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Electrodynamic Paradigm of Antisymmetric Mechanical Lorentz Momentum Tensor

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ABSTRACT: Antisymmetric mechanical Lorentz force tensor $\mathbf{f}^{\mu\nu}$ contains Newton's second law of motion $\mathbf{F} = m\mathbf{a}$ as temporal component [1]. This naturally implies that Newton's first law of motion $\mathbf{P} = m\mathbf{v}$ is the temporal component of antisymmetric mechanical Lorentz momentum tensor $\mathbf{P}^{\mu\nu}$ whose spatial component is Coriolis momentum $m(\mathbf{x} \times \boldsymbol{\omega})$. In this way, we have complete framework of mechanical momentum electrodynamics that obeys principle of relativity, conservation law and symmetry. This model contains Newton's first and 2^{nd} law of motion and action and reaction concept. In this paper, we shall present the basic framework of mechanical model of momentum electrodynamics with its dual formulation. The transformation of Lorentz momentum tensor in noninertial coordinate metric based on single transformation law (STL) for 4-vectors and tensors predicts the zero-point origin of Plank's constant, angular momentum and 7D wave of Plank's constant in conservation law. Matrix method and Einstein convention method are employed where matrix method gives zero conservation and Einstein method predicts 7D wave supplemented with conservation law. As a consequence, antisymmetric electromagnetic Lorentz momentum dynamics and antisymmetric gravitational momentum dynamics will be directly mentioned to avoid repetition of calculations. These are the marvelous achievements in the field of relativistic dynamics.

Key Words: Antisymmetric Lorentz momentum tensor, mechanical momentum electrodynamic, 4D wave of Newtonian momentum, wave of Coriolis momentum, wave of Lorentz momentum.

Contents

1. Introduction

Discovery of antisymmetric Lorentz force tensor-based theory of electrodynamics, gravitation and mechanical electrodynamics [1-3], predicts the existence of corresponding models of antisymmetric Lorentz momentum tensor theories. Einstein's special relativity provided us the concept of 4-dimensional momentum $\mathbf{p}^{\mu}$ but it has very limited scope in the context of spacetime physics where tensors are needed to explore the complete picture of dynamics. Perhaps this is the reason that Einstein's general relativity or theory of gravitation doesn't include it as an integral part and switched towards the concept of stress-energy tensor. Furthermore, general relativity cannot be derived from special relativity in a simple way. Gravito-electromagnetism (GEM) and Lorentz invariant theory of gravitation [4-11] are actually attempts to look for the tensorial origin of general relativity and gravitation.

After the development of antisymmetric mechanical Lorentz force (MLF) electrodynamics that includes Newton's 2^{nd} law of motion as an integral part, led to the idea to procure Newton's first law of motion as a natural part of theory. Integration of MLF tensor with respect to time gives us the framework of antisymmetric Lorentz momentum tensor (LMT) where newton's first law of motion $\mathbf{P} = m\mathbf{v}$ appears as a temporal component of LMT whose spatial component is Coriolis momentum. In elementary

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mechanics, we define force as the rate of change of linear momentum so in the present context, rate of change of Lorentz momentum is equal to Lorentz force. This concept is equally valid in the derivation of electromagnetic Lorentz momentum dynamics and gravitational Lorentz momentum.

It is important to note that electromagnetic momentum dynamics cannot be obtained from electromagnetic field tensor $\mathbf{F}^{\mu\nu}$ multiplying it by time because $q\mathbf{E}t$ is electrical momentum but $q\mathbf{B}t$ doesn't represent magnetic momentum. In the case of Lorentz force electrodynamics $q\mathbf{E}t$ and $q(\mathbf{v}\times\mathbf{B})t$ clearly represent the expressions for electric and magnetic momentum respectively. Anyhow, Maxwellian electrodynamics can provide us lower order theory of electrodynamics in terms of fields but not as momentum theory whereas Lorentz force-based electrodynamics not only gives Lorentz momentum but also leads to the existence of antisymmetric Lorentz position tensor. The concept of antisymmetric gravitational Lorentz momentum tensor will open a new window to look in the foundations of gravitation physics. In this paper, basic framework of Lorentz momentum dynamics is presented in the language of electrodynamics. It consists of Lorentz momentum field, Momentum Maxwell's equations, momentum conservation law, wave equations of linear momentum, Coriolis momentum and Lorentz momentum. It is very important to point out that Newton's first law and 2^{nd} law of motion are the inbuilt concepts in this model. The dual framework Lorentz momentum have the same pattern. The summary of the results is formulated in table-1. The transformation of Lorentz momentum tensor in noninertial coordinate metric based on single transformation law (STL) for 4-vectors and tensors predicts the origin of Plank's constant, angular momentum and 7D wave of Plank's constant along with classical conservation law. As a consequence of this model, relations for antisymmetric electromagnetic Lorentz momentum tensor dynamics and antisymmetric gravitational Lorentz momentum tensor dynamics with their duals will be presented in tables-2 and 3.

Notations in this model are adopted according to modern approach of relativity. Greek alphabets μ, ν, α, β, ...runs from 0 to 3 and Latin letters i, j, k, ... from 1 to 3. Comma (,) denote partial differentiation e. g. $\mathbf{F}_{,0}=\frac{\partial\mathbf{F}}{\partial t}$ Partial derivative of Force field w. r. t. time, $\mathbf{F}_{,1}=\frac{\partial\mathbf{F}}{\partial\mathbf{x}}$ Partial derivative of force w. r. t. x-axis, $\mathbf{F}_{,2}=\frac{\partial\mathbf{E}}{\partial\mathbf{y}}$ Partial derivative of force w. r. t. y-axis, $\mathbf{F}_{,3}=\frac{\partial\mathbf{F}}{\partial\mathbf{z}}$ Partial derivative of force w. r. t. z-axis, $\mathbf{f}^{\mu\nu}{}_{,\nu}$ means 4-dimensional or spacetime partial derivative of Lorentz force tensor.

1.1. Electrodynamic Paradigm of Antisymmetric Mechanical Lorentz Momentum Dynamics

Antisymmetric mechanical Lorentz momentum tensor $\mathbf{P}^{\mu\nu}$ whose components are linear momentum and Coriolis momentum that incorporate the whole story of mechanical momentum electrodynamics.

Structure of Antisymmetric Lorentz Momentum Tensor

$$\mathbf{P}^{\mu\nu}=\begin{bmatrix}0 & \mathbf{p}^1 & \mathbf{p}^2 & \mathbf{p}^3\\ -\mathbf{p}^1 & 0 & m(\mathbf{x}\times\boldsymbol{\omega})^3 & -m(\mathbf{x}\times\boldsymbol{\omega})^2\\ -\mathbf{p}^2 & -m(\mathbf{x}\times\boldsymbol{\omega})^3 & 0 & m(\mathbf{x}\times\boldsymbol{\omega})^1\\ -\mathbf{p}^3 & m(\mathbf{x}\times\boldsymbol{\omega})^2 & -m(\mathbf{x}\times\boldsymbol{\omega})^1 & 0\end{bmatrix} \tag{1.1}$$

$$\mathbf{P}^{0i}=\mathbf{p}^i \quad \mathbf{P}^{ij}=\varepsilon^{ijk}m(\mathbf{x}\times\boldsymbol{\omega})_k t \tag{1.2}$$

Linear Momentum as Newton's first Law of motion

$$\mathbf{P}^{0i}=\mathbf{p}^i \tag{1.3}$$

Spatial Component of Lorentz Momentum

$$\mathbf{P}^{i\nu}=[m\,(\mathbf{x}\times\boldsymbol{\omega})]-[m\,(\mathbf{x}\times\boldsymbol{\omega})+\mathbf{p}] \tag{1.4}$$

Antisymmetry of Lorentz Momentum Tensor

$$\mathbf{P}^{\mu\nu}=[m\,(\mathbf{x}\times\boldsymbol{\omega})+\mathbf{p}]\text{–}[m\,(\mathbf{x}\times\boldsymbol{\omega})+\mathbf{p}] \tag{1.5}$$

Gauss's law of Momentum is equal to divergence of linear momentum

$$\mathbf{P}^{0i}{}_{,i}=\nabla.\mathbf{p} \tag{1.6}$$

Ampere's Law of Momentum

$$\nabla \times (m\mathbf{x} \times \boldsymbol{\omega}) = \mathbf{P}^{i\nu}{}_{,\nu} + \frac{\partial}{\partial t}\mathbf{p} \tag{1.7}$$

The curl of Coriolis momentum is equal to momentum density and time varying linear momentum or Newtonian force

Conservation Law of Lorentz Momentum

$$\mathbf{P}^{\mu\nu}{}_{,\nu\mu} = \nabla.\mathbf{p}^{I}{}_{,0} + \nabla.[m(\nabla \times \mathbf{x} \times \boldsymbol{\omega}) - \mathbf{p}^{i}{}_{,0}] = 0 \tag{1.8}$$

Structure of Dual Momentum Field Tensor

$$*\mathbf{P}^{\mu\nu} = \begin{bmatrix} 0 & m(\mathbf{x} \times \boldsymbol{\omega})^1 & m(\mathbf{x} \times \boldsymbol{\omega})^2 & m(\mathbf{x} \times \boldsymbol{\omega})^3 \\ -m(\mathbf{x} \times \boldsymbol{\omega})^1 & 0 & -\mathbf{p}^3 & \mathbf{p}^2 \\ -m(\mathbf{x} \times \boldsymbol{\omega})^2 & \mathbf{p}^3 & 0 & -\mathbf{p}^1 \\ -m(\mathbf{x} \times \boldsymbol{\omega})^1 & -\mathbf{p}^2 & \mathbf{p}^1 & 0 \end{bmatrix} \tag{1.9}$$

$$m\left(\mathbf{x} \times \boldsymbol{\omega}\right) \rightarrow -\mathbf{p}^i \quad \mathbf{p}^i \rightarrow m\left(\mathbf{x} \times \boldsymbol{\omega}\right) \tag{1.10}$$

Coriolis Momentum

$$*\mathbf{P}^{0i} = m\left(\mathbf{x} \times \boldsymbol{\omega}\right)^i \tag{1.11}$$

Spatial Component of Dual Lorentz Momentum

$$*\mathbf{P}^{i\nu} = [m\left(\mathbf{x} \times \boldsymbol{\omega}\right)] - [m\left(\mathbf{x} \times \boldsymbol{\omega}\right) + \mathbf{p}] \tag{1.12}$$

Antisymmetry of Dual Lorentz Momentum Tensor

$$*\mathbf{P}^{\mu\nu} = [m\left(\mathbf{x} \times \boldsymbol{\omega}\right) + \mathbf{p}] - [m\left(\mathbf{x} \times \boldsymbol{\omega}\right) + \mathbf{p}] = 0 \tag{1.13}$$

Gauss's law of Dual Momentum as divergence of Coriolis momentum

$$*\mathbf{P}^{0i}{}_{,i} = \nabla.m\left(\mathbf{x} \times \boldsymbol{\omega}\right) \tag{1.14}$$

Faraday's Momentum law

$$(\nabla \times \mathbf{p}) = - * \mathbf{P}^{i\nu}{}_{,\nu} - \frac{\partial}{\partial t}m(\mathbf{x} \times \boldsymbol{\omega}) \tag{1.15}$$

The curl of linear momentum is equal to dual momentum density and time varying momentum or Coriolis force

Dual Conservation Law of Momentum

$$*\mathbf{P}^{\mu\nu}{}_{,\nu\mu} = \nabla.m(\mathbf{x} \times \boldsymbol{\omega})_{,0} + \nabla.[\nabla \times \mathbf{p} - m(\mathbf{x} \times \boldsymbol{\omega})_{,0}] = 0 \tag{1.16}$$

Wave Equation of Coriolis Momentum

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)[m\left(\mathbf{x} \times \boldsymbol{\omega}\right)] = 0 \tag{1.17}$$

Wave Equation of linear Momentum

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\mathbf{p} = 0 \tag{1.18}$$

Wave Equation of Lorentz Momentum

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)[\mathbf{p} + m\left(\mathbf{x} \times \boldsymbol{\omega}\right)] = 0 \tag{1.19}$$

Table-1

Summary of Results of Antisymmetric Mechanical Lorentz Momentum Dynamics

Antisymmetric Lorentz Momentum Dynamics	**Mathematical Representation**
Nomenclature	**Mathematical Relations**
Linear Momentum as Newton's first Law	$\mathbf{P}^{0i} = \mathbf{p}^i$
Spatial Component of Lorentz Momentum	$\mathbf{P}^{i\nu} = [m(\mathbf{x}\times\boldsymbol{\omega})] - [m(\mathbf{x}\times\boldsymbol{\omega}) + \mathbf{p}]$
Antisymmetry of Lorentz Momentum Tensor	$\mathbf{P}^{\mu\nu} = [m(\mathbf{x}\times\boldsymbol{\omega}) + \mathbf{p}] - [m(\mathbf{x}\times\boldsymbol{\omega}) + \mathbf{p}] = 0$
Gauss's law of Momentum	$\mathbf{P}^{0i}{}_{,i} = \nabla.\mathbf{p}$
Ampere's Momentum law	$\mathbf{P}^{i\nu}{}_{,\nu} = [m(\nabla\times\mathbf{x}\times\boldsymbol{\omega}) - \mathbf{p}^i{}_{,0}]$
Conservation Law of Momentum	$\mathbf{P}^{\mu\nu}{}_{,\nu\mu} = \nabla.\mathbf{p}^i{}_{,0} + \nabla.[m(\nabla\times\mathbf{x}\times\boldsymbol{\omega}) - \mathbf{p}^i{}_{,0}] = 0$
Wave Equation of linear Momentum	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\mathbf{p} = 0$
Wave Equation of Coriolis Momentum	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)[(m\mathbf{x}\times\boldsymbol{\omega})] = 0$
Wave Equation of Lorentz Momentum	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)[\mathbf{p} + (m\mathbf{x}\times\boldsymbol{\omega})] = 0$
Obeys principle of relativity	Obeys principle of relativity

Summary of Results of Dual of Antisymmetric Mechanical Lorentz Momentum Dynamics

Dual of Antisym. Lorentz Momentum Dynamics	**Mathematical Representation**
Nomenclature	**Mathematical Relations**
Coriolis Momentum	$*\mathbf{P}^{0i} = m(\mathbf{x}\times\boldsymbol{\omega})^i$
Dual of Spatial Component of Lorentz Momentum	$*\mathbf{P}^{i\nu} = [m(\mathbf{x}\times\boldsymbol{\omega})] - [m(\mathbf{x}\times\boldsymbol{\omega}) + \mathbf{p}]$
Dual of Antisym. of Lorentz Momentum Tensor	$*\mathbf{P}^{\mu\nu} = [m(\mathbf{x}\times\boldsymbol{\omega}) + \mathbf{p}] - [m(\mathbf{x}\times\boldsymbol{\omega}) + \mathbf{p}] = 0$
Gauss's law of Dual Momentum	$*\mathbf{P}^{0i}{}_{,i} = \nabla.m(\mathbf{x}\times\boldsymbol{\omega})$
Faraday's Momentum law	$*\mathbf{P}^{i\nu}{}_{,\nu} = [(\nabla\times\mathbf{p}) - m(\mathbf{x}\times\boldsymbol{\omega})_{,0}]$
Gauss's law of Dual Momentum	$*\mathbf{P}^{0i}{}_{,i} = \nabla.m(\mathbf{x}\times\boldsymbol{\omega})$

Form invariance of the model can be seen by utilizing [12] and references therein.

Dual of Conservation Law of Momentum	$*\mathbf{P}^{\mu\nu}{}_{,\nu\mu} = \nabla.m(\mathbf{x}\times\boldsymbol{\omega})_{,0} + \nabla.[\nabla\times\mathbf{p} - m(\mathbf{x}\times\boldsymbol{\omega})_{,0}] = 0$
Wave Equation of Coriolis Momentum	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)[m(\mathbf{x}\times\boldsymbol{\omega})] = 0$
Wave Equation of Linear Momentum	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\mathbf{p} = 0$
Wave Equation of Lorentz Momentum	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)[\mathbf{p} + m(\mathbf{x}\times\boldsymbol{\omega})] = 0$

2. Derivation of Antisymmetric Mechanical Lorentz Force Tensor Dynamics

From elementary mechanics we know that rate of change of linear momentum is equal to force. Similarly, rate of change of antisymmetric mechanical Lorentz momentum tensor is equal to antisymmetric mechanical Lorentz force tensor. Since there is issue of dimensional inconsistency therefore mechanical Lorentz momentum dynamics becomes antisymmetric Lorentz force dynamics. One can check that the results of table-2 are equal to the rate of change of results of table-1.

Table-2

Antisymmetric Mechanical Lorentz Force Tensor Dynamics

Antisymmetric Lorentz Force Dynamics	**Mathematical Representation**
Lorentz Force Field Tensor $\mathbf{f}^{\mu\nu}$	$\begin{bmatrix} 0 & \mathbf{F}^1 & \mathbf{F}^2 & \mathbf{F}^3 \\ -\mathbf{F}^1 & 0 & (\mathbf{P}\times\boldsymbol{\omega})^3 & -(\mathbf{P}\times\boldsymbol{\omega})^2 \\ -\mathbf{F}^2 & -(\mathbf{P}\times\boldsymbol{\omega})^3 & 0 & (\mathbf{P}\times\boldsymbol{\omega})^1 \\ -\mathbf{F}^3 & (\mathbf{P}\times\boldsymbol{\omega})^2 & -(\mathbf{P}\times\boldsymbol{\omega})^1 & 0 \end{bmatrix}$ $\mathbf{f}^{0i} = \mathbf{F}^i \; \mathbf{f}^{ij} = \varepsilon^{ijk}(\mathbf{P}\times\boldsymbol{\omega})_k$
Newtonian Force Field	$\mathbf{f}^{0i} = \mathbf{F}^i$
Spatial Inertial Force Field	$\mathbf{f}^{i\nu} = (\mathbf{P}\times\boldsymbol{\omega}) - [\mathbf{F} + (\mathbf{P}\times\boldsymbol{\omega})]$
Antisymmetry of Mechanical Lorentz Force Tensor	$\mathbf{f}^{\mu\nu} = [\mathbf{F} + (\mathbf{P}\times\boldsymbol{\omega})] - [\mathbf{F} + (\mathbf{P}\times\boldsymbol{\omega})] = 0$
Mechanical Lorentz Force Maxwell's Equations	Formulae of Lorentz Force Maxwell's Equations
Gaussian Force Law	$\mathbf{f}^{0i}{}_{,i} = \nabla.\mathbf{F}$
Amperian Force Law	$\mathbf{f}^{i\nu}{}_{,\nu} = [(\nabla\times\mathbf{P}\times\boldsymbol{\omega}) - \mathbf{F}_{,0}]$
Conservation Law for Inertial Force	$\mathbf{f}^{\mu\nu}{}_{,\nu\mu} = \nabla.\mathbf{F}_{,0} + [\nabla.\nabla\times(\mathbf{P}\times\boldsymbol{\omega}) - \mathbf{F}_{,0}] = 0$
Newtonian Force Wave Equation	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\mathbf{F} = 0$
Coriolis Force Wave Equation	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)[(\mathbf{P}\times\boldsymbol{\omega})] = 0$

Wave Equation of Mechanical Lorentz Force	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)[\mathbf{F} + (\mathbf{P}\times\boldsymbol{\omega})] = 0$

Dual of Antisymmetric Lorentz Force Dynamics	**Mathematical Representation**
Dual of Lorentz Tensor $*\mathbf{f}^{\mu\nu}$	$\begin{bmatrix} 0 & (\mathbf{P}\times\boldsymbol{\omega})^1 & (\mathbf{P}\times\boldsymbol{\omega})^2 & (\mathbf{P}\times\boldsymbol{\omega})^3 \\ -(\mathbf{P}\times\boldsymbol{\omega})^1 & 0 & -\mathbf{F}^3 & \mathbf{F}^2 \\ -(\mathbf{P}\times\boldsymbol{\omega})^2 & \mathbf{F}^3 & 0 & -\mathbf{F}^1 \\ -(\mathbf{P}\times\boldsymbol{\omega})^3 & -\mathbf{F}^2 & \mathbf{F}^1 & 0 \end{bmatrix}$ $(\mathbf{P}\times\boldsymbol{\omega}) \to -\mathbf{F}^i$ $\mathbf{F}^i \to (\mathbf{P}\times\boldsymbol{\omega})$
Coriolis Force Field	$*\mathbf{f}^{0i} = (\mathbf{P}\times\boldsymbol{\omega})$
Dual of Spatial Inertial Force Field	$*\mathbf{f}^{i\nu} = (\mathbf{P}\times\boldsymbol{\omega}) - [(\mathbf{P}\times\boldsymbol{\omega}) - \mathbf{F}]$
Dual of Antisymmetric Lorentz Force Tensor	$*\mathbf{f}^{\mu\nu} = [\mathbf{F} + (\mathbf{P}\times\boldsymbol{\omega})] - [\mathbf{F} + (\mathbf{P}\times\boldsymbol{\omega})] = 0$
Mechanical Lorentz Force Maxwell's Equations	Formulae of Dual Mechanical Maxwell's Equations
Dual of Gauss's Law	$*\mathbf{f}^{0i}{}_{,i} = \nabla.(\mathbf{P}\times\boldsymbol{\omega})$
Faraday's Force Law	$*\mathbf{f}^{i\nu}{}_{,\nu} = -(\nabla\times\mathbf{F}) - \frac{\partial}{\partial t}[(\mathbf{P}\times\boldsymbol{\omega})]$
Dual Conservation Law	$*\mathbf{f}^{\mu\nu}{}_{,\nu\mu} = \nabla.(\mathbf{P}\times\boldsymbol{\omega})_{,0} - (\nabla.\nabla\times\mathbf{F}) - \nabla.(\mathbf{P}\times\boldsymbol{\omega})_{,0} = 0$
Coriolis Force Wave Equation	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)(\mathbf{P}\times\boldsymbol{\omega}) = 0$
Newtonian Force Wave Equation	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\mathbf{F} = 0$
Wave Equation of Mechanical Lorentz Force	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)[\mathbf{F} + (\mathbf{P}\times\boldsymbol{\omega})] = 0$

3. Antisymmetric Lorentz Momentum Electrodynamics in Noninertial Coordinate Metric

We have calculated the transformation of antisymmetric mechanical momentum electrodynamics in noninertial coordinate metric in an earlier paper so we present the results directly in table-3. Interested readers can see the detailed calculations there. We have employed color scheme to distinguish between classical theory by blue and noninertial effects by red color.

Table-3

Antisymmetric Mechanical Lorentz Momentum Dynamics in Noninertial Coordinate Metric

$$[\mathbf{g}^{\mu}_{\alpha}] = \begin{bmatrix} 1 & -\mathbf{x}_1 & -\mathbf{x}_2 & -\mathbf{x}_3 \\ -\mathbf{x}_1 & 1 & 0 & 0 \\ -\mathbf{x}_2 & 0 & 1 & 0 \\ -\mathbf{x}_3 & 0 & 0 & 1 \end{bmatrix}$$

Mechanical Lorentz Momentum Dynamics	**Mathematical Representation**
Transformation Law of MLM Tensor	$\mathbf{P}^{\mu'\nu'} = \mathbf{g}^{\mu'}_{\alpha}\mathbf{P}^{\alpha\nu'}$
Zero-point Plank's constant	$\mathbf{P}^{0'0'} = \mathbf{x}.\mathbf{p}$
Zero-point Negative Plank's constant	$\mathbf{P}^{i'i'} = -\mathbf{x}.\mathbf{p}$
Zero-point Action and reaction of Plank's constant	$\mathbf{P}^{\mu'\mu'} = -\mathbf{x}.\mathbf{p} - \mathbf{x}.\mathbf{p} = 0$
Temporal Component as 4D Newtonian Momentum	$\mathbf{P}^{0'\nu'} = \mathbf{p} + [\mathbf{x}.\mathbf{p}]$
Spatial Component of MLM Tensor	$\mathbf{P}^{i'\nu'} = [(m\mathbf{x}\times\boldsymbol{\omega}) - \mathbf{p}] - [\mathbf{x}.\mathbf{p} + (m\mathbf{x}\times\boldsymbol{\omega})]$
MLM Tensor as Coriolis Angular momentum	$\mathbf{P}^{\mu'\nu'} = \mathbf{x}\times(m\mathbf{x}\times\boldsymbol{\omega}) = 0$
Transformation Law for MLM Maxwell's Eqs.	$\mathbf{P}^{\mu'\nu'}{}_{,\nu'} = \mathbf{g}^{\mu'}_{\alpha}\mathbf{P}^{\alpha\nu'}{}_{,\nu'}$
Zero-Point MLM Maxwell's Equations	$\mathbf{P}^{\mu'\mu'}{}_{,\mu'} = \mathbf{x}.\mathbf{p}_{,0} - \nabla[\mathbf{x}.\mathbf{p}]$
Mechanical Momentum Gauss's law	$\mathbf{P}^{0'\nu'}{}_{,\nu'} = \nabla.\mathbf{p} - \mathbf{x}.[\nabla\times m(\mathbf{x}\times\boldsymbol{\omega}) - \mathbf{p}_{,0}]$
Mechanical Ampere's Momentum law	$\mathbf{P}^{i'\nu'}{}_{,\nu'} = m[(\nabla\times\mathbf{x}\times\boldsymbol{\omega}) - \mathbf{v}_{,0}] - \nabla[\mathbf{x}.\mathbf{p}]$
Transformation of Conservation Law	$\mathbf{P}^{\mu'\nu'}{}_{,\nu'\mu'} = \mathbf{g}^{\mu'}_{\alpha}\mathbf{P}^{\alpha\nu'}{}_{,\nu'\mu'}$
MLM Conservation Law by Matrix Method	$\mathbf{P}^{\mu'\nu'}{}_{,\nu'\mu'} = \nabla[\mathbf{x}.\mathbf{p}_{,0}] - \nabla[\mathbf{x}.\mathbf{P}_{,0}] = 0$
Conservation Law by Usual Method	$\mathbf{P}^{\mu'\nu'}{}_{,\nu'\mu'} = \mathbf{p}^{\mu\nu}{}_{,\nu\mu} + [\frac{\partial^2}{\partial t^2} - \nabla^2][\mathbf{x}.\mathbf{p}]$

Dual of Antisymmetric Mechanical Lorentz Momentum Dynamics in Noninertial Coordinate Metric

Mechanical Lorentz Momentum Dynamics	**Mathematical Representation**
Transformation of Dual of MLM Tensor	$*\mathbf{P}^{\mu'\nu'} = \mathbf{g}^{\mu'}_{\alpha} * \mathbf{P}^{\alpha\nu'}$
Zero-Point Coriolis Plank's Constant	$*\mathbf{P}^{0'0'} = \mathbf{x}.(m\mathbf{x}\times\boldsymbol{\omega}) = 0$

Zero-Point Negative Coriolis Plank's constant	$*\mathbf{P}^{i'i'}=-\mathbf{x}.\left(m\mathbf{x}\times\boldsymbol{\omega}\right)=0$
Zero-Point Balance of Coriolis Plank's constant	$*\mathbf{P}^{\mu'\mu'}=\mathbf{x}.\left(m\mathbf{x}\times\boldsymbol{\omega}\right)-\mathbf{x}.\left(m\mathbf{x}\times\boldsymbol{\omega}\right)=0$
Temporal Component of 4D ML Coriolis Momentum	$*\mathbf{P}^{0'\nu'}=\left(m\mathbf{x}\times\boldsymbol{\omega}\right)-\left[\left(\mathbf{x}\times\mathbf{p}\right)\right]$
Spatial Component of Dual MLM Tensor	$*\mathbf{P}^{i'\nu'}=\left[\mathbf{p}+\left(m\mathbf{x}\times\boldsymbol{\omega}\right)\right]-\mathbf{p}$
Dual of MLM Tensor as Angular Momentum	$*\mathbf{P}^{\mu'\nu'}=-\mathbf{x}\times\mathbf{p}$
Transformation Law Dual MLM Maxwell's Eqs.	$*\mathbf{P}^{\mu'\nu'}{}_{,\nu'}=\mathbf{g}^{\mu'}_{\alpha}*\mathbf{P}^{\alpha\nu'}{}_{,\nu'}$
Zero-Point Max. Eq. as Coriolis Energy-Coriolis Mom.	$*\mathbf{p}^{\mu'\mu'}{}_{,\mu'}=\mathbf{x}.(m\mathbf{x}\times\boldsymbol{\omega})_{,0}-\nabla\left[\mathbf{x}.(m\mathbf{x}\times\boldsymbol{\omega})\right]=0$
Dual of Gaussian Momentum law	$*\mathbf{P}^{0'\nu'}{}_{,\nu'}=\nabla.(m\mathbf{x}\times\boldsymbol{\omega})+\mathbf{x}.[(\nabla\times\mathbf{p})+(m\mathbf{x}\times\boldsymbol{\omega})_{,0}]$
Faraday's Law of Momentum	$*\mathbf{p}^{i'\nu'}{}_{,\nu'}=-[(\nabla\times\mathbf{p})+(m\mathbf{x}\times\boldsymbol{\omega})_{,0}]-\nabla\left[\mathbf{x}.\left(m\mathbf{x}\times\boldsymbol{\omega}\right)\right]$
Transformation of Dual Conservation Law	$*\mathbf{P}^{\mu'\nu'}{}_{,\nu'\mu'}=\mathbf{g}^{\mu'}_{\alpha}*\mathbf{P}^{\alpha\nu'}{}_{,\nu'\mu'}$
Dual of MLM Conservation Law by Matrix method	$*\mathbf{P}^{\mu'\nu'}{}_{,\nu'\mu'}=\nabla\left[\mathbf{x}.m\left(\mathbf{x}\times\boldsymbol{\omega}\right)_{,0}\right]-\nabla\left[\mathbf{x}.m\left(\mathbf{x}\times\boldsymbol{\omega}\right)_{,0}\right]=0$
Dual Conservation Law by Usual Method	$*\mathbf{P}^{\mu'\nu'}{}_{,\nu'\mu'}=*\mathbf{P}^{\mu\nu}{}_{,\nu\mu}+[\frac{\partial^2}{\partial t^2}-\nabla^2][\mathbf{x}.\left(m\mathbf{x}\times\boldsymbol{\omega}\right)]$

From table-3, we mention only some important results

i. Zero-point origin of Plank's constant

Zero-point Plank's constant

$$\mathbf{P}^{0'0'}=\mathbf{x}.\mathbf{p} \tag{3.1}$$

The dimensions of $\mathbf{x}.\mathbf{P}$ are that of Plank's constant or scalar angular momentum.

ii. Origin of Angular Momentum

$$*\mathbf{P}^{\mu'\nu'}=-\mathbf{x}\times\mathbf{p} \tag{3.2}$$

Dual of antisymmetric Lorentz momentum is transformed in to vector angular momentum with minus sign

iii. Zero-Point Momentum Maxwell's Equations as Unification of Energy and Momentum

$$\mathbf{P}^{\mu'\mu'}{}_{,\mu'}=\mathbf{x}.\mathbf{p}_{,0}-\nabla\left[\mathbf{x}.\mathbf{p}\right] \tag{3.3}$$

Note that the unit of $\mathbf{x}.\mathbf{p}_{,0}$ is that of energy and that of $-\nabla\left[\mathbf{x}.\mathbf{p}\right]$ is momentum. As the negative of gradient of energy is force so the negative of gradient of Plank's constant is momentum unlike special relativistic relation for energy and momentum where the time component mc of 4-momentum is coined in terms of energy/speed of light. One can see that our expression for energy-momentum is natural.

vi. Existence of 7D wave of Plank's constant Supplemented with Conservation Law
Conservation Law by Usual Method

$$\mathbf{P}^{\mu'\nu'}{}_{,\nu'\mu'} = \mathbf{p}^{\mu\nu}{}_{,\nu\mu} + \left[\frac{\partial^2}{\partial t^2} - \nabla^2\right][\mathbf{x}.\mathbf{p}] \tag{3.4}$$

Here $[\frac{\partial^2}{\partial t^2} - \nabla^2]$ is 7D wave operator and is an invariant quantity.
CONSEQUENCES:
1. Existence of Antisymmetric Gravitational Lorentz Momentum Tensor
As the mathematical framework of gravitational momentum tensor is same as that of mechanical momentum tensor so we write the summary of results with dual in tabe-4.
Table-4
Summary of Results of Antisymmetric Gravitational Lorentz Momentum Dynamics

Antisym. Gravitational Lorentz Momen. Tensor	**Mathematical Representation**
Antisymmetric Gravitational Lorentz Momentum Tensor $\mathbf{G}^{\mu\nu}$t	$mt\begin{bmatrix} 0 & \mathbf{g}^1 & \mathbf{g}^2 & \mathbf{g}^3 \\ -\mathbf{g}^1 & 0 & (\mathbf{g}t\times\boldsymbol{\omega})^3 & -(\mathbf{g}t\times\boldsymbol{\omega})^2 \\ -\mathbf{g}^2 & -(\mathbf{g}t\times\boldsymbol{\omega})^3 & 0 & (\mathbf{g}t\times\boldsymbol{\omega})^1 \\ -\mathbf{g}^3 & (\mathbf{g}t\times\boldsymbol{\omega})^2 & -(\mathbf{g}t\times\boldsymbol{\omega})^1 & 0 \end{bmatrix}$ $\mathbf{G}^{0i}t = m\mathbf{g}^i t$ $\mathbf{G}^{ij}t = mt\varepsilon^{ijk}(\mathbf{g}t\times\boldsymbol{\omega})_k$
Newtonian Gravitational Momentum Field	$\mathbf{G}^{0i}t = m\mathbf{g}^i t$
Spatial Component of Grav. Lorentz Momentum	$\mathbf{G}^{i\nu}t = m(\mathbf{g}t\times\boldsymbol{\omega})t - m[\mathbf{g}+(\mathbf{g}t\times\boldsymbol{\omega})]t$
Antisymmetry of Gravitational Lorentz Momentum	$\mathbf{G}^{\mu\nu}t = m[\mathbf{g}+(\mathbf{g}t\times\boldsymbol{\omega})]t - m[\mathbf{g}+(\mathbf{g}t\times\boldsymbol{\omega})]t = 0$
Gravitational Momentum Maxwell's Equations	**Mathematical Representation**
Gaussian Momentum Law for Gravitation	$\mathbf{G}^{0i}{}_{,i}t = \nabla.m\mathbf{g}t$
Amperian Momentum Law of Gravitation	$\mathbf{G}^{i\nu}{}_{,\nu}t = m[(\nabla\times\mathbf{g}t\times\boldsymbol{\omega}) - \mathbf{g}_{,0}]t$
Grav. Momen. Maxwell's Equations in Tensor Form	$\mathbf{G}^{\mu\nu}{}_{,\nu}t = \nabla.m\mathbf{g}t + m[(\nabla\times\mathbf{g}t\times\boldsymbol{\omega}) - \mathbf{g}_{,0}]t$
Gravitational Momentum Wave Equation	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)m\mathbf{g}t = 0$
Gravitational Coriolis Momentum Wave Equation	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)[m(\mathbf{g}t\times\boldsymbol{\omega})]t = 0$

Summary of Results of dual of Antisymmetric Gravitational Lorentz Momentum Dynamics

Gravitational Lorentz Momentum Wave Equation	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) m\left[\mathbf{g} + (\mathbf{g}t \times \boldsymbol{\omega})\right] t = 0$
Obeys principle of relativity	Obeys principle of relativity

Antisym. Gravitational Lorentz Momen. Tensor	**Mathematical Representation**
Antisymmetric Dual of Gravitational Lorentz Momentum Tensor $*\mathbf{G}^{\mu\nu}$t	$mt \begin{bmatrix} 0 & (\mathbf{g}t\times\boldsymbol{\omega})^1 & (\mathbf{g}t\times\boldsymbol{\omega})^2 & (\mathbf{g}t\times\boldsymbol{\omega})^3 \\ -(\mathbf{g}t\times\boldsymbol{\omega})^1 & 0 & -\mathbf{g}^3 & \mathbf{g}^2 \\ -(\mathbf{g}t\times\boldsymbol{\omega})^2 & \mathbf{g}^3 & 0 & -\mathbf{g}^1 \\ -(\mathbf{g}t\times\boldsymbol{\omega})^3 & -\mathbf{g}^2 & \mathbf{g}^1 & 0 \end{bmatrix} *$ $m\mathbf{g}^i t \to m(\mathbf{g}t\times\boldsymbol{\omega})t$, $m(\mathbf{g}t\times\boldsymbol{\omega})t \to -m\ \mathbf{g}it$
Gravitational Coriolis Momentum	$*\mathbf{G}^{0i}t = m(\mathbf{g}t\times\boldsymbol{\omega})t$
Spatial Component of Dual Grav. Lorentz moment.	$*\mathbf{G}^{i\nu}t = m\left(\mathbf{g}t\times\boldsymbol{\omega}\right)t - m\left[\mathbf{g} + (\mathbf{g}t\times\boldsymbol{\omega})\right]t$
Antisymmetry of Dual of Grav. Lorentz Momentum	$*\mathbf{G}^{\mu\nu}t = m\left[\mathbf{g} + (\mathbf{g}t\times\boldsymbol{\omega})\right]t - m\left[\mathbf{g} + (\mathbf{g}t\times\boldsymbol{\omega})\right]t = 0$
Gravitational Momentum Maxwell's Equations	**Formulae**
Dual of Gaussian Momentum law of gravitation	$*\mathbf{G}^{0i}{}_{,i}t = \nabla.m(\mathbf{g}t\times\boldsymbol{\omega})t$
Faraday's Momentum Law of Gravitation	$*\mathbf{G}^{i\nu}{}_{,\nu}t = -(\nabla\times m\mathbf{g}t) - \frac{\partial}{\partial t}m[(\mathbf{g}t\times\boldsymbol{\omega})]t$
Dual Grav. Momentum Maxwell's Eqs. in Tensor	$*\mathbf{G}^{\mu\nu}{}_{,\nu}t = \nabla.m\mathbf{g}t + m\left[\nabla\times(\mathbf{g}t\times\boldsymbol{\omega}) - \mathbf{g}_{,0}\right]t$
Gravitational Coriolis Momentum Wave Equation	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\left[m\left(\mathbf{g}t\times\boldsymbol{\omega}\right)\right]t = 0$
Gravitational Momentum Wave Equation	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) m\mathbf{g}t = 0$
Dual of Grav. Lorentz Momentum Wave Equation	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) m\left[\mathbf{g} + (\mathbf{g}t\times\boldsymbol{\omega})\right]t = 0$
Obeys principle of relativity	Obeys principle of relativity

2. Existence of Antisymmetric Electromagnetic Lorentz Momentum Tensor

Table-5

Summary of Antisymmetric Electromagnetic Lorentz Momentum Electrodynamics

Antisymmet. Lorentz Momentum Electrodynamics	**Mathematical representation**
Structure of Lorentz Momentum Tensor $\mathbf{P}^{\mu\nu}$	$qt\begin{bmatrix} 0 & \mathbf{E}^1 & \mathbf{E}^2 & \mathbf{E}^3 \\ -\mathbf{E}^1 & 0 & (\mathbf{v}\times\mathbf{B})^3 & -(\mathbf{v}\times\mathbf{B})^2 \\ -\mathbf{E}^2 & -(\mathbf{v}\times\mathbf{B})^3 & 0 & (\mathbf{v}\times\mathbf{B})^1 \\ -\mathbf{E}^3 & (\mathbf{v}\times\mathbf{B})^2 & -(\mathbf{v}\times\mathbf{B})^1 & 0 \end{bmatrix}$ $\mathbf{P}^{0i} = q\mathbf{E}^i t$ $\mathbf{P}^{ij} = q\varepsilon^{ijk}(\mathbf{v}\times\mathbf{B})_k t$
Electric Momentum Field	$\mathbf{P}^{0i} = q\mathbf{E}t$
Magnetic Momentum Field	$\mathbf{p}^{i\nu} = -q\mathbf{E}t + q[(\mathbf{v}\times\mathbf{B}) - (\mathbf{v}\times\mathbf{B})]t$
Antisymmetry of Lorentz Momentum Tensor	$\mathbf{P}^{\mu\nu} = q[\mathbf{E} + (\mathbf{v}\times\mathbf{B})]t - q[\mathbf{E} + (\mathbf{v}\times\mathbf{B})]t = 0$
Lorentz Momentum Maxwell's Equations	**Formulae of Momentum Maxwell's Equations**
Gaussian Momentum Law	$\mathbf{P}^{0i}{}_{,i} = \nabla.q\mathbf{E}t$
Amperian Momentum Law	$\mathbf{P}^{i\nu}{}_{,\nu} = q\left[\nabla\times(\mathbf{v}\times\mathbf{B}) - \mathbf{E}_{,0}\right]t$
Lorentz Momen. Maxwell's equations in Tensor Form	$\mathbf{P}^{\mu\nu}{}_{,\nu} = \nabla.q\mathbf{E}t + q\left[(\nabla\times\mathbf{v}\times\mathbf{B}) - \mathbf{E}_{,0}\right]t$
Lorentz Momentum Conservation Law in Tensor Form	$\mathbf{P}^{\mu\nu}{}_{,\nu\mu} = \nabla.q\mathbf{E}_{,0}t + \left[\nabla.q\left(\nabla\times\mathbf{v}\times\mathbf{B}\right) - q\mathbf{E}_{,0}\right]t = 0$
Electric Momentum Wave Equation	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) q\mathbf{E}t = 0$
Magnetic Momentum Wave Equation	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\left[q(\mathbf{v}\times\mathbf{B})\right]t = 0$
Lorentz Momentum Wave Equation	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) q\left[\mathbf{E} + (\mathbf{v}\times\mathbf{B})\right]t = 0$
Loretz Momentum Poynting Vector	$\mathbf{S} = q^2\left[\mathbf{E}\times(\mathbf{v}\times\mathbf{B})\right]t$
Obeys principle of relativity	Obeys principle of relativity

Dual of Antisymmetric Electromagnetic Lorentz Momentum Electrodynamics

Dual Antisymmet. Momentum Electrodynamics	**Mathematical Representation**
Dual of Antisymmetric Lorentz Momentum Tensor $*\mathbf{P}^{\mu\nu}$	$qt\begin{bmatrix} 0 & (\mathbf{v}\times\mathbf{B})^1 & (\mathbf{v}\times\mathbf{B})^2 & (\mathbf{v}\times\mathbf{B})^3 \\ -(\mathbf{v}\times\mathbf{B})^1 & 0 & -\mathbf{E}^3 & \mathbf{E}^2 \\ -(\mathbf{v}\times\mathbf{B})^2 & \mathbf{E}^3 & 0 & -\mathbf{E}^1 \\ -(\mathbf{v}\times\mathbf{B})^3 & -\mathbf{E}^2 & \mathbf{E}^1 & 0 \end{bmatrix}$ $q\mathbf{E}t \to q(\mathbf{v}\times\mathbf{B})^i t$ $q(\mathbf{v}\times\mathbf{B})^i t \to -\, q\mathbf{E}^i t$

Dual of Electric Momentum or Coriolis Momentum	$*\mathbf{P}^{0i} = q\left(\mathbf{v}\times\mathbf{B}\right)t$
Dual of Magnetic Momentum Field	$*\mathbf{P}^{i\nu} = q\mathbf{E}t -\ q[\mathbf{E}+(\mathbf{v}\times\mathbf{B})]t$
Antisymmetry of Dual Lorentz Momentum Tensor	$*\mathbf{P}^{\mu\nu} = q[\mathbf{E}+(\mathbf{v}\times\mathbf{B})]t - q[\mathbf{E}+(\mathbf{v}\times\mathbf{B})]t = 0$
Dual of Lorentz Momentum Maxwell's Equations	**Formulae of Momentum Maxwell's Equations**
Gaussian Force Law for Magnetic Momentum	$*\mathbf{P}^{0i}{}_{,i} = \nabla.q(\mathbf{v}\times\mathbf{B})t$
Faraday's Momentum Law	$*\mathbf{P}^{i\nu}{}_{,\nu} = -(\nabla\times q\mathbf{E}t) - \frac{\partial}{\partial t}\mathrm{q}[(\mathbf{v}\times\mathbf{B})\,t]$
Dual Lorentz Momen. Maxwell's equations in Tensor	$*\mathbf{P}^{\mu\nu}{}_{,\nu} = \nabla.q(\mathbf{v}\times\mathbf{B})t - [(\nabla\times q\mathbf{E}t) + \frac{\partial}{\partial t}q\left(\mathbf{v}\times\mathbf{B}\right)t]$
Dual Lorentz Momentum Conservation Law	$*\mathbf{P}^{\mu\nu}{}_{,\nu\mu} = \frac{\partial}{\partial t}q\nabla.\mathbf{v}\times\mathbf{B}t\ -\ \nabla.\nabla\times q\mathbf{E}t - \frac{\partial}{\partial t}\nabla.q\mathbf{v}\times\mathbf{B}t = 0$
Magnetic Momentum Wave Equation	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)[q(\mathbf{v}\times\mathbf{B})]\,t = 0$
Electric Momentum Wave Equation	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)q\mathbf{E}t = 0$
Dual Lorentz Momentum Wave Equation	$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)q\left[\mathbf{E}+(\mathbf{v}\times\mathbf{B})\right]t = 0$
Dual Loretz Momentum Poynting Vector	$\mathbf{S} = q^2\left[\mathbf{E}\times(\mathbf{v}\times\mathbf{B})\right]t$
Obeys principle of relativity	Obeys principle of relativity

4. Discussion and Comparison

One of the outstanding predictions of antisymmetric mechanical Lorentz momentum tensor is the presence of Newton's first and 2^{nd} law of motion and the concept of action and reaction. Tensorial nature of momentum has extended the scope of fundamental relations in classical dynamics as compared to special relativistic ideas of dynamics. The dawn of force dynamic theories of electrodynamics, gravitation and mechanical electrodynamics via antisymmetric Lorentz force tensor naturally led towards the more fundamental dynamics viz. antisymmetric Lorentz momentum dynamics in the context of same theories. It is entirely new framework and there doesn't exist a single example in the contemporary world of relativity theory and electrodynamics. The transformation of Lorentz momentum dynamics in noninertial coordinate metric based on STL for 4-vectors and tensors tells us about the origin of Plank's constant which is scalar angular momentum, vector angular momentum, zero-point Momentum Maxwell's equations as unification of energy and momentum unlike special relativistic interpretation. The existence of 7D wave of Plank's constant supplemented with classical conservation law is very strange result. These ideas are leading towards relativistic origin of quantum theory without any artificial attempt. We hope that our contemporaries will promote the ideas in a deeper insight as well as in applications.

Mechanical Momentum Maxwell's Equations

Gauss's law of Momentum

$$\mathbf{P}^{0i}{}_{,i} = \nabla.\mathbf{p} \tag{4.1}$$

Ampere's Law of Momentum

$$(\nabla \times m\mathbf{x} \times \boldsymbol{\omega}) = \mathbf{P}^{i\nu}{}_{,\nu} + \frac{\partial}{\partial t}\mathbf{p} \tag{4.2}$$

Gauss's law of Dual Momentum

$$*\mathbf{P}^{0i}{}_{,i} = \nabla.(m\mathbf{x} \times \boldsymbol{\omega}) \tag{4.3}$$

Faraday's Momentum law

$$(\nabla \times \mathbf{p}) = *\mathbf{P}^{i\nu}{}_{,\nu} - \frac{\partial}{\partial t}(m\mathbf{x} \times \boldsymbol{\omega}) \tag{4.4}$$

Wave Equation of Coriolis Momentum

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)[(m\mathbf{x} \times \boldsymbol{\omega})] = 0 \tag{4.5}$$

Wave Equation of linear Momentum

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\mathbf{p} = 0 \tag{4.6}$$

Wave Equation of Lorentz Momentum

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)[\mathbf{p} + (m\mathbf{x} \times \boldsymbol{\omega})] = 0 \tag{4.7}$$

Mechanical Lorentz Momentum Maxwell's Equations in Noninertial Coordinate Metric
Gauss's Law of Momentum

$$\mathbf{P}^{0'\nu'}{}_{,\nu'} = \nabla.\mathbf{p} + \mathbf{x}.\mathbf{F} \tag{4.8}$$

Ampere's Law of Momentum

$$(\nabla \times m\mathbf{x} \times \boldsymbol{\omega}) = \mathbf{P}^{i'\nu'}{}_{,\nu'} + \mathbf{p}_{,0} + \nabla[\mathbf{x}.\mathbf{p}] \tag{4.9}$$

Dual Gauss's law for Momentum

$$*\mathbf{P}^{0'\nu'}{}_{,\nu'} = \nabla.(m\mathbf{x} \times \boldsymbol{\omega}) + \mathbf{x}.[(\nabla \times \mathbf{p})] \tag{4.10}$$

Faraday's Law of Momentum

$$(\nabla \times \mathbf{p}) = - * \mathbf{p}^{i'\nu'}{}_{,\nu'} - \left[(m\mathbf{x} \times \boldsymbol{\omega})_{,0}\right] \tag{4.11}$$

5. Conclusion

Electrodynamic paradigm for antisymmetric mechanical Lorentz momentum tensor is entirely a new contribution in the history of relativistic theory of dynamics along with gravitational momentum dynamics and electrodynamic momentum theory. These models obey principle of relativity, conservation law and symmetry. This model contains Newton's laws of motion as an intrinsic property which is a basic requirement of any generalized physical theory. Our next model is about the electrodynamic paradigm for antisymmetric Lorentz position tensor which is the fuel tank for these models and higher order theories of inertia, gravitation and electrodynamics.

Dedicated to the ideal of my Artist and His all students.
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