^{\{15\}}$	$0.24102^{\{3\}}$	$0.30908^{\{8\}}$	$0.41023^{\{16\}}$	$0.04624^{\{1\}}$	$0.23985^{\{2\}}$	$0.31668^{\{11\}}$	$0.25837^{\{4\}}$
	MRE($\hat{\gamma}$)	$0.19797^{\{5\}}$	$0.22409^{\{7\}}$	$0.23811^{\{12\}}$	$0.22609^{\{9\}}$	$0.25104^{\{14\}}$	$0.2465^{\{13\}}$	$0.20105^{\{6\}}$	$0.22561^{\{8\}}$	$0.28407^{\{15\}}$	$0.16569^{\{2\}}$	$0.23375^{\{11\}}$	$0.35416^{\{16\}}$	$0.03691^{\{1\}}$	$0.1826^{\{3\}}$	$0.22741^{\{10\}}$	$0.19367^{\{4\}}$
	D_{abs}	$0.01717^{\{4\}}$	$0.01744^{\{7\}}$	$0.0176^{\{9\}}$	$0.01737^{\{6\}}$	$0.01678^{\{2\}}$	$0.01736^{\{5\}}$	$0.01752^{\{8\}}$	$0.01705^{\{3\}}$	$0.01821^{\{10\}}$	$0.01932^{\{12\}}$	$0.0195^{\{13\}}$	$0.01975^{\{14\}}$	$0.01638^{\{1\}}$	$0.02169^{\{16\}}$	$0.01852^{\{11\}}$	$0.02152^{\{15\}}$
	D_{max}	$0.02763^{\{2\}}$	$0.02843^{\{8\}}$	$0.02902^{\{9\}}$	$0.02815^{\{5\}}$	$0.02796^{\{4\}}$	$0.02831^{\{7\}}$	$0.02816^{\{6\}}$	$0.02788^{\{3\}}$	$0.03062^{\{12\}}$	$0.03039^{\{11\}}$	$0.0314^{\{13\}}$	$0.03465^{\{16\}}$	$0.02414^{\{1\}}$	$0.03408^{\{15\}}$	$0.02993^{\{10\}}$	$0.03398^{\{14\}}$
	ASAE	$0.00984^{\{2\}}$	$0.0098^{\{8\}}$	$0.01014^{\{9\}}$	$0.00961^{\{5\}}$	$0.01035^{\{4\}}$	$0.01008^{\{7\}}$	$0.00949^{\{6\}}$	$0.00958^{\{3\}}$	$0.01138^{\{12\}}$	$0.01345^{\{11\}}$	$0.01219^{\{13\}}$	$0.01529^{\{16\}}$	$0.01245^{\{1\}}$	$0.01483^{\{15\}}$	$0.01148^{\{10\}}$	$0.01486^{\{14\}}$
	$\sum Ranks$	$54^{\{4\}}$	$78^{\{9\}}$	$99^{\{14\}}$	$56^{\{6\}}$	$97^{\{13\}}$	$86^{\{10\}}$	$51^{\{2\}}$	$55^{\{5\}}$	$121^{\{15\}}$	$52^{\{3\}}$	$94^{\{12\}}$	$142^{\{16\}}$	$20^{\{1\}}$	$60^{\{7\}}$	$91^{\{11\}}$	$68^{\{8\}}$
350	BIAS($\hat{\delta}$)	$0.34168^{\{9\}}$	$0.35966^{\{12\}}$	$0.38558^{\{13\}}$	$0.30264^{\{5\}}$	$0.38801^{\{14\}}$	$0.34234^{\{10\}}$	$0.33995^{\{8\}}$	$0.348^{\{11\}}$	$0.40587^{\{15\}}$	$0.28709^{\{3\}}$	$0.32955^{\{6\}}$	$0.50316^{\{16\}}$	$0.05015^{\{1\}}$	$0.29429^{\{4\}}$	$0.33572^{\{7\}}$	$0.27147^{\{2\}}$
	BIAS($\hat{\gamma}$)	$0.31433^{\{3\}}$	$0.35902^{\{8\}}$	$0.38484^{\{12\}}$	$0.32693^{\{5\}}$	$0.40549^{\{14\}}$	$0.39516^{\{13\}}$	$0.34447^{\{7\}}$	$0.36446^{\{11\}}$	$0.45342^{\{15\}}$	$0.29622^{\{2\}}$	$0.3591^{\{9\}}$	$0.68102^{\{16\}}$	$0.06224^{\{1\}}$	$0.33303^{\{6\}}$	$0.35964^{\{10\}}$	$0.31549^{\{4\}}$
	MSE($\hat{\delta}$)	$0.19774^{\{11\}}$	$0.2006^{\{12\}}$	$0.22736^{\{14\}}$	$0.14239^{\{4\}}$	$0.22087^{\{13\}}$	$0.16978^{\{7\}}$	$0.18063^{\{9\}}$	$0.18873^{\{10\}}$	$0.24093^{\{15\}}$	$0.14384^{\{5\}}$	$0.16656^{\{6\}}$	$0.33007^{\{16\}}$	$0.00558^{\{1\}}$	$0.12783^{\{3\}}$	$0.17348^{\{8\}}$	$0.12051^{\{2\}}$
	MSE($\hat{\gamma}$)	$0.15925^{\{2\}}$	$0.20587^{\{8\}}$	$0.23793^{\{12\}}$	$0.18^{\{4\}}$	$0.26251^{\{14\}}$	$0.24998^{\{13\}}$	$0.1938^{\{6\}}$	$0.22019^{\{10\}}$	$0.35021^{\{15\}}$	$0.16467^{\{3\}}$	$0.22066^{\{11\}}$	$0.74386^{\{16\}}$	$0.00844^{\{1\}}$	$0.19025^{\{5\}}$	$0.20982^{\{9\}}$	$0.20294^{\{7\}}$
	MRE($\hat{\delta}$)	$0.27335^{\{9\}}$	$0.28773^{\{12\}}$	$0.30847^{\{13\}}$	$0.24211^{\{5\}}$	$0.31041^{\{14\}}$	$0.27387^{\{10\}}$	$0.27196^{\{8\}}$	$0.2784^{\{11\}}$	$0.32469^{\{15\}}$	$0.22967^{\{3\}}$	$0.26364^{\{6\}}$	$0.40253^{\{16\}}$	$0.04012^{\{1\}}$	$0.23543^{\{4\}}$	$0.26858^{\{7\}}$	$0.21718^{\{2\}}$
	MRE($\hat{\gamma}$)	$0.15717^{\{3\}}$	$0.17951^{\{8\}}$	$0.19242^{\{12\}}$	$0.16347^{\{5\}}$	$0.20274^{\{14\}}$	$0.19758^{\{13\}}$	$0.17223^{\{7\}}$	$0.18223^{\{11\}}$	$0.22671^{\{15\}}$	$0.14811^{\{2\}}$	$0.17955^{\{9\}}$	$0.34051^{\{16\}}$	$0.03112^{\{1\}}$	$0.16651^{\{6\}}$	$0.17982^{\{10\}}$	$0.15775^{\{4\}}$
	D_{abs}	$0.01327^{\{4\}}$	$0.01332^{\{5.5\}}$	$0.01322^{\{3\}}$	$0.01259^{\{1\}}$	$0.01332^{\{5.5\}}$	$0.01339^{\{8\}}$	$0.01337^{\{7\}}$	$0.01348^{\{9\}}$	$0.01351^{\{10\}}$	$0.01536^{\{14\}}$	$0.01481^{\{12\}}$	$0.01532^{\{13\}}$	$0.01263^{\{2\}}$	$0.01753^{\{16\}}$	$0.01445^{\{11\}}$	$0.01701^{\{15\}}$
	D_{max}	$0.02154^{\{3\}}$	$0.0218^{\{5\}}$	$0.02216^{\{8\}}$	$0.02059^{\{2\}}$	$0.02222^{\{9\}}$	$0.02197^{\{6\}}$	$0.02177^{\{4\}}$	$0.02209^{\{7\}}$	$0.02293^{\{10\}}$	$0.02438^{\{13\}}$	$0.02395^{\{12\}}$	$0.02755^{\{15\}}$	$0.01867^{\{1\}}$	$0.02779^{\{16\}}$	$0.02355^{\{11\}}$	$0.02691^{\{14\}}$
	ASAE	$0.00698^{\{3\}}$	$0.00689^{\{5\}}$	$0.00729^{\{8\}}$	$0.00691^{\{2\}}$	$0.00732^{\{9\}}$	$0.00725^{\{6\}}$	$0.00686^{\{4\}}$	$0.00673^{\{7\}}$	$0.00793^{\{10\}}$	$0.0096^{\{13\}}$	$0.00867^{\{12\}}$	$0.01192^{\{15\}}$	$0.00873^{\{1\}}$	$0.01067^{\{16\}}$	$0.00814^{\{11\}}$	$0.01078^{\{14\}}$
	$\sum Ranks$	$49^{\{3\}}$	$73.5^{\{7\}}$	$94^{\{13\}}$	$35^{\{2\}}$	$105.5^{\{14\}}$	$86^{\{12\}}$	$58^{\{4.5\}}$	$81^{\{9\}}$	$119^{\{15\}}$	$58^{\{4.5\}}$	$82^{\{10\}}$	$140^{\{16\}}$	$21^{\{1\}}$	$74^{\{8\}}$	$83^{\{11\}}$	$65^{\{6\}}$
450	BIAS($\hat{\delta}$)	$0.31636^{\{10\}}$	$0.31513^{\{9\}}$	$0.36831^{\{13\}}$	$0.28358^{\{5\}}$	$0.36963^{\{14\}}$	$0.31646^{\{11\}}$	$0.31475^{\{8\}}$	$0.3387^{\{12\}}$	$0.38838^{\{15\}}$	$0.27278^{\{4\}}$	$0.31134^{\{7\}}$	$0.49323^{\{16\}}$	$0.0477^{\{1\}}$	$0.27265^{\{3\}}$	$0.29376^{\{6\}}$	$0.22931^{\{2\}}$
	BIAS($\hat{\gamma}$)	$0.28837^{\{4\}}$	$0.3084^{\{8\}}$	$0.36467^{\{13\}}$	$0.29478^{\{5\}}$	$0.36908^{\{14\}}$	$0.35496^{\{12\}}$	$0.30381^{\{7\}}$	$0.32198^{\{10\}}$	$0.41054^{\{15\}}$	$0.28137^{\{3\}}$	$0.325^{\{11\}}$	$0.66076^{\{16\}}$	$0.05783^{\{1\}}$	$0.29914^{\{6\}}$	$0.31352^{\{9\}}$	$0.26906^{\{2\}}$
	MSE($\hat{\delta}$)	$0.1632^{\{11\}}$	$0.1599^{\{9\}}$	$0.20945^{\{14\}}$	$0.13241^{\{5\}}$	$0.20505^{\{13\}}$	$0.15184^{\{7\}}$	$0.16213^{\{10\}}$	$0.18244^{\{12\}}$	$0.22413^{\{15\}}$	$0.12776^{\{4\}}$	$0.15192^{\{8\}}$	$0.31544^{\{16\}}$	$0.00525^{\{1\}}$	$0.11694^{\{3\}}$	$0.13652^{\{6\}}$	$0.08946^{\{2\}}$
	MSE($\hat{\gamma}$)	$0.12889^{\{2\}}$	$0.15054^{\{7\}}$	$0.21441^{\{14\}}$	$0.14495^{\{3\}}$	$0.214^{\{13\}}$	$0.20897^{\{12\}}$	$0.14673^{\{5\}}$	$0.16112^{\{8\}}$	$0.27614^{\{15\}}$	$0.14612^{\{4\}}$	$0.17492^{\{11\}}$	$0.69464^{\{16\}}$	$0.00797^{\{1\}}$	$0.16447^{\{10\}}$	$0.16433^{\{9\}}$	$0.15052^{\{6\}}$
	MRE($\hat{\delta}$)	$0.25309^{\{10\}}$	$0.25211^{\{9\}}$	$0.29465^{\{13\}}$	$0.22687^{\{5\}}$	$0.29571^{\{14\}}$	$0.25317^{\{11\}}$	$0.2518^{\{8\}}$	$0.27096^{\{12\}}$	$0.3107^{\{15\}}$	$0.21823^{\{4\}}$	$0.24907^{\{7\}}$	$0.39458^{\{16\}}$	$0.03816^{\{1\}}$	$0.21812^{\{3\}}$	$0.23501^{\{6\}}$	$0.18344^{\{2\}}$
	MRE($\hat{\gamma}$)	$0.14419^{\{4\}}$	$0.1542^{\{8\}}$	$0.18234^{\{13\}}$	$0.14739^{\{5\}}$	$0.18454^{\{14\}}$	$0.17748^{\{12\}}$	$0.15191^{\{7\}}$	$0.16099^{\{10\}}$	$0.20527^{\{15\}}$	$0.14068^{\{3\}}$	$0.1625^{\{11\}}$	$0.33038^{\{16\}}$	$0.02891^{\{1\}}$	$0.14957^{\{6\}}$	$0.15676^{\{9\}}$	$0.13453^{\{2\}}$
	D_{abs}	$0.01126^{\{2\}}$	$0.01187^{\{6\}}$	$0.01226^{\{10\}}$	$0.01161^{\{3\}}$	$0.01224^{\{9\}}$	$0.01207^{\{7\}}$	$0.0118^{\{5\}}$	$0.01169^{\{4\}}$	$0.0122^{\{8\}}$	$0.01356^{\{13\}}$	$0.01301^{\{11.5\}}$	$0.0143^{\{14\}}$	$0.01119^{\{1\}}$	$0.01491^{\{15\}}$	$0.01301^{\{11.5\}}$	$0.0152^{\{16\}}$
	D_{max}	$0.01839^{\{2\}}$	$0.01936^{\{6\}}$	$0.02035^{\{8\}}$	$0.01889^{\{3\}}$	$0.02056^{\{9\}}$	$0.01982^{\{7\}}$	$0.01922^{\{4.5\}}$	$0.01922^{\{4.5\}}$	$0.02079^{\{10\}}$	$0.02167^{\{13\}}$	$0.02103^{\{11.5\}}$	$0.0262^{\{16\}}$	$0.01658^{\{1\}}$	$0.02381^{\{14\}}$	$0.02103^{\{11.5\}}$	$0.02391^{\{15\}}$
	ASAE	$0.00594^{\{2\}}$	$0.00596^{\{6\}}$	$0.00632^{\{8\}}$	$0.00588^{\{3\}}$	$0.0063^{\{9\}}$	$0.00635^{\{7\}}$	$0.00577^{\{4.5\}}$	$0.00589^{\{4.5\}}$	$0.00705^{\{10\}}$	$0.00815^{\{13\}}$	$0.0075^{\{11.5\}}$	$0.01079^{\{16\}}$	$0.00751^{\{1\}}$	$0.00909^{\{14\}}$	$0.00703^{\{11.5\}}$	$0.00908^{\{15\}}$
	$\sum Ranks$	$49^{\{3\}}$	$67^{\{7\}}$	$105^{\{13\}}$	$36^{\{2\}}$	$106^{\{14\}}$	$87^{\{11\}}$	$55.5^{\{4\}}$	$75.5^{\{9\}}$	$118^{\{15\}}$	$61^{\{5.5\}}$	$89^{\{12\}}$	$142^{\{16\}}$	$20^{\{1\}}$	$75^{\{8\}}$	$77^{\{10\}}$	$61^{\{5.5\}}$
600	BIAS($\hat{\delta}$)	$0.29229^{\{10\}}$	$0.29667^{\{11\}}$	$0.3284^{\{13\}}$	$0.25017^{\{4\}}$	$0.34394^{\{14\}}$	$0.2879^{\{8\}}$	$0.28041^{\{6\}}$	$0.30639^{\{12\}}$	$0.36598^{\{15\}}$	$0.26037^{\{5\}}$	$0.29228^{\{9\}}$	$0.48236^{\{16\}}$	$0.0438^{\{1\}}$	$0.23764^{\{3\}}$	$0.28622^{\{7\}}$	$0.22222^{\{2\}}$
	BIAS($\hat{\gamma}$)	$0.26397^{\{4\}}$	$0.27959^{\{8\}}$	$0.30932^{\{13\}}$	$0.26135^{\{3\}}$	$0.34222^{\{14\}}$	$0.30713^{\{11\}}$	$0.26479^{\{7\}}$	$0.2962^{\{10\}}$	$0.38215^{\{15\}}$	$0.25343^{\{2\}}$	$0.30731^{\{12\}}$	$0.62396^{\{16\}}$	$0.05284^{\{1\}}$	$0.26411^{\{5\}}$	$0.29375^{\{9\}}$	$0.26446^{\{6\}}$
	MSE($\hat{\delta}$)	$0.14525^{\{10\}}$	$0.14558^{\{11\}}$	$0.17581^{\{13\}}$	$0.10221^{\{4\}}$	$0.18459^{\{14\}}$	$0.1316^{\{8\}}$	$0.12922^{\{6\}}$	$0.1523^{\{12\}}$	$0.19969^{\{15\}}$	$0.11619^{\{5\}}$	$0.13144^{\{7\}}$	$0.30852^{\{16\}}$	$0.0044^{\{1\}}$	$0.09317^{\{3\}}$	$0.13301^{\{9\}}$	$0.08261^{\{2\}}$
	MSE($\hat{\gamma}$)	$0.10623^{\{2\}}$	$0.1246^{\{6\}}$	$0.15464^{\{12\}}$	$0.11254^{\{5\}}$	$0.17933^{\{14\}}$	$0.15477^{\{13\}}$	$0.10796^{\{3\}}$	$0.14024^{\{10\}}$	$0.24171^{\{15\}}$	$0.10886^{\{4\}}$	$0.15262^{\{11\}}$	$0.62521^{\{16\}}$	$0.00609^{\{1\}}$	$0.13282^{\{7\}}$	$0.13994^{\{9\}}$	$0.13971^{\{8\}}$
	MRE($\hat{\delta}$)	$0.23383^{\{9.5\}}$	$0.23733^{\{11\}}$	$0.26272^{\{13\}}$	$0.20014^{\{4\}}$	$0.27515^{\{14\}}$	$0.23032^{\{8\}}$	$0.22433^{\{6\}}$	$0.24511^{\{12\}}$	$0.29278^{\{15\}}$	$0.2083^{\{5\}}$	$0.23383^{\{9.5\}}$	$0.38589^{\{16\}}$	$0.03504^{\{1\}}$	$0.19011^{\{3\}}$	$0.22898^{\{7\}}$	$0.17777^{\{2\}}$
	MRE($\hat{\gamma}$)	$0.13199^{\{4\}}$	$0.1398^{\{8\}}$	$0.15466^{\{13\}}$	$0.13068^{\{3\}}$	$0.17111^{\{14\}}$	$0.15356^{\{11\}}$	$0.13239^{\{7\}}$	$0.1481^{\{10\}}$	$0.19107^{\{15\}}$	$0.12671^{\{2\}}$	$0.15366^{\{12\}}$	$0.31198^{\{16\}}$	$0.02642^{\{1\}}$	$0.13205^{\{5\}}$	$0.14688^{\{9\}}$	$0.13223^{\{6\}}$
	D_{abs}	$0.01007^{\{5\}}$	$0.01005^{\{4\}}$	$0.0104^{\{9\}}$	$0.00985^{\{3\}}$	$0.01035^{\{7\}}$	$0.01037^{\{8\}}$	$0.00984^{\{2\}}$	$0.01024^{\{6\}}$	$0.01057^{\{10\}}$	$0.01193^{\{13\}}$	$0.0115^{\{12\}}$	$0.01307^{\{15\}}$	$0.00965^{\{1\}}$	$0.01388^{\{16\}}$	$0.01105^{\{11\}}$	$0.013^{\{14\}}$
	D_{max}	$0.01646^{\{4\}}$	$0.01657^{\{5\}}$	$0.0174^{\{8\}}$	$0.01607^{\{2\}}$	$0.01756^{\{9\}}$	$0.0171^{\{7\}}$	$0.01619^{\{3\}}$	$0.01697^{\{6\}}$	$0.01826^{\{11\}}$	$0.01916^{\{13\}}$	$0.0188^{\{12\}}$	$0.02427^{\{16\}}$	$0.01434^{\{1\}}$	$0.02202^{\{15\}}$	$0.01809^{\{10\}}$	$0.02059^{\{14\}}$
	ASAE	$0.00502^{\{4\}}$	$0.00497^{\{5\}}$	$0.00532^{\{8\}}$	$0.00501^{\{2\}}$	$0.00533^{\{9\}}$	$0.00534^{\{7\}}$	$0.00483^{\{3\}}$	$0.00497^{\{6\}}$	$0.00594^{\{11\}}$	$0.00684^{\{13\}}$	$0.00636^{\{12\}}$	$0.00979^{\{16\}}$	$0.00615^{\{1\}}$	$0.00787^{\{15\}}$	$0.00592^{\{10\}}$	$0.00751^{\{14\}}$
	$\sum Ranks$	$53.5^{\{4\}}$	$66^{\{6\}}$	$100^{\{13\}}$	$32^{\{2\}}$	$107^{\{14\}}$	$82^{\{11\}}$	$41^{\{3\}}$	$81^{\{10\}}$	$121^{\{15\}}$	$62^{\{5\}}$	$96.5^{\{12\}}$	$143^{\{16\}}$	$19^{\{1\}}$	$72^{\{8\}}$	$80^{\{9\}}$	$68^{\{7\}}$

Table 6: Numerical values of simulation measures for $\delta = 0.75,\ \gamma = 1.5$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	PCE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
25	BIAS($\hat{\delta}$)	$0.27074^{\{6\}}$	$0.27582^{\{7\}}$	$0.27659^{\{8\}}$	$0.28096^{\{9\}}$	$0.28832^{\{11\}}$	$0.29115^{\{12\}}$	$0.29241^{\{14\}}$	$0.2869^{\{10\}}$	$0.29427^{\{15\}}$	$0.21819^{\{2\}}$	$0.26857^{\{5\}}$	$0.29874^{\{16\}}$	$0.10207^{\{1\}}$	$0.2402^{\{3\}}$	$0.29176^{\{13\}}$	$0.25514^{\{4\}}$
	BIAS($\hat{\gamma}$)	$0.45474^{\{6\}}$	$0.49699^{\{8\}}$	$0.48342^{\{7\}}$	$0.50867^{\{10\}}$	$0.52891^{\{14\}}$	$0.52696^{\{13\}}$	$0.49776^{\{9\}}$	$0.51128^{\{11\}}$	$0.52964^{\{15\}}$	$0.30036^{\{2\}}$	$0.445^{\{5\}}$	$0.54575^{\{16\}}$	$0.12064^{\{1\}}$	$0.39444^{\{3\}}$	$0.51297^{\{12\}}$	$0.40354^{\{4\}}$
	MSE($\hat{\delta}$)	$0.10462^{\{8\}}$	$0.10442^{\{7\}}$	$0.10411^{\{6\}}$	$0.10556^{\{9\}}$	$0.11044^{\{11\}}$	$0.11248^{\{12\}}$	$0.11666^{\{16\}}$	$0.10915^{\{10\}}$	$0.11359^{\{14\}}$	$0.07461^{\{2\}}$	$0.09964^{\{5\}}$	$0.11636^{\{15\}}$	$0.01711^{\{1\}}$	$0.08153^{\{3\}}$	$0.11349^{\{13\}}$	$0.08854^{\{4\}}$
	MSE($\hat{\gamma}$)	$0.34456^{\{5\}}$	$0.40731^{\{9\}}$	$0.38079^{\{7\}}$	$0.41514^{\{10\}}$	$0.44337^{\{14\}}$	$0.4354^{\{13\}}$	$0.39362^{\{8\}}$	$0.41728^{\{11\}}$	$0.44604^{\{15\}}$	$0.18947^{\{2\}}$	$0.3518^{\{6\}}$	$0.46421^{\{16\}}$	$0.03198^{\{1\}}$	$0.25809^{\{4\}}$	$0.41824^{\{12\}}$	$0.24913^{\{3\}}$
	MRE($\hat{\delta}$)	$0.36098^{\{6\}}$	$0.36776^{\{7\}}$	$0.36879^{\{8\}}$	$0.37461^{\{9\}}$	$0.38442^{\{11\}}$	$0.3882^{\{12\}}$	$0.38988^{\{14\}}$	$0.38253^{\{10\}}$	$0.39235^{\{15\}}$	$0.29092^{\{2\}}$	$0.3581^{\{5\}}$	$0.39832^{\{16\}}$	$0.13609^{\{1\}}$	$0.32026^{\{3\}}$	$0.38901^{\{13\}}$	$0.34019^{\{4\}}$
	MRE($\hat{\gamma}$)	$0.30316^{\{6\}}$	$0.33132^{\{8\}}$	$0.32228^{\{7\}}$	$0.33911^{\{10\}}$	$0.3526^{\{14\}}$	$0.35131^{\{13\}}$	$0.33184^{\{9\}}$	$0.34085^{\{11\}}$	$0.3531^{\{15\}}$	$0.20024^{\{2\}}$	$0.29667^{\{5\}}$	$0.36383^{\{16\}}$	$0.08042^{\{1\}}$	$0.26296^{\{3\}}$	$0.34198^{\{12\}}$	$0.26903^{\{4\}}$
	D_{abs}	$0.0446^{\{4\}}$	$0.04374^{\{1\}}$	$0.04403^{\{2\}}$	$0.04436^{\{3\}}$	$0.04545^{\{7\}}$	$0.04536^{\{6\}}$	$0.04474^{\{5\}}$	$0.04567^{\{8\}}$	$0.04606^{\{9\}}$	$0.05179^{\{14\}}$	$0.04885^{\{12\}}$	$0.04898^{\{13\}}$	$0.04743^{\{11\}}$	$0.05567^{\{15\}}$	$0.04615^{\{10\}}$	$0.0571^{\{16\}}$
	D_{max}	$0.06902^{\{3\}}$	$0.06815^{\{2\}}$	$0.06911^{\{4\}}$	$0.06797^{\{1\}}$	$0.0706^{\{8\}}$	$0.07024^{\{7\}}$	$0.0698^{\{6\}}$	$0.07081^{\{9\}}$	$0.07254^{\{11\}}$	$0.07877^{\{14\}}$	$0.07518^{\{12\}}$	$0.07677^{\{13\}}$	$0.06968^{\{5\}}$	$0.08196^{\{15\}}$	$0.07082^{\{10\}}$	$0.08417^{\{16\}}$
	ASAE	$0.03631^{\{3\}}$	$0.03513^{\{2\}}$	$0.03599^{\{4\}}$	$0.03389^{\{1\}}$	$0.03572^{\{8\}}$	$0.03409^{\{7\}}$	$0.03523^{\{6\}}$	$0.03408^{\{9\}}$	$0.03859^{\{11\}}$	$0.04736^{\{14\}}$	$0.04206^{\{12\}}$	$0.04514^{\{13\}}$	$0.04747^{\{5\}}$	$0.05094^{\{15\}}$	$0.0389^{\{10\}}$	$0.05111^{\{16\}}$
	$\sum Ranks$	$52^{\{2\}}$	$53^{\{3.5\}}$	$56^{\{5\}}$	$62^{\{6\}}$	$96^{\{13\}}$	$91^{\{12\}}$	$86^{\{11\}}$	$82^{\{10\}}$	$118^{\{15\}}$	$53^{\{3.5\}}$	$66^{\{8\}}$	$133^{\{16\}}$	$36^{\{1\}}$	$64^{\{7\}}$	$105^{\{14\}}$	$71^{\{9\}}$
100	BIAS($\hat{\delta}$)	$0.23589^{\{5\}}$	$0.25164^{\{12\}}$	$0.24594^{\{11\}}$	$0.24004^{\{7\}}$	$0.25977^{\{14\}}$	$0.25605^{\{13\}}$	$0.24008^{\{8\}}$	$0.24317^{\{10\}}$	$0.26976^{\{15\}}$	$0.20059^{\{2\}}$	$0.23662^{\{6\}}$	$0.29101^{\{16\}}$	$0.07048^{\{1\}}$	$0.23068^{\{4\}}$	$0.24239^{\{9\}}$	$0.20936^{\{3\}}$
	BIAS($\hat{\gamma}$)	$0.35407^{\{4\}}$	$0.39783^{\{10\}}$	$0.38895^{\{8\}}$	$0.39791^{\{11\}}$	$0.42644^{\{13\}}$	$0.45089^{\{15\}}$	$0.36575^{\{5\}}$	$0.3877^{\{7\}}$	$0.44057^{\{14\}}$	$0.27884^{\{2\}}$	$0.39025^{\{9\}}$	$0.51081^{\{16\}}$	$0.08072^{\{1\}}$	$0.36897^{\{6\}}$	$0.40631^{\{12\}}$	$0.34128^{\{3\}}$
	MSE($\hat{\delta}$)	$0.0828^{\{7\}}$	$0.09035^{\{13\}}$	$0.08663^{\{11\}}$	$0.08038^{\{5\}}$	$0.09367^{\{14\}}$	$0.09017^{\{12\}}$	$0.08331^{\{8\}}$	$0.08455^{\{10\}}$	$0.09839^{\{15\}}$	$0.06724^{\{3\}}$	$0.08085^{\{6\}}$	$0.11148^{\{16\}}$	$0.00921^{\{1\}}$	$0.07238^{\{4\}}$	$0.08368^{\{9\}}$	$0.0624^{\{2\}}$
	MSE($\hat{\gamma}$)	$0.20851^{\{4\}}$	$0.26514^{\{11\}}$	$0.25316^{\{7\}}$	$0.26053^{\{10\}}$	$0.30319^{\{13\}}$	$0.33166^{\{15\}}$	$0.22541^{\{6\}}$	$0.25434^{\{8\}}$	$0.31819^{\{14\}}$	$0.15528^{\{2\}}$	$0.25849^{\{9\}}$	$0.41898^{\{16\}}$	$0.01387^{\{1\}}$	$0.21226^{\{5\}}$	$0.28115^{\{12\}}$	$0.19634^{\{3\}}$
	MRE($\hat{\delta}$)	$0.31451^{\{5\}}$	$0.33552^{\{12\}}$	$0.32792^{\{11\}}$	$0.32005^{\{7\}}$	$0.34636^{\{14\}}$	$0.3414^{\{13\}}$	$0.32011^{\{8\}}$	$0.32423^{\{10\}}$	$0.35968^{\{15\}}$	$0.26746^{\{2\}}$	$0.3155^{\{6\}}$	$0.38801^{\{16\}}$	$0.09398^{\{1\}}$	$0.30757^{\{4\}}$	$0.32318^{\{9\}}$	$0.27915^{\{3\}}$
	MRE($\hat{\gamma}$)	$0.23605^{\{4\}}$	$0.26522^{\{10\}}$	$0.2593^{\{8\}}$	$0.26527^{\{11\}}$	$0.28429^{\{13\}}$	$0.30059^{\{15\}}$	$0.24383^{\{5\}}$	$0.25847^{\{7\}}$	$0.29371^{\{14\}}$	$0.18589^{\{2\}}$	$0.26016^{\{9\}}$	$0.34054^{\{16\}}$	$0.05381^{\{1\}}$	$0.24598^{\{6\}}$	$0.27088^{\{12\}}$	$0.22752^{\{3\}}$
	D_{abs}	$0.02468^{\{7\}}$	$0.02505^{\{9\}}$	$0.0249^{\{8\}}$	$0.02442^{\{4\}}$	$0.02462^{\{5\}}$	$0.02464^{\{6\}}$	$0.02398^{\{2\}}$	$0.02428^{\{3\}}$	$0.02519^{\{10\}}$	$0.02817^{\{14\}}$	$0.02695^{\{12\}}$	$0.02609^{\{11\}}$	$0.02358^{\{1\}}$	$0.02988^{\{16\}}$	$0.02707^{\{13\}}$	$0.02944^{\{15\}}$
	D_{max}	$0.03981^{\{5\}}$	$0.04065^{\{8\}}$	$0.0407^{\{9\}}$	$0.03972^{\{4\}}$	$0.04047^{\{7\}}$	$0.04028^{\{6\}}$	$0.0389^{\{2\}}$	$0.03947^{\{3\}}$	$0.04186^{\{10\}}$	$0.04422^{\{13\}}$	$0.0432^{\{11\}}$	$0.0444^{\{14\}}$	$0.03507^{\{1\}}$	$0.04722^{\{16\}}$	$0.04337^{\{12\}}$	$0.04626^{\{15\}}$
	ASAE	$0.0146^{\{5\}}$	$0.01457^{\{8\}}$	$0.01547^{\{9\}}$	$0.01443^{\{4\}}$	$0.01542^{\{7\}}$	$0.01513^{\{6\}}$	$0.01426^{\{2\}}$	$0.01452^{\{3\}}$	$0.01681^{\{10\}}$	$0.01989^{\{13\}}$	$0.01854^{\{11\}}$	$0.02132^{\{14\}}$	$0.01859^{\{1\}}$	$0.02173^{\{16\}}$	$0.01744^{\{12\}}$	$0.02144^{\{15\}}$
	$\sum Ranks$	$46^{\{3\}}$	$89^{\{11\}}$	$81^{\{10\}}$	$61^{\{5.5\}}$	$100^{\{13\}}$	$101^{\{14\}}$	$45^{\{2\}}$	$61^{\{5.5\}}$	$116^{\{15\}}$	$53^{\{4\}}$	$79^{\{9\}}$	$135^{\{16\}}$	$20^{\{1\}}$	$77^{\{8\}}$	$98^{\{12\}}$	$62^{\{7\}}$
200	BIAS($\hat{\delta}$)	$0.21873^{\{9\}}$	$0.21922^{\{10\}}$	$0.229^{\{14\}}$	$0.19565^{\{4\}}$	$0.22711^{\{13\}}$	$0.22319^{\{12\}}$	$0.20698^{\{6\}}$	$0.21984^{\{11\}}$	$0.24264^{\{15\}}$	$0.1922^{\{2\}}$	$0.20994^{\{7\}}$	$0.28187^{\{16\}}$	$0.05919^{\{1\}}$	$0.19741^{\{5\}}$	$0.21392^{\{8\}}$	$0.19401^{\{3\}}$
	BIAS($\hat{\gamma}$)	$0.29225^{\{5\}}$	$0.30057^{\{6\}}$	$0.31671^{\{10\}}$	$0.30078^{\{7\}}$	$0.33068^{\{13\}}$	$0.3617^{\{14\}}$	$0.28655^{\{3\}}$	$0.30248^{\{8\}}$	$0.36646^{\{15\}}$	$0.26114^{\{2\}}$	$0.31696^{\{11\}}$	$0.48964^{\{16\}}$	$0.07502^{\{1\}}$	$0.3124^{\{9\}}$	$0.31837^{\{12\}}$	$0.2872^{\{4\}}$
	MSE($\hat{\delta}$)	$0.07448^{\{12\}}$	$0.07327^{\{10\}}$	$0.07837^{\{14\}}$	$0.05698^{\{4\}}$	$0.07582^{\{13\}}$	$0.07259^{\{9\}}$	$0.06791^{\{7\}}$	$0.07414^{\{11\}}$	$0.08486^{\{15\}}$	$0.06454^{\{5\}}$	$0.06648^{\{6\}}$	$0.10778^{\{16\}}$	$0.00747^{\{1\}}$	$0.05633^{\{3\}}$	$0.06949^{\{8\}}$	$0.05511^{\{2\}}$
	MSE($\hat{\gamma}$)	$0.13631^{\{5\}}$	$0.14484^{\{6\}}$	$0.16687^{\{10\}}$	$0.1534^{\{8\}}$	$0.18318^{\{13\}}$	$0.21833^{\{14\}}$	$0.13468^{\{4\}}$	$0.15331^{\{7\}}$	$0.22931^{\{15\}}$	$0.13042^{\{2\}}$	$0.1728^{\{11\}}$	$0.40026^{\{16\}}$	$0.01197^{\{1\}}$	$0.16498^{\{9\}}$	$0.17491^{\{12\}}$	$0.13421^{\{3\}}$
	MRE($\hat{\delta}$)	$0.29163^{\{9\}}$	$0.29229^{\{10\}}$	$0.30533^{\{14\}}$	$0.26087^{\{4\}}$	$0.30281^{\{13\}}$	$0.29759^{\{12\}}$	$0.27598^{\{6\}}$	$0.29312^{\{11\}}$	$0.32352^{\{15\}}$	$0.25627^{\{2\}}$	$0.27992^{\{7\}}$	$0.37582^{\{16\}}$	$0.07891^{\{1\}}$	$0.26321^{\{5\}}$	$0.28522^{\{8\}}$	$0.25867^{\{3\}}$
	MRE($\hat{\gamma}$)	$0.19483^{\{5\}}$	$0.20038^{\{6\}}$	$0.21114^{\{10\}}$	$0.20052^{\{7\}}$	$0.22045^{\{13\}}$	$0.24114^{\{14\}}$	$0.19103^{\{3\}}$	$0.20165^{\{8\}}$	$0.24431^{\{15\}}$	$0.17409^{\{2\}}$	$0.21131^{\{11\}}$	$0.32643^{\{16\}}$	$0.05001^{\{1\}}$	$0.20826^{\{9\}}$	$0.21225^{\{12\}}$	$0.19147^{\{4\}}$
	D_{abs}	$0.018^{\{6\}}$	$0.01695^{\{1\}}$	$0.01781^{\{5\}}$	$0.01759^{\{4\}}$	$0.01802^{\{8\}}$	$0.019^{\{10\}}$	$0.01734^{\{3\}}$	$0.01801^{\{7\}}$	$0.01842^{\{9\}}$	$0.02087^{\{13\}}$	$0.01966^{\{12\}}$	$0.02091^{\{14\}}$	$0.01722^{\{2\}}$	$0.0229^{\{16\}}$	$0.0195^{\{11\}}$	$0.02229^{\{15\}}$
	D_{max}	$0.02961^{\{6\}}$	$0.02803^{\{2\}}$	$0.02955^{\{5\}}$	$0.02889^{\{4\}}$	$0.03005^{\{8\}}$	$0.03139^{\{10\}}$	$0.02839^{\{3\}}$	$0.0298^{\{7\}}$	$0.03128^{\{9\}}$	$0.0333^{\{13\}}$	$0.03204^{\{12\}}$	$0.0367^{\{16\}}$	$0.02578^{\{1\}}$	$0.03654^{\{15\}}$	$0.03188^{\{11\}}$	$0.03577^{\{14\}}$
	ASAE	$0.00952^{\{6\}}$	$0.00974^{\{2\}}$	$0.0102^{\{5\}}$	$0.00951^{\{4\}}$	$0.00985^{\{8\}}$	$0.01007^{\{10\}}$	$0.00925^{\{3\}}$	$0.00947^{\{7\}}$	$0.0112^{\{9\}}$	$0.01293^{\{13\}}$	$0.01179^{\{12\}}$	$0.01599^{\{16\}}$	$0.01188^{\{1\}}$	$0.0145^{\{15\}}$	$0.01123^{\{11\}}$	$0.01438^{\{14\}}$
	$\sum Ranks$	$61^{\{6\}}$	$56^{\{5\}}$	$90^{\{11\}}$	$45^{\{3\}}$	$100^{\{13\}}$	$102^{\{14\}}$	$36^{\{2\}}$	$72^{\{8\}}$	$117^{\{15\}}$	$54^{\{4\}}$	$88^{\{10\}}$	$142^{\{16\}}$	$21^{\{1\}}$	$86^{\{9\}}$	$92^{\{12\}}$	$62^{\{7\}}$
350	BIAS($\hat{\delta}$)	$0.18319^{\{10\}}$	$0.17676^{\{5\}}$	$0.20038^{\{13\}}$	$0.15915^{\{2\}}$	$0.20534^{\{14\}}$	$0.18631^{\{11\}}$	$0.18016^{\{9\}}$	$0.1879^{\{12\}}$	$0.22244^{\{15\}}$	$0.16013^{\{3\}}$	$0.17887^{\{8\}}$	$0.27573^{\{16\}}$	$0.05327^{\{1\}}$	$0.16934^{\{4\}}$	$0.17874^{\{7\}}$	$0.1772^{\{6\}}$
	BIAS($\hat{\gamma}$)	$0.22083^{\{3\}}$	$0.23518^{\{6\}}$	$0.25578^{\{11\}}$	$0.2259^{\{5\}}$	$0.26821^{\{13\}}$	$0.28096^{\{14\}}$	$0.22436^{\{4\}}$	$0.24301^{\{8\}}$	$0.29557^{\{15\}}$	$0.21769^{\{2\}}$	$0.25822^{\{12\}}$	$0.44155^{\{16\}}$	$0.06478^{\{1\}}$	$0.23769^{\{7\}}$	$0.24529^{\{9\}}$	$0.24946^{\{10\}}$
	MSE($\hat{\delta}$)	$0.05725^{\{11\}}$	$0.05062^{\{7\}}$	$0.06352^{\{13\}}$	$0.04007^{\{2\}}$	$0.06628^{\{14\}}$	$0.05361^{\{9\}}$	$0.05497^{\{10\}}$	$0.05779^{\{12\}}$	$0.07773^{\{15\}}$	$0.04585^{\{4\}}$	$0.04929^{\{6\}}$	$0.10319^{\{16\}}$	$0.00631^{\{1\}}$	$0.04539^{\{3\}}$	$0.05121^{\{8\}}$	$0.04767^{\{5\}}$
	MSE($\hat{\gamma}$)	$0.07813^{\{2\}}$	$0.09002^{\{6\}}$	$0.106^{\{11\}}$	$0.08547^{\{4\}}$	$0.11585^{\{13\}}$	$0.13545^{\{14\}}$	$0.08035^{\{3\}}$	$0.0975^{\{8\}}$	$0.14542^{\{15\}}$	$0.08951^{\{5\}}$	$0.11177^{\{12\}}$	$0.32596^{\{16\}}$	$0.00962^{\{1\}}$	$0.09742^{\{7\}}$	$0.09898^{\{9\}}$	$0.10044^{\{10\}}$
	MRE($\hat{\delta}$)	$0.24425^{\{10\}}$	$0.23568^{\{5\}}$	$0.26718^{\{13\}}$	$0.2122^{\{2\}}$	$0.27379^{\{14\}}$	$0.24842^{\{11\}}$	$0.24021^{\{9\}}$	$0.25053^{\{12\}}$	$0.29659^{\{15\}}$	$0.21351^{\{3\}}$	$0.23849^{\{8\}}$	$0.36764^{\{16\}}$	$0.07103^{\{1\}}$	$0.22578^{\{4\}}$	$0.23832^{\{7\}}$	$0.23627^{\{6\}}$
	MRE($\hat{\gamma}$)	$0.14722^{\{3\}}$	$0.15678^{\{6\}}$	$0.17052^{\{11\}}$	$0.1506^{\{5\}}$	$0.17881^{\{13\}}$	$0.18731^{\{14\}}$	$0.14957^{\{4\}}$	$0.16201^{\{8\}}$	$0.19704^{\{15\}}$	$0.14513^{\{2\}}$	$0.17215^{\{12\}}$	$0.29437^{\{16\}}$	$0.04319^{\{1\}}$	$0.15846^{\{7\}}$	$0.16353^{\{9\}}$	$0.16631^{\{10\}}$
	D_{abs}	$0.01348^{\{4\}}$	$0.01329^{\{3\}}$	$0.01353^{\{5\}}$	$0.01319^{\{2\}}$	$0.01413^{\{8\}}$	$0.01416^{\{9\}}$	$0.01379^{\{7\}}$	$0.01361^{\{6\}}$	$0.01435^{\{10\}}$	$0.01599^{\{13\}}$	$0.0154^{\{12\}}$	$0.01615^{\{14\}}$	$0.01242^{\{1\}}$	$0.01757^{\{16\}}$	$0.01491^{\{11\}}$	$0.01726^{\{15\}}$
	D_{max}	$0.02225^{\{4\}}$	$0.0221^{\{3\}}$	$0.02288^{\{7\}}$	$0.02159^{\{2\}}$	$0.02379^{\{9\}}$	$0.02355^{\{8\}}$	$0.02266^{\{5\}}$	$0.02269^{\{6\}}$	$0.0245^{\{10\}}$	$0.0257^{\{13\}}$	$0.02526^{\{12\}}$	$0.02903^{\{16\}}$	$0.01874^{\{1\}}$	$0.02826^{\{15\}}$	$0.02451^{\{11\}}$	$0.02802^{\{14\}}$
	ASAE	$0.00667^{\{4\}}$	$0.00678^{\{3\}}$	$0.00717^{\{7\}}$	$0.00662^{\{2\}}$	$0.0073^{\{9\}}$	$0.00726^{\{8\}}$	$0.00659^{\{5\}}$	$0.00675^{\{6\}}$	$0.00794^{\{10\}}$	$0.00916^{\{13\}}$	$0.00858^{\{12\}}$	$0.01195^{\{16\}}$	$0.00841^{\{1\}}$	$0.01031^{\{15\}}$	$0.00801^{\{11\}}$	$0.01011^{\{14\}}$
	$\sum Ranks$	$50^{\{4\}}$	$46^{\{3\}}$	$90^{\{10.5\}}$	$26^{\{2\}}$	$106^{\{14\}}$	$97^{\{13\}}$	$52^{\{5\}}$	$76^{\{7\}}$	$119^{\{15\}}$	$58^{\{6\}}$	$94^{\{12\}}$	$142^{\{16\}}$	$19^{\{1\}}$	$78^{\{8\}}$	$81^{\{9\}}$	$90^{\{10.5\}}$
450	BIAS($\hat{\delta}$)	$0.15363^{\{4\}}$	$0.17448^{\{12\}}$	$0.18969^{\{14\}}$	$0.14477^{\{2\}}$	$0.18795^{\{13\}}$	$0.16722^{\{11\}}$	$0.15804^{\{6\}}$	$0.16637^{\{10\}}$	$0.20759^{\{15\}}$	$0.15215^{\{3\}}$	$0.16002^{\{7\}}$	$0.26274^{\{16\}}$	$0.04556^{\{1\}}$	$0.16608^{\{9\}}$	$0.15776^{\{5\}}$	$0.16423^{\{8\}}$
	BIAS($\hat{\gamma}$)	$0.19504^{\{3\}}$	$0.21065^{\{7\}}$	$0.23088^{\{10\}}$	$0.19968^{\{4\}}$	$0.23628^{\{13\}}$	$0.24828^{\{14\}}$	$0.1929^{\{2\}}$	$0.21044^{\{6\}}$	$0.25284^{\{15\}}$	$0.20262^{\{5\}}$	$0.22289^{\{9\}}$	$0.41647^{\{16\}}$	$0.05675^{\{1\}}$	$0.23114^{\{11\}}$	$0.21389^{\{8\}}$	$0.23482^{\{12\}}$
	MSE($\hat{\delta}$)	$0.04075^{\{5\}}$	$0.04952^{\{12\}}$	$0.05905^{\{14\}}$	$0.03323^{\{2\}}$	$0.05813^{\{13\}}$	$0.0427^{\{8\}}$	$0.0433^{\{10\}}$	$0.04699^{\{11\}}$	$0.06929^{\{15\}}$	$0.03936^{\{3\}}$	$0.04175^{\{7\}}$	$0.09631^{\{16\}}$	$0.00498^{\{1\}}$	$0.04318^{\{9\}}$	$0.04055^{\{4\}}$	$0.04139^{\{6\}}$
	MSE($\hat{\gamma}$)	$0.06181^{\{3\}}$	$0.0673^{\{5\}}$	$0.08406^{\{9\}}$	$0.06503^{\{4\}}$	$0.09052^{\{12\}}$	$0.10362^{\{15\}}$	$0.06022^{\{2\}}$	$0.07035^{\{6\}}$	$0.10167^{\{14\}}$	$0.07506^{\{7\}}$	$0.08626^{\{11\}}$	$0.29272^{\{16\}}$	$0.00779^{\{1\}}$	$0.08495^{\{10\}}$	$0.07642^{\{8\}}$	$0.09152^{\{13\}}$
	MRE($\hat{\delta}$)	$0.20485^{\{4\}}$	$0.23264^{\{12\}}$	$0.25292^{\{14\}}$	$0.19302^{\{2\}}$	$0.2506^{\{13\}}$	$0.22297^{\{11\}}$	$0.21072^{\{6\}}$	$0.22182^{\{10\}}$	$0.27679^{\{15\}}$	$0.20287^{\{3\}}$	$0.21336^{\{7\}}$	$0.35032^{\{16\}}$	$0.06074^{\{1\}}$	$0.22143^{\{9\}}$	$0.21034^{\{5\}}$	$0.21898^{\{8\}}$
	MRE($\hat{\gamma}$)	$0.13003^{\{3\}}$	$0.14043^{\{7\}}$	$0.15392^{\{10\}}$	$0.13312^{\{4\}}$	$0.15752^{\{13\}}$	$0.16552^{\{14\}}$	$0.1286^{\{2\}}$	$0.1403^{\{6\}}$	$0.16856^{\{15\}}$	$0.13508^{\{5\}}$	$0.1486^{\{9\}}$	$0.27765^{\{16\}}$	$0.03783^{\{1\}}$	$0.1541^{\{11\}}$	$0.1426^{\{8\}}$	$0.15655^{\{12\}}$
	D_{abs}	$0.01168^{\{3\}}$	$0.01188^{\{4\}}$	$0.0126^{\{9\}}$	$0.01149^{\{2\}}$	$0.01223^{\{7\}}$	$0.01275^{\{10\}}$	$0.0121^{\{5\}}$	$0.01217^{\{6\}}$	$0.01237^{\{8\}}$	$0.01449^{\{13\}}$	$0.0139^{\{12\}}$	$0.01481^{\{14\}}$	$0.01106^{\{1\}}$	$0.01513^{\{15\}}$	$0.0135^{\{11\}}$	$0.01537^{\{16\}}$
	D_{max}	$0.01927^{\{3\}}$	$0.01993^{\{4\}}$	$0.02125^{\{9\}}$	$0.01898^{\{2\}}$	$0.02065^{\{7\}}$	$0.02121^{\{8\}}$	$0.02003^{\{5\}}$	$0.02022^{\{6\}}$	$0.02126^{\{10\}}$	$0.02344^{\{13\}}$	$0.02279^{\{12\}}$	$0.0268^{\{16\}}$	$0.01672^{\{1\}}$	$0.02476^{\{14\}}$	$0.02209^{\{11\}}$	$0.02511^{\{15\}}$
	ASAE	$0.00566^{\{3\}}$	$0.00585^{\{4\}}$	$0.00613^{\{9\}}$	$0.00573^{\{2\}}$	$0.00631^{\{7\}}$	$0.0062^{\{8\}}$	$0.00566^{\{5\}}$	$0.00582^{\{6\}}$	$0.00695^{\{10\}}$	$0.00787^{\{13\}}$	$0.00733^{\{12\}}$	$0.01058^{\{16\}}$	$0.00718^{\{1\}}$	$0.00875^{\{14\}}$	$0.00694^{\{11\}}$	$0.00871^{\{15\}}$
	$\sum Ranks$	$29^{\{3\}}$	$68^{\{7\}}$	$95^{\{10\}}$	$25^{\{2\}}$	$99^{\{12\}}$	$98^{\{11\}}$	$40^{\{4\}}$	$65^{\{5.5\}}$	$117^{\{15\}}$	$65^{\{5.5\}}$	$86^{\{9\}}$	$142^{\{16\}}$	$19^{\{1\}}$	$103^{\{13\}}$	$69^{\{8\}}$	$104^{\{14\}}$
600	BIAS($\hat{\delta}$)	$0.13776^{\{5\}}$	$0.14219^{\{7\}}$	$0.17502^{\{14\}}$	$0.12809^{\{2\}}$	$0.16288^{\{13\}}$	$0.15103^{\{11\}}$	$0.1433^{\{8\}}$	$0.14687^{\{9\}}$	$0.18914^{\{15\}}$	$0.141^{\{6\}}$	$0.14738^{\{10\}}$	$0.25555^{\{16\}}$	$0.04456^{\{1\}}$	$0.15381^{\{12\}}$	$0.13751^{\{4\}}$	$0.13467^{\{3\}}$
	BIAS($\hat{\gamma}$)	$0.16723^{\{2\}}$	$0.17765^{\{4\}}$	$0.20753^{\{12\}}$	$0.17523^{\{3\}}$	$0.2031^{\{11\}}$	$0.21961^{\{14\}}$	$0.17913^{\{6\}}$	$0.18251^{\{7\}}$	$0.23643^{\{15\}}$	$0.18398^{\{8\}}$	$0.19735^{\{9\}}$	$0.39671^{\{16\}}$	$0.05377^{\{1\}}$	$0.20848^{\{13\}}$	$0.17876^{\{5\}}$	$0.19811^{\{10\}}$
	MSE($\hat{\delta}$)	$0.03306^{\{5\}}$	$0.03552^{\{8.5\}}$	$0.05019^{\{14\}}$	$0.02626^{\{2\}}$	$0.04393^{\{13\}}$	$0.03552^{\{8.5\}}$	$0.0369^{\{10\}}$	$0.03783^{\{12\}}$	$0.05806^{\{15\}}$	$0.03474^{\{6\}}$	$0.03513^{\{7\}}$	$0.09258^{\{16\}}$	$0.00487^{\{1\}}$	$0.037^{\{11\}}$	$0.03057^{\{4\}}$	$0.02824^{\{3\}}$
	MSE($\hat{\gamma}$)	$0.04339^{\{2\}}$	$0.0537^{\{6\}}$	$0.06753^{\{12\}}$	$0.05098^{\{4\}}$	$0.06548^{\{10\}}$	$0.08013^{\{14\}}$	$0.05131^{\{5\}}$	$0.05373^{\{7\}}$	$0.09147^{\{15\}}$	$0.05936^{\{8\}}$	$0.06433^{\{9\}}$	$0.26998^{\{16\}}$	$0.00699^{\{1\}}$	$0.06946^{\{13\}}$	$0.05076^{\{3\}}$	$0.0671^{\{11\}}$
	MRE($\hat{\delta}$)	$0.18368^{\{5\}}$	$0.18959^{\{7\}}$	$0.23336^{\{14\}}$	$0.17079^{\{2\}}$	$0.21718^{\{13\}}$	$0.20137^{\{11\}}$	$0.19106^{\{8\}}$	$0.19583^{\{9\}}$	$0.25219^{\{15\}}$	$0.18801^{\{6\}}$	$0.1965^{\{10\}}$	$0.34074^{\{16\}}$	$0.05941^{\{1\}}$	$0.20508^{\{12\}}$	$0.18335^{\{4\}}$	$0.17955^{\{3\}}$
	MRE($\hat{\gamma}$)	$0.11149^{\{2\}}$	$0.11843^{\{4\}}$	$0.13836^{\{12\}}$	$0.11682^{\{3\}}$	$0.1354^{\{11\}}$	$0.14641^{\{14\}}$	$0.11942^{\{6\}}$	$0.12168^{\{7\}}$	$0.15762^{\{15\}}$	$0.12265^{\{8\}}$	$0.13157^{\{9\}}$	$0.26447^{\{16\}}$	$0.03585^{\{1\}}$	$0.13899^{\{13\}}$	$0.11917^{\{5\}}$	$0.13207^{\{10\}}$
	D_{abs}	$0.01052^{\{7\}}$	$0.01002^{\{2\}}$	$0.01028^{\{4\}}$	$0.01021^{\{3\}}$	$0.01062^{\{8\}}$	$0.01125^{\{11\}}$	$0.01043^{\{6\}}$	$0.01032^{\{5\}}$	$0.01077^{\{9\}}$	$0.01236^{\{13\}}$	$0.01192^{\{12\}}$	$0.01334^{\{14\}}$	$0.00979^{\{1\}}$	$0.01421^{\{16\}}$	$0.01104^{\{10\}}$	$0.01381^{\{15\}}$
	D_{max}	$0.01729^{\{5\}}$	$0.01672^{\{2\}}$	$0.01756^{\{7\}}$	$0.01688^{\{3\}}$	$0.0179^{\{8\}}$	$0.01877^{\{11\}}$	$0.01738^{\{6\}}$	$0.01718^{\{4\}}$	$0.01867^{\{10\}}$	$0.0202^{\{13\}}$	$0.01966^{\{12\}}$	$0.02463^{\{16\}}$	$0.01483^{\{1\}}$	$0.02312^{\{15\}}$	$0.01825^{\{9\}}$	$0.02235^{\{14\}}$
	ASAE	$0.00477^{\{5\}}$	$0.0049^{\{2\}}$	$0.00529^{\{7\}}$	$0.00479^{\{3\}}$	$0.0052^{\{8\}}$	$0.00523^{\{11\}}$	$0.0047^{\{6\}}$	$0.00483^{\{4\}}$	$0.00582^{\{10\}}$	$0.00666^{\{13\}}$	$0.00607^{\{12\}}$	$0.00972^{\{16\}}$	$0.00589^{\{1\}}$	$0.00753^{\{15\}}$	$0.00574^{\{9\}}$	$0.00742^{\{14\}}$
	$\sum Ranks$	$35^{\{3\}}$	$45.5^{\{4\}}$	$97^{\{12\}}$	$25^{\{2\}}$	$93^{\{11\}}$	$101.5^{\{13\}}$	$56^{\{6\}}$	$64^{\{7\}}$	$119^{\{14\}}$	$81^{\{8\}}$	$90^{\{10\}}$	$142^{\{16\}}$	$19^{\{1\}}$	$120^{\{15\}}$	$53^{\{5\}}$	$83^{\{9\}}$

Table 7: Numerical values of simulation measures for $\delta = 0.5,\ \gamma = 1.0$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	PCE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
25	BIAS($\hat{\delta}$)	$0.17963^{\{8\}}$	$0.17533^{\{6\}}$	$0.18409^{\{11\}}$	$0.18096^{\{10\}}$	$0.18618^{\{13\}}$	$0.18889^{\{15\}}$	$0.18001^{\{9\}}$	$0.1853^{\{12\}}$	$0.18756^{\{14\}}$	$0.15109^{\{2\}}$	$0.17357^{\{4\}}$	$0.19108^{\{16\}}$	$0.08044^{\{1\}}$	$0.17498^{\{5\}}$	$0.17885^{\{7\}}$	$0.16852^{\{3\}}$
	BIAS($\hat{\gamma}$)	$0.31543^{\{8\}}$	$0.30319^{\{5\}}$	$0.32013^{\{10\}}$	$0.34115^{\{14\}}$	$0.34375^{\{15\}}$	$0.34581^{\{16\}}$	$0.31833^{\{9\}}$	$0.32532^{\{11\}}$	$0.33614^{\{12\}}$	$0.21539^{\{2\}}$	$0.2988^{\{4\}}$	$0.33949^{\{13\}}$	$0.10574^{\{1\}}$	$0.30837^{\{6\}}$	$0.31386^{\{7\}}$	$0.27743^{\{3\}}$
	MSE($\hat{\delta}$)	$0.04478^{\{9\}}$	$0.043^{\{6\}}$	$0.04706^{\{13\}}$	$0.04381^{\{7\}}$	$0.0465^{\{11\}}$	$0.04753^{\{14\}}$	$0.04546^{\{10\}}$	$0.04705^{\{12\}}$	$0.04845^{\{15\}}$	$0.03588^{\{2\}}$	$0.04277^{\{5\}}$	$0.05018^{\{16\}}$	$0.0104^{\{1\}}$	$0.04066^{\{4\}}$	$0.04463^{\{8\}}$	$0.04005^{\{3\}}$
	MSE($\hat{\gamma}$)	$0.16254^{\{9\}}$	$0.15175^{\{4\}}$	$0.16744^{\{10\}}$	$0.18358^{\{13\}}$	$0.18551^{\{16\}}$	$0.1851^{\{15\}}$	$0.16157^{\{8\}}$	$0.17059^{\{11\}}$	$0.1804^{\{12\}}$	$0.09253^{\{2\}}$	$0.15585^{\{6\}}$	$0.18367^{\{14\}}$	$0.02118^{\{1\}}$	$0.15415^{\{5\}}$	$0.1581^{\{7\}}$	$0.12861^{\{3\}}$
	MRE($\hat{\delta}$)	$0.35927^{\{8\}}$	$0.35065^{\{6\}}$	$0.36818^{\{11\}}$	$0.36192^{\{10\}}$	$0.37236^{\{13\}}$	$0.37778^{\{15\}}$	$0.36002^{\{9\}}$	$0.37059^{\{12\}}$	$0.37513^{\{14\}}$	$0.30218^{\{2\}}$	$0.34713^{\{4\}}$	$0.38217^{\{16\}}$	$0.16087^{\{1\}}$	$0.34996^{\{5\}}$	$0.35769^{\{7\}}$	$0.33703^{\{3\}}$
	MRE($\hat{\gamma}$)	$0.31543^{\{8\}}$	$0.30319^{\{5\}}$	$0.32013^{\{10\}}$	$0.34115^{\{14\}}$	$0.34375^{\{15\}}$	$0.34581^{\{16\}}$	$0.31833^{\{9\}}$	$0.32532^{\{11\}}$	$0.33614^{\{12\}}$	$0.21539^{\{2\}}$	$0.2988^{\{4\}}$	$0.33949^{\{13\}}$	$0.10574^{\{1\}}$	$0.30837^{\{6\}}$	$0.31386^{\{7\}}$	$0.27743^{\{3\}}$
	D_{abs}	$0.04433^{\{2\}}$	$0.045^{\{4\}}$	$0.04531^{\{5\}}$	$0.04668^{\{7\}}$	$0.04688^{\{9\}}$	$0.04673^{\{8\}}$	$0.04429^{\{1\}}$	$0.0446^{\{3\}}$	$0.04691^{\{10\}}$	$0.05535^{\{14\}}$	$0.04842^{\{13\}}$	$0.04715^{\{11\}}$	$0.0466^{\{6\}}$	$0.05636^{\{16\}}$	$0.04808^{\{12\}}$	$0.05623^{\{15\}}$
	D_{max}	$0.06957^{\{4\}}$	$0.06995^{\{5\}}$	$0.07124^{\{6\}}$	$0.0715^{\{7\}}$	$0.07258^{\{9\}}$	$0.07238^{\{8\}}$	$0.06909^{\{2\}}$	$0.06925^{\{3\}}$	$0.07379^{\{11\}}$	$0.084^{\{15\}}$	$0.07493^{\{13\}}$	$0.07432^{\{12\}}$	$0.06879^{\{1\}}$	$0.08479^{\{16\}}$	$0.07365^{\{10\}}$	$0.08346^{\{14\}}$
	ASAE	$0.03726^{\{4\}}$	$0.03504^{\{5\}}$	$0.03715^{\{6\}}$	$0.03422^{\{7\}}$	$0.03587^{\{9\}}$	$0.03402^{\{8\}}$	$0.03418^{\{2\}}$	$0.0341^{\{3\}}$	$0.03852^{\{11\}}$	$0.04678^{\{15\}}$	$0.04199^{\{13\}}$	$0.04484^{\{12\}}$	$0.04594^{\{1\}}$	$0.05064^{\{16\}}$	$0.03914^{\{10\}}$	$0.04955^{\{14\}}$
	$\sum Ranks$	$64^{\{6.5\}}$	$46^{\{2\}}$	$83^{\{11\}}$	$86^{\{12\}}$	$107^{\{13\}}$	$108^{\{14\}}$	$60^{\{4\}}$	$77^{\{9\}}$	$109^{\{15\}}$	$55^{\{3\}}$	$64^{\{6.5\}}$	$123^{\{16\}}$	$26^{\{1\}}$	$79^{\{10\}}$	$75^{\{8\}}$	$62^{\{5\}}$
100	BIAS($\hat{\delta}$)	$0.15521^{\{7\}}$	$0.159^{\{11\}}$	$0.15892^{\{10\}}$	$0.14817^{\{3\}}$	$0.15998^{\{13\}}$	$0.16699^{\{14\}}$	$0.15623^{\{8\}}$	$0.15478^{\{6\}}$	$0.17086^{\{15\}}$	$0.13343^{\{2\}}$	$0.15925^{\{12\}}$	$0.18548^{\{16\}}$	$0.06284^{\{1\}}$	$0.14926^{\{4\}}$	$0.15683^{\{9\}}$	$0.1521^{\{5\}}$
	BIAS($\hat{\gamma}$)	$0.23255^{\{3\}}$	$0.24461^{\{9\}}$	$0.24051^{\{7\}}$	$0.25342^{\{10\}}$	$0.25911^{\{12\}}$	$0.29223^{\{15\}}$	$0.23552^{\{4\}}$	$0.2393^{\{6\}}$	$0.2645^{\{14\}}$	$0.19447^{\{2\}}$	$0.25536^{\{11\}}$	$0.32499^{\{16\}}$	$0.08226^{\{1\}}$	$0.23762^{\{5\}}$	$0.26384^{\{13\}}$	$0.24442^{\{8\}}$
	MSE($\hat{\delta}$)	$0.03651^{\{7.5\}}$	$0.03756^{\{11\}}$	$0.03808^{\{13\}}$	$0.03162^{\{3\}}$	$0.03776^{\{12\}}$	$0.03996^{\{14\}}$	$0.03751^{\{10\}}$	$0.03651^{\{7.5\}}$	$0.04217^{\{15\}}$	$0.03021^{\{2\}}$	$0.03713^{\{9\}}$	$0.04653^{\{16\}}$	$0.0075^{\{1\}}$	$0.03249^{\{4\}}$	$0.03629^{\{6\}}$	$0.03268^{\{5\}}$
	MSE($\hat{\gamma}$)	$0.09111^{\{4\}}$	$0.09831^{\{8\}}$	$0.09936^{\{9\}}$	$0.10766^{\{10\}}$	$0.11319^{\{12\}}$	$0.14153^{\{15\}}$	$0.09507^{\{7\}}$	$0.09385^{\{6\}}$	$0.11571^{\{13\}}$	$0.07665^{\{2\}}$	$0.11228^{\{11\}}$	$0.17393^{\{16\}}$	$0.01362^{\{1\}}$	$0.09069^{\{3\}}$	$0.11811^{\{14\}}$	$0.09384^{\{5\}}$
	MRE($\hat{\delta}$)	$0.31042^{\{7\}}$	$0.31799^{\{11\}}$	$0.31784^{\{10\}}$	$0.29635^{\{3\}}$	$0.31997^{\{13\}}$	$0.33399^{\{14\}}$	$0.31247^{\{8\}}$	$0.30957^{\{6\}}$	$0.34172^{\{15\}}$	$0.26686^{\{2\}}$	$0.31849^{\{12\}}$	$0.37096^{\{16\}}$	$0.12569^{\{1\}}$	$0.29853^{\{4\}}$	$0.31366^{\{9\}}$	$0.3042^{\{5\}}$
	MRE($\hat{\gamma}$)	$0.23255^{\{3\}}$	$0.24461^{\{9\}}$	$0.24051^{\{7\}}$	$0.25342^{\{10\}}$	$0.25911^{\{12\}}$	$0.29223^{\{15\}}$	$0.23552^{\{4\}}$	$0.2393^{\{6\}}$	$0.2645^{\{14\}}$	$0.19447^{\{2\}}$	$0.25536^{\{11\}}$	$0.32499^{\{16\}}$	$0.08226^{\{1\}}$	$0.23762^{\{5\}}$	$0.26384^{\{13\}}$	$0.24442^{\{8\}}$
	D_{abs}	$0.02537^{\{8\}}$	$0.02505^{\{6\}}$	$0.02549^{\{9\}}$	$0.02436^{\{3\}}$	$0.02434^{\{2\}}$	$0.02556^{\{10\}}$	$0.02359^{\{1\}}$	$0.0251^{\{7\}}$	$0.02501^{\{5\}}$	$0.02822^{\{14\}}$	$0.02628^{\{12\}}$	$0.0269^{\{13\}}$	$0.0244^{\{4\}}$	$0.03039^{\{15\}}$	$0.02592^{\{11\}}$	$0.03139^{\{16\}}$
	D_{max}	$0.04119^{\{7\}}$	$0.04094^{\{6\}}$	$0.04163^{\{9\}}$	$0.0397^{\{3\}}$	$0.04022^{\{4\}}$	$0.04216^{\{10\}}$	$0.03854^{\{2\}}$	$0.04077^{\{5\}}$	$0.04146^{\{8\}}$	$0.04447^{\{13\}}$	$0.04301^{\{12\}}$	$0.04556^{\{14\}}$	$0.03674^{\{1\}}$	$0.04829^{\{15\}}$	$0.04226^{\{11\}}$	$0.04979^{\{16\}}$
	ASAE	$0.0144^{\{7\}}$	$0.01489^{\{6\}}$	$0.01563^{\{9\}}$	$0.01437^{\{3\}}$	$0.01553^{\{4\}}$	$0.01491^{\{10\}}$	$0.01415^{\{2\}}$	$0.01453^{\{5\}}$	$0.01666^{\{8\}}$	$0.0197^{\{13\}}$	$0.0181^{\{12\}}$	$0.02155^{\{14\}}$	$0.01843^{\{1\}}$	$0.02074^{\{15\}}$	$0.0169^{\{11\}}$	$0.02178^{\{16\}}$
	$\sum Ranks$	$49.5^{\{4\}}$	$76^{\{8\}}$	$82^{\{9\}}$	$47^{\{3\}}$	$87^{\{11\}}$	$113^{\{15\}}$	$45^{\{2\}}$	$53.5^{\{6\}}$	$108^{\{14\}}$	$52^{\{5\}}$	$101^{\{13\}}$	$138^{\{16\}}$	$23^{\{1\}}$	$69^{\{7\}}$	$96^{\{12\}}$	$84^{\{10\}}$
200	BIAS($\hat{\delta}$)	$0.13674^{\{10\}}$	$0.13229^{\{7\}}$	$0.1404^{\{11\}}$	$0.12391^{\{3\}}$	$0.14374^{\{14\}}$	$0.14368^{\{13\}}$	$0.14056^{\{12\}}$	$0.13129^{\{5\}}$	$0.14735^{\{15\}}$	$0.12032^{\{2\}}$	$0.13403^{\{8\}}$	$0.18224^{\{16\}}$	$0.05211^{\{1\}}$	$0.13413^{\{9\}}$	$0.13197^{\{6\}}$	$0.12695^{\{4\}}$
	BIAS($\hat{\gamma}$)	$0.17925^{\{3\}}$	$0.18706^{\{5\}}$	$0.19395^{\{7\}}$	$0.19485^{\{8\}}$	$0.20977^{\{12\}}$	$0.2499^{\{15\}}$	$0.18867^{\{6\}}$	$0.18325^{\{4\}}$	$0.21677^{\{14\}}$	$0.17613^{\{2\}}$	$0.2012^{\{10\}}$	$0.29906^{\{16\}}$	$0.06755^{\{1\}}$	$0.21166^{\{13\}}$	$0.203^{\{11\}}$	$0.20002^{\{9\}}$
	MSE($\hat{\delta}$)	$0.02961^{\{10\}}$	$0.02838^{\{9\}}$	$0.0305^{\{11\}}$	$0.0241^{\{2\}}$	$0.03151^{\{14\}}$	$0.03059^{\{12\}}$	$0.03142^{\{13\}}$	$0.02717^{\{6\}}$	$0.03276^{\{15\}}$	$0.02464^{\{4\}}$	$0.02815^{\{8\}}$	$0.04543^{\{16\}}$	$0.00549^{\{1\}}$	$0.02765^{\{7\}}$	$0.0271^{\{5\}}$	$0.0244^{\{3\}}$
	MSE($\hat{\gamma}$)	$0.05159^{\{2\}}$	$0.06028^{\{6\}}$	$0.06136^{\{7\}}$	$0.06523^{\{9\}}$	$0.0742^{\{13\}}$	$0.10727^{\{15\}}$	$0.05822^{\{4\}}$	$0.05549^{\{3\}}$	$0.0775^{\{14\}}$	$0.06024^{\{5\}}$	$0.06935^{\{11\}}$	$0.14903^{\{16\}}$	$0.00928^{\{1\}}$	$0.07285^{\{12\}}$	$0.06852^{\{10\}}$	$0.06397^{\{8\}}$
	MRE($\hat{\delta}$)	$0.27347^{\{10\}}$	$0.26457^{\{7\}}$	$0.2808^{\{11\}}$	$0.24782^{\{3\}}$	$0.28748^{\{14\}}$	$0.28736^{\{13\}}$	$0.28112^{\{12\}}$	$0.26258^{\{5\}}$	$0.29471^{\{15\}}$	$0.24065^{\{2\}}$	$0.26807^{\{8\}}$	$0.36447^{\{16\}}$	$0.10422^{\{1\}}$	$0.26827^{\{9\}}$	$0.26393^{\{6\}}$	$0.25391^{\{4\}}$
	MRE($\hat{\gamma}$)	$0.17925^{\{3\}}$	$0.18706^{\{5\}}$	$0.19395^{\{7\}}$	$0.19485^{\{8\}}$	$0.20977^{\{12\}}$	$0.2499^{\{15\}}$	$0.18867^{\{6\}}$	$0.18325^{\{4\}}$	$0.21677^{\{14\}}$	$0.17613^{\{2\}}$	$0.2012^{\{10\}}$	$0.29906^{\{16\}}$	$0.06755^{\{1\}}$	$0.21166^{\{13\}}$	$0.203^{\{11\}}$	$0.20002^{\{9\}}$
	D_{abs}	$0.01698^{\{2\}}$	$0.01844^{\{8\}}$	$0.0179^{\{4\}}$	$0.01792^{\{5\}}$	$0.01835^{\{7\}}$	$0.01952^{\{10\}}$	$0.01812^{\{6\}}$	$0.01764^{\{3\}}$	$0.01849^{\{9\}}$	$0.02113^{\{14\}}$	$0.02034^{\{13\}}$	$0.02031^{\{12\}}$	$0.01673^{\{1\}}$	$0.02304^{\{16\}}$	$0.01986^{\{11\}}$	$0.02244^{\{15\}}$
	D_{max}	$0.02843^{\{2\}}$	$0.03051^{\{7\}}$	$0.02993^{\{5\}}$	$0.02954^{\{4\}}$	$0.0307^{\{8\}}$	$0.03255^{\{10\}}$	$0.03019^{\{6\}}$	$0.02932^{\{3\}}$	$0.03123^{\{9\}}$	$0.03423^{\{13\}}$	$0.03351^{\{12\}}$	$0.0358^{\{14\}}$	$0.02556^{\{1\}}$	$0.03739^{\{16\}}$	$0.03265^{\{11\}}$	$0.03622^{\{15\}}$
	ASAE	$0.00944^{\{2\}}$	$0.00955^{\{7\}}$	$0.00987^{\{5\}}$	$0.00928^{\{4\}}$	$0.0102^{\{8\}}$	$0.00995^{\{10\}}$	$0.00907^{\{6\}}$	$0.00952^{\{3\}}$	$0.01106^{\{9\}}$	$0.01292^{\{13\}}$	$0.01179^{\{12\}}$	$0.01526^{\{14\}}$	$0.01162^{\{1\}}$	$0.01399^{\{16\}}$	$0.01119^{\{11\}}$	$0.01417^{\{15\}}$
	$\sum Ranks$	$45^{\{4\}}$	$59^{\{6\}}$	$69^{\{8\}}$	$44^{\{3\}}$	$102^{\{12\}}$	$110^{\{14\}}$	$66^{\{7\}}$	$37^{\{2\}}$	$114^{\{15\}}$	$57^{\{5\}}$	$92^{\{11\}}$	$138^{\{16\}}$	$19^{\{1\}}$	$109^{\{13\}}$	$81^{\{9\}}$	$82^{\{10\}}$
350	BIAS($\hat{\delta}$)	$0.10271^{\{3\}}$	$0.10398^{\{4\}}$	$0.11741^{\{10\}}$	$0.09817^{\{2\}}$	$0.11778^{\{11\}}$	$0.12155^{\{14\}}$	$0.10845^{\{6\}}$	$0.11195^{\{9\}}$	$0.13295^{\{15\}}$	$0.11064^{\{8\}}$	$0.10418^{\{5\}}$	$0.1726^{\{16\}}$	$0.04811^{\{1\}}$	$0.11862^{\{12\}}$	$0.10886^{\{7\}}$	$0.1192^{\{13\}}$
	BIAS($\hat{\gamma}$)	$0.13485^{\{2\}}$	$0.14374^{\{3\}}$	$0.1519^{\{7\}}$	$0.14584^{\{4\}}$	$0.16455^{\{11\}}$	$0.19373^{\{15\}}$	$0.14792^{\{5\}}$	$0.14945^{\{6\}}$	$0.1723^{\{12\}}$	$0.15341^{\{8\}}$	$0.15511^{\{9\}}$	$0.26671^{\{16\}}$	$0.06242^{\{1\}}$	$0.17767^{\{14\}}$	$0.15936^{\{10\}}$	$0.17406^{\{13\}}$
	MSE($\hat{\delta}$)	$0.01832^{\{4\}}$	$0.01837^{\{5\}}$	$0.02282^{\{14\}}$	$0.01517^{\{2\}}$	$0.02257^{\{12\}}$	$0.02259^{\{13\}}$	$0.02007^{\{7\}}$	$0.02059^{\{8\}}$	$0.02822^{\{15\}}$	$0.02173^{\{9\}}$	$0.01753^{\{3\}}$	$0.04164^{\{16\}}$	$0.00489^{\{1\}}$	$0.02212^{\{11\}}$	$0.01926^{\{6\}}$	$0.02177^{\{10\}}$
	MSE($\hat{\gamma}$)	$0.02917^{\{2\}}$	$0.03404^{\{3\}}$	$0.03683^{\{7\}}$	$0.03533^{\{5\}}$	$0.04439^{\{11\}}$	$0.06332^{\{15\}}$	$0.03528^{\{4\}}$	$0.03581^{\{6\}}$	$0.04697^{\{12\}}$	$0.04304^{\{10\}}$	$0.04222^{\{8\}}$	$0.1174^{\{16\}}$	$0.0082^{\{1\}}$	$0.05253^{\{14\}}$	$0.04296^{\{9\}}$	$0.04771^{\{13\}}$
	MRE($\hat{\delta}$)	$0.20543^{\{3\}}$	$0.20796^{\{4\}}$	$0.23482^{\{10\}}$	$0.19633^{\{2\}}$	$0.23556^{\{11\}}$	$0.2431^{\{14\}}$	$0.2169^{\{6\}}$	$0.2239^{\{9\}}$	$0.26589^{\{15\}}$	$0.22128^{\{8\}}$	$0.20837^{\{5\}}$	$0.34521^{\{16\}}$	$0.09622^{\{1\}}$	$0.23723^{\{12\}}$	$0.21772^{\{7\}}$	$0.2384^{\{13\}}$
	MRE($\hat{\gamma}$)	$0.13485^{\{2\}}$	$0.14374^{\{3\}}$	$0.1519^{\{7\}}$	$0.14584^{\{4\}}$	$0.16455^{\{11\}}$	$0.19373^{\{15\}}$	$0.14792^{\{5\}}$	$0.14945^{\{6\}}$	$0.1723^{\{12\}}$	$0.15341^{\{8\}}$	$0.15511^{\{9\}}$	$0.26671^{\{16\}}$	$0.06242^{\{1\}}$	$0.17767^{\{14\}}$	$0.15936^{\{10\}}$	$0.17406^{\{13\}}$
	D_{abs}	$0.01339^{\{4\}}$	$0.01333^{\{3\}}$	$0.01373^{\{6\}}$	$0.01306^{\{2\}}$	$0.01417^{\{9\}}$	$0.01491^{\{10\}}$	$0.01374^{\{7\}}$	$0.01363^{\{5\}}$	$0.01394^{\{8\}}$	$0.01638^{\{13\}}$	$0.01528^{\{11\}}$	$0.01686^{\{14\}}$	$0.01278^{\{1\}}$	$0.01792^{\{15\}}$	$0.0156^{\{12\}}$	$0.01881^{\{16\}}$
	D_{max}	$0.02223^{\{3.5\}}$	$0.02223^{\{3.5\}}$	$0.0233^{\{7\}}$	$0.02181^{\{2\}}$	$0.02401^{\{9\}}$	$0.02515^{\{10\}}$	$0.02292^{\{6\}}$	$0.0228^{\{5\}}$	$0.0239^{\{8\}}$	$0.02692^{\{13\}}$	$0.02523^{\{11\}}$	$0.03049^{\{15\}}$	$0.01977^{\{1\}}$	$0.02942^{\{14\}}$	$0.02574^{\{12\}}$	$0.03074^{\{16\}}$
	ASAE	$0.00655^{\{3.5\}}$	$0.00674^{\{3.5\}}$	$0.00709^{\{7\}}$	$0.00657^{\{2\}}$	$0.00705^{\{9\}}$	$0.0073^{\{10\}}$	$0.00634^{\{6\}}$	$0.00672^{\{5\}}$	$0.00782^{\{8\}}$	$0.00909^{\{13\}}$	$0.00829^{\{11\}}$	$0.01204^{\{15\}}$	$0.00816^{\{1\}}$	$0.01003^{\{14\}}$	$0.00794^{\{12\}}$	$0.01018^{\{16\}}$
	$\sum Ranks$	$25.5^{\{2\}}$	$33.5^{\{4\}}$	$75^{\{8\}}$	$26^{\{3\}}$	$91^{\{11\}}$	$114^{\{13\}}$	$47^{\{5\}}$	$58^{\{6\}}$	$106^{\{12\}}$	$90^{\{10\}}$	$73^{\{7\}}$	$141^{\{16\}}$	$19^{\{1\}}$	$120^{\{14\}}$	$83^{\{9\}}$	$122^{\{15\}}$
450	BIAS($\hat{\delta}$)	$0.09343^{\{3\}}$	$0.09641^{\{4\}}$	$0.10667^{\{10\}}$	$0.08286^{\{2\}}$	$0.1067^{\{11\}}$	$0.11099^{\{13\}}$	$0.09721^{\{5\}}$	$0.09867^{\{7\}}$	$0.11743^{\{15\}}$	$0.09742^{\{6\}}$	$0.10002^{\{8\}}$	$0.16484^{\{16\}}$	$0.04662^{\{1\}}$	$0.11228^{\{14\}}$	$0.10122^{\{9\}}$	$0.1081^{\{12\}}$
	BIAS($\hat{\gamma}$)	$0.12328^{\{3\}}$	$0.12866^{\{5\}}$	$0.14032^{\{8\}}$	$0.12102^{\{2\}}$	$0.1426^{\{9\}}$	$0.17446^{\{15\}}$	$0.13024^{\{6\}}$	$0.12806^{\{4\}}$	$0.15394^{\{13\}}$	$0.13298^{\{7\}}$	$0.1459^{\{11\}}$	$0.25395^{\{16\}}$	$0.05847^{\{1\}}$	$0.16511^{\{14\}}$	$0.14463^{\{10\}}$	$0.15072^{\{12\}}$
	MSE($\hat{\delta}$)	$0.01548^{\{3\}}$	$0.01595^{\{4\}}$	$0.01924^{\{12\}}$	$0.01127^{\{2\}}$	$0.01905^{\{11\}}$	$0.01936^{\{13\}}$	$0.01657^{\{5\}}$	$0.01697^{\{8\}}$	$0.02235^{\{15\}}$	$0.01723^{\{9\}}$	$0.01685^{\{7\}}$	$0.03913^{\{16\}}$	$0.0045^{\{1\}}$	$0.01943^{\{14\}}$	$0.0167^{\{6\}}$	$0.01848^{\{10\}}$
	MSE($\hat{\gamma}$)	$0.02463^{\{2\}}$	$0.02754^{\{5\}}$	$0.03225^{\{7\}}$	$0.0247^{\{3\}}$	$0.03387^{\{9\}}$	$0.05128^{\{15\}}$	$0.02766^{\{6\}}$	$0.02634^{\{4\}}$	$0.03818^{\{13\}}$	$0.03263^{\{8\}}$	$0.03591^{\{11\}}$	$0.1124^{\{16\}}$	$0.00686^{\{1\}}$	$0.04399^{\{14\}}$	$0.03408^{\{10\}}$	$0.03681^{\{12\}}$
	MRE($\hat{\delta}$)	$0.18687^{\{3\}}$	$0.19282^{\{4\}}$	$0.21335^{\{10\}}$	$0.16573^{\{2\}}$	$0.21341^{\{11\}}$	$0.22197^{\{13\}}$	$0.19442^{\{5\}}$	$0.19735^{\{7\}}$	$0.23486^{\{15\}}$	$0.19484^{\{6\}}$	$0.20005^{\{8\}}$	$0.32967^{\{16\}}$	$0.09324^{\{1\}}$	$0.22457^{\{14\}}$	$0.20244^{\{9\}}$	$0.21619^{\{12\}}$
	MRE($\hat{\gamma}$)	$0.12328^{\{3\}}$	$0.12866^{\{5\}}$	$0.14032^{\{8\}}$	$0.12102^{\{2\}}$	$0.1426^{\{9\}}$	$0.17446^{\{15\}}$	$0.13024^{\{6\}}$	$0.12806^{\{4\}}$	$0.15394^{\{13\}}$	$0.13298^{\{7\}}$	$0.1459^{\{11\}}$	$0.25395^{\{16\}}$	$0.05847^{\{1\}}$	$0.16511^{\{14\}}$	$0.14463^{\{10\}}$	$0.15072^{\{12\}}$
	D_{abs}	$0.01217^{\{5\}}$	$0.01189^{\{3.5\}}$	$0.01234^{\{8\}}$	$0.01185^{\{2\}}$	$0.01233^{\{6.5\}}$	$0.01358^{\{11\}}$	$0.01233^{\{6.5\}}$	$0.01189^{\{3.5\}}$	$0.01283^{\{9\}}$	$0.01468^{\{13\}}$	$0.01431^{\{12\}}$	$0.01551^{\{14\}}$	$0.01129^{\{1\}}$	$0.01612^{\{16\}}$	$0.01326^{\{10\}}$	$0.01553^{\{15\}}$
	D_{max}	$0.02022^{\{5\}}$	$0.01993^{\{3.5\}}$	$0.02095^{\{7.5\}}$	$0.01955^{\{2\}}$	$0.02095^{\{7.5\}}$	$0.02287^{\{11\}}$	$0.0205^{\{6\}}$	$0.01993^{\{3.5\}}$	$0.02189^{\{9\}}$	$0.02398^{\{13\}}$	$0.02356^{\{12\}}$	$0.02818^{\{16\}}$	$0.0176^{\{1\}}$	$0.02674^{\{15\}}$	$0.02216^{\{10\}}$	$0.02558^{\{14\}}$
	ASAE	$0.0057^{\{5\}}$	$0.00575^{\{3.5\}}$	$0.00611^{\{7.5\}}$	$0.00558^{\{2\}}$	$0.00604^{\{7.5\}}$	$0.00621^{\{11\}}$	$0.00558^{\{6\}}$	$0.00572^{\{3.5\}}$	$0.00679^{\{9\}}$	$0.00779^{\{13\}}$	$0.00707^{\{12\}}$	$0.01078^{\{16\}}$	$0.00688^{\{1\}}$	$0.00861^{\{15\}}$	$0.00658^{\{10\}}$	$0.00845^{\{14\}}$
	$\sum Ranks$	$30^{\{3\}}$	$39^{\{4\}}$	$77.5^{\{7\}}$	$18^{\{1\}}$	$80^{\{8\}}$	$114^{\{14\}}$	$47.5^{\{6\}}$	$45^{\{5\}}$	$112^{\{12\}}$	$82^{\{9\}}$	$92^{\{11\}}$	$142^{\{16\}}$	$19^{\{2\}}$	$130^{\{15\}}$	$83^{\{10\}}$	$113^{\{13\}}$
600	BIAS($\hat{\delta}$)	$0.08047^{\{3\}}$	$0.0859^{\{6\}}$	$0.09671^{\{12\}}$	$0.07655^{\{2\}}$	$0.09797^{\{13\}}$	$0.09581^{\{11\}}$	$0.08413^{\{4\}}$	$0.08595^{\{7\}}$	$0.10983^{\{15\}}$	$0.08619^{\{8\}}$	$0.08731^{\{9\}}$	$0.16095^{\{16\}}$	$0.04254^{\{1\}}$	$0.09924^{\{14\}}$	$0.08579^{\{5\}}$	$0.09564^{\{10\}}$
	BIAS($\hat{\gamma}$)	$0.10846^{\{2\}}$	$0.11454^{\{6\}}$	$0.12235^{\{8\}}$	$0.1107^{\{3\}}$	$0.12861^{\{11\}}$	$0.15142^{\{15\}}$	$0.11282^{\{5\}}$	$0.11262^{\{4\}}$	$0.13765^{\{12\}}$	$0.11656^{\{7\}}$	$0.12294^{\{9\}}$	$0.24698^{\{16\}}$	$0.05489^{\{1\}}$	$0.1428^{\{14\}}$	$0.12423^{\{10\}}$	$0.13825^{\{13\}}$
	MSE($\hat{\delta}$)	$0.01134^{\{3\}}$	$0.01272^{\{7\}}$	$0.0162^{\{13\}}$	$0.00921^{\{2\}}$	$0.01606^{\{12\}}$	$0.01463^{\{11\}}$	$0.01164^{\{4\}}$	$0.01236^{\{6\}}$	$0.02005^{\{15\}}$	$0.01356^{\{9\}}$	$0.01273^{\{8\}}$	$0.03753^{\{16\}}$	$0.00397^{\{1\}}$	$0.01639^{\{14\}}$	$0.01204^{\{5\}}$	$0.01457^{\{10\}}$
	MSE($\hat{\gamma}$)	$0.01907^{\{2\}}$	$0.02093^{\{6\}}$	$0.02408^{\{7\}}$	$0.0193^{\{3\}}$	$0.02636^{\{11\}}$	$0.04163^{\{15\}}$	$0.0201^{\{5\}}$	$0.02^{\{4\}}$	$0.0288^{\{12\}}$	$0.02425^{\{8\}}$	$0.02477^{\{9\}}$	$0.10458^{\{16\}}$	$0.00611^{\{1\}}$	$0.0336^{\{14\}}$	$0.02525^{\{10\}}$	$0.03247^{\{13\}}$
	MRE($\hat{\delta}$)	$0.16093^{\{3\}}$	$0.17179^{\{6\}}$	$0.19342^{\{12\}}$	$0.1531^{\{2\}}$	$0.19594^{\{13\}}$	$0.19163^{\{11\}}$	$0.16825^{\{4\}}$	$0.17191^{\{7\}}$	$0.21966^{\{15\}}$	$0.17239^{\{8\}}$	$0.17461^{\{9\}}$	$0.3219^{\{16\}}$	$0.08509^{\{1\}}$	$0.19848^{\{14\}}$	$0.17157^{\{5\}}$	$0.19127^{\{10\}}$
	MRE($\hat{\gamma}$)	$0.10846^{\{2\}}$	$0.11454^{\{6\}}$	$0.12235^{\{8\}}$	$0.1107^{\{3\}}$	$0.12861^{\{11\}}$	$0.15142^{\{15\}}$	$0.11282^{\{5\}}$	$0.11262^{\{4\}}$	$0.13765^{\{12\}}$	$0.11656^{\{7\}}$	$0.12294^{\{9\}}$	$0.24698^{\{16\}}$	$0.05489^{\{1\}}$	$0.1428^{\{14\}}$	$0.12423^{\{10\}}$	$0.13825^{\{13\}}$
	D_{abs}	$0.01067^{\{6\}}$	$0.01044^{\{2\}}$	$0.01093^{\{9\}}$	$0.01046^{\{3.5\}}$	$0.0107^{\{7\}}$	$0.01171^{\{11\}}$	$0.01046^{\{3.5\}}$	$0.01047^{\{5\}}$	$0.0109^{\{8\}}$	$0.0128^{\{13\}}$	$0.01153^{\{10\}}$	$0.01376^{\{14\}}$	$0.01035^{\{1\}}$	$0.01447^{\{16\}}$	$0.01178^{\{12\}}$	$0.01394^{\{15\}}$
	D_{max}	$0.01762^{\{6\}}$	$0.01756^{\{5\}}$	$0.01845^{\{8\}}$	$0.0174^{\{2\}}$	$0.01824^{\{7\}}$	$0.01987^{\{12\}}$	$0.01749^{\{3\}}$	$0.01754^{\{4\}}$	$0.01883^{\{9\}}$	$0.02095^{\{13\}}$	$0.01923^{\{10\}}$	$0.02508^{\{16\}}$	$0.01619^{\{1\}}$	$0.02373^{\{15\}}$	$0.01958^{\{11\}}$	$0.02297^{\{14\}}$
	ASAE	$0.00466^{\{6\}}$	$0.0047^{\{5\}}$	$0.00501^{\{8\}}$	$0.00461^{\{2\}}$	$0.00503^{\{7\}}$	$0.00519^{\{12\}}$	$0.00467^{\{3\}}$	$0.00478^{\{4\}}$	$0.00571^{\{9\}}$	$0.00663^{\{13\}}$	$0.00604^{\{10\}}$	$0.00934^{\{16\}}$	$0.00573^{\{1\}}$	$0.00745^{\{15\}}$	$0.0056^{\{11\}}$	$0.00706^{\{14\}}$
	$\sum Ranks$	$29^{\{3\}}$	$48^{\{6\}}$	$83^{\{8\}}$	$21.5^{\{2\}}$	$92^{\{11\}}$	$109^{\{13\}}$	$36.5^{\{4\}}$	$46^{\{5\}}$	$108^{\{12\}}$	$86^{\{10\}}$	$85^{\{9\}}$	$142^{\{16\}}$	$19^{\{1\}}$	$130^{\{15\}}$	$77^{\{7\}}$	$112^{\{14\}}$

Table 8: Numerical values of simulation measures for $\delta = 1.0,\ \gamma = 0.5$.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	PCE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
25	BIAS($\hat{\delta}$)	$0.35862^{\{4\}}$	$0.37589^{\{8\}}$	$0.36236^{\{5\}}$	$0.39476^{\{12\}}$	$0.3721^{\{6\}}$	$0.38897^{\{10\}}$	$0.37713^{\{9\}}$	$0.39382^{\{11\}}$	$0.39583^{\{13\}}$	$0.18017^{\{2\}}$	$0.33746^{\{3\}}$	$0.42717^{\{16\}}$	$0.02524^{\{1\}}$	$0.40217^{\{14\}}$	$0.37215^{\{7\}}$	$0.42043^{\{15\}}$
	BIAS($\hat{\gamma}$)	$0.15534^{\{4\}}$	$0.17321^{\{7\}}$	$0.17708^{\{9\}}$	$0.18433^{\{12\}}$	$0.18678^{\{13\}}$	$0.1774^{\{10\}}$	$0.15865^{\{5\}}$	$0.17973^{\{11\}}$	$0.17675^{\{8\}}$	$0.09838^{\{2\}}$	$0.14725^{\{3\}}$	$0.19504^{\{14\}}$	$0.05351^{\{1\}}$	$0.19675^{\{15\}}$	$0.16646^{\{6\}}$	$0.21929^{\{16\}}$
	MSE($\hat{\delta}$)	$0.17643^{\{6\}}$	$0.18951^{\{8\}}$	$0.17135^{\{4\}}$	$0.20168^{\{11\}}$	$0.17633^{\{5\}}$	$0.19775^{\{10\}}$	$0.19362^{\{9\}}$	$0.20187^{\{12\}}$	$0.20443^{\{14\}}$	$0.08489^{\{2\}}$	$0.16987^{\{3\}}$	$0.236^{\{16\}}$	$0.00237^{\{1\}}$	$0.20361^{\{13\}}$	$0.18589^{\{7\}}$	$0.21819^{\{15\}}$
	MSE($\hat{\gamma}$)	$0.03997^{\{3\}}$	$0.04852^{\{7\}}$	$0.05156^{\{11\}}$	$0.05333^{\{12\}}$	$0.05412^{\{13\}}$	$0.04895^{\{8\}}$	$0.04109^{\{5\}}$	$0.05028^{\{10\}}$	$0.04933^{\{9\}}$	$0.02048^{\{2\}}$	$0.03999^{\{4\}}$	$0.05803^{\{14\}}$	$0.005^{\{1\}}$	$0.05814^{\{15\}}$	$0.04419^{\{6\}}$	$0.06594^{\{16\}}$
	MRE($\hat{\delta}$)	$0.35862^{\{4\}}$	$0.37589^{\{8\}}$	$0.36236^{\{5\}}$	$0.39476^{\{12\}}$	$0.3721^{\{6\}}$	$0.38897^{\{10\}}$	$0.37713^{\{9\}}$	$0.39382^{\{11\}}$	$0.39583^{\{13\}}$	$0.18017^{\{2\}}$	$0.33746^{\{3\}}$	$0.42717^{\{16\}}$	$0.02524^{\{1\}}$	$0.40217^{\{14\}}$	$0.37215^{\{7\}}$	$0.42043^{\{15\}}$
	MRE($\hat{\gamma}$)	$0.31068^{\{4\}}$	$0.34641^{\{7\}}$	$0.35416^{\{9\}}$	$0.36865^{\{12\}}$	$0.37355^{\{13\}}$	$0.3548^{\{10\}}$	$0.3173^{\{5\}}$	$0.35947^{\{11\}}$	$0.35349^{\{8\}}$	$0.19676^{\{2\}}$	$0.29449^{\{3\}}$	$0.39007^{\{14\}}$	$0.10702^{\{1\}}$	$0.39351^{\{15\}}$	$0.33291^{\{6\}}$	$0.43858^{\{16\}}$
	D_{abs}	$0.0429^{\{7\}}$	$0.04248^{\{5\}}$	$0.04241^{\{3.5\}}$	$0.04276^{\{6\}}$	$0.04222^{\{2\}}$	$0.04536^{\{10\}}$	$0.04241^{\{3.5\}}$	$0.04145^{\{1\}}$	$0.04383^{\{8\}}$	$0.05433^{\{15\}}$	$0.04839^{\{13\}}$	$0.04479^{\{9\}}$	$0.04676^{\{12\}}$	$0.05361^{\{14\}}$	$0.04583^{\{11\}}$	$0.05922^{\{16\}}$
	D_{max}	$0.06619^{\{6\}}$	$0.06589^{\{5\}}$	$0.06675^{\{7\}}$	$0.06546^{\{3\}}$	$0.06557^{\{4\}}$	$0.06967^{\{11\}}$	$0.06543^{\{2\}}$	$0.06466^{\{1\}}$	$0.0687^{\{9\}}$	$0.08024^{\{14\}}$	$0.07259^{\{13\}}$	$0.07139^{\{12\}}$	$0.06837^{\{8\}}$	$0.08273^{\{15\}}$	$0.06933^{\{10\}}$	$0.09073^{\{16\}}$
	ASAE	$0.03632^{\{6\}}$	$0.03496^{\{5\}}$	$0.03608^{\{7\}}$	$0.0336^{\{3\}}$	$0.0354^{\{4\}}$	$0.03495^{\{11\}}$	$0.03457^{\{2\}}$	$0.03397^{\{1\}}$	$0.0379^{\{9\}}$	$0.04974^{\{14\}}$	$0.04333^{\{13\}}$	$0.04479^{\{12\}}$	$0.0479^{\{8\}}$	$0.0518^{\{15\}}$	$0.03916^{\{10\}}$	$0.0514^{\{16\}}$
	$\sum Ranks$	$46^{\{2\}}$	$60^{\{6\}}$	$60.5^{\{7\}}$	$81^{\{11\}}$	$68^{\{8\}}$	$83^{\{12\}}$	$50.5^{\{3\}}$	$70^{\{9.5\}}$	$91^{\{13\}}$	$55^{\{4\}}$	$56^{\{5\}}$	$123^{\{14\}}$	$39^{\{1\}}$	$131^{\{15\}}$	$70^{\{9.5\}}$	$140^{\{16\}}$
100	BIAS($\hat{\delta}$)	$0.32372^{\{4\}}$	$0.34391^{\{8\}}$	$0.3535^{\{13\}}$	$0.34846^{\{12\}}$	$0.35786^{\{14\}}$	$0.34664^{\{11\}}$	$0.34403^{\{9\}}$	$0.34406^{\{10\}}$	$0.3615^{\{15\}}$	$0.18^{\{2\}}$	$0.30876^{\{3\}}$	$0.39401^{\{16\}}$	$0.02039^{\{1\}}$	$0.33126^{\{6\}}$	$0.33611^{\{7\}}$	$0.33096^{\{5\}}$
	BIAS($\hat{\gamma}$)	$0.12022^{\{3\}}$	$0.13484^{\{6\}}$	$0.14683^{\{10\}}$	$0.14602^{\{9\}}$	$0.14904^{\{11\}}$	$0.1495^{\{12\}}$	$0.12808^{\{5\}}$	$0.13571^{\{7\}}$	$0.15056^{\{13\}}$	$0.08249^{\{2\}}$	$0.12731^{\{4\}}$	$0.17992^{\{16\}}$	$0.02704^{\{1\}}$	$0.1674^{\{15\}}$	$0.13752^{\{8\}}$	$0.1627^{\{14\}}$
	MSE($\hat{\delta}$)	$0.15438^{\{6\}}$	$0.16615^{\{10\}}$	$0.16934^{\{13\}}$	$0.16662^{\{11\}}$	$0.17323^{\{14\}}$	$0.16235^{\{8\}}$	$0.16756^{\{12\}}$	$0.16576^{\{9\}}$	$0.17917^{\{15\}}$	$0.07729^{\{2\}}$	$0.13977^{\{3\}}$	$0.20322^{\{16\}}$	$0.00171^{\{1\}}$	$0.14608^{\{5\}}$	$0.15958^{\{7\}}$	$0.14538^{\{4\}}$
	MSE($\hat{\gamma}$)	$0.02496^{\{3\}}$	$0.03053^{\{6\}}$	$0.0349^{\{9\}}$	$0.03549^{\{10\}}$	$0.03696^{\{11\}}$	$0.03712^{\{12\}}$	$0.02721^{\{4\}}$	$0.03099^{\{7\}}$	$0.03767^{\{13\}}$	$0.01655^{\{2\}}$	$0.02918^{\{5\}}$	$0.05209^{\{16\}}$	$0.00131^{\{1\}}$	$0.04662^{\{15\}}$	$0.03229^{\{8\}}$	$0.04484^{\{14\}}$
	MRE($\hat{\delta}$)	$0.32372^{\{4\}}$	$0.34391^{\{8\}}$	$0.3535^{\{13\}}$	$0.34846^{\{12\}}$	$0.35786^{\{14\}}$	$0.34664^{\{11\}}$	$0.34403^{\{9\}}$	$0.34406^{\{10\}}$	$0.3615^{\{15\}}$	$0.18^{\{2\}}$	$0.30876^{\{3\}}$	$0.39401^{\{16\}}$	$0.02039^{\{1\}}$	$0.33126^{\{6\}}$	$0.33611^{\{7\}}$	$0.33096^{\{5\}}$
	MRE($\hat{\gamma}$)	$0.24044^{\{3\}}$	$0.26969^{\{6\}}$	$0.29367^{\{10\}}$	$0.29203^{\{9\}}$	$0.29808^{\{11\}}$	$0.29899^{\{12\}}$	$0.25617^{\{5\}}$	$0.27143^{\{7\}}$	$0.30113^{\{13\}}$	$0.16499^{\{2\}}$	$0.25462^{\{4\}}$	$0.35984^{\{16\}}$	$0.05409^{\{1\}}$	$0.3348^{\{15\}}$	$0.27503^{\{8\}}$	$0.32541^{\{14\}}$
	D_{abs}	$0.02387^{\{6\}}$	$0.02248^{\{1\}}$	$0.02442^{\{9\}}$	$0.02322^{\{2\}}$	$0.02403^{\{8\}}$	$0.02477^{\{10\}}$	$0.02392^{\{7\}}$	$0.02344^{\{4\}}$	$0.02374^{\{5\}}$	$0.02833^{\{14\}}$	$0.02668^{\{13\}}$	$0.02649^{\{12\}}$	$0.02328^{\{3\}}$	$0.02951^{\{15\}}$	$0.02607^{\{11\}}$	$0.02982^{\{16\}}$
	D_{max}	$0.03814^{\{5\}}$	$0.03662^{\{2\}}$	$0.03985^{\{9\}}$	$0.03788^{\{3\}}$	$0.03936^{\{8\}}$	$0.04009^{\{10\}}$	$0.03837^{\{6\}}$	$0.03805^{\{4\}}$	$0.03914^{\{7\}}$	$0.04317^{\{13\}}$	$0.04238^{\{12\}}$	$0.04459^{\{14\}}$	$0.03403^{\{1\}}$	$0.048^{\{15\}}$	$0.0415^{\{11\}}$	$0.04822^{\{16\}}$
	ASAE	$0.01485^{\{5\}}$	$0.01534^{\{2\}}$	$0.01554^{\{9\}}$	$0.01474^{\{3\}}$	$0.01545^{\{8\}}$	$0.01513^{\{10\}}$	$0.0146^{\{6\}}$	$0.01482^{\{4\}}$	$0.01642^{\{7\}}$	$0.02048^{\{13\}}$	$0.0184^{\{12\}}$	$0.02054^{\{14\}}$	$0.01903^{\{1\}}$	$0.02209^{\{15\}}$	$0.01696^{\{11\}}$	$0.02201^{\{16\}}$
	$\sum Ranks$	$38^{\{2\}}$	$53^{\{4\}}$	$94^{\{11\}}$	$70^{\{8\}}$	$98^{\{12\}}$	$91^{\{10\}}$	$58^{\{5.5\}}$	$61^{\{7\}}$	$105^{\{14\}}$	$52^{\{3\}}$	$58^{\{5.5\}}$	$136^{\{16\}}$	$22^{\{1\}}$	$108^{\{15\}}$	$77^{\{9\}}$	$103^{\{13\}}$
200	BIAS($\hat{\delta}$)	$0.2915^{\{7\}}$	$0.30351^{\{10\}}$	$0.33935^{\{14\}}$	$0.28692^{\{6\}}$	$0.31626^{\{13\}}$	$0.30904^{\{12\}}$	$0.29841^{\{8\}}$	$0.30061^{\{9\}}$	$0.34198^{\{15\}}$	$0.17492^{\{2\}}$	$0.28385^{\{5\}}$	$0.38899^{\{16\}}$	$0.01695^{\{1\}}$	$0.28326^{\{4\}}$	$0.3062^{\{11\}}$	$0.27228^{\{3\}}$
	BIAS($\hat{\gamma}$)	$0.09366^{\{3\}}$	$0.10475^{\{5\}}$	$0.11501^{\{10\}}$	$0.10713^{\{7\}}$	$0.1183^{\{11\}}$	$0.12323^{\{13\}}$	$0.10244^{\{4\}}$	$0.10495^{\{6\}}$	$0.12832^{\{14\}}$	$0.07515^{\{2\}}$	$0.10874^{\{8\}}$	$0.16267^{\{16\}}$	$0.02024^{\{1\}}$	$0.13319^{\{15\}}$	$0.11319^{\{9\}}$	$0.12139^{\{12\}}$
	MSE($\hat{\delta}$)	$0.13401^{\{7\}}$	$0.13796^{\{11\}}$	$0.16685^{\{15\}}$	$0.12043^{\{5\}}$	$0.14506^{\{13\}}$	$0.13691^{\{10\}}$	$0.13528^{\{8\}}$	$0.13595^{\{9\}}$	$0.16605^{\{14\}}$	$0.07295^{\{2\}}$	$0.12279^{\{6\}}$	$0.19691^{\{16\}}$	$0.00138^{\{1\}}$	$0.11097^{\{4\}}$	$0.13959^{\{12\}}$	$0.10593^{\{3\}}$
	MSE($\hat{\gamma}$)	$0.01461^{\{3\}}$	$0.01841^{\{5\}}$	$0.02161^{\{8\}}$	$0.01886^{\{6\}}$	$0.02369^{\{11\}}$	$0.0257^{\{12\}}$	$0.0181^{\{4\}}$	$0.01909^{\{7\}}$	$0.0279^{\{13\}}$	$0.01382^{\{2\}}$	$0.02167^{\{9\}}$	$0.04215^{\{16\}}$	$0.00077^{\{1\}}$	$0.03197^{\{15\}}$	$0.02181^{\{10\}}$	$0.02809^{\{14\}}$
	MRE($\hat{\delta}$)	$0.2915^{\{7\}}$	$0.30351^{\{10\}}$	$0.33935^{\{14\}}$	$0.28692^{\{6\}}$	$0.31626^{\{13\}}$	$0.30904^{\{12\}}$	$0.29841^{\{8\}}$	$0.30061^{\{9\}}$	$0.34198^{\{15\}}$	$0.17492^{\{2\}}$	$0.28385^{\{5\}}$	$0.38899^{\{16\}}$	$0.01695^{\{1\}}$	$0.28326^{\{4\}}$	$0.3062^{\{11\}}$	$0.27228^{\{3\}}$
	MRE($\hat{\gamma}$)	$0.18732^{\{3\}}$	$0.20949^{\{5\}}$	$0.23002^{\{10\}}$	$0.21427^{\{7\}}$	$0.23659^{\{11\}}$	$0.24646^{\{13\}}$	$0.20487^{\{4\}}$	$0.2099^{\{6\}}$	$0.25664^{\{14\}}$	$0.1503^{\{2\}}$	$0.21747^{\{8\}}$	$0.32535^{\{16\}}$	$0.04047^{\{1\}}$	$0.26638^{\{15\}}$	$0.22637^{\{9\}}$	$0.24278^{\{12\}}$
	D_{abs}	$0.01739^{\{4\}}$	$0.0166^{\{1\}}$	$0.01766^{\{9\}}$	$0.01763^{\{8\}}$	$0.01754^{\{7\}}$	$0.01836^{\{10\}}$	$0.01744^{\{6\}}$	$0.01706^{\{3\}}$	$0.01741^{\{5\}}$	$0.02121^{\{14\}}$	$0.01926^{\{12\}}$	$0.01947^{\{13\}}$	$0.01694^{\{2\}}$	$0.0222^{\{15\}}$	$0.01889^{\{11\}}$	$0.02306^{\{16\}}$
	D_{max}	$0.02813^{\{4\}}$	$0.02738^{\{2\}}$	$0.02936^{\{9\}}$	$0.02876^{\{6\}}$	$0.02913^{\{7\}}$	$0.03011^{\{10\}}$	$0.02843^{\{5\}}$	$0.02794^{\{3\}}$	$0.02928^{\{8\}}$	$0.03279^{\{13\}}$	$0.03123^{\{12\}}$	$0.03414^{\{14\}}$	$0.02486^{\{1\}}$	$0.03622^{\{15\}}$	$0.0308^{\{11\}}$	$0.0371^{\{16\}}$
	ASAE	$0.00964^{\{4\}}$	$0.00966^{\{2\}}$	$0.01009^{\{9\}}$	$0.00947^{\{6\}}$	$0.01034^{\{7\}}$	$0.00994^{\{10\}}$	$0.00954^{\{5\}}$	$0.00955^{\{3\}}$	$0.01115^{\{8\}}$	$0.01299^{\{13\}}$	$0.01203^{\{12\}}$	$0.01497^{\{14\}}$	$0.0122^{\{1\}}$	$0.01476^{\{15\}}$	$0.01133^{\{11\}}$	$0.01479^{\{16\}}$
	$\sum Ranks$	$42^{\{2\}}$	$54^{\{6\}}$	$96^{\{12\}}$	$52^{\{4.5\}}$	$94^{\{10\}}$	$98^{\{13\}}$	$49^{\{3\}}$	$55^{\{7\}}$	$107^{\{15\}}$	$52^{\{4.5\}}$	$76^{\{8\}}$	$139^{\{16\}}$	$21^{\{1\}}$	$101^{\{14\}}$	$94^{\{10\}}$	$94^{\{10\}}$
350	BIAS($\hat{\delta}$)	$0.2451^{\{5\}}$	$0.26856^{\{12\}}$	$0.29395^{\{14\}}$	$0.24638^{\{6\}}$	$0.28126^{\{13\}}$	$0.26547^{\{11\}}$	$0.25722^{\{8\}}$	$0.25933^{\{9\}}$	$0.30792^{\{15\}}$	$0.17294^{\{2\}}$	$0.24766^{\{7\}}$	$0.38289^{\{16\}}$	$0.01566^{\{1\}}$	$0.2323^{\{3\}}$	$0.26377^{\{10\}}$	$0.23913^{\{4\}}$
	BIAS($\hat{\gamma}$)	$0.07431^{\{3\}}$	$0.08374^{\{7\}}$	$0.09389^{\{11\}}$	$0.08261^{\{6\}}$	$0.08954^{\{10\}}$	$0.09659^{\{12\}}$	$0.0803^{\{4\}}$	$0.08166^{\{5\}}$	$0.10405^{\{15\}}$	$0.06653^{\{2\}}$	$0.08696^{\{8\}}$	$0.15894^{\{16\}}$	$0.01631^{\{1\}}$	$0.09913^{\{13\}}$	$0.08824^{\{9\}}$	$0.10031^{\{14\}}$
	MSE($\hat{\delta}$)	$0.09911^{\{7\}}$	$0.11944^{\{12\}}$	$0.13187^{\{14\}}$	$0.09609^{\{5\}}$	$0.12146^{\{13\}}$	$0.10464^{\{8\}}$	$0.11036^{\{11\}}$	$0.10581^{\{9\}}$	$0.14389^{\{15\}}$	$0.06745^{\{2\}}$	$0.09643^{\{6\}}$	$0.19261^{\{16\}}$	$0.00115^{\{1\}}$	$0.07882^{\{3\}}$	$0.10793^{\{10\}}$	$0.08626^{\{4\}}$
	MSE($\hat{\gamma}$)	$0.00876^{\{2\}}$	$0.01163^{\{7\}}$	$0.01433^{\{11\}}$	$0.01125^{\{6\}}$	$0.01309^{\{9\}}$	$0.01555^{\{12\}}$	$0.01061^{\{5\}}$	$0.01047^{\{4\}}$	$0.01849^{\{13\}}$	$0.01034^{\{3\}}$	$0.01318^{\{10\}}$	$0.04087^{\{16\}}$	$0.00054^{\{1\}}$	$0.01874^{\{14\}}$	$0.013^{\{8\}}$	$0.01897^{\{15\}}$
	MRE($\hat{\delta}$)	$0.2451^{\{5\}}$	$0.26856^{\{12\}}$	$0.29395^{\{14\}}$	$0.24638^{\{6\}}$	$0.28126^{\{13\}}$	$0.26547^{\{11\}}$	$0.25722^{\{8\}}$	$0.25933^{\{9\}}$	$0.30792^{\{15\}}$	$0.17294^{\{2\}}$	$0.24766^{\{7\}}$	$0.38289^{\{16\}}$	$0.01566^{\{1\}}$	$0.2323^{\{3\}}$	$0.26377^{\{10\}}$	$0.23913^{\{4\}}$
	MRE($\hat{\gamma}$)	$0.14861^{\{3\}}$	$0.16749^{\{7\}}$	$0.18778^{\{11\}}$	$0.16522^{\{6\}}$	$0.17908^{\{10\}}$	$0.19317^{\{12\}}$	$0.1606^{\{4\}}$	$0.16332^{\{5\}}$	$0.20811^{\{15\}}$	$0.13306^{\{2\}}$	$0.17391^{\{8\}}$	$0.31788^{\{16\}}$	$0.03262^{\{1\}}$	$0.19827^{\{13\}}$	$0.17649^{\{9\}}$	$0.20061^{\{14\}}$
	D_{abs}	$0.0129^{\{1\}}$	$0.01346^{\{6\}}$	$0.01354^{\{8\}}$	$0.01333^{\{4\}}$	$0.01348^{\{7\}}$	$0.01376^{\{9\}}$	$0.01339^{\{5\}}$	$0.01301^{\{2\}}$	$0.01426^{\{10\}}$	$0.01556^{\{13\}}$	$0.01538^{\{12\}}$	$0.01593^{\{14\}}$	$0.0133^{\{3\}}$	$0.01799^{\{15\}}$	$0.01498^{\{11\}}$	$0.01894^{\{16\}}$
	D_{max}	$0.02104^{\{2\}}$	$0.02212^{\{6\}}$	$0.02282^{\{9\}}$	$0.02184^{\{4\}}$	$0.02253^{\{7\}}$	$0.02273^{\{8\}}$	$0.02189^{\{5\}}$	$0.02155^{\{3\}}$	$0.02417^{\{10\}}$	$0.02444^{\{12\}}$	$0.02489^{\{13\}}$	$0.02898^{\{15\}}$	$0.01955^{\{1\}}$	$0.02888^{\{14\}}$	$0.02436^{\{11\}}$	$0.03026^{\{16\}}$
	ASAE	$0.00683^{\{2\}}$	$0.00688^{\{6\}}$	$0.00734^{\{9\}}$	$0.00679^{\{4\}}$	$0.00717^{\{7\}}$	$0.00738^{\{8\}}$	$0.00665^{\{5\}}$	$0.00672^{\{3\}}$	$0.00798^{\{10\}}$	$0.00941^{\{12\}}$	$0.00867^{\{13\}}$	$0.0125^{\{15\}}$	$0.00879^{\{1\}}$	$0.01064^{\{14\}}$	$0.00807^{\{11\}}$	$0.01074^{\{16\}}$
	$\sum Ranks$	$32^{\{2\}}$	$74^{\{7\}}$	$99^{\{13\}}$	$46^{\{3\}}$	$88^{\{9.5\}}$	$91^{\{11\}}$	$51^{\{5.5\}}$	$48^{\{4\}}$	$117^{\{15\}}$	$51^{\{5.5\}}$	$82^{\{8\}}$	$141^{\{16\}}$	$22^{\{1\}}$	$92^{\{12\}}$	$88^{\{9.5\}}$	$102^{\{14\}}$
450	BIAS($\hat{\delta}$)	$0.22709^{\{7\}}$	$0.25229^{\{12\}}$	$0.28075^{\{14\}}$	$0.20975^{\{4\}}$	$0.2722^{\{13\}}$	$0.24296^{\{10\}}$	$0.24154^{\{9\}}$	$0.24596^{\{11\}}$	$0.30011^{\{15\}}$	$0.17029^{\{2\}}$	$0.22316^{\{6\}}$	$0.37094^{\{16\}}$	$0.01554^{\{1\}}$	$0.21309^{\{5\}}$	$0.23256^{\{8\}}$	$0.20766^{\{3\}}$
	BIAS($\hat{\gamma}$)	$0.06766^{\{3\}}$	$0.0777^{\{9\}}$	$0.08374^{\{12\}}$	$0.07131^{\{4\}}$	$0.08712^{\{13\}}$	$0.0822^{\{10\}}$	$0.07413^{\{5\}}$	$0.07518^{\{7\}}$	$0.09848^{\{15\}}$	$0.06482^{\{2\}}$	$0.07457^{\{6\}}$	$0.15113^{\{16\}}$	$0.01429^{\{1\}}$	$0.08344^{\{11\}}$	$0.07659^{\{8\}}$	$0.0876^{\{14\}}$
	MSE($\hat{\delta}$)	$0.08808^{\{7\}}$	$0.10095^{\{11\}}$	$0.12558^{\{14\}}$	$0.06951^{\{4\}}$	$0.11395^{\{13\}}$	$0.094^{\{9\}}$	$0.09512^{\{10\}}$	$0.10183^{\{12\}}$	$0.13769^{\{15\}}$	$0.06348^{\{2\}}$	$0.0814^{\{6\}}$	$0.18584^{\{16\}}$	$0.00106^{\{1\}}$	$0.06853^{\{3\}}$	$0.08864^{\{8\}}$	$0.07083^{\{5\}}$
	MSE($\hat{\gamma}$)	$0.0072^{\{2\}}$	$0.00962^{\{8\}}$	$0.01092^{\{10\}}$	$0.00834^{\{3\}}$	$0.01189^{\{12\}}$	$0.01105^{\{11\}}$	$0.00849^{\{4\}}$	$0.0091^{\{5\}}$	$0.01613^{\{15\}}$	$0.00931^{\{6\}}$	$0.0094^{\{7\}}$	$0.03791^{\{16\}}$	$0.00041^{\{1\}}$	$0.01383^{\{13\}}$	$0.00979^{\{9\}}$	$0.01517^{\{14\}}$
	MRE($\hat{\delta}$)	$0.22709^{\{7\}}$	$0.25229^{\{12\}}$	$0.28075^{\{14\}}$	$0.20975^{\{4\}}$	$0.2722^{\{13\}}$	$0.24296^{\{10\}}$	$0.24154^{\{9\}}$	$0.24596^{\{11\}}$	$0.30011^{\{15\}}$	$0.17029^{\{2\}}$	$0.22316^{\{6\}}$	$0.37094^{\{16\}}$	$0.01554^{\{1\}}$	$0.21309^{\{5\}}$	$0.23256^{\{8\}}$	$0.20766^{\{3\}}$
	MRE($\hat{\gamma}$)	$0.13531^{\{3\}}$	$0.15541^{\{9\}}$	$0.16747^{\{12\}}$	$0.14262^{\{4\}}$	$0.17423^{\{13\}}$	$0.1644^{\{10\}}$	$0.14827^{\{5\}}$	$0.15035^{\{7\}}$	$0.19697^{\{15\}}$	$0.12964^{\{2\}}$	$0.14913^{\{6\}}$	$0.30225^{\{16\}}$	$0.02859^{\{1\}}$	$0.16689^{\{11\}}$	$0.15319^{\{8\}}$	$0.1752^{\{14\}}$
	D_{abs}	$0.01136^{\{2\}}$	$0.01167^{\{4\}}$	$0.01166^{\{3\}}$	$0.01179^{\{5\}}$	$0.01241^{\{10\}}$	$0.01224^{\{9\}}$	$0.01197^{\{6\}}$	$0.01213^{\{7\}}$	$0.01222^{\{8\}}$	$0.0141^{\{13\}}$	$0.01368^{\{12\}}$	$0.01418^{\{14\}}$	$0.01131^{\{1\}}$	$0.01604^{\{15\}}$	$0.01286^{\{11\}}$	$0.01682^{\{16\}}$
	D_{max}	$0.01866^{\{2\}}$	$0.01939^{\{4\}}$	$0.01967^{\{5\}}$	$0.0193^{\{3\}}$	$0.02086^{\{9\}}$	$0.02031^{\{8\}}$	$0.01971^{\{6\}}$	$0.02005^{\{7\}}$	$0.02102^{\{10\}}$	$0.02229^{\{13\}}$	$0.0222^{\{12\}}$	$0.02595^{\{15\}}$	$0.01662^{\{1\}}$	$0.02576^{\{14\}}$	$0.02104^{\{11\}}$	$0.02681^{\{16\}}$
	ASAE	$0.0058^{\{2\}}$	$0.00588^{\{4\}}$	$0.00625^{\{5\}}$	$0.00576^{\{3\}}$	$0.00636^{\{9\}}$	$0.00625^{\{8\}}$	$0.00572^{\{6\}}$	$0.00589^{\{7\}}$	$0.00702^{\{10\}}$	$0.0079^{\{13\}}$	$0.00744^{\{12\}}$	$0.01072^{\{15\}}$	$0.00739^{\{1\}}$	$0.00886^{\{14\}}$	$0.00694^{\{11\}}$	$0.00913^{\{16\}}$
	$\sum Ranks$	$36^{\{3\}}$	$73^{\{7.5\}}$	$90^{\{11\}}$	$33^{\{2\}}$	$104^{\{14\}}$	$84^{\{10\}}$	$55^{\{4.5\}}$	$72^{\{6\}}$	$118^{\{15\}}$	$55^{\{4.5\}}$	$73^{\{7.5\}}$	$141^{\{16\}}$	$19^{\{1\}}$	$91^{\{12\}}$	$80^{\{9\}}$	$100^{\{13\}}$
600	BIAS($\hat{\delta}$)	$0.20223^{\{6\}}$	$0.22351^{\{12\}}$	$0.26086^{\{14\}}$	$0.19481^{\{5\}}$	$0.24021^{\{13\}}$	$0.21228^{\{10\}}$	$0.21027^{\{9\}}$	$0.21509^{\{11\}}$	$0.27664^{\{15\}}$	$0.15549^{\{2\}}$	$0.20678^{\{7\}}$	$0.35239^{\{16\}}$	$0.0134^{\{1\}}$	$0.19321^{\{4\}}$	$0.2073^{\{8\}}$	$0.17133^{\{3\}}$
	BIAS($\hat{\gamma}$)	$0.0596^{\{3\}}$	$0.06722^{\{9\}}$	$0.07755^{\{14\}}$	$0.06344^{\{5\}}$	$0.07415^{\{12\}}$	$0.07524^{\{13\}}$	$0.06326^{\{4\}}$	$0.06461^{\{7\}}$	$0.08895^{\{15\}}$	$0.05463^{\{2\}}$	$0.06635^{\{8\}}$	$0.13744^{\{16\}}$	$0.01181^{\{1\}}$	$0.07243^{\{11\}}$	$0.06409^{\{6\}}$	$0.06931^{\{10\}}$
	MSE($\hat{\delta}$)	$0.07114^{\{9\}}$	$0.08258^{\{12\}}$	$0.10818^{\{14\}}$	$0.06296^{\{5\}}$	$0.0962^{\{13\}}$	$0.07082^{\{7\}}$	$0.07475^{\{10\}}$	$0.07959^{\{11\}}$	$0.1176^{\{15\}}$	$0.05057^{\{2\}}$	$0.07052^{\{6\}}$	$0.17171^{\{16\}}$	$0.00068^{\{1\}}$	$0.06006^{\{4\}}$	$0.07112^{\{8\}}$	$0.05089^{\{3\}}$
	MSE($\hat{\gamma}$)	$0.00565^{\{2\}}$	$0.00712^{\{8\}}$	$0.00922^{\{11\}}$	$0.00659^{\{7\}}$	$0.00883^{\{10\}}$	$0.00937^{\{12\}}$	$0.00642^{\{4\}}$	$0.00647^{\{5\}}$	$0.01281^{\{15\}}$	$0.00614^{\{3\}}$	$0.00718^{\{9\}}$	$0.03226^{\{16\}}$	$0.00026^{\{1\}}$	$0.0108^{\{14\}}$	$0.0065^{\{6\}}$	$0.00957^{\{13\}}$
	MRE($\hat{\delta}$)	$0.20223^{\{6\}}$	$0.22351^{\{12\}}$	$0.26086^{\{14\}}$	$0.19481^{\{5\}}$	$0.24021^{\{13\}}$	$0.21228^{\{10\}}$	$0.21027^{\{9\}}$	$0.21509^{\{11\}}$	$0.27664^{\{15\}}$	$0.15549^{\{2\}}$	$0.20678^{\{7\}}$	$0.35239^{\{16\}}$	$0.0134^{\{1\}}$	$0.19321^{\{4\}}$	$0.2073^{\{8\}}$	$0.17133^{\{3\}}$
	MRE($\hat{\gamma}$)	$0.11921^{\{3\}}$	$0.13445^{\{9\}}$	$0.15509^{\{14\}}$	$0.12688^{\{5\}}$	$0.14829^{\{12\}}$	$0.15049^{\{13\}}$	$0.12651^{\{4\}}$	$0.12922^{\{7\}}$	$0.17791^{\{15\}}$	$0.10926^{\{2\}}$	$0.1327^{\{8\}}$	$0.27487^{\{16\}}$	$0.02363^{\{1\}}$	$0.14485^{\{11\}}$	$0.12819^{\{6\}}$	$0.13862^{\{10\}}$
	D_{abs}	$0.00972^{\{2\}}$	$0.01063^{\{8\}}$	$0.01035^{\{4\}}$	$0.0103^{\{3\}}$	$0.01057^{\{7\}}$	$0.01074^{\{9\}}$	$0.01036^{\{5\}}$	$0.01052^{\{6\}}$	$0.01094^{\{11\}}$	$0.0122^{\{13\}}$	$0.0117^{\{12\}}$	$0.01306^{\{14\}}$	$0.00958^{\{1\}}$	$0.0142^{\{16\}}$	$0.01086^{\{10\}}$	$0.01419^{\{15\}}$
	D_{max}	$0.016^{\{2\}}$	$0.01761^{\{7\}}$	$0.01753^{\{6\}}$	$0.0169^{\{3\}}$	$0.01779^{\{9\}}$	$0.01772^{\{8\}}$	$0.01704^{\{4\}}$	$0.01732^{\{5\}}$	$0.0188^{\{11\}}$	$0.0193^{\{13\}}$	$0.01898^{\{12\}}$	$0.02384^{\{16\}}$	$0.01408^{\{1\}}$	$0.02269^{\{15\}}$	$0.01783^{\{10\}}$	$0.02252^{\{14\}}$
	ASAE	$0.0049^{\{2\}}$	$0.00492^{\{7\}}$	$0.00528^{\{6\}}$	$0.00491^{\{3\}}$	$0.00527^{\{9\}}$	$0.00528^{\{8\}}$	$0.0048^{\{4\}}$	$0.00491^{\{5\}}$	$0.00604^{\{11\}}$	$0.00673^{\{13\}}$	$0.00612^{\{12\}}$	$0.00948^{\{16\}}$	$0.0062^{\{1\}}$	$0.00776^{\{15\}}$	$0.00579^{\{10\}}$	$0.00754^{\{14\}}$
	$\sum Ranks$	$35^{\{2\}}$	$82^{\{9\}}$	$98^{\{14\}}$	$41^{\{3\}}$	$95^{\{13\}}$	$90^{\{11\}}$	$50^{\{4\}}$	$67^{\{6\}}$	$122^{\{15\}}$	$52^{\{5\}}$	$80^{\{8\}}$	$142^{\{16\}}$	$20^{\{1\}}$	$94^{\{12\}}$	$71^{\{7\}}$	$85^{\{10\}}$

Table 9: Partial and overall ranks for all estimation methods of our proposed model.

Parameter	n	MLE	ADE	CVME	MPSE	OLSE	PCE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
$\delta = 0.25,\ \gamma = 0.75$	25	10.0	3.0	6.0	13.0	7.0	8.0	2.0	11.0	9.0	4.0	5.0	14.0	1.0	16.0	12.0	15.0
	100	2.0	3.0	13.0	4.0	11.0	12.0	9.0	8.0	10.0	5.0	6.0	15.0	1.0	16.0	7.0	14.0
	200	2.0	6.0	10.0	3.0	11.0	14.0	4.0	7.0	12.0	5.0	8.0	16.0	1.0	13.0	9.0	15.0
	350	2.0	4.0	10.0	3.0	11.0	14.0	5.5	8.0	12.0	7.0	5.5	16.0	1.0	15.0	9.0	13.0
	450	2.0	4.0	11.0	3.0	10.0	14.0	5.0	8.0	12.0	9.0	7.0	16.0	1.0	15.0	6.0	13.0
	600	3.0	5.0	9.0	2.0	10.0	14.0	4.0	7.0	12.0	11.0	8.0	16.0	1.0	15.0	6.0	13.0
$\delta = 1.25,\ \gamma = 2.0$	25	5.0	8.0	2.5	12.0	15.0	11.0	4.0	9.0	13.0	2.5	7.0	16.0	1.0	6.0	14.0	10.0
	100	2.0	11.0	9.0	5.0	12.0	13.0	3.0	10.0	15.0	4.0	8.0	16.0	1.0	7.0	14.0	6.0
	200	4.0	9.0	14.0	6.0	13.0	10.0	2.0	5.0	15.0	3.0	12.0	16.0	1.0	7.0	11.0	8.0
	350	3.0	7.0	13.0	2.0	14.0	12.0	4.5	9.0	15.0	4.5	10.0	16.0	1.0	8.0	11.0	6.0
	450	3.0	7.0	13.0	2.0	14.0	11.0	4.0	9.0	15.0	5.5	12.0	16.0	1.0	8.0	10.0	5.5
	600	4.0	6.0	13.0	2.0	14.0	11.0	3.0	10.0	15.0	5.0	12.0	16.0	1.0	8.0	9.0	7.0
$\delta = 0.75,\ \gamma = 1.5$	25	2.0	3.5	5.0	6.0	13.0	12.0	11.0	10.0	15.0	3.5	8.0	16.0	1.0	7.0	14.0	9.0
	100	3.0	11.0	10.0	5.5	13.0	14.0	2.0	5.5	15.0	4.0	9.0	16.0	1.0	8.0	12.0	7.0
	200	6.0	5.0	11.0	3.0	13.0	14.0	2.0	8.0	15.0	4.0	10.0	16.0	1.0	9.0	12.0	7.0
	350	4.0	3.0	10.5	2.0	14.0	13.0	5.0	7.0	15.0	6.0	12.0	16.0	1.0	8.0	9.0	10.5
	450	3.0	7.0	10.0	2.0	12.0	11.0	4.0	5.5	15.0	5.5	9.0	16.0	1.0	13.0	8.0	14.0
	600	3.0	4.0	12.0	2.0	11.0	13.0	6.0	7.0	14.0	8.0	10.0	16.0	1.0	15.0	5.0	9.0
$\delta = 0.5,\ \gamma = 1.0$	25	6.5	2.0	11.0	12.0	13.0	14.0	4.0	9.0	15.0	3.0	6.5	16.0	1.0	10.0	8.0	5.0
	100	4.0	8.0	9.0	3.0	11.0	15.0	2.0	6.0	14.0	5.0	13.0	16.0	1.0	7.0	12.0	10.0
	200	4.0	6.0	8.0	3.0	12.0	14.0	7.0	2.0	15.0	5.0	11.0	16.0	1.0	13.0	9.0	10.0
	350	2.0	4.0	8.0	3.0	11.0	13.0	5.0	6.0	12.0	10.0	7.0	16.0	1.0	14.0	9.0	15.0
	450	3.0	4.0	7.0	1.0	8.0	14.0	6.0	5.0	12.0	9.0	11.0	16.0	2.0	15.0	10.0	13.0
	600	3.0	6.0	8.0	2.0	11.0	13.0	4.0	5.0	12.0	10.0	9.0	16.0	1.0	15.0	7.0	14.0
$\delta = 1.0,\ \gamma = 0.5$	25	2.0	6.0	7.0	11.0	8.0	12.0	3.0	9.5	13.0	4.0	5.0	14.0	1.0	15.0	9.5	16.0
	100	2.0	4.0	11.0	8.0	12.0	10.0	5.5	7.0	14.0	3.0	5.5	16.0	1.0	15.0	9.0	13.0
	200	2.0	6.0	12.0	4.5	10.0	13.0	3.0	7.0	15.0	4.5	8.0	16.0	1.0	14.0	10.0	10.0
	350	2.0	7.0	13.0	3.0	9.5	11.0	5.5	4.0	15.0	5.5	8.0	16.0	1.0	12.0	9.5	14.0
	450	3.0	7.5	11.0	2.0	14.0	10.0	4.5	6.0	15.0	4.5	7.5	16.0	1.0	12.0	9.0	13.0
	600	2.0	9.0	14.0	3.0	13.0	11.0	4.0	6.0	15.0	5.0	8.0	16.0	1.0	12.0	7.0	10.0
$\sum$ Ranks		98.5	176.0	301.0	133.0	350.5	371.0	133.5	216.5	411.0	165.0	258.0	475.0	31.0	348.0	287.0	325.0
Overall Rank		2	6	10	3	13	14	4	7	15	5	8	16	1	12	9	11

6. Real Data Analysis

To demonstrate the effectiveness of the proposed model, we have utilized two real-world data sets from reliability engineering. The subsequent evaluation, application, and analysis of our model are detailed below.

We selected several existing models that share similar characteristics and use Topp-Leone and half-logistic (HL) base functions to compare our model. The models chosen for this comparison include:

- Type II Topp Leone half logistic ($T_{II}TLHL$) distribution defined by [11].
- The exponentiated generalized standard HLD (EGSHLD), developed by Cordeiro et al. [18].
- The exponentiated HLD (EHL), proposed by Seo and Kang [19].
- Topp Leone Frechet (TLF) distribution defined by [20].
- The Type-II half logistic Weibull distribution (HLWD), created by Hassan et al. [21].
- The PHLD, developed by Krishnarani [22].
- The Poisson generalized HLD (PGHLD), introduced by Muhammad and Liu [23].

These models were selected for their relevance and compatibility with the HLD base function framework, providing a robust basis for comparison with our proposed model.

First data:

A complete set of skewed-to-right data contains the 50 items put into use at t = 0 (time=t) and failure times in weeks [24] are given:

1.578, 1.582, 1.858, 2.595, 2.710, 2.899, 2.940, 3.087, 3.669, 3.848, 4.740, 4.848, 5.170, 5.783, 5.866, 5.872, 6.152, 6.406, 6.839, 7.042, 7.187, 7.262, 7.466, 7.479, 7.481, 8.292, 8.443, 8.475, 8.587, 9.053, 9.172, 9.229, 9.352, 10.046, 11.182,11.270, 11.490, 11.623, 11.848, 12.695, 14.369, 14.812, 15.662, 16.296, 16.410, 17.181, 17.675, 19.742, 29.022, 29.047

We applied basic exploratory data analysis techniques to examine the dataset under study, with the results displayed in Figure 9. The left side shows a box plot, while the right side presents a total-time-on-test (TTT) plot. From the box plot, we observed that the data are skewed right. To understand the

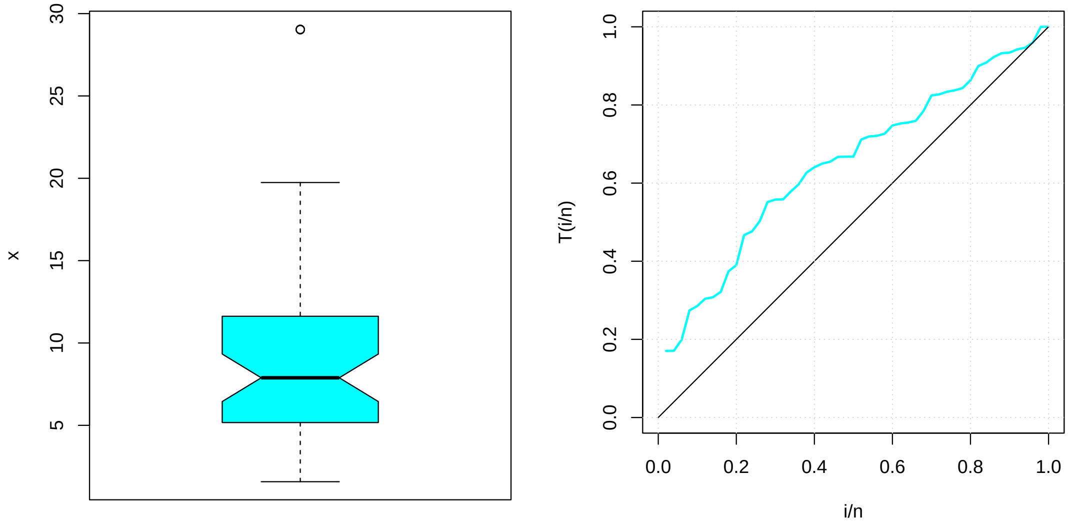

Figure 9: Box and TTT plots of first data

nature of the hazard function, we used the TTT plot as described by Aarset [25]. The TTT plot reveals a concave curve, indicating that the hazard function is increasing.

Table 10: MLEs with their SE of the first data

Model	parameter	SE	parameter	SE	parameter	SE
$ET_{II}TLHL(\delta,\gamma)$	0.7073	0.4975	0.2689	0.1232	–	–
TTLHL(δ)	0.1263	0.0179	–	–	–	–
EGSHL(β,δ)	0.1856	0.0284	2.2517	0.4861	–	–
EHL(λ,θ)	0.2155	0.0282	1.9810	0.4146	–	–
PGHL(β,θ,λ)	0.0786	0.0421	1.9147	0.3373	6.5895	4.2551
TLF(θ,δ)	17.8838	3.0508	0.7910	0.0635	–	–
HLW(λ,θ,δ)	0.0609	0.1177	1.2769	0.5711	1.9682	1.5795
PHL(β,θ)	0.0587	0.0248	1.3796	0.1534	–	–

Table 11: Model adequacy and goodness of fit test statistics for the first data

Models	-2logL	AIC	BIC	CAIC	HQIC	KS	p(KS)	CVM	p(CVM)	AD	p(AD)
$ET_{II}TLHL$	304.2370	308.2370	312.0611	308.4819	309.6932	0.0633	0.9807	0.0300	0.9773	0.2148	0.9858
TTLHL	310.7232	312.7232	314.6352	312.8032	313.4513	0.1666	0.1109	0.2813	0.1527	1.5198	0.1719
EGSHL	304.2670	308.4067	312.8911	308.5119	309.8233	0.0702	0.9516	0.0355	0.9564	0.2490	0.9709
EHL	304.4564	308.4564	312.2804	308.7013	309.9126	0.0720	0.9410	0.0306	0.9752	0.2368	0.9769
PGHL	304.7506	310.7506	316.4867	311.2506	312.9349	0.0690	0.9579	0.0288	0.9808	0.2146	0.9859
TLF	315.0641	319.0641	322.8882	319.3090	320.5203	0.1489	0.1967	0.2276	0.2201	1.3481	0.2172
HLW	304.0419	310.0419	315.7779	310.5419	312.2262	0.0665	0.9693	0.0333	0.9653	0.2419	0.9744
PHL	307.2536	311.2536	315.0776	311.4985	312.7098	0.0943	0.7304	0.0570	0.8352	0.4197	0.8283

Second data:

The following skewed to the right, a dataset presents the time between failures for repairable item
1.43, 0.11, 0.71, 0.77, 2.63, 1.49, 3.46, 2.46, 0.59, 0.74, 1.23, 0.94, 4.36, 0.40, 1.74, 4.73, 2.23, 0.45, 0.70, 1.06, 1.46, 0.30, 1.82, 2.37, 0.63, 1.23, 1.24, 1.97, 1.86, 1.17

Similarly, the results are presented in Figure 12 for the second dataset. A box plot is displayed on the left, and a TTT plot is shown on the right. The box plot indicates that the data are right-skewed. The TTT plot also shows a convex curve, indicating that the distribution of the data points has an increasing hazard function.

The parameters (δ,γ) of the proposed distribution are estimated using the ML method, implemented

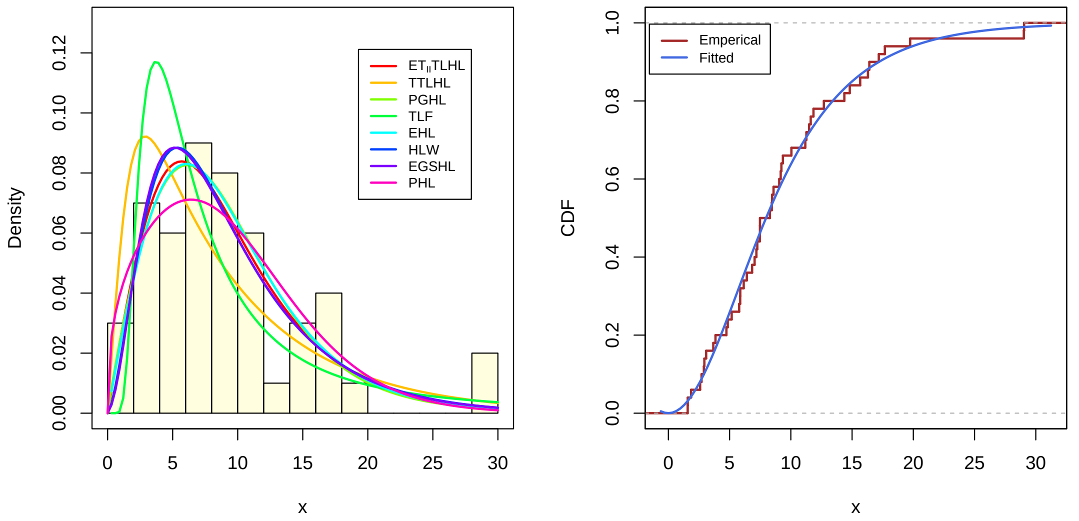

Figure 10: PDF plots of the models under comparison and CDF of $ET_{II}TLHLD$ for the first data

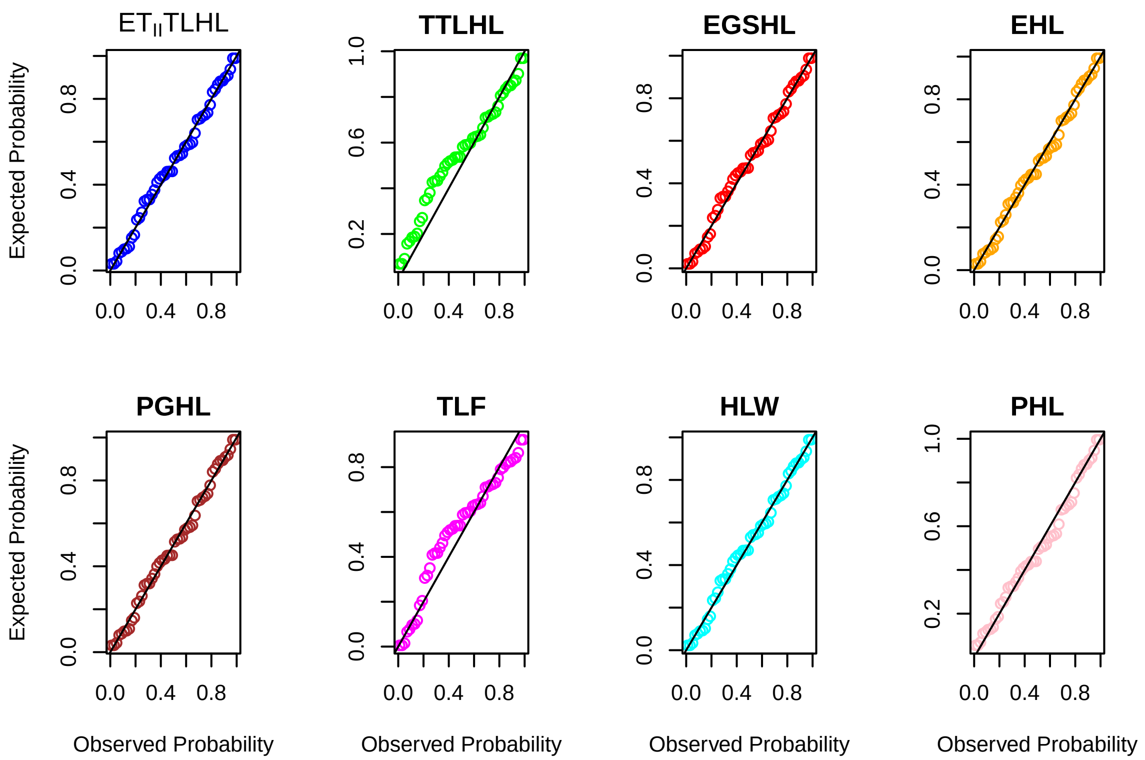

Figure 11: PP plots of all models under study for first data

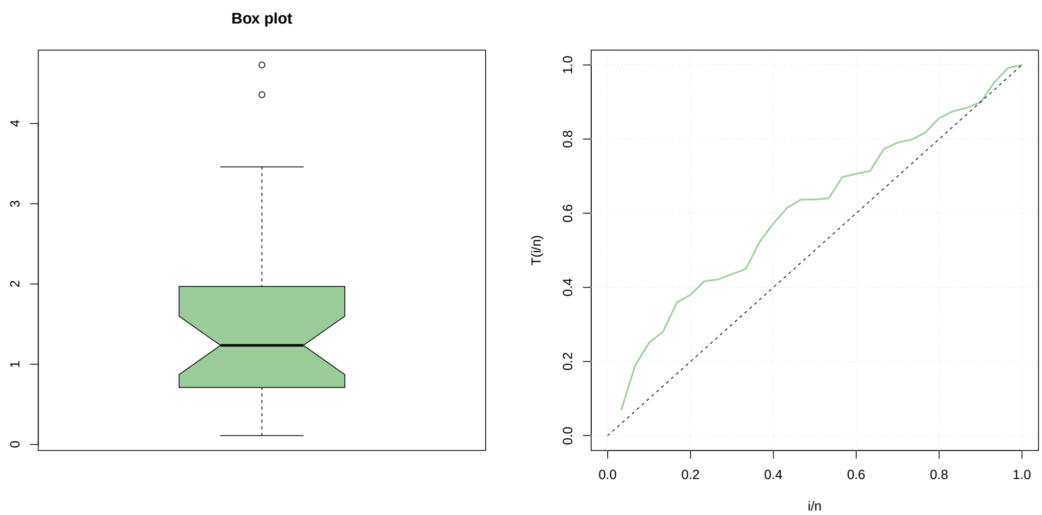

Figure 12: Box and TTT plots of second data

Table 12: MLEs with their SE of the second data

Model	parameter	SE	parameter	SE	parameter	SE
$ET_{II}TLHL(\delta,\gamma)$	0.3669	0.3073	2.5377	1.5463	–	–
TTLHL(δ)	1.4617	0.2669	–	–	–	–
EGSHL(β,δ)	1.1795	0.2552	1.5614	0.4095	–	–
EHL(λ,θ)	1.1516	0.2041	1.5601	0.3997	–	–
PGHL(β,θ,λ)	0.3204	0.7997	1.5787	0.5350	8.3332	24.7711
TLF(θ,δ)	1.4990	0.3091	0.7966	0.1063	–	–
HLW(λ,θ,δ)	0.4271	0.5437	1.3906	0.7733	1.3781	1.2515
PHL(β,θ)	0.7623	0.1670	1.2368	0.1802	–	–

Table 13: Model adequacy and goodness of fit test statistics for the second data

Models	-2logL	AIC	BIC	CAIC	HQIC	KS	p(KS)	CVM	p(CVM)	AD	p(AD)
$ET_{II}TLHL$	79.2242	83.2242	86.0266	83.6380	84.1207	0.0700	0.9985	0.0166	0.9993	0.1245	0.9998
TTLHL	81.4030	83.4030	84.8042	83.5364	83.8513	0.1427	0.5744	0.1019	0.5784	0.6792	0.5751
EGSHL	79.5589	83.5589	86.3613	83.9727	84.4554	0.0782	0.9930	0.0226	0.9947	0.1700	0.9966
EHL	79.5062	83.5062	86.3086	83.9200	84.4027	0.0784	0.9928	0.0210	0.9966	0.1619	0.9976
PGHL	79.6249	85.6249	89.8285	86.4820	86.9696	0.0713	0.998	0.0194	0.9979	0.1484	0.9988
TLF	88.0817	92.0817	94.8841	92.4955	92.9782	0.1320	0.6731	0.1207	0.4949	0.8077	0.4744
HLW	79.2894	85.1194	89.3230	85.9765	86.4641	0.0769	0.9913	0.0169	0.9903	0.1296	0.9906
PHL	80.4345	84.4345	87.2369	84.8483	85.3310	0.0793	0.9916	0.0309	0.9750	0.2434	0.9737

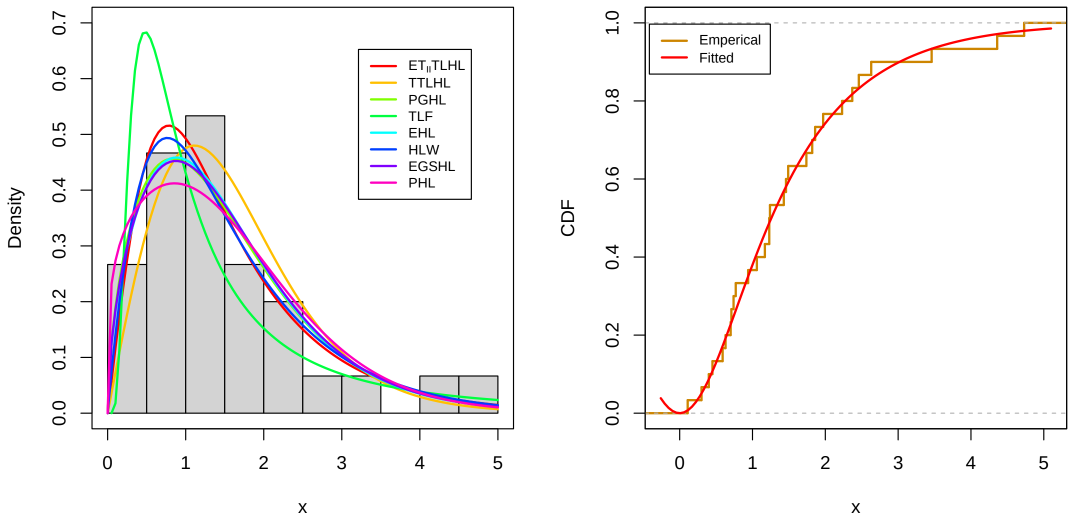

Figure 13: PDF plots of the models under comparison and CDF $ET_{II}TLHLD$ of the second data

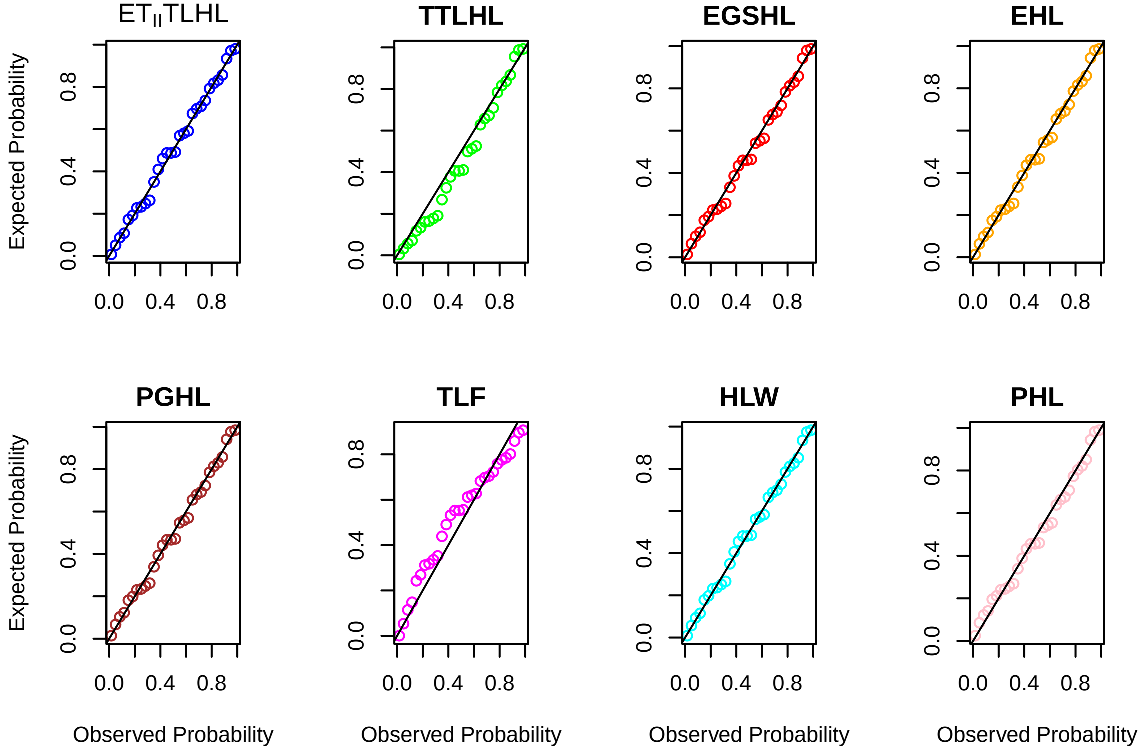

Figure 14: PP plots of the models under comparison for the second data

through the 'maxLik' package as described by Henningsen and Toomet [26]. This estimation is performed using R programming software [27], with further details available in Peng [28]. Additionally, the aforementioned candidate distributions and the $ET_{II}TLHLD$ are fitted to the datasets under study. The MLEs and their corresponding standard errors (SEs) are calculated, with the results presented in Tables 10 and 12, respectively.

Model Selection

To determine the best model among the candidates, we used several standard criteria: log-likelihood $(-2\log L)$, Akaike's information criterion (AIC), Bayesian information criterion (BIC), corrected AIC (CAIC), Hannan-Quinn information criterion (HQIC) and goodness-of-fit test statistics, including Kolmogorov-Smirnov (KS) test, AD test, and CVM test. The corresponding p-values for these tests are denoted as $p(KS)$, $p(AD)$, and $p(CVM)$, respectively. The best model is identified by the smallest values of $(-2\log L)$, AIC, BIC, CAIC, HQIC, KS, AD, and CVM, and the largest p-values, as recommended by Burnham and Anderson [29].

The results in Tables 11 and 13 show that the proposed $ET_{II}TLHLD$ outperforms the other candidate models. This conclusion is further supported by the graphical representations of the fitted PDF and CDF shown in Figures 10 and 13, where the $ET_{II}TLHLD$ provides the best fit for the data. Additionally, we have displayed the probability-probability (PP) plots of all models under study in Figures 11 and 14, respectively, for both data sets. It is observed that the proposed model yields better fits.

To evaluate the validity of the model, we have calculated the KS distances are 0.0635 and 0.1020 between the fitted distribution function and empirical distribution function, and its corresponding $p-values$ are 0.7812 and 0.9822, respectively, for both datasets (see other metrics AD and CVM also), using the parameters estimated by the ML method. These results showed that the $ET_{II}TLHLD$ fits the data very well.

We calculated the KS distances between the fitted distribution function and the empirical distribution function for both datasets to evaluate the model's validity. The KS distances are 0.0635 and 0.1020, with corresponding p-values of 0.7812 and 0.9822, respectively. These calculations used the parameters estimated by the ML method. The results indicate that the $ET_{II}TLHLD$ model fits the data very well (see other metrics, AD and CVM, also in Tables 11 and 13).

7. Conclusion

This study successfully extended the Type II Topp-Leone half-logistic distribution by incorporating a scale parameter, significantly enhancing its flexibility and applicability for modeling real-world datasets. The newly introduced extended Type II Topp-Leone half-logistic distribution encompasses a comprehensive set of derived statistical properties, including the hazard function, reliability function, odds function, quantile function, order statistics, and entropy. This provides a robust theoretical framework that deepens our understanding of this versatile distribution. Our parameter estimation process, which utilizes 16 different strategies and is validated through extensive simulations across varying sample sizes and metrics, confirms the consistency and accuracy of the estimates. Notably, the Kolmogorov method excelled by producing the most reliable and accurate estimates. The practical utility of the extended model was demonstrated through its application to two real-life datasets, particularly in analyzing the failure times of repairable items in reliability engineering. The results highlighted the model's exceptional performance, as it substantially outperformed existing models in terms of fit. This superior performance underscores the model's potential to significantly enhance maintenance scheduling and reliability assessments, offering a powerful and versatile tool for reliability engineers. This study broadens the scope of statistical modeling in reliability engineering. It paves the way for future research to explore and exploit the potential of this refined distribution in other complex phenomena across various fields.

Data availability

The corresponding author can provide the datasets created and/or studied during the current work upon reasonable request.

Authors contributions

All authors have worked equally to write and review the manuscript.

Competing interests

The authors declare no competing interests.

References

1. M. A. Meraou, Noriah M. Al-Kandari, and Raqab Z. Mohammad. Fundamental properties of the characteristic function using the compound poisson distribution as the sum of the gamma model. *Modern Journal of Statistics*, 1(1):49–57, 2025.
2. S. O. Bashiru, A. M. Isa, A. A. Khalaf, M. A. Khaleel, K. C. Arum, and C. L. Anioke. A hybrid cosine inverse lomax-g family of distributions with applications in medical and engineering data. *Nigerian Journal of Technological Development*, 22(1):261–278, 2025.
3. E. R. Ibrahim, K. Sadaf, A. Najwan, J. Farrukh, G. H. Gholamhossein, and M. Elgarhy. Modeling to covid-19 and cancer data: Using the generalized burr-hatke model. *Journal of Radiation Research and Applied Sciences*, 17(3):100972, 2024.
4. A. A. Bhat, S. P. Ahmad, A. M. Gemeay, A. H. Muse, M. E. Bakr, and O. S. Balogun. A novel extension of half-logistic distribution with statistical inference, estimation and applications. *Scientific Reports*, 14:4326, 2024.
5. S. M. Alghamdi, M. Shrahili, A.S. Hassan, A.M. Gemeay, I. Elbatal, and M. Elgarhy. Statistical inference of the half logistic modified kies exponential model with modeling to engineering data. *Symmetry*, 15:586, 2023.
6. R.A. Aldallal and A. Hussam. The new extended-x exponentiated inverted weibull distribution: Statistical inference and application to carbon data. *Applied Mathematics & Information Sciences*, 18(4):737–747, 2024.
7. M. Elgarhy, A. Johannssen, and M. Kayid. An extended rayleigh weibull model with actuarial measures and applications. *Heliyon*, 10:e32143, 2024.
8. M. Khalaf and A. Khaleel. The new strange generalized rayleigh family: Characteristics and applications to covid-19 data. *Iraqi Journal For Computer Science and Mathematics*, 5(3):92–107, 2024.
9. O. B. Sule and O. Y. Halid. On the properties and applications of topp-leone gompertz inverse rayleigh distribution. *Reliability: theory & applications*, 18(4):1032–1045, 2023.
10. A. M. Gemeay, T. Moakofi, O. S. Balogun, E. Ozkan, and M. M. Hossain. Analyzing real data by a new heavy-tailed statistical model. *Modern Journal of Statistics*, 1(1):1–24, 2025.
11. O. D. Adubisi and C. E. Adubisi. Novel distribution for modeling uncensored and censored survival time data and regression model. *Reliability: Theory & Applications*, 18(3 (74)):808–824, 2023.
12. Sule Omeiza Bashiru, Alaa Abdulrahman Khalaf, Alhaji Modu Isa, and Aishatu Kaigama. On modeling of biomedical data with exponentiated gompertz inverse rayleigh distribution. *Reliability: Theory & Applications*, 19(3 (79)):460–475, September 2024.
13. C. K. Onyekwere, O. C. Aguwa, and O. J. Obulezi. An updated lindley distribution: Properties, estimation, acceptance sampling, actuarial risk assessment and applications. *Innovation in Statistics and Probability*, 1(1):1–27, 2025.
14. Q. N. Husain, A. S. Qaddoori, N. A. Noori, K. N. Abdullah, A. A. Suleiman, and O. S. Balogun. New expansion of chen distribution according to the nitrosophic logic using the gompertz family. *Innovation in Statistics and Probability*, 1(1):60–75, 2025.
15. Amal Hassan, Diaa S. Metwally, Hatem Elsayed Semary, Sid Ahmed Benchiha, Ahmed M. Gemeay, and Mohamed Elgarhy. Improved estimation based on ranked set sampling for the chris-jerry distribution with application to engineering data. *Computational Journal of Mathematical and Statistical Sciences*, pages –, 2025.
16. T. M. Adegoke, O. M. Oladoja, O. B. Sule, A. A. Mustapha, D. E. Aderupatan, and L. C. Nzei. Topp-leone inverse gompertz distribution: Properties and different estimations techniques and applications. *Pakistan Journal of Statistics*, 39(4):433–456, 2023.
17. M. G. M. Ghazal, Y. A. Tashkandy, O. S. Balogun, and M. E. Bakr. Exponentiated extended extreme value distribution: Properties, estimation, and applications in applied fields. *AIMS Mathematics*, 9(7):17634–17656, 2024.
18. G. M. Cordeiro, T. A. N. de Andrade, M. Bourguignon, and F. G. Silva. The exponentiated generalized standardized half-logistic distribution. *International Journal of Statistics and Probability*, 6(3):24–42, 2017.
19. J. Seo and S. Kang. Notes on the exponentiated half logistic distribution. *Applied Mathematical Modelling*, 39(21):6491–6500, 2015.
20. L. P. Sapkota. Topp-leone fréchet distribution with theory and application. *Janapriya Journal of Interdisciplinary Studies (Jjis)*, 10(1):65–80, 2021.
21. A. S. Hassan, M. Elgarhy, M. A. ul Haq, and S. Alrajhi. On type ii half logistic weibull distribution with applications. *Math. Theory Modeling*, 19:49–63, 2019.

22. S. D. Krishnarani. On a power transformation of half-logistic distribution. *Journal of Probability and Statistics*, 2016(1):1–10, 2016.

23. M. Muhammad and L. Liu. A new extension of the generalized half logistic distribution with applications to real data. *Entropy*, 21(4):339, 2019.

24. DN. P. Murthy, M. Xie, and R. Jiang. *Weibull models*. John Wiley & Sons, 2004.

25. M. V. Aarset. How to identify a bathtub hazard rate. *IEEE transactions on reliability*, 36(1):106–108, 1987.

26. A. Henningsen and O. Toomet. maxlik: A package for maximum likelihood estimation in r. *Computational Statistics*, 26(3):443–458, 2011.

27. R Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria, 2024.

28. R. D. Peng. *R programming for data science*. Leanpub Victoria, BC, Canada, 2016.

29. K. P. Burnham and D. R. Anderson. *Model selection and multimodel inference: a practical information-theoretic approach*. Springer, 2002.

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