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Vikor Method Based Supplier Selection Under Nonlinear Diophantine Rough Fuzzy Environment

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ABSTRACT: Uncertainty and imprecision are inherent in real-world decision-making problems, requiring robust mathematical frameworks for accurate modeling. This paper introduces a novel approach by integrating vikor method with Non-Linear Diophantine Rough Fuzzy Sets (NLDRFS) to enhance information representation and processing in uncertain environments. The proposed model leverages the flexibility of rough fuzzy sets to handle vagueness, while the vikor method provide a smooth, nonlinear aggregation mechanism that improves decision-making efficiency. Furthermore, the Diophantine structure introduces a new dimension of mathematical rigor, allowing a more precise approximation of uncertainty. We explore theoretical properties, develop algebraic operations, and illustrate practical applications in multi-criteria decision-making (MCDM) and pattern recognition. Comparative analyses demonstrate that our proposed framework outperforms existing rough fuzzy set models in handling complex uncertainties.

Key Words: Rough fuzzy sets, vikor method, Diophantine fuzzy sets, nonlinear aggregation, decision-making.

Contents

1. Introduction

In real-life decision-making, dealing with vague and uncertain data has become a significant challenge. To address the complexities and uncertainties inherent in real-world problems, Zadeh [1] introduced the concept of fuzzy sets (FS). While fuzzy sets were a breakthrough, they did not account for the possibility of negative opinions or uncertainty from experts, which created limitations in tackling real-life problems. To overcome this, Atanassov [2] extended fuzzy sets by proposing intuitionistic fuzzy sets (IFS). This new approach introduced a mechanism where the sum of membership and non-membership degrees is constrained between 0 and 1, providing a more comprehensive way to represent uncertainty and expert opinions.In some real-life decision-making scenarios, the sum of the degree of membership and non-membership, as determined by experts, can exceed one. To address this issue, Yager [3] introduced the concept of Pythagorean fuzzy sets (PFS). PFS allows for membership and non-membership degrees, where the sum of their squares is less than or equal to 1. However, even with this approach, there are situations where the sum of the squares can exceed 1 , and these cases cannot be fully addressed by PFS theory. To tackle this limitation, Yager [4] proposed the q-rung orthopair fuzzy set (q-ROFS), which introduces a more flexible condition. The key advantage of q-ROFS is that the value of q can be determined by the decision makers (DMs), giving them more flexibility in defining the membership and non-membership degrees. To address these limitations, Riaz and Hashmi [5] introduced the concept of Linear Diophantine Fuzzy Set (LDFS) by incorporating reference parameters into the definition of fuzzy sets. These reference parameters allow LDFS to overcome the constraints faced by traditional fuzzy sets,

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offering a more flexible and efficient method for handling uncertain data. The introduction of reference parameters significantly broadens the scope of theoretical knowledge, making it more inclusive and applicable to a wider range of problems. Information measures such as distance, similarity, and entropy play a crucial role in various fields, including medical diagnosis, clustering, and pattern recognition. The distance measure helps differentiate between two fuzzy sets, enabling us to assess how far apart they are. The similarity measure is particularly useful for dealing with vague and uncertain data, as it helps determine the degree of similarity between fuzzy concepts, guiding us towards a final decision. The entropy measure not only gauges the degree of uncertainty or fuzziness in fuzzy sets, but it is also widely used for attribute weighting in multi-criteria decision-making (MCDM) problems. Over the years, these measures—distance, similarity, and entropy—have gained increasing attention from researchers. They are now recognized as essential tools for quantifying and managing uncertain information, making them invaluable in decision-making processes and various applications across different domains. Atanassov [6] was the first to introduce a distance measure for intuitionistic fuzzy sets (IFS). Later, Burillo and Bustince [7] expanded on this by providing distance measures and entropy measures for IFSs. Szmidt and Kacprzyk [8] introduced new definitions for distance measures applied to IFSs. Grzegorzewski [9] further refined these measures by defining distance measures for IFSs based on the Hausdorff metric. Wang and Xin [10] introduced several new intuitionistic fuzzy distance measures to address various needs. In addition, Hung and Yang [11] developed multiple fuzzy similarity measures for IFSs, including two new similarity measures. Iancu [12] proposed similarity measures for IFSs based on min-max operators. Gohain et al. [13] defined a novel distance measure grounded in the concept of information within IFSs, which they applied to pattern recognition, medical diagnosis, and even the decision-making process for selecting face masks during the COVID-19 pandemic. Lastly, Joshi and Kumar [14] proposed an α-ordered entropy measure for IFSs, demonstrating its application in multi-attribute decision-making (MADM) problems. Thao [15] developed an entropy measure for intuitionistic fuzzy sets (IFS) based on divergence measures, using this entropy to determine criteria weights for solving multi-criteria decision-making (MCDM) problems. Zhang and Xu [16] introduced a distance measure for Pythagorean fuzzy sets (PFSs) and extended the TOPSIS method to work with PFSs. Building on this, Peng et al. [17] proposed distance, similarity, and entropy measures for PFSs, exploring their interrelationships and applying them to areas such as pattern recognition, clustering analysis, and medical diagnosis. Biswas and Sarkar [18] defined an entropy measure for PFSs based on distance measures, applying the proposed entropy to determine the weights of criteria in decision-making processes. Hussian and Yang [19] introduced distance and similarity measures for PFSs, again using the Hausdorff metric, and applied these measures to the PF-TOPSIS method. Ejegwa [20] provided an axiomatic definition for distance and similarity measures in Pythagorean fuzzy sets (PFSs), and extended several existing distance and similarity measures from intuitionistic fuzzy sets (IFS) to PFS. Sarkar and Biswas [21] proposed new distance and entropy measures for PFSs, developing an entropy-based weight model to determine criteria weights for Pythagorean fuzzy multi-criteria decision-making (PF-MCDM) problems. Wan et al. [22] introduced an entropy measure for PFSs to calculate attribute weights. Additionally, Yang and Hussain [23] defined Pythagorean fuzzy entropy measures using probabilistic-type, distance, and min-max operators. Thao and Smarandache [24] introduced an entropy measure for Pythagorean fuzzy sets (PFS), extending the entropy measure of intuitionistic fuzzy sets (IFS), and used this proposed entropy to calculate weights in the COPRAS method. Xu et al. [25] provided a new definition for the entropy measure in PFS and applied the entropy weight formula to solve multi-criteria decision-making (MCDM) problems. Peng and Liu [26] proposed several q-rung orthopair fuzzy information measures, including distance, similarity, entropy, and inclusion measures, and explored the relationships between them. They also applied the similarity measure to fields like pattern recognition, clustering analysis, and medical diagnosis. Peng and Dai [27] presented formulas for distance and similarity measures, along with two algorithms to solve q-ROF decision-making problems using CODAS and multi-parametric similarity measures. Pinar and Boran [28] developed a new distance measure for q-rung orthopair fuzzy sets (q-ROFS) and applied it in q-ROF TOPSIS and q-ROF ELECTRE methods. Verma [29] introduced an α-ordered entropy measure for q-ROFS, using it to determine attribute weights for solving MCDM problems. Liu et al. [30] defined entropy measures for q-ROFS to obtain attribute weights, further enhancing decision-making processes. In the context of Linear Diophantine Fuzzy Sets (LDFS), Mohammad et al. [31] and Gül and Aydoğdu [32] independently introduced the Euclidean and

Hamming distance measures, both in parallel. Additionally, Mohammad et al. [31] generalized these distance measures and proposed several similarity measures for LDFS, applying them to medical diagnosis problems. Gül and Aydoğdu [32] introduced the first entropy measure for LDFS and utilized this entropy measure to calculate attribute weights in a novel extension of the LDFS-TOPSIS method. The extension of LDFSs has been further explored by various researchers. For instance, Riaz et al. [33] defined spherical LDFSs, while Mahmood et al. [34] introduced interval-valued LDFSs. Almagrabi et al. [35] proposed q-LDFSs and aggregation operators for q-LDSS. Kamacı [36] defined complex LDFSs and proposed similarity measures for these complex LDFSs. Ashraf et al. [37] proposed a generalization of q-LDFSs, which they called spherical q-LDFSs. Additionally, Riaz et al. [38] introduced prioritized aggregation operators for LDFS to solve the problem of selecting the best third-party reverse logistics provider. Alnoor et al. [39] extended linear Diophantine fuzzy rough sets (LDFRSs) to multi-criteria decision-making (MCDM) and applied them to the issue of sustainable transportation. Over the years, many MCDM methods have been developed to address real-life problems involving different alternatives and multiple criteria in the decision-making process. These methods play a crucial role in helping decision-makers navigate complex choices with diverse factors. Fuzzy logic has been widely utilized in the literature to address the uncertainties that arise during the decision-making process in multi-criteria decision-making (MCDM) methods [40], [41], [42], [43], [44], [45], [46], [47]. One of the key challenges in MCDM problems is determining the importance or weight of each attribute when evaluating the criteria. In the literature, there are two main types of weighting methods. The subjective weighting method involves assigning weights based on the decision-makers' preferences and judgments, usually represented numerically. On the other hand, the objective weighting method determines the weights through mathematical models, without considering the decision-makers' personal preferences. One widely-used objective weighting method is entropy. In the entropy-based method, the importance of an attribute is determined by the degree of dispersion or variation in the evaluation of different alternatives. One of the well-known multi-criteria decision-making (MCDM) methods is VIKOR (VIsekriterijumska optimizacija i KOmpromisno Resenje). VIKOR focuses on proposing compromise solutions that help decision-makers reach a final decision in MCDM problems, especially when dealing with non-commensurable and conflicting criteria [48]. The method aims to maximize the "group utility" of the majority while minimizing the individual regret of the "opponent," ultimately providing more reasonable and balanced sorting results. The rest of the material is organized in the following fashion:In Sect. 2, the main ideas are explained. In Section 3, we define the operational laws of NLDRF data. In Sect 4, we define the MCDM method and develops the numerical example for teaching excellence assessment of global sequence in advanced occupational schools. Some conclusions and future work are presented in Sect. 5.

2. Backgroud

Definition 2.1 *[30] Let us consider that $\Phi \neq X$ and by a fuzzy set $\gamma = \left\{ \begin{array}{c} \langle x, \mu_{\gamma(x)} \rangle \\ : x \in X \end{array} \right\}$, $\mu_{\gamma(x)}$ is a mapping from X to $[0,1]$ present membership task of an component x in X .*

Definition 2.2 *[31] Let $G_1 = [\xi_1, \chi_1]$ and $G_2 = [\xi_2, \chi_2]$ be two IFNs, $\lambda > 0$, then*

$G_1 \oplus G_2 = [(\xi_1 + \xi_2 - \xi_1 \xi_2), (\chi_1, \chi_2)]$,
$G_1 \otimes G_2 = [(\xi_1 \xi_2), (\chi_1 + \chi_2 - \chi_1 \chi_2)]$,
$\lambda G_1 = [1 - (1 - \xi_1)^\lambda, \chi_1]$,
$G_1^\lambda = [(\xi_1^-)^\lambda, 1 - (1 - \chi_1^-)^\lambda]$.

Definition 2.3 *[45] Let $G_1 = \langle [\xi_1, \chi_1], r_1 \rangle$ and $G_2 = \langle [\xi_2, \chi_2], r_2 \rangle$ be two CFNs, $\lambda > 0$, then*

$G_1 \oplus G_2 = \left\langle [(\xi_1^- + \xi_2^- - \xi_1^- \xi_2^-), ((\chi_1^+ + \chi_2^+ - \chi_1^+ \chi_2^+))], (r_1, r_2) \right\rangle$;
$G_1 \otimes G_2 = \left\langle [(\xi_1^- \xi_2^-), (\chi_1^+ \chi_2^+)], r_1, r_2 \right\rangle$;
$\lambda G_1 = \left\langle [1 - (1 - \xi_1^-)^\lambda, 1 - (1 - \chi_1^+)^\lambda], r_1 \right\rangle$;
$G_1^\lambda = \left\langle [(\xi_1^-)^\lambda, (\chi_1^-)^\lambda], 1 - (1 - r_1^-)^\lambda \right\rangle$.

Definition 2.4 *[45]Let $G_1 = \langle [\xi_1, \chi_1], r_1 \rangle$ be the CFNs, then the score function is presented as $H = \frac{\langle [\xi_1 + \chi_1] - r_1 \rangle}{3}$.*

3. Non Linear Diophantine Rough Fuzzy Set

Definition 3.1 *Let R be refrence set. A non linear diophantine fuzzy rough set* **D** *is defined as* $D=\{< Q, \overbrace{{}^{\varphi}u_D(Q)}, \overbrace{{}^{\varphi}\beta_D(Q)} >< \overbrace{{}^{\varphi}\eta_D(Q)}, \overbrace{{}^{\varphi}\delta_D(Q)} : Q\in R >,$
$< Q, \underbrace{{}^{\varphi}u_D(Q)}, \underbrace{{}^{\varphi}\beta_D(Q)} >< \underbrace{{}^{\varphi}\eta_D(Q)}, \underbrace{{}^{\varphi}\delta_D(Q)} : Q\in R >\}, \varphi = 1,2....n,$

where ${}^{\varphi}u_D(Q), {}^{\varphi}\beta_D(Q), {}^{\varphi}\eta_D(Q), {}^{\varphi}\delta_D(Q)\epsilon[0,1]$ *and* ${}^{\varphi}u_D(Q)$ *is membership* ${}^{\varphi}\beta_D(Q)$ *non membership* ${}^{\varphi}\eta_D(Q), {}^{\varphi}\delta_D(Q)$ *is parameter . This functions fulfill the limitation*

$$0 \leq \overbrace{{}^{\varphi}\eta_D(Q){}^{\varphi}u_D(Q)} + \overbrace{{}^{\varphi}\delta_D(Q){}^{\varphi}\beta_D(Q)}, \underbrace{{}^{\varphi}\eta_D(Q){}^{\varphi}u_D(Q)} + \underbrace{{}^{\varphi}\delta_D(Q){}^{\varphi}\beta_D(Q)} \leq 1.$$

Definition 3.2 *Let* $D = \left\{ \begin{array}{c} < Q, \overbrace{{}^{\varphi}u_D(Q)}, \overbrace{{}^{\varphi}\beta_D(Q)} > \\ < \overbrace{{}^{\varphi}\eta_D(Q)}, \overbrace{{}^{\varphi}\delta_D(Q)} : Q\in R >, \\ < Q, \underbrace{{}^{\varphi}u_D(Q)}, \underbrace{{}^{\varphi}\beta_D(Q)} > \\ < \underbrace{{}^{\varphi}\eta_D(Q)}, \underbrace{{}^{\varphi}\delta_D(Q)} : Q\in R > \end{array} \right\}$, *and* $\hat{G} = \left\{ \begin{array}{c} < Q, \overbrace{{}^{\varphi}u_{\hat{G}}(Q)}, \overbrace{{}^{\varphi}\beta_{\hat{G}}(Q)} > \\ < \overbrace{{}^{\varphi}\eta_{\hat{G}}(Q)}, \overbrace{{}^{\varphi}\delta_{\hat{G}}(Q)} : Q\in R >, \\ < Q, \underbrace{{}^{\varphi}u_{\hat{G}}(Q)}, \underbrace{{}^{\varphi}\beta_{\hat{G}}(Q)} > \\ < \underbrace{{}^{\varphi}\eta_{\hat{G}}(Q)}, \underbrace{{}^{\varphi}\delta_{\hat{G}}(Q)} : Q\in R > \end{array} \right\}$ *be two NLDRFS on R then*

1: $D^c = \left\{ \begin{array}{c} < Q, \overbrace{{}^{\varphi}u_D(Q)}, \overbrace{{}^{\varphi}\beta_D(Q)} > \\ < \overbrace{{}^{\varphi}\eta_D(Q)}, \overbrace{{}^{\varphi}\delta_D(Q)} : Q\in R >, \\ < Q, \underbrace{{}^{\varphi}u_D(Q)}, \underbrace{{}^{\varphi}\beta_D(Q)} > \\ < \underbrace{{}^{\varphi}\eta_D(Q)}, \underbrace{{}^{\varphi}\delta_D(Q)} : Q\in R > \end{array} \right\}$

2:$D = \hat{G}$ *if and only if*

$$\overbrace{{}^{\varphi}u_{\hat{G}}(Q)} = \overbrace{{}^{\varphi}u_D(Q)}, \overbrace{{}^{\varphi}\beta_{\hat{G}}(Q)} = \overbrace{{}^{\varphi}\beta_D(Q)}, \overbrace{{}^{\varphi}\eta_{\hat{G}}(Q)} = \overbrace{{}^{\varphi}\eta_D(Q)}, \overbrace{{}^{\varphi}\delta_{\hat{G}}(Q)} = \overbrace{{}^{\varphi}\delta_D(Q)}, \underbrace{{}^{\varphi}u_D(Q)} = \underbrace{{}^{\varphi}u_{\hat{G}}(Q)}$$
$$\underbrace{{}^{\varphi}\beta_{\hat{G}}(Q)} = \underbrace{{}^{\varphi}\beta_D(Q)}, \underbrace{{}^{\varphi}\eta_{\hat{G}}(Q)} = \underbrace{{}^{\varphi}\eta_D(Q)}, \underbrace{{}^{\varphi}\delta_{\hat{G}}(Q)} = \underbrace{{}^{\varphi}\delta_D(Q)}$$

3:$D \sqsubseteq \hat{G}$ if and only if

$$\overbrace{{}^{\varphi}u_{\hat{G}}(Q)} \geq \overbrace{{}^{\varphi}u_D(Q)}, \overbrace{{}^{\varphi}\beta_{\hat{G}}(Q)} \geq \overbrace{{}^{\varphi}\beta_D(Q)}, \overbrace{{}^{\varphi}\eta_{\hat{G}}(Q)} \geq \overbrace{{}^{\varphi}\eta_D(Q)}, \overbrace{{}^{\varphi}\delta_{\hat{G}}(Q)} \geq \overbrace{{}^{\varphi}\delta_D(Q)}, \underbrace{{}^{\varphi}u_D(Q)} \geq \underbrace{{}^{\varphi}u_{\hat{G}}(Q)}$$
$$\underbrace{{}^{\varphi}\beta_{\hat{G}}(Q)} \geq \underbrace{{}^{\varphi}\beta_D(Q)}, \underbrace{{}^{\varphi}\eta_{\hat{G}}(Q)} \geq \underbrace{{}^{\varphi}\eta_D(Q)}, \underbrace{{}^{\varphi}\delta_{\hat{G}}(Q)} \geq \underbrace{{}^{\varphi}\delta_D(Q)}$$

$$4{:}D\cup\hat{G} = \left\{ \left(\begin{array}{c} Q, \left\langle \begin{array}{c} \max(\overbrace{{}^{\varphi}u_{\hat{G}}(Q)}, \overbrace{{}^{\varphi}u_D(Q)}) \\ \min(\overbrace{{}^{\varphi}\beta_{\hat{G}}(Q)}, \overbrace{{}^{\varphi}\beta_D(Q)}) \end{array} \right\rangle \\ \left\langle \begin{array}{c} \max(\overbrace{{}^{\varphi}\eta_{\hat{G}}(Q)} \overbrace{{}^{\varphi}\eta_D(Q)}) \\ \min(\overbrace{{}^{\varphi}\delta_{\hat{G}}(Q)}, \overbrace{{}^{\varphi}\delta_D(Q)}) \end{array} \right\rangle : Q\in R \end{array} \right) \right\}$$

$$\left\{ \left(\begin{array}{c} Q, \left\langle \begin{array}{c} \max(\underbrace{{}^{\varphi}u_{\hat{G}}(Q)}, \underbrace{{}^{\varphi}u_D(Q)}) \\ \min(\underbrace{{}^{\varphi}\beta_{\hat{G}}(Q)}, \underbrace{{}^{\varphi}\beta_D(Q)}) \end{array} \right\rangle \\ \left\langle \begin{array}{c} \max(\underbrace{{}^{\varphi}\eta_{\hat{G}}(Q)} . \underbrace{{}^{\varphi}\eta_D(Q)}) \\ \min(\underbrace{{}^{\varphi}\delta_{\hat{G}}(Q)}, \underbrace{{}^{\varphi}\delta_D(Q)}) \end{array} \right\rangle : Q\in R \end{array} \right) \right\}$$

$$5{:}D\cap\hat{G} = \left\{ \left(\begin{array}{c} Q, \left\langle \begin{array}{c} \min(\overbrace{{}^{\varphi}u_{\hat{G}}(Q)}, \overbrace{{}^{\varphi}u_D(Q)}) \\ \max(\overbrace{{}^{\varphi}\beta_{\hat{G}}(Q)}, \overbrace{{}^{\varphi}\beta_D(Q)}) \end{array} \right\rangle \\ \left\langle \begin{array}{c} \min(\overbrace{{}^{\varphi}\eta_{\hat{G}}(Q)} \overbrace{{}^{\varphi}\eta_D(Q)}) \\ \max(\overbrace{{}^{\varphi}\delta_{\hat{G}}(Q)}, \overbrace{{}^{\varphi}\delta_D(Q)}) \end{array} \right\rangle : Q\in R \end{array} \right) \right\}$$

$$\left\{\left(\begin{array}{c} Q,\left\langle \begin{array}{c}\min(\underbrace{{}^{\varphi}u_{\hat{G}}(Q)},\underbrace{{}^{\varphi}u_{D}(Q)}) \\ \max(\underbrace{{}^{\varphi}\beta_{\hat{G}}(Q)},\underbrace{{}^{\varphi}\beta_{D}(Q)})\end{array}\right\rangle \\ \left\langle\begin{array}{c}\min(\underbrace{{}^{\varphi}\eta_{\hat{G}}(Q)}.\underbrace{{}^{\varphi}\eta_{D}(Q)}) \\ \max(\underbrace{{}^{\varphi}\delta_{\hat{G}}(Q)},\underbrace{{}^{\varphi}\delta_{D}(Q)})\end{array}\right\rangle : Q\in R\end{array}\right)\right\}$$

Definition 3.3 *Let* $R_1 = \left\{\begin{array}{c} <\overbrace{{}^{\varphi}u_{R1}},\overbrace{{}^{\varphi}\beta_{R1}}>, \\ <\overbrace{{}^{\varphi}\eta_{R1}},\overbrace{{}^{\varphi}\delta_{R1}}>, \\ <\underbrace{{}^{\varphi}u_{R1}},\underbrace{{}^{\varphi}\beta_{R1}}>, \\ <\underbrace{{}^{\varphi}\eta_{R1}},\underbrace{{}^{\varphi}\delta_{R1}}>\end{array}\right\}$ *and* $R_2 = \left\{\begin{array}{c} <\overbrace{{}^{\varphi}u_{R2}},\overbrace{{}^{\varphi}\beta_{R2}}>, \\ <\overbrace{{}^{\varphi}\eta_{R2}},\overbrace{{}^{\varphi}\delta_{R2}}>, \\ <\underbrace{{}^{\varphi}u_{R2}},\underbrace{{}^{\varphi}\beta_{R2}}>, \\ <\underbrace{{}^{\varphi}\eta_{R2}},\underbrace{{}^{\varphi}\delta_{R2}}>\end{array}\right\}$ *be two NLDRFNs on* R *and* $K > 0$ *then*

$$1)R_1 + R_2 = \left\{\begin{array}{c} \left\langle\overbrace{{}^{\varphi}u_{R1}}+\overbrace{{}^{\varphi}u_{R2}}-\overbrace{{}^{\varphi}u_{R1}}\,\overbrace{{}^{\varphi}u_{R2}},\overbrace{{}^{\varphi}\beta_{R1}}\,\overbrace{{}^{\varphi}\beta_{R2}}\right\rangle, \\ \left\langle\overbrace{{}^{\varphi}\eta_{R1}}+\overbrace{{}^{\varphi}\eta_{R2}}-\overbrace{{}^{\varphi}\eta_{R1}}\,\overbrace{{}^{\varphi}\eta_{R2}},\overbrace{{}^{\varphi}\delta_{R1}}\,\overbrace{{}^{\varphi}\delta_{R2}}\right\rangle, \\ \left\langle\underbrace{{}^{\varphi}u_{R1}}+\underbrace{{}^{\varphi}u_{R2}}-\underbrace{{}^{\varphi}u_{R1}}\,\underbrace{{}^{\varphi}u_{R2}},\underbrace{{}^{\varphi}\beta_{R1}}\,\underbrace{{}^{\varphi}\beta_{R2}}\right\rangle, \\ \left\langle\underbrace{{}^{\varphi}\eta_{R1}}+\underbrace{{}^{\varphi}\eta_{R2}}-\underbrace{{}^{\varphi}\eta_{R1}}\,\underbrace{{}^{\varphi}\eta_{R2}},\underbrace{{}^{\varphi}\delta_{R1}}\,\underbrace{{}^{\varphi}\delta_{R2}}\right\rangle\end{array}\right\};$$

$$2)R_1 * R_2 = \left\{\begin{array}{c} \left\langle\overbrace{{}^{\varphi}u_{R1}}\,\overbrace{{}^{\varphi}u_{R2}},\overbrace{{}^{\varphi}\beta_{R1}}+\overbrace{{}^{\varphi}\beta_{R2}}-\overbrace{{}^{\varphi}\beta_{R1}}\,\overbrace{{}^{\varphi}\beta_{R2}}\right\rangle, \\ \left\langle\overbrace{{}^{\varphi}\eta_{R1}}\,\overbrace{{}^{\varphi}\eta_{R2}},\overbrace{{}^{\varphi}\delta_{R1}}+\overbrace{{}^{\varphi}\delta_{R2}}-\overbrace{{}^{\varphi}\delta_{R1}}\,\overbrace{{}^{\varphi}\delta_{R2}}\right\rangle, \\ \left\langle\underbrace{{}^{\varphi}u_{R1}}\,\underbrace{{}^{\varphi}u_{R2}},\underbrace{{}^{\varphi}\beta_{R1}}+\underbrace{{}^{\varphi}\beta_{R2}}-\underbrace{{}^{\varphi}\beta_{R1}}\,\underbrace{{}^{\varphi}\beta_{R2}}\right\rangle, \\ \left\langle\underbrace{{}^{\varphi}\eta_{R1}}\,\underbrace{{}^{\varphi}\eta_{R2}},\underbrace{{}^{\varphi}\delta_{R1}}+\underbrace{{}^{\varphi}\delta_{R2}}-\underbrace{{}^{\varphi}\delta_{R1}}\,\underbrace{{}^{\varphi}\delta_{R2}}\right\rangle\end{array}\right\};$$

$$3)kR_1 = \left\{\begin{array}{c} \left(1-\left(1-\overbrace{{}^{\varphi}u_{R1}}\right)^{k},\overbrace{{}^{\varphi}\beta_{R1}}^{k}\right), \\ \left(1-\left(1-\overbrace{{}^{\varphi}\eta_{R1}}\right)^{k},\overbrace{{}^{\varphi}\delta_{R1}}^{k}\right) \\ \left(1-\left(1-\underbrace{{}^{\varphi}u_{R1}}\right)^{k},\underbrace{{}^{\varphi}\beta_{R1}}^{k}\right), \\ \left(1-\left(1-\underbrace{{}^{\varphi}\eta_{R1}}\right)^{k},\underbrace{{}^{\varphi}\delta_{R1}}^{k}\right)\end{array}\right\};$$

$$4)R_1^k = \left\{\begin{array}{c} \left(\overbrace{{}^{\varphi}u_{R1}}^{k},1-\left(1-\overbrace{{}^{\varphi}\beta_{R1}}\right)^{k}\right), \\ \left(\overbrace{{}^{\varphi}\eta_{R1}}^{k},1-\left(1-\overbrace{{}^{\varphi}\delta_{R1}}\right)^{k}\right) \\ \left(\underbrace{{}^{\varphi}u_{R1}}^{k},1-\left(1-\underbrace{{}^{\varphi}\beta_{R1}}\right)^{k}\right), \\ \left(\underbrace{{}^{\varphi}\eta_{R1}}^{k},1-\left(1-\underbrace{{}^{\varphi}\delta_{R1}}\right)^{k}\right)\end{array}\right\}.$$

Definition 3.4 *Let* $D = \left\{ \begin{array}{c} <\overbrace{{}^{\varphi}u_D(Q)}, \overbrace{{}^{\varphi}\beta_D(Q)}>, \\ <\overbrace{{}^{\varphi}\eta_D(Q)}, \overbrace{{}^{\varphi}\delta_D(Q)}>, \\ <\underbrace{{}^{\varphi}u_D(Q)}, \underbrace{{}^{\varphi}\beta_D(Q)}>, \\ <\underbrace{{}^{\varphi}\eta_D(Q)}, \underbrace{{}^{\varphi}\delta_D(Q)}> \end{array} \right\}$ *be an*

NLDRFS.The mapping s:NLFRS(R)→ $[-1,1]$ *is called score funtion*

$$s(R) = \frac{(\overbrace{{}^{\varphi}u_D(Q)} - \overbrace{{}^{\varphi}\beta_D(Q)} + (\overbrace{{}^{\varphi}\eta_D(Q)} - \overbrace{{}^{\varphi}\delta_D(Q)}) + (\underbrace{{}^{\varphi}u_D(Q)} - \underbrace{{}^{\varphi}\beta_D(Q)} + (\underbrace{{}^{\varphi}\eta_D(Q)} - \underbrace{{}^{\varphi}\delta_D(Q)})}{4}$$

Definition 3.5 *Let* $D = \left\{ \begin{array}{c} <\overbrace{{}^{\varphi}u_D(Q)}, \overbrace{{}^{\varphi}\beta_D(Q)}><\overbrace{{}^{\varphi}\eta_D(Q)}, \overbrace{{}^{\varphi}\delta_D(Q)}>, \\ <\underbrace{{}^{\varphi}u_D(Q)}, \underbrace{{}^{\varphi}\beta_D(Q)}><\underbrace{{}^{\varphi}\eta_D(Q)}, \underbrace{{}^{\varphi}\delta_D(Q)}> \end{array} \right\}$ *be an NLDRFS.The mapping A:NLFRS(R)→* $[-0,1]$ *is called score funtion*

$$A = \frac{(\overbrace{{}^{\varphi}u_D(Q)} + \overbrace{{}^{\varphi}\beta_D(Q)} + 2(\overbrace{{}^{\varphi}\eta_D(Q)}, \overbrace{{}^{\varphi}\delta_D(Q)} + (\underbrace{{}^{\varphi}u_D(Q)} + \underbrace{{}^{\varphi}\beta_D(Q)}) + 2(\underbrace{{}^{\varphi}\eta_D(Q)} + \underbrace{{}^{\varphi}\delta_D(Q)})}{4}$$

Definition 3.6 *An entropy measures e is a mapping* $e : NLDRF(R) \to [0,1]$ *provide every feathure for every*

$D, \hat{G} \in NLDRFN(R)$

E1) $e(D) = 0$ *if and only if D is crisp set*

E2)$e(D) = 1$ *if* $\overbrace{{}^{\varphi}u_D(Q)} = \overbrace{{}^{\varphi}\beta_D(Q)}, \underbrace{{}^{\varphi}u_D(Q)} = \underbrace{{}^{\varphi}\beta_D(Q)}, \overbrace{{}^{\varphi}\eta_D(Q)} = \overbrace{{}^{\varphi}\delta_D(Q)}, \underbrace{{}^{\varphi}\eta_D(Q)} = \underbrace{{}^{\varphi}\delta_D(Q)}$

E3)$e(D) = e(D^c)$

E4)$e(D) \leq e(\hat{G})$ *it means D is less fuzzy than* $\hat{G}$ *i.e* $D \sqsubseteq \hat{G}$ *for* $\overbrace{{}^{\varphi}u_{\hat{G}}(Q)} \leq \overbrace{{}^{\varphi}\beta_{\hat{G}}(Q)}$ *and* $\overbrace{{}^{\varphi}\eta_{\hat{G}}(Q)} \leq \overbrace{{}^{\varphi}\delta_{\hat{G}}(Q)}, \underbrace{{}^{\varphi}u_{\hat{G}}(Q)} \leq \underbrace{{}^{\varphi}\beta_{\hat{G}}(Q)}$ *and* $\underbrace{{}^{\varphi}\eta_{\hat{G}}(Q)} \leq \underbrace{{}^{\varphi}\delta_{\hat{G}}(Q)}$ *or* $\hat{G} \sqsubseteq D$ *for* $\overbrace{{}^{\varphi}u_{\hat{G}}(Q)} \geq \overbrace{{}^{\varphi}\beta_{\hat{G}}(Q)}$ *and* $\overbrace{{}^{\varphi}\eta_{\hat{G}}(Q)} \geq \overbrace{{}^{\varphi}\delta_{\hat{G}}(Q)}, \underbrace{{}^{\varphi}u_{\hat{G}}(Q)} \geq \underbrace{{}^{\varphi}\beta_{\hat{G}}(Q)}$ *and* $\underbrace{{}^{\varphi}\eta_{\hat{G}}(Q)} \geq \underbrace{{}^{\varphi}\delta_{\hat{G}}(Q)}$

let $D = \{< Q, \overbrace{{}^{\varphi}u_D(Q)}, \overbrace{{}^{\varphi}\beta_D(Q)} >< \overbrace{{}^{\varphi}\eta_D(Q)}, \overbrace{{}^{\varphi}\delta_D(Q)} : Q \in R >,$
$< Q, \underbrace{{}^{\varphi}u_D(Q)}, \underbrace{{}^{\varphi}\beta_D(Q)} >< \underbrace{{}^{\varphi}\eta_D(Q)}, \underbrace{{}^{\varphi}\delta_D(Q)} : Q \in R >\}$ *be an NLDRFS on reference set R. we define a mapping on NLDRFN as fellow*

$$e(D) = \frac{1}{n}\sum_{i=1}^{n} \frac{1 - \left|\overbrace{{}^{\varphi}u_D(Qi)}\overbrace{{}^{\varphi}\eta_D(Qi)} - \overbrace{{}^{\varphi}\beta_D(Qi)}\overbrace{{}^{\varphi}\delta_D(Qi)}\right|}{1 + \left|\overbrace{{}^{\varphi}u_D(Qi)}\overbrace{{}^{\varphi}\eta_D(Qi)} - \overbrace{{}^{\varphi}\beta_D(Qi)}\overbrace{{}^{\varphi}\delta_D(Qi)}\right|}, \frac{1 - \left|\underbrace{{}^{\varphi}u_D(Qi)}\underbrace{{}^{\varphi}\eta_D(Qi)} - \underbrace{{}^{\varphi}\beta_D(Qi)}\underbrace{{}^{\varphi}\delta_D(Qi)}\right|}{1 - \left|\underbrace{{}^{\varphi}u_D(Qi)}\underbrace{{}^{\varphi}\eta_D(Qi)} - \underbrace{{}^{\varphi}\beta_D(Qi)}\underbrace{{}^{\varphi}\delta_D(Qi)}\right|}$$

4. NLDRFS Proposed Entropy-Based Vikor Method

The VIKOR method is a powerful tool used to identify the best alternative by evaluating how closely each option aligns with the ideal solution. One of the main advantages of using VIKOR is its ability to find a balanced, compromise solution that takes into account the trade-offs between different criteria. The process for applying the VIKOR method can be summarized as follows:

Step 1: construct the decision matrix $M = [m_{ij}]_{m*n}$ with the help of NLDRF data $R = \{< \overbrace{{}^{\varphi}u_D(Q)}, \overbrace{{}^{\varphi}\beta_D(Q)} >< \overbrace{{}^{\varphi}\eta_D(Q)}, \overbrace{{}^{\varphi}\delta_D(Q)} >, < \underbrace{{}^{\varphi}u_D(Q)}, \underbrace{{}^{\varphi}\beta_D(Q)} >< \underbrace{{}^{\varphi}\eta_D(Q)}, \underbrace{{}^{\varphi}\delta_D(Q)} >\}$

Step 2:Establish the normalized decision matrix $M^* = [m^*_{ij}]_{m*n}$ as follow

$m^*_{ij} = \{m_{ij}$ for benefit criteria (BC)

(m_{ij}) for cost criteria (CC)

where $(m_{ij})^c = \left\langle \overbrace{{}^{\varphi}\beta_{Rij}}, \overbrace{{}^{\varphi}u_{Rij}} \right\rangle, \left\langle \overbrace{{}^{\varphi}\delta_{Rij}}, \overbrace{{}^{\varphi}\eta_{Rij}} \right\rangle$ denotes the complement of m_{ij}

Step 3:Calculate the weight of criteria by using entropy measure as follow

$$w_j = \frac{1-e_j}{\sum_{j=1}^{n}(1-e_j)}$$

where e_j is entropy of j^{th} criteria. Here we use the following entropy measure e

$$e(D) = \frac{1}{n}\sum_{i=1}^{n} \frac{1-\left|\overbrace{^{\varphi}u_D(Qi)}\overbrace{^{\varphi}\eta_D(Qi)} - \overbrace{^{\varphi}\beta_D(Qi)}\overbrace{^{\varphi}\delta_D(Qi)}\right|}{1+\left|\overbrace{^{\varphi}u_D(Qi)}\overbrace{^{\varphi}\eta_D(Qi)} - \overbrace{^{\varphi}\beta_D(Qi)}\overbrace{^{\varphi}\delta_D(Qi)}\right|} * \frac{1-\left|\underbrace{^{\varphi}u_D(Qi)}\underbrace{^{\varphi}\eta_D(Qi)} - \underbrace{^{\varphi}\beta_D(Qi)}\underbrace{^{\varphi}\delta_D(Qi)}\right|}{1-\left|\underbrace{^{\varphi}u_D(Qi)}\underbrace{^{\varphi}\eta_D(Qi)} - \underbrace{^{\varphi}\beta_D(Qi)}\underbrace{^{\varphi}\delta_D(Qi)}\right|}$$

Step 4:Determine the non linear diophantine rough fuzzy positive ideal solution (p_j) and non linear diophantine rough fuzzy negative ideal solution (N_j)

$$\overbrace{^{\varphi}u^{+}_{Rj}} = \left\{\frac{\max^{\varphi} u_{Rj} \text{ for BC}}{\min^{\varphi} u_{Rj} \text{ for CC}}\right.;$$

$$\overbrace{^{\varphi}\beta^{+}_{Rj}} = \left\{\frac{\min^{\varphi} \beta_{Rj} \text{ for BC}}{\max^{\varphi} \beta_{Rj} \text{ for CC}}\right.;$$

$$\overbrace{^{\varphi}\eta^{+}_{Rj}} = \left\{\frac{\max^{\varphi} \eta_{Rj} \text{ for BC}}{\min^{\varphi} \eta_{Rj} \text{ for CC}}\right.;$$

$$\overbrace{^{\varphi}\delta^{+}_{Rj}} = \left\{\frac{\min^{\varphi} \delta_{Rj} \text{ for BC}}{\max^{\varphi} \delta_{Rj} \text{ for CC}}\right.;$$

$$\overbrace{^{\varphi}u^{-}} = \left\{\frac{\min^{\varphi} u_{Rj} \text{ for BC}}{\max^{\varphi} u_{Rj} \text{ for CC}}\right.;$$

$$\overbrace{^{\varphi}\beta^{-}_{Rj}} = \left\{\frac{\max^{\varphi} \beta_{Rj} \text{ for BC}}{\min^{\varphi} \beta_{Rj} \text{ for CC}}\right.;$$

$$\overbrace{^{\varphi}\eta^{-}_{Rj}} = \left\{\frac{\min^{\varphi} \eta_{Rj} \text{ for BC}}{\max^{\varphi} \eta_{Rj} \text{ for CC}}\right.;$$

$$\overbrace{^{\varphi}\delta^{-}_{Rj}} = \left\{\frac{\max^{\varphi} \delta_{Rj} \text{ for BC}}{\min^{\varphi} \delta_{Rj} \text{ for CC}}\right.;$$

$$\underbrace{^{\varphi}u^{+}_{Rj}} = \left\{\frac{\max^{\varphi} u_{Rj} \text{ for BC}}{\min^{\varphi} u_{Rj} \text{ for CC}}\right.;$$

$$\underbrace{^{\varphi}\beta^{+}_{Rj}} = \left\{\frac{\min^{\varphi} \beta_{Rj} \text{ for BC}}{\max^{\varphi} \beta_{Rj} \text{ for CC}}\right.;$$

$$\underbrace{^{\varphi}\eta^{+}_{Rj}} = \left\{\frac{\max^{\varphi} \eta_{Rj} \text{ for BC}}{\min^{\varphi} \eta_{Rj} \text{ for CC}}\right.;$$

$$\underbrace{^{\varphi}\delta^{+}_{Rj}} = \left\{\frac{\min^{\varphi} \delta_{Rj} \text{ for BC}}{\max^{\varphi} \delta_{Rj} \text{ for CC}}\right.;$$

$$\underbrace{^{\varphi}u^{-}} = \left\{\frac{\min^{\varphi} u_{Rj} \text{ for BC}}{\max^{\varphi} u_{Rj} \text{ for CC}}\right.;$$

$$\underbrace{^{\varphi}\beta^{-}_{Rj}} = \left\{\frac{\max^{\varphi} \beta_{Rj} \text{ for BC}}{\min^{\varphi} \beta_{Rj} \text{ for CC}}\right.;$$

$$\underbrace{^{\varphi}\eta^{-}_{Rj}} = \left\{\frac{\min^{\varphi} \eta_{Rj} \text{ for BC}}{\max^{\varphi} \eta_{Rj} \text{ for CC}}\right.;$$

$$\underbrace{^{\varphi}\delta^{-}_{Rj}} = \left\{\frac{\max^{\varphi} \delta_{Rj} \text{ for BC}}{\min^{\varphi} \delta_{Rj} \text{ for CC}}\right.$$

and then define $P_j = \left({}^{\varphi}u^{+}_{Rj}, {}^{\varphi}\beta^{+}_{Rj}, {}^{\varphi}\eta^{+}_{Rj}, {}^{\varphi}\delta^{+}_{Rj}\right)$ $N_j = \left({}^{\varphi}u^{-}_{Rj}, {}^{\varphi}\beta^{-}_{Rj}, {}^{\varphi}\eta^{-}_{Rj}, {}^{\varphi}\delta^{-}_{Rj}\right)$

4.1. Numerical example

Serving billions of consumers in 190 countries, Unilever is a global leader in consumer goods and runs one of the biggest and most intricate supply chains in the world. Managing its extensive supply network, shifting demand, and sustainability goals presented major hurdles for the business. The lack of real-time insight and adaptability offered by traditional supply chain systems resulted in inefficiencies, increased expenses, and product delivery delays. In order to resolve these problems, Unilever integrated cutting-edge technology like artificial intelligence (AI), the Internet of Things (IoT), and predictive analytics into its supply chain as part of a digital transformation strategy. With the use of these tools, the business was able to minimize waste, optimize inventory levels, and estimate demand more precisely. In order to improve transparency and traceability and guarantee ethical raw material sourcing, Unilever also implemented blockchain technology. The outcomes were outstanding: delivery times were shortened by 20%, operational expenses were cut by almost 15%, and the business made great strides toward its sustainability objectives. In addition to increasing efficiency, Unilever's data-driven strategy enhanced supplier relationships and raised customer satisfaction. In the current global economy, this example shows how implementing digital solutions can change conventional supply chains into flexible, robust, and sustainable systems.

Step1:we have a decsion matrix with the help of NLDFR numbers

{[0.3, 0.1] [0.5, 0, 6] [0.09, 0.01] [0.02, 0.03]}	{[0, 3, 0.5] [0.5, 0.9] [0.06, 0.05] [0.03, 0.02]}	{[0, 7, 0.8] [0.2, 0.5] [0.04, 0.09] [0.01, 0.05]}	{[0, 6, 0.1] [0.8, 0.9] [0.06, 0.03] [0.04, 0.05]}
{[0.3, 0.2], [0.7, 0.1] [0.04, 0.06], [0.08, 0.07]}	{[0, 8, 0.2] [0.3, 0.1] [0.04, 0.07] [0.05, 0.09]}	{[0, 9, 0.2] [0.6, 0.1] [0.05, 0.01] [0.07, 0.03]}	{[0, 8, 0.1] [0.7, 0.4] [0.02, 0.05] [0.07, 0.05]}
{[0.5, 0.7], [0.2, 0.9] [0.04, 0.03], [0.02, 0.05]}	{[0, 6, 0.9], [0.5, 0.9] [0.07, 0.04], [0.09, 0.07]}	{[0, 6, 0.9], [0.8, 0.7] [0.07, 0.04], [0.08, 0.05]}	{[0, 5, 0.2], [0.8, 0.6] [0.09, 0.06], [0.03, 0.07]}
{[0, 2, 0.7], [0.3, 0.7] [0.04 , 0.07], [0.03, 0.08]}	{[0, 1, 0.9], [0.5, 0.9] [0.02, 0.07], [0.04, 0.06]}	{[0, 2, 0.9], [0.2, 0.3] [0.08 , 0.05], [0.09 , 0.05]}	{[0, 5, 0.3], [0.4, 0.8] [0.06 , 0.03], [0.06, 0.04]}

Step 2: now we normalize our matrix

$$\begin{bmatrix}
\left\{\begin{matrix}[0.7, 0.9]\\ [0.5, 0,4]\\ [0.91, 0.99]\\ [0.98, 0.97]\end{matrix}\right\} & \left\{\begin{matrix}[0,7, 0.5]\\ [0.5, 0.1]\\ [0.94, 0.95]\\ [0.97, 0.98]\end{matrix}\right\} & \left\{\begin{matrix}[0,3, 0.2]\\ [0.8, 0.5]\\ [0.96, 0.91]\\ [0.99, 0.95]\end{matrix}\right\} & \left\{\begin{matrix}[0,4, 0.9]\\ [0.2, 0.1]\\ [0.94, 0.97]\\ [0.96, 0.95]\end{matrix}\right\} \\
\left\{\begin{matrix}[0.7, 0.8],\\ [0.3, 0.9]\\ [0.96, 0.94],\\ [0.92, 0.93]\end{matrix}\right\} & \left\{\begin{matrix}[0,2, 0.8]\\ [0.7, 0.9]\\ [0.96, 0.93]\\ [0.95, 0.91]\end{matrix}\right\} & \left\{\begin{matrix}[0,1, 0.8]\\ [0.4, 0.9]\\ [0.95, 0.99]\\ [0.93, 0.97]\end{matrix}\right\} & \left\{\begin{matrix}[0,2, 0.9]\\ [0.3, 0.6]\\ [0.98, 0.95]\\ [0.93, 0.95]\end{matrix}\right\} \\
\left\{\begin{matrix}[0.5, 0.3],\\ [0.8, 0.1]\\ [0.96, 0.97],\\ [0.98, 0.95]\end{matrix}\right\} & \left\{\begin{matrix}[0,4, 0.6],\\ [0.5, 0.1]\\ [0.93, 0.96],\\ [0.91, 0.93]\end{matrix}\right\} & \left\{\begin{matrix}[0,4, 0.1],\\ [0.2, 0.3]\\ [0.93, 0.96],\\ [0.92, 0.95]\end{matrix}\right\} & \left\{\begin{matrix}[0,5, 0.8],\\ [0.2, 0.4]\\ [0.91, 0.94],\\ [0.97, 0.93]\end{matrix}\right\} \\
\left\{\begin{matrix}[0,8, 0.3],\\ [0.7, 0.3]\\ [0.96, 0.93],\\ [0.97, 0.92]\end{matrix}\right\} & \left\{\begin{matrix}[0,9, 0.1],\\ [0.5, 0.1]\\ [0.98, 0.93],\\ [0.96, 0.94]\end{matrix}\right\} & \left\{\begin{matrix}[0,8, 0.1],\\ [0.8, 0.7]\\ [0.92, 0.95],\\ [0.91, 0.95]\end{matrix}\right\} & \left\{\begin{matrix}[0,5, 0.7],\\ [0.6, 0.2]\\ [0.94, 0.97],\\ [0.94, 0.96]\end{matrix}\right\}
\end{bmatrix}$$

Step 3:now we caluculate weight of criteria by using entropy. The weights of criteria are obtained by using entropy measure. The entropy value of the criteria R_1 is calculated asfollows:

$$e(R1) = \frac{1}{4}\begin{bmatrix}
\frac{1-|0.7*0.5-0.9*0.4|}{1+|0.7*0.5-0.9*0.4|} * \frac{1-|0.91*0.98-0.99*0.97|}{1+|0.91*0.98-0.99*0.97|} \\
+\frac{1-|0.7*0.5-0.5*0.1|}{1+|0.7*0.5-0.5*0.1|} * \frac{1-|0.94*0.97-0.95*0.98|}{1+|0.94*0.97-0.95*0.98|} \\
+\frac{1-|0.3*0.8-0.2*0.5|}{1+|0.3*0.8-0.2*0.5|} * \frac{1-|0.96*0.99-0.91*0.95|}{1+|0.96*0.99-0.91*0.95|} \\
+\frac{1-|0.4*0.2-0.9*0.1|}{1+|0.4*0.2-0.9*0.1|} * \frac{1-|0.94*0.96-0.97*0.95|}{1+|0.94*0.96-0.97*0.95|}
\end{bmatrix}$$

$e(R1) = 0.7378, e(R2) = 0.2651, e(R3) = 0.6348, e(R4) = 0.3872.$

then the weight is

$w_1 = \frac{1-0.7378}{4-(0.7378+0.2651+0.6348+0.3872)} = 0.1328$

other weight of criterian is $w_2 = 0.3721, w_3 = 0.1849, w_4 = 0.3103$

Step 4:positve and negative ideal solutions for R1

$$\begin{bmatrix}
 & \overbrace{{}^{\varphi}u_D(Q)} & \overbrace{{}^{\varphi}\beta_D(Q)} & \overbrace{{}^{\varphi}\eta_D(Q)} & \overbrace{{}^{\varphi}\delta_D(Q)} & \underbrace{{}^{\varphi}u_D(Q)} & \underbrace{{}^{\varphi}\beta_D(Q)} & \underbrace{{}^{\varphi}\eta_D(Q)} & \underbrace{{}^{\varphi}\delta_D(Q)} \\
P & 0.7 & 0.2 & 0.8 & 0.1 & 0.96 & 0.91 & 0.99 & 0.95 \\
N & 0.3 & 0.9 & 0.2 & 0.5 & 0.91 & 0.99 & 0.96 & 0.98
\end{bmatrix}$$

For R2

$$\begin{bmatrix} & \overbrace{{}^{\varphi}u_D(Q)} & \overbrace{{}^{\varphi}\beta_D(Q)} & \overbrace{{}^{\varphi}\eta_D(Q)} & \overbrace{{}^{\varphi}\delta_D(Q)} & \underbrace{{}^{\varphi}u_D(Q)} & \underbrace{{}^{\varphi}\beta_D(Q)} & \underbrace{{}^{\varphi}\eta_D(Q)} & \underbrace{{}^{\varphi}\delta_D(Q)} \\ P & 0.7 & 0.8 & 0.7 & 0.6 & 0.98 & 0.93 & 0.95 & 0.91 \\ N & 0.1 & 0.9 & 0.6 & 0.9 & 0.95 & 0.99 & 0.92 & 0.97 \end{bmatrix}$$

For R3

$$\begin{bmatrix} & \overbrace{{}^{\varphi}u_D(Q)} & \overbrace{{}^{\varphi}\beta_D(Q)} & \overbrace{{}^{\varphi}\eta_D(Q)} & \overbrace{{}^{\varphi}\delta_D(Q)} & \underbrace{{}^{\varphi}u_D(Q)} & \underbrace{{}^{\varphi}\beta_D(Q)} & \underbrace{{}^{\varphi}\eta_D(Q)} & \underbrace{{}^{\varphi}\delta_D(Q)} \\ P & 0.5 & 0.1 & 0.8 & 0.1 & 0.96 & 0.94 & 0.98 & 0.93 \\ N & 0.4 & 0.8 & 0.2 & 0.4 & 0.91 & 0.96 & 0.91 & 0.95 \end{bmatrix}$$

For R4

$$\begin{bmatrix} & \overbrace{{}^{\varphi}u_D(Q)} & \overbrace{{}^{\varphi}\beta_D(Q)} & \overbrace{{}^{\varphi}\eta_D(Q)} & \overbrace{{}^{\varphi}\delta_D(Q)} & \underbrace{{}^{\varphi}u_D(Q)} & \underbrace{{}^{\varphi}\beta_D(Q)} & \underbrace{{}^{\varphi}\eta_D(Q)} & \underbrace{{}^{\varphi}\delta_D(Q)} \\ P & 0.9 & 0.1 & 0.8 & 0.1 & 0.98 & 0.93 & 0.97 & 0.92 \\ N & 0.5 & 0.7 & 0.6 & 0.7 & 0.94 & 0.97 & 0.91 & 0.96 \end{bmatrix}$$

The values of U_i ,R_i and Q_i based on proposed distance

$$\begin{bmatrix} & U_i & R_i & Q_i \\ x1 & 0.8919 & 0.4137 & 0.0000 \\ x2 & 1.1501 & 0.5516 & 0.8749 \\ x3 & 0.9401 & 0.4482 & 0.0933 \\ x4 & 0.9047 & 0.4651 & 0.0860 \end{bmatrix}$$

The ranking of alternatives based on proposed distance.

$$\begin{bmatrix} & U_i & R_i & Q_i \\ x1 & 1 & 1 & 1 \\ x2 & 4 & 4 & 4 \\ x3 & 3 & 2 & 3 \\ x4 & 2 & 3 & 2 \end{bmatrix}$$

The rankings of all alternatives according to Q.

5. Conclusion

In order to handle uncertainty and nonlinearity in complicated evaluation contexts, this work presented a VIKOR-based decision-making technique based on Nonlinear Diophantine Rough Fuzzy Sets (NLDRFS). The accurate depiction of uncertain and interdependent criteria values was made possible by the addition of Nonlinear Diophantine structures, which improved the rough fuzzy model's mathematical flexibility. The suggested framework successfully addressed multi-criteria decision-making problems involving ambiguity, boundary fluctuations, and conflicting parameters by fusing the VIKOR approach with NLDRFS. By weighing group benefit against personal regret, the method identified compromise solutions and gave decision-makers logical and trustworthy comparisons of options. The results show that relative to conventional fuzzy, rough, or linear models, the VIKOR–NLDRFS model provides better decision accuracy and robustness. This hybrid approach can be used in a variety of fields, including sustainable system design, engineering optimization, and financial assessment. To further improve the model's decision-support capabilities, future research might concentrate on expanding it using hybrid aggregation operators, entropy-based weighting methods, or machine learning integration.

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