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Simulations by Using the Real-Time Finite Element Method

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ABSTRACT: This paper provides an overview of the various applications of Finite Element Method (FEM) technology across a broad range of practical uses. While it is difficult to cover every aspect comprehensively, FEM technology has significantly enhanced the field of numerical mathematics, earning praise for its effectiveness and versatility in fundamental analysis. Although traditional FEM component software typically requires separate mathematical analysis and individual calculations, rapid advancements in scientific research and practical applications over recent decades have motivated experts to further explore the growing trend of employing these techniques, which rely on FEM-based models (i.e., specific component technique models). The limitations imposed on different sectors—based on the pace of scientific progress—demand a high level of expertise, often requiring numerical analysis tests. Various methodologies for implementing FEM technology have been suggested, depending on the specific needs of different fields. Despite its challenges, FEM technology remains one of the most advanced and reliable tools globally, offering precise and rapid calculations for a wide range of complex tasks, provided through comprehensive and accurate software programs.

Keywords: FEM, interactive, real-time, simulation, survey.

Contents

1. Introduction

This study model focuses on core engineering practices, exploring various options to identify the most effective solution for delivering accurate and reliable simulations. The approach improves upon simpler models, such as those based on hand calculations and basic charts, by leveraging more advanced techniques and high-performance computing. These improvements are especially significant when working with complex models, involving advanced algorithms and substantial computational resources. Modelers collect and analyze data on existing methodologies, adapting and refining them to enhance performance and solve real-world problems.

Modern computational tools, hardware, and software are essential in addressing complex engineering challenges that, just a few decades ago, were nearly impossible to tackle. The advancements in hardware now allow for the analysis of physical systems in almost any stage of their development, with simulations running efficiently on modern machines. These technological breakthroughs enable the modeling of diverse physical processes in real-time, helping to further refine engineering designs and improve accuracy
The Finite Element Method (FEM) is one such technique that has evolved significantly in recent years. It is now automated and widely used for simulating complex systems and structures, incorporating self-adaptive capabilities and handling numerous constraints. As a mathematical tool, FEM heavily relies on modern computing technology. Finite Element Analysis (FEA) computations, which are closely related to

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FEM, still operate in a "detached" manner, meaning that various stages of the computation are processed sequentially [1].

In this approach, the three main components of an FEA process-preprocessing, solving, and postprocessing-are carried out in distinct stages. During preprocessing, a plan for the problem is established, and during postprocessing, the results are analyzed.

The growth of hardware capabilities over the last few decades has played a crucial role in enabling more sophisticated, real-time simulations based on FEM models. This progress has opened new doors for simulation applications across various domains, from design optimization to testing systems and entertainment [2].

while-many-experts-acknowledge-the-promise-of-these-technologies-there-are-still-challenges-to-overcome-particularly-regarding-the-development-of-more-refined-and-advanced-simulation-techniques.-despite-these-hurdles-ongoing-progress-continues-to-push-the-boundaries-of-what-is-possible-in-real-time-computational-modeling.-as-such-continuous-simulation-remains-a-key-area-of-focus-for-future-advancements-in-this-field.

2. Real Time Simulation Procedure

In various practical domains, the concept of "real-time simulation" is interpreted in different ways. Generally, this term refers to a process where results are generated within a defined time frame. According to the standard "DIN ISO/IEC 2382" [1], real-time simulation is seen as a sequence of calculations that provides outcomes within a specified duration. However, the exact definition can vary depending on the application, as the processing time may differ from the actual real-time (i.e., the clock time). In some situations, the simulation might take longer than the real-time, while in other cases, it may be much quicker than expected, depending on specific requirements.

Typically, real-time simulation refers to the time required for computations to produce results, which is generally measured against the clock time. The process might be slightly shorter than real time, allowing for synchronization. However, in some instances, simulations may exceed real time, depending on the complexity and demands of the simulation task [1].

Key factors influencing real-time simulation in engineering based on Finite Element Method (FEM) include [2]:

- The use of advanced computer hardware capable of handling complex tasks and enabling high-speed processing.
- The development of optimized software, including algorithm improvements for faster calculations.
- Advancements in various methodologies and designs for fundamental analysis.
- Despite these advancements, it's important to note that improvements in hardware and software aim to optimize the simulation speed but do not guarantee uniform performance across all scenarios. For example, just as the performance of a vehicle can be improved with better engine design, simulation speed can also be enhanced by better computing hardware. This follows the concept of "Moore's Law," which suggests that computational power continues to improve with each technological generation [2].
- Software development has similarly progressed, enhancing user experience and operational efficiency. The ultimate goal is to determine the optimal relationship between computational speed and precision. For example, just like reducing the complexity of a circuit to improve performance while maintaining the same input-output behavior, the challenge is to minimize computation time while preserving accuracy.
- In this context, the paper will not dive into hardware advancements but will briefly mention one notable development: the shift from using Central Processing Units (CPUs) to Graphics Processing Units (GPUs) for computations. GPUs offer significant parallel processing capabilities, which makes them ideal for handling large-scale simulations [3, 4]. This shift involves addressing challenges like efficiently transferring the necessary data to the GPU [5].

- The main focus of this study will be on the third aspect: the integration of strategies to balance the trade-off between computation accuracy and processing power.

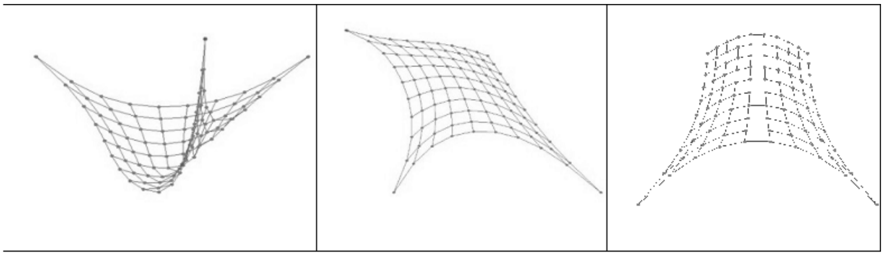


Figure 1: Fabric shape scheme such as mass–spring framework for intelligent recreation [7].

3. Lump–Spring Networks

Mass–spring models are widely used in engineering practices because they simplistically simulate physical behaviors associating mass elements to each other by springs. This idea was introduced in the early times by Hrennikoff in 1941 [6], but it has greatly developed since, and mainly because of the development of computational methods. This method usually uses a set of springs, operating in one direction, (forces are transmitted in their length only) and so complexity of model is reduced. This simple approach is effective in many realistic simulations, since it permits forces to be moved from here to there in any location of the system. One such example is given in Fig. 1, which shows how mass-spring models are used to model more complex systems such as nonlinear materials via numerical simulations [7].

An overwhelming advantage of mass–spring models is their ease of implementation computationally, which has led to their wide use in areas such as biomechanics and medical simulations. Thesemodels can be adequate if precision demands can be relaxed, as they yield physically based descriptions of the behavior, also for complex, nonlinear systems. In the medical context, mass-spring systems are common for the simulation of biological tissues, including surgery or other medical interventions [8–13], the latter even considering GPUs for better speed [14].

Mass–spring systems are especially versatile in applications where materials exhibit simple behaviors, such as when bending strength is minimal. For example, they are effective in simulating the interaction between materials and external forces. O'Connor and Stevens [15] developed models to find optimal spring strengths for simulating contact between materials, contributing to realistic simulations in areas like surface interactions. In a similar vein, Zhang et al. [16] explored how mass-spring systems could model shallow surfaces and tissues, introducing methods to simulate surface stiffness through exponential spring behavior.

While mass–spring models are flexible, one significant challenge lies in how to distribute mass and define relationships between masses to adequately represent the physical behavior of a system. Bianchi et al. [17] addressed this by applying genetic algorithms to optimize the arrangement of masses in a way that improved the model's ability to simulate physical interactions. They developed a method to reduce errors in physical representation, using a reference model as a benchmark. However, the method still faces limitations when dealing with complex nonlinear behavior, as demonstrated by the need for accurate geometric descriptions in certain cases.

A key observation from the work of Bianchi et al. [18] is that homogeneous mass-spring models (i.e., using uniform springs) often fail to capture the material's true behavior, especially in nonlinear or anisotropic systems. This issue has been addressed by Bourguignon and Cani [19], who proposed techniques for managing anisotropy in mass–spring systems.

4. Modular Command Collection Approaches

By adopting the Finite Elements (FE) solutions in a dense sampling along the computational domain that permits complete axial analysis under different conditions. But, solving the resulting equations is still an intricate task which consumes more time of the total process of FEM. This fact has led some researchers to search for less computationally demanding approaches by using simplified, low-order approximations that still yield acceptable accuracy in capturing the essential system response.

Concurrently, improvements in MBS allow the combined modelling of rigid-body motions along with flexible structures deformations within the body oriented local coordinate systems. The potential of including components of Multi-Body Systems (MBS) (which can cope with strong nonlinearities, various multiphysics couplings, and cross-coupling) has been well demonstrated through MBS–FEM co-simulation methods [20–23]. Despite the merits of such coupling, the method requires significantly more computational resources due to the complex data transfer and evaluation process between the coupled models.

To alleviate this burden and significantly improve simulation efficiency, model order reduction techniques are frequently applied to flexible components within MBS frameworks. It is important to note that achieving continuous, high-fidelity reproduction of all dynamics is often not prioritized in these scenarios. Instead, the emphasis lies in optimizing computational performance, particularly given that MBS models are widely utilized in mission-critical applications, where accurate prediction of system responses—such as precession effects—is of paramount importance, especially in high-precision or large-scale engineering tasks.

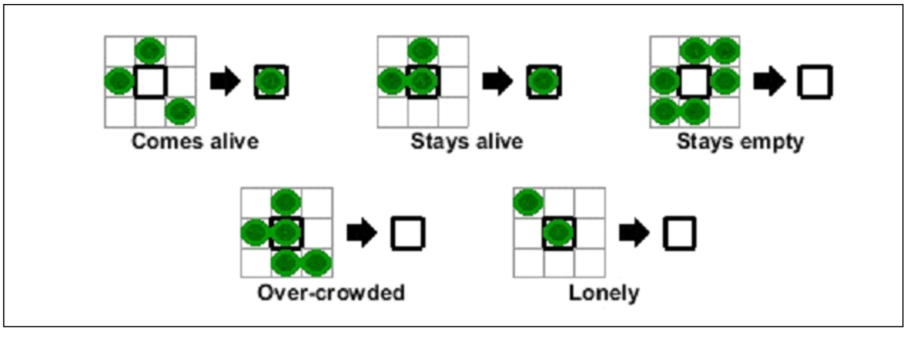


Figure 2: Modular case of decreased limited component (FE): Screen-shots from recreation of a stencil activity [24].

The application of variable reduction techniques to simplify complex models in MBS frameworks relies heavily on specific directional dependencies and configurations. This principle is essential for executing computationally demanding simulations before running full-scale analyses. During a pre-processing phase, modal reductions are typically implemented, which not only streamline the problem setup but also help isolate critical deformation behaviors by reducing the degrees of freedom (DOFs) in large-scale FE models. For example, models with approximately 10^5 stochastic DOFs can be condensed to just a few hundred or fewer DOFs within targeted regions, significantly improving computational efficiency. The use of reduced-order models derived from modal decomposition helps alleviate the computational burden while still preserving the structural fidelity required for evaluating dynamic behaviors such as deformations. In practical applications, global reduction techniques are already established in FEM, where modal superposition methods are frequently employed to capture essential transient responses [24]. Within FEM, specific modal components are extracted based on kinetic constraints and used for detailed analysis under defined boundary conditions. These methodologies also apply when examining structural fluctuations, particularly through eigenfrequency analyses. As illustrated in Figure 2, structural impacts are analyzed during rear-end vehicle collisions. The original, full-scale FE model may contain around 300,000 DOFs, yet, through modal reduction, this complexity can be condensed to as few as ten DOFs, maintaining accurate dynamic simulation with minimal computational expense during repeated simulation cycles. To further improve flexibility, MBS structures often integrate modal reduction techniques such as the

Craig–Bampton method [25], which effectively separates the system's DOFs into boundary and internal components. Boundary DOFs correspond to nodes at the interface of the FE model and the simulation environment, facilitating the interaction with external constraints and dynamic loads. In this process, fixed-interface modes and constraint modes are computed. Constraint modes represent static deformation patterns when boundary DOFs are displaced while keeping the remaining DOFs fixed. Normal modes are determined through a standard eigenvalue analysis with boundary DOFs constrained. However, this reduction method does not yield a purely orthogonal basis, so an orthogonalization process is often applied to preserve numerical consistency. A trade-off of orthogonalization is the potential loss of physical interpretation of certain modes, which may hinder the assignment of distinct damping parameters to individual modes. Typically, modal reduction is based on the system's undeformed configuration, making these methods suitable for linear analyses and small deformation assumptions. Nonetheless, recent advancements have extended these approaches to address moderate geometric nonlinearities. One such approach, implemented within the commercial software SIMPACK [26], introduces a numerical stiffness matrix that distinguishes between forces causing large displacements but small stresses (representing structural flexibility) and those inducing small displacements but large stresses (representing system stiffness) [27]. This numerical stiffness matrix is normalized for each force component and adjusted based on the magnitude of applied forces, yielding an overall stiffness representation that combines both linear and nonlinear effects. Further developments by Marinkovic and Zehn [28,29] introduced mode-dependent stiffness modifications computed through nonlinear assessments of the deformed structure prior to simulation. This technique applies numerical stiffness corrections to modal components based on deformation patterns, enabling dynamic simulations to reflect updated stiffness characteristics as the structure deforms.

In some cases, local geometric nonlinearities are managed through subdomain-specific corrections. For example, techniques like modal warping have been proposed to enhance the accuracy of reduced models in the presence of significant local deformations. Choi and Ko [30] monitor localized rotations occurring during simulations and adjust modal displacements accordingly to better represent the deformed configuration. Similarly, Marinkovic and Zehn [29,31] identify regions with prominent localized deformations and incorporate rotational corrections into the global displacement field using the linear modal basis. This improves the representation of deformed geometries, as demonstrated in simulations of girder beams and vehicle rear structures. Notably, these geometric corrections introduce minimal computational overhead, maintaining the overall efficiency of the reduced-order model, which is particularly advantageous for large-scale simulations.

An alternative hybrid approach was proposed by Zehn and Marinkovic [32], involving partial reduction of the system where the majority is represented by a reduced model, while regions with significant nonlinearities remain in their full FE form. This allows the accurate capture of localized nonlinear behavior without compromising the global computational efficiency. The concept of reduced-space modeling for enhancing simulation speed and achieving real-time response rates has also been widely adopted in computer graphics. Introduced by Pentland and Williams [33], reduced models have been applied across various fields, including real-time deformation simulations with collision handling [34], dynamic animations of trees under strong winds [35], and skeletal-driven character animations in interactive gaming environments [36].

5. Neural-Networks "FEM"-Based Learning

A particularly effective approach for model order reduction—sometimes regarded as an "indirect" technique—relies on using FEM simulation results to train neural networks for rapid predictions. In this method, computationally intensive and time-consuming simulations are carried out in advance, prior to the actual application of the model. The key advantage of this technique lies in its ability to handle complex nonlinear structural deformations, addressing both numerical and physical uncertainties. The data generated from detailed FEM simulations are used to train neural networks, enabling fast and reliable approximations during subsequent simulations.

An early application of this approach was demonstrated by Hambli et al. [37], who developed a neural-network-based simulation to predict deformation in a tennis ball-racket impact scenario. Their model incorporated various input parameters, such as impact angle, ball velocity, and contact location, while the corresponding outputs included ball deformation patterns and associated impact forces. The framework was integrated with a real-time experimental setup that included a force measurement system.

Similarly, Ordaz-Hernandez et al. [38] used a neural network with the training by multi-layer perceptrons to develop a reduced order model to predict the complex and nonlinear fracture behavior in wooden beams. C'ojbašic' et al. [39] also investigated a similar framework for structural analysis. In another example, Torano et al. [40] incorporated fuzzy logic, neural networks, and 3-D FEM analyses to develop an effective prediction model to measure performance in the massive river dam foundations, particularly those considered for a considerable degree of drilling process. Computed output was made available to users under Virtual Reality Modeling Language (VRML) interfaces to facilitate visualization and interpretation.

Morooka et al. [41] presents an alternative approach to model structural deformation by neural networks that are trained to learn the relation between external loads and corresponding distortions. Their study presented the possibility of real-time simulations and the closeness of outputs produced by neural networks and the one by the full FEM analysis.

Tzong-Ming et al. [42] utilized Ansys for FEM simulations and Matlab 17b to develop and train neural networks for structural prediction tasks. In their setup, the predicted deformations of structures with reduced mesh representations were translated into visual outputs using virtual reality modules.

Such reduced-order modeling techniques hold significant promise for control systems and real-time regulation, where computational efficiency is essential without sacrificing accuracy. As shown by Kalkkuhl et al. [43], integrating FEM-based neural networks within control loops enhances performance in monitoring and regulating structural behavior. Additionally, Runge et al. [44] employed neural networks trained with FEM data to estimate key parameters and predict behavior in delicate automation systems, both during simulation and real-time control.

6. Simulation Outcomes of "FE" Modular for "Real-Time" Utilizations

Although using full-scale Finite Element (FE) models with complete degrees of freedom (DOFs) offers unparalleled accuracy for structural representation, implementing these models for real-time or continuous simulations remains highly challenging. Even with moderate model sizes, the resulting system of equations is often too large to allow efficient real-time simulations using conventional computational resources, especially when nonlinearities are present. Consequently, numerous strategies have been proposed to restructure such problems, seeking a balance between maintaining sufficient accuracy and improving computational efficiency, particularly for continuous or interactive simulation environments. The effectiveness of these methods largely depends on the specific application area.

With the advent of FEM-based computer simulations integrated into Augmented Reality (AR) environments, there has been a noticeable push toward reducing computational loads. In scenarios where full-scale FE models are still employed, simplifications such as linear approximations become necessary, primarily because accurate, fully nonlinear simulations demand substantial computational effort. To this day, linear FE models remain prevalent in applications where linear assumptions sufficiently meet simulation objectives.

For instance, Huang et al. [45] developed an innovative system integrating sensor data with continuous FEA updates projected within an AR environment. Their method involved real-time monitoring of distributed loads via remote sensing, while the FEA component provided instantaneous structural response predictions. The incorporation of linear and semi-passive dynamics enabled efficient precomputed estimations of stiffness matrices, allowing the overlay of FEA results directly onto physical structures, enhancing user interpretation.

Similarly, Fiorentino et al. [46] presented a comparable framework where FE simulation results were superimposed onto physical objects, enabling users to interactively visualize boundary conditions and evaluate stress and strain distributions. In another example, Cerracchio et al. [47] applied linear FE modeling for continuous full-field deformation reconstruction using limited sets of discrete strain data, which was obtained through in-situ structural monitoring. Their approach was successfully applied to hybrid material systems subjected to mechanical, thermal, and residual stress effects.

The trade-off between computational efficiency and mechanical realism is also prominent in biomedical simulation, where accurate modeling of soft tissues is essential. Early research by Bro-Nielsen and Cotin

[48] combined linear elasticity with simplified tetrahedral elements to reduce computational demand. To enhance performance, simulations were restricted to surface nodes, minimizing internal computation. Additionally, precomputed stiffness matrices were inverted prior to simulation, significantly reducing numerical workload. Cotin et al. [49] further advanced this approach, proposing precompiled deformation libraries based on linear FE analysis for real-time soft tissue simulation.

Expanding on these principles, Pincinbono et al. [50] developed a bio-mechanical framework integrating linear FE analysis with anisotropic material behavior. They introduced additional surface layers with isotropic properties to approximate realistic surface stiffness while maintaining computational efficiency. This also addressed interaction dynamics with external devices, including force feedback and haptic systems, enhancing realism for medical simulation applications. By the late 1990s and early 2000s, several comprehensive review articles highlighted advancements in this domain [51–53].

Despite the critical importance of incorporating nonlinearities into simulations, doing so significantly increases computational effort, particularly for full-scale FE models. Commercial FEA software generally employs either updated or total Lagrangian formulations [24], with the key distinction being their treatment of reference configurations. These formulations provide high accuracy but at the cost of increased computational complexity. For example, Székely et al. [54] applied a total Lagrangian approach to model soft tissue deformations for endoscopic surgery simulations, utilizing brute-force methods to capture complex, tri-axial deformations of biological structures, such as the uterus. Although this provided realistic deformation behavior, computational demands were substantial.

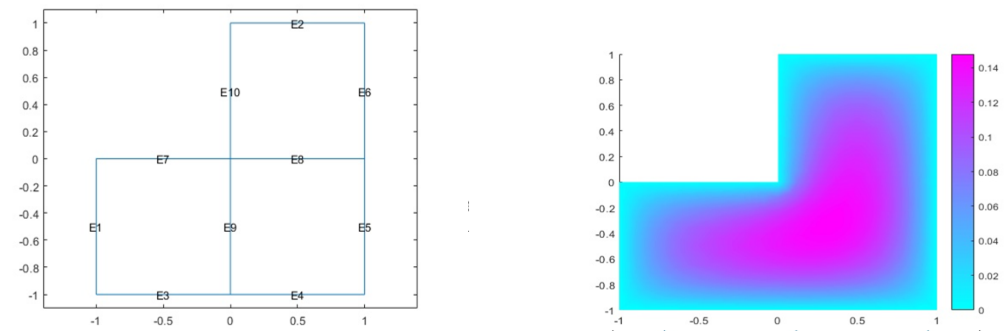
Alternative methodologies have been proposed to limit computational burden while maintaining acceptable accuracy. Heng et al. [55] introduced a hybrid dense FE modeling approach, partitioning the model into regions of interest and peripheral zones. The central zone utilized high-fidelity, nonlinear FE modeling to capture detailed behaviors, while outer regions were modeled with simplified, linear, geometry-stable FE representations. This methodology was successfully applied to training simulations for patellar endoscopic procedures.

In parallel, Dulong et al. [56] proposed interactive workflows that combined designer input with virtual models for iterative design optimization. During the planning phase, intensive nonlinear FE simulations were conducted to characterize system behavior under various load conditions. These precomputed results were subsequently incorporated into real-time simulations, analogous to data-driven approaches utilizing neural networks for rapid predictive modeling.

The computational challenges posed by fully nonlinear FE simulations have led to the development of innovative FE formulations and numerical solvers. Time integration schemes are critical in this context. Many researchers have highlighted the advantages of implicit schemes, which reduce the need for small time-steps and avoid iterative corrections in nonlinear scenarios [57,58]. However, these benefits are often offset by stability limitations, particularly when simulating low-stiffness materials such as biological tissues, which inherently necessitate smaller time-steps due to internal damping effects.

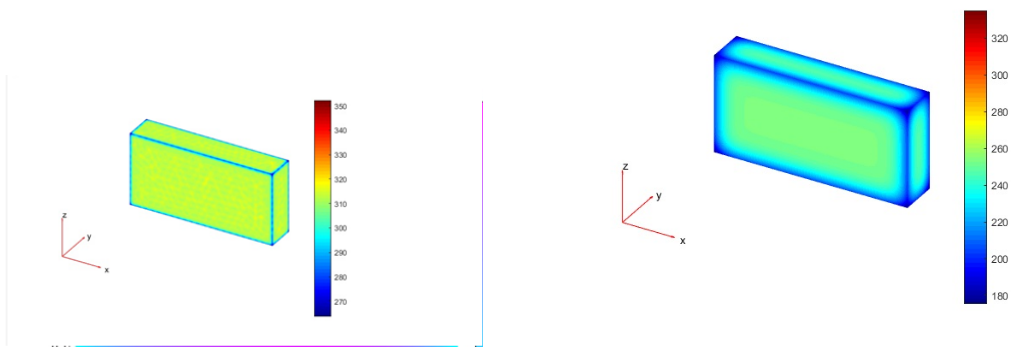
Real-time simulations targeting applications like surgical training require response times within human-perceivable limits, typically ranging from 2 to 10 seconds. Larger time-steps may simplify numerical integration but increase the complexity of solving the coupled system of equations at each step. Implicit schemes provide robust stability but at the expense of computational cost, while explicit methods offer faster evaluations with trade-offs in accuracy.

The need for efficient, nonlinear FE formulations has driven interest in methods such as co-rotational FE modeling [60,63,64], which address large, rigid-body rotations by introducing local coordinate systems that decouple rigid motion from elastic deformation. This allows for the efficient simulation of geometrically nonlinear effects while preserving computational performance.



(a) (b)

Figure 3: Reproduction of FEM scheme: (a) Trigonometry as well a level portrayal for an FEM work Example1; (b) screen captures about intuitive reproduction of FE work Example2 [63].



(a) (b)

Figure 4: Three Dimensional FEM model: (a) Geometry, FEM of Example3, (b)) Geometry, FEM of Example4 [54]

7. Conclusions

FEM-based real-time simulation has progressively expanded across numerous disciplines, including engineering, entertainment, and virtual environments. Although this field is still considered to be in its developmental phase, significant progress has already been made to overcome the computational challenges associated with executing real-time simulations using currently available hardware. This paper presents a selective, representative overview of developments in the domain. Providing an exhaustive review of all contributions would be impractical due to the volume of existing work. Therefore, the focus here is to highlight key methodologies and perspectives that aim to balance the conflicting requirements of achieving high numerical efficiency while preserving the accuracy essential for real-time simulations.

Among the available modeling approaches, mass–spring systems offer simplicity and computational efficiency but often at the expense of accuracy and structural reliability. Though now considered somewhat outdated, this technique remains applicable in specific scenarios where precision is not the primary concern, such as for preliminary estimations of structural damage or for simplified interactive models where user interaction is a priority.

Model order reduction techniques have demonstrated substantial benefits not only within the scope of MBS (multi-body systems) for engineering purposes but have also found applications in interactive simulations within computer graphics and animation. Despite these successes, the primary limitation of these techniques remains their inherent restriction to linear deformations. To address this, methods incorporating numerical stiffness adjustments and modal warping have been proposed as intermediate solutions. However, to date, there is no unified approach that combines both strategies to leverage their mutual advantages, which presents an opportunity for further research.

Additionally, utilizing FEM-based neural network training has emerged as an indirect yet effective approach for model reduction. This data-driven method shows promise for identifying and predicting nonlinear behaviors within complex systems, though its accuracy is heavily dependent on the diversity and comprehensiveness of the deformation data used during the neural network training phase.

The most ambitious solutions in this domain aim to maintain the full complexity and nonlinear damage behaviors inherent to complete FE models. Technological advancements over the past two decades have inspired researchers to explore innovative strategies to meet these computational demands. Among the more promising approaches is the use of enhanced co-rotational FEM formulations, also referred to by some as "geometrically distorted stiffness" methods, in combination with coupled mesh strategies. These techniques enable the reuse of pre-computed linear stiffness matrices for individual elements while the coupled mesh approach allows for efficient handling of coarse FE meshes using sophisticated algorithms.

Reflecting on the progress made, it is reasonable to anticipate that further innovative methodologies will emerge, advancing the boundaries of real-time FEM-based simulation. Moreover, with continued adherence to Moore's Law, which predicts exponential improvements in computational hardware performance, and the optimization of algorithms to better utilize such hardware, the future potential of real-time FEM applications in structural analysis is considerable.

It is also important to recognize that related disciplines—such as Computational Fluid Dynamics (CFD) and particle-based physics simulations—face similar challenges concerning real-time performance. Interest in real-time simulations leveraging both standard and cutting-edge hardware solutions is growing rapidly across these fields. Furthermore, advancements in these domains are increasingly converging, laying the groundwork for real-time, high-fidelity simulations that can accurately capture coupled-field effects, a significant milestone in achieving comprehensive, interactive simulations across complex physical systems.

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