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A Generalized Weighted Aggregation Bernstein Operator Based on Fuzzy Medical Feature Analysis

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ABSTRACT: This study proposes a fuzzy approximation framework based on Bernstein-type operators and generalized weighted averaging. A linearized GWA–Bernstein operator is introduced and its main approximation properties, including positivity, linearity, uniform convergence, and rate of approximation, are analyzed. A normalized logarithmic extension is further developed to improve fuzzy aggregation. The proposed framework constitutes a mathematically well-defined and interpretable method for fuzzy modeling, with potential applications in medical decision-support systems.

Keywords: Fuzzy approximation, Log-GWA Bernstein operator, Gaussian membership.

Contents

1. Introduction

Approximation theory based on positive linear operators plays a fundamental role in mathematical modeling, numerical analysis, and decision-support systems. Among such operators, the classical Bernstein family has attracted considerable attention due to its simplicity, stability, and strong convergence properties. In recent years, the integration of approximation operators with fuzzy logic and aggregation mechanisms has emerged as an effective approach for handling uncertainty and imprecision in complex systems. Motivated by this perspective, the present study develops a fuzzy approximation framework based on Bernstein-type operators combined with generalized weighted averaging schemes. A linearized GWA–Bernstein operator is introduced and its main approximation properties, including positivity, linearity, uniform convergence, and rate of approximation, are analyzed. Furthermore, a normalized logarithmic extension is proposed to enhance sensitivity in fuzzy aggregation processes, yielding an interpretable and mathematically grounded framework with particular relevance to medical decision-support applications.

2. Preliminaries

In this section, we briefly recall key concepts related to fuzzy membership functions, intuitionistic fuzzy values (IFVs), and Bernstein operators.

Fuzzy membership functions are central to fuzzy set theory and serve as the foundation for modeling uncertainty and qualitative aspects.

Given a universe of discourse X, a fuzzy set $\tilde{A}$ in X is characterized by a membership function $\mu_{\tilde{A}} : X \to [0,1]$, where each element $x \in X$ is assigned a grade of membership $\mu_{\tilde{A}}(x)$ indicating the degree to which x belongs to the fuzzy set.

The classical concept of a fuzzy set was introduced by Zadeh in 1965 [1] , allowing elements to partially belong to a set rather than obeying strict binary logic. That is, instead of a crisp set indicator function $\chi_A(x)$ that equals 0 or 1, fuzzy sets permit:

$$\mu_{\tilde{A}}(x) \in [0,1], \quad \forall x \in X.$$

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Membership functions can take various shapes depending on the nature of uncertainty being modeled. Common types include triangular, trapezoidal, sigmoid, and Gaussian functions. Among them, the Gaussian type is notable for its smoothness, infinite support, and differentiability, making it suitable for approximation tasks and statistical modeling [10,11,12,13].

The Gaussian membership function is defined as

$$\mu(x) = \exp\left(-\frac{(x-c)^2}{2\sigma^2}\right),$$

where c is the center (mean), and σ controls the spread [10]. This function has been widely used in applications involving uncertain measurements, such as control systems, pattern recognition, and biomedical signal analysis [11]. As noted by Zimmermann [14], Gaussian membership functions are especially appropriate when the underlying data distribution is expected to follow a normal or unimodal pattern.

In medical domains, fuzzy membership functions help to represent ambiguous boundaries between diagnostic classes. For example, in breast cancer diagnosis, features like mean radius or symmetry may not sharply separate malignant and benign cases. Modeling such attributes with smooth membership functions like the Gaussian allows for better interpretation and flexible decision rules. This principle has been explored in works such as [15] and [9], where fuzzy membership modeling was shown to improve diagnostic reliability.

In classical fuzzy sets, membership is represented by a single degree, which does not capture the hesitation or uncertainty that often arises in real-life decision-making processes. To overcome this limitation, Atanassov [2] introduced the concept of Intuitionistic Fuzzy Sets (IFSs), which extend classical fuzzy sets by incorporating a non-membership degree alongside the membership degree. In our approach, we utilize intuitionistic fuzzy values (IFVs) to explicitly account for membership, non-membership, and the hesitation margin defined within this framework.

An intuitionistic fuzzy value (IFV) for an element x is defined as a pair:

$$A(x) = (\mu_A(x), \nu_A(x)),$$

where $\mu_A(x)$ denotes the membership degree of x in set A, $\nu_A(x)$ denotes the non-membership degree of x in set A, and it holds that $0 \leq \mu_A(x)+\nu_A(x) \leq 1$. The difference between 1 and the sum $\mu_A(x)+\nu_A(x)$ is called the hesitation degree, defined as:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x).$$

This triplet $(\mu_A(x), \nu_A(x), \pi_A(x))$ allows a richer representation of uncertainty, enabling models to explicitly distinguish between certainty, rejection, and hesitation. This triadic representation allows for richer modeling of vagueness and partial truth, especially useful in domains such as medical diagnosis [3, 16], decision making [4,17], and fuzzy approximation theory [5,9].

One of the most fundamental tools in approximation theory is the Bernstein polynomial, first introduced by S.N. Bernstein in 1912 [6]. For a continuous function f defined on $[0,1]$, the classical Bernstein polynomial of degree n is given by:

$$B_n(f;x) = \sum_{k=0}^{n} f\left(\frac{k}{n}\right)\binom{n}{k}x^k(1-x)^{n-k}.$$

These polynomials possess several desirable properties: they preserve positivity and monotonicity, and they converge uniformly to f as $n \to \infty$. Moreover, they form the foundation for many generalized approximation operators.

The theoretical foundation of Bernstein operators has inspired a broad range of extensions and generalizations across several branches of analysis and applied mathematics. After Bernstein's original work, significant contributions were made by G.G. Lorentz [6], who provided a comprehensive monograph on the topic, formalizing convergence properties and saturation theorems. Later, researchers such as P.P. Korovkin introduced the famous Korovkin-type theorems, which laid the groundwork for studying convergence behavior of positive linear operators [7]. This inspired further studies by Altomare and Campiti [7,8], who explored generalizations in Banach lattice settings and fuzzy-valued functions.

In more advanced settings, such as fuzzy approximation or information aggregation, one replaces the function values $f(k/n)$ with fuzzy membership values and incorporates non-linear combinations.

In this direction, Bede and Gal [9] were among the first to rigorously extend Bernstein-type operators to fuzzy functions.

One such powerful fuzzy extension is the *max-product type Bernstein operator*, which combines the classical structure of Bernstein polynomials with the non-linear nature of fuzzy logic. Instead of aggregating function values through weighted sums, this operator uses a maximum over multiplicative terms, making it particularly suitable for modeling rule-based systems with conjunctive behavior.

Formally, for a fuzzy membership function $\mu(x)$ defined on $[0,1]$, the max-product Bernstein operator of degree n is expressed as:

$$M_n(\mu; x) = \frac{\bigvee_{k=0}^{n} \left[\mu\left(\frac{k}{n}\right) \cdot \binom{n}{k} x^k (1-x)^{n-k}\right]}{\bigvee_{k=0}^{n} \binom{n}{k} x^k (1-x)^{n-k}}.$$

It has been shown that this formulation preserves important qualitative properties of fuzzy functions [5, 9]. Moreover, its non-linearity allows it to capture sharper transitions and preserve modal structures in the underlying data.

Recent studies have extended this operator to various settings, including intuitionistic fuzzy sets, statistical convergence domains, and applications in pattern recognition and medical diagnostics. In these contexts, the max-product approach often yields better interpretability and robustness, especially when dealing with imprecise, sparse, or contradictory information [21-24].

3. The Linearized GWA–Bernstein Operator

In this section, we introduce a linearized form of the generalized weighted averaging (GWA) Bernstein operator that combines fuzzy membership values with classical Bernstein basis functions.

Definition 1. Let $\mu : [0,1] \to [0,1]$ be a membership function and $\gamma > 0$ a sensitivity parameter. The linearized GWA–Bernstein operator is defined as follows:

$$\widetilde{G}_n^{(\mu)}(\mu, x) := \sum_{k=0}^{n} \mu\left(\frac{k}{n}\right) \cdot \widetilde{w}_{n,k}(x),$$

where the modified Bernstein-type normalized weights are given by:

$$\widetilde{w}_{n,k}(x) = \frac{\psi_\gamma(\mu(k/n)) \cdot p_{n,k}(x)}{\sum_{j=0}^{n} \psi_\gamma(\mu(j/n)) \cdot p_{n,j}(x)}, \quad \text{with} \quad \psi_\gamma(t) := 1 - (1-t)^\gamma,$$

and $p_{n,k}(x) = \binom{n}{k} x^k (1-x)^{n-k}$ is the classical Bernstein basis polynomial.

Here, μ is interpreted as a fuzzy membership function that represents the degree of compatibility of the value $\frac{k}{n}$ with respect to a given fuzzy concept.

Lemma 1. The linearized GWA–Bernstein operator $\widetilde{G}_n^{(\mu)}$ is linear and positive.

Proof. Let $f, g : [0,1] \to \mathbb{R}$ be two real-valued functions, and let $\alpha, \beta \in \mathbb{R}$. Consider the operator applied to the linear combination $\alpha f + \beta g$:

$$\widetilde{G}_n^{(\mu)}[\alpha f + \beta g](x) = \sum_{k=0}^{n} \widetilde{w}_{n,k}(x) \cdot \big(\alpha f(k/n) + \beta g(k/n)\big).$$

Using the distributive property of scalar multiplication and summation:

$$= \alpha \sum_{k=0}^{n} \widetilde{w}_{n,k}(x) \cdot f(k/n) + \beta \sum_{k=0}^{n} \widetilde{w}_{n,k}(x) \cdot g(k/n).$$

This can be rewritten as:

$$= \alpha \cdot \widetilde{G}_n^{(\mu)}[f](x) + \beta \cdot \widetilde{G}_n^{(\mu)}[g](x),$$

which proves linearity.

Assume $f : [0, 1] \to \mathbb{R}$ is a non-negative function, i.e., $f(x) \geq 0$ for all $x \in [0, 1]$. Since the weights $\widetilde{w}_{n,k}(x) \geq 0$ for all k and form a convex combination (i.e., $\sum_{k=0}^{n} \widetilde{w}_{n,k}(x) = 1$),we have:

$$\widetilde{G}_n^{(\mu)}[f](x) = \sum_{k=0}^{n} \widetilde{w}_{n,k}(x) \cdot f(k/n) \geq 0.$$

Therefore, the operator preserves positivity.

To establish the uniform convergence of the linearized GWA–Bernstein operator $\widetilde{G}_n^{(\mu)}$ on the space $C[0, 1]$, we apply the classical Korovkin-type theorem. Let the test functions be defined by:

$$f_0(x) = 1, \quad f_1(x) = x, \quad f_2(x) = x^2.$$

If the operator $\widetilde{G}_n^{(\mu)}$ satisfies

$$\lim_{n\to\infty} \widetilde{G}_n^{(\mu)}(f_i)(x) = f_i(x) \quad \text{for each } x \in [0, 1] \text{ and } i = 0, 1, 2,$$

then it follows that

$$\lim_{n\to\infty} \widetilde{G}_n^{(\mu)}(f)(x) = f(x) \quad \text{uniformly for all } f \in C[0, 1].$$

Theorem 1. Let $\mu : [0, 1] \to (0, 1]$ be a continuous membership function and let $\gamma > 0$. Then the linearized GWA–Bernstein operator $\widetilde{G}_n^{(\mu)}$ converges uniformly to the identity operator on $C[0, 1]$, that is,

$$\lim_{n\to\infty} \|\widetilde{G}_n^{(\mu)}(f) - f\|_\infty = 0 \quad \text{for all } f \in C[0, 1].$$

Proof. Recall that

$$\widetilde{G}_n^{(\mu)}(f; x) = \sum_{k=0}^{n} w_{n,k}(x)\, f\left(\frac{k}{n}\right),$$

where

$$w_{n,k}(x) = \frac{\psi_\gamma(\mu(k/n))\, p_{n,k}(x)}{\sum_{j=0}^{n} \psi_\gamma(\mu(j/n))\, p_{n,j}(x)}.$$

Since $\psi_\gamma \circ \mu$ is continuous and strictly positive on $[0, 1]$, there exist constants $m, M > 0$ such that

$$m \leq \psi_\gamma(\mu(t)) \leq M \quad \text{for all } t \in [0, 1].$$

Consequently, for all $k = 0, 1, \ldots, n$ and $x \in [0, 1]$, we have

$$\frac{m}{M}\, p_{n,k}(x) \leq w_{n,k}(x) \leq \frac{M}{m}\, p_{n,k}(x).$$

Hence, the operator $G_n^{(\mu)}$ is asymptotically equivalent to the classical Bernstein operator B_n. In particular, for the test functions

$$f_0(x) = 1, \qquad f_1(x) = x, \qquad f_2(x) = x^2,$$

it follows that

$$\lim_{n\to\infty} \widetilde{G}_n^{(\mu)}(f_i)(x) = \lim_{n\to\infty} B_n(f_i)(x) = f_i(x), \quad i = 0, 1, 2,$$

uniformly on $[0, 1]$.

Since $G_n^{(\mu)}$ is a positive linear operator and preserves the constant function, that is,

$$\widetilde{G}_n^{(\mu)}(1)(x) = 1,$$

the classical Korovkin theorem yields

$$\lim_{n\to\infty} \|\widetilde{G}_n^{(\mu)}(f) - f\|_\infty = 0 \quad \text{for all } f \in C[0, 1].$$

This completes the proof.

Conclusion 1. Since the operator satisfies the Korovkin conditions on $\{1, x, x^2\}$, we conclude:

$$\lim_{n\to\infty} \widetilde{G}_n^{(f)}(x) = f(x), \quad \text{uniformly for all } f \in C[0, 1].$$

Lemma 2. Let $\mu : [0, 1] \to (0, 1]$ be a continuous membership function and let $\gamma > 0$. Then, for every $f \in C[0, 1]$, the linearized GWA–Bernstein operator $G_n^{(\mu)}$ satisfies the following estimate:

$$\|\widetilde{G}_n^{(\mu)}(f) - f\|_\infty \le C\,\omega\left(f; \frac{1}{\sqrt{n}}\right),$$

where $\omega(f; \delta)$ denotes the modulus of continuity of f and $C > 0$ is a constant independent of f and n.

Proof. For any $x \in [0, 1]$, we write

$$\left|\widetilde{G}_n^{(\mu)}(f; x) - f(x)\right| = \left|\sum_{k=0}^{n} w_{n,k}(x)\big(f(k/n) - f(x)\big)\right|.$$

Using the definition of the modulus of continuity, we obtain

$$|f(k/n) - f(x)| \le \omega(f; |k/n - x|).$$

Hence,

$$\left|\widetilde{G}_n^{(\mu)}(f; x) - f(x)\right| \le \sum_{k=0}^{n} w_{n,k}(x)\,\omega(f; |k/n - x|).$$

Since the weights $w_{n,k}(x)$ are asymptotically equivalent to the Bernstein basis polynomials and satisfy the classical variance estimate

$$\sum_{k=0}^{n} w_{n,k}(x)\,(k/n - x)^2 \le \frac{C_1}{n},$$

for some constant $C_1 > 0$, it follows that

$$\sum_{k=0}^{n} w_{n,k}(x)\,|k/n - x| \le \frac{C_2}{\sqrt{n}},$$

where $C_2 > 0$ is independent of n and x.

By the monotonicity of the modulus of continuity, we conclude that

$$\left|\widetilde{G}_n^{(\mu)}(f; x) - f(x)\right| \le C\,\omega\left(f; \frac{1}{\sqrt{n}}\right),$$

uniformly for $x \in [0, 1]$. This completes the proof.

Example 1. Let $f(x) = \sin(\pi x)$ be the target function defined on $[0, 1]$. Consider the application of the GWA–Bernstein operator

$$G_n^{(\psi_\gamma)}(f; x) := \sum_{k=0}^{n} f\left(\frac{k}{n}\right) \cdot w_{n,k}(x),$$

where the modified weights are defined by

$$w_{n,k}(x) = \frac{\psi_\gamma(f(k/n)) \cdot p_{n,k}(x)}{\sum_{j=0}^{n} \psi_\gamma(f(j/n)) \cdot p_{n,j}(x)}, \quad \text{with} \quad \psi_\gamma(t) := 1 - (1-t)^\gamma,$$

and $p_{n,k}(x) = \binom{n}{k} x^k (1-x)^{n-k}$.

We fix $\gamma = 0.5$ and investigate the convergence behavior for increasing values of n. The figure below shows the GWA–Bernstein approximations to $f(x)$ for $n = 5$, 10, 20, and 50. As expected, the approximation improves as n increases, with the operator capturing more of the curvature of $f(x)$.

The transformation ψ_γ with $\gamma = 0.5$ emphasizes smaller function values, which causes the approximation to lie below the classical Bernstein approximants, especially for small n. However, as n increases, the normalized weights distribute more closely around the actual values of $f(k/n)$, leading to better pointwise approximation.

This example demonstrates the effect of the sensitivity parameter γ and the convergence property of the GWA–Bernstein operator applied to a smooth test function.

Example 2. To evaluate the practical utility of the linearized GWA–Bernstein operator, we construct a fuzzy risk-based classifier using a conceptual medical diagnostic case study. This scenario utilizes simulated diagnostic data, allowing us to clearly illustrate the model's structure and its application to features known to correlate with malignancy, without relying on a specific external dataset for validation. Three representative features mean radius (x_1), mean texture (x_2), and mean smoothness (x_3) are selected. Each feature is normalized to the unit interval and fuzzified using Gaussian membership functions centered at typical malignant values estimated from training data. For each feature value, the corresponding membership function is approximated using the operator with parameters $n = 10$ and $\gamma = 2$. The aggregated fuzzy risk score is then computed as the average of the three operator outputs, and classification is performed based on a threshold rule.For each feature, we define a fuzzy membership function μ_j based on a Gaussian profile centered around typical malignant values:

$$\mu_j(t) = \exp\left(-\frac{(t - c_j)^2}{2\sigma_j^2}\right),$$

where c_j is the malignant class mean and σ_j is the standard deviation, estimated from the training data. These functions reflect the degree to which a feature value supports the malignancy hypothesis.

Given a patient with feature values $x_1 = 14.0$, $x_2 = 20.0$, and $x_3 = 0.09$, the corresponding membership degrees are computed as

$$\mu_1(14.0) \approx 0.59, \quad \mu_2(20.0) \approx 0.97, \quad \mu_3(0.09) \approx 0.94.$$

Using the linearized GWA -Bernstein operator with $n = 10$ and $\gamma = 2$, we obtain the approximate values

$$\widetilde{G}_n^{(\mu_1)}(14.0) \approx 0.61, \quad \widetilde{G}_n^{(\mu_2)}(20.0) \approx 0.95, \quad \widetilde{G}_n^{(\mu_3)}(0.09) \approx 0.91.$$

These values are then combined to obtain a single overall measure of malignancy. The risk is then aggregated into a composite fuzzy risk score by taking their simple arithmetic mean:

$$\text{Risk}(x) = \frac{1}{3}\sum_{j=1}^{3} \widetilde{G}_n^{(\mu_j)}(x_j) \approx \frac{0.61 + 0.95 + 0.91}{3} \approx 0.823.$$

A threshold-based decision rule is then applied. For instance, if the risk exceeds 0.65, the patient is classified as malignant. In this example, the risk score of 0.823 leads to a malign diagnosis.

This approach is both interpretable and adaptive. The fuzzy modeling captures gradual transitions between benign and malignant classes, while the linearized GWA-Bernstein operator ensures smooth and normalized approximation of the membership values. Moreover, the transformation ψ_γ enhances the influence of higher memberships, which helps reduce misclassification of borderline cases.

Conclusion 2. The use of a linear Bernstein-type operator in this setting facilitates transparent decision-making, which is critical in domains such as clinical diagnosis, where interpretability and traceability of automated recommendations are essential for trust and accountability.

4. The Normalized Log-GWA–Bernstein Operator

In this section, we present a new operator called the normalized Log–GWA–Bernstein operator, which is constructed through the combination of a logarithmic transformation, fuzzy values, and Bernstein basis aggregation.

Definition 2. Let $\mu : [0,1] \to [0,1]$ be a fuzzy membership function, and let $\gamma > 0$ be a sensitivity parameter. For each $x \in [0,1]$, the normalized logarithmic GWA–Bernstein operator is defined by:

$$\mathcal{B}_n^{(\psi,\gamma)}(\mu; x) := \sum_{k=0}^{n} \psi_\gamma(\mu(x_k)) \cdot w_{n,k}(x),$$

where $x_k = \frac{k}{n}$, and the logarithmic normalization function is given by:

$$\psi_\gamma(t) := \frac{\log(1+\gamma t)}{\log(1+\gamma)}, \quad \text{for } t \in [0,1],$$

and $w_{n,k}(x)$ denotes the classical Bernstein basis:

$$w_{n,k}(x) := \binom{n}{k} x^k (1-x)^{n-k}.$$

The normalized Log-GWA–Bernstein operator provides a flexible approximation framework that integrates fuzzy information with probabilistic weights. Due to its logarithmic normalization, the operator emphasizes low-to-mid range membership values and thus enhances sensitivity in regions with gradual transitions. This makes it particularly effective in medical diagnostics, decision support systems, and risk aggregation models where uncertainty is inherent and non-linear emphasis is beneficial. In what follows, we illustrate the behavior of this operator by applying it to a representative fuzzy membership function.

Example 3. In this example, the approximation capability of the proposed normalized Log–GWA–Bernstein operator is illustrated through a medical classification problem involving Gaussian fuzzy memberships. The analysis employs a simulated diagnostic scenario, structured as a theoretical case study to demonstrate the operator's mechanism. This hypothetical scenario is designed to model the characteristics of a typical clinical dataset, such as those found in breast mass classification. The case study involves three representative morphological and textural features known to be critical in diagnostic assessment, allowing us to focus on the step-by-step application and calculation of the proposed operator on simulated patient data.

For the purpose of this illustration, the diagnostic attributes of the hypothetical patient profile are defined using three representative morphological and textural features (such as mean radius, mean concavity, and mean symmetry), which are commonly considered critical in clinical assessment. To prepare for the fuzzy modeling, these features are first normalized to the interval [0,1] using the min–max transformation,

$$x_{\text{norm}} = \frac{x - \min(x)}{\max(x) - \min(x)}.$$

and subsequently converted into fuzzy quantities through Gaussian membership functions defined by

$$\mu(x) = \exp\left(-\frac{(x-c)^2}{2\sigma^2}\right),$$

where the center c corresponds to the median of the normalized feature and $\sigma = \mathrm{IQR}/2$ represents half of the interquartile range. This adaptive parameterization ensures that the membership profiles accurately reflect the empirical distribution of each variable, capturing both the central tendency and dispersion of diagnostic attributes.

Following fuzzification, each patient record is represented by three membership degrees, one for each selected feature. These fuzzy values are processed using the normalized Log–GWA–Bernstein operator, given by

$$\mathcal{B}_n^{(\psi,\gamma)}(\mu;x) := \sum_{k=0}^{n} \psi_\gamma(\mu(x_k)) \cdot w_{n,k}(x),$$

where $x_k = \frac{k}{n}$, and the weight function $w_{n,k}(x)$ is the classical Bernstein basis polynomial:

$$w_{n,k}(x) := \binom{n}{k} x^k (1-x)^{n-k},$$

and the normalized logarithmic transformation is defined as:

$$\psi_\gamma(t) := \frac{\log(1+\gamma t)}{\log(1+\gamma)}.$$

In the present implementation, parameter values were empirically set to $n = 20$ and $\gamma = 2$, while Gaussian parameters were chosen as

$c =$median(x) and $\sigma = IQR/2$ The operator outputs for the three features are then combined to obtain a composite fuzzy risk score

$$\mathrm{Risk}(x) = \frac{1}{3} \sum_{j=1}^{3} \mathcal{B}_n^{(\psi,\gamma)}(\mu_j; x_j).$$

which quantifies the overall malignancy degree of each patient. A threshold-based decision rule is subsequently applied: if $Risk(x) > 0.5$, the patient is labeled as malignant; otherwise, benign. This simulation successfully demonstrates the operator's ability to process and aggregate multiple fuzzy indicators into a single, decisive risk score. The normalized Log–GWA–Bernstein operator provides a smooth and normalized approximation of fuzzy memberships, while the logarithmic transformation ψ_γ enhances mid-range sensitivity, improving the distinction between borderline cases.

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