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A Study On Independence Number of Deg-Centric Graphs

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ABSTRACT: The deg-centric graph of a simple, connected graph G, denoted by G_d, is a graph constructed from G such that $V(G_d) = V(G)$ and $E(G_d) = \{v_iv_j : d_G(v_i, v_j) \leq deg_G(v_i)\}$. A set S is independent in a graph G if any pair of vertices $u, v \in S$ are nonadjacent in G. Maximum independent sets in G will also be called $\alpha-$sets in G. The independence number $\alpha(G)$ of a graph G is the cardinality of an $\alpha-$set in G. Thus, an independent set S in G is an $\alpha-$ set whenever $|S| = \alpha(G)$. This paper presents the independence number of the deg-centric graphs. Also, investigate the properties and structural characteristics of this type of graph.

Keywords: Distance, degree, deg-centric graph, independence number, independent set.

Contents

1. Introduction

For a basic terminology of graph theory, we refer to [1]. For further topics on graph classes, [4]. Throughout this paper, a graph is assumed to be simple, connected, and undirected. The number of edges of a graph G is denoted by $\varepsilon(G)$. Recall that the distance between two distinct vertices v_i and v_j of G, denoted by $d_G(v_i, v_j)$, is the length of the shortest path joining them. The eccentricity of a vertex $v_i \in V(G)$, denoted by $e(v_i)$, is the farthest distance from v_i to some vertex of G. A particular type of newly derived graphs based on the vertex degrees and distances in graphs called *deg-centric graphs* have been introduced in (see [2]) as follows, The *degree centric graph* or *deg-centric graph* of a graph G is the graph G_d with $V(G_d) = V(G)$ and $E(G_d) = \{v_iv_j : d_G(v_i, v_j) \leq deg_G(v_i)\}$ (see [2]). Let G be a graph and G_d be the deg-centric graph of G. Then, the successive iteration *deg-centric graph* of G, denoted by G_{d^k}, is defined as the derived graph obtained by taking the deg-centric graph successively k times, that is, $G_{d^k} = ((G_d)_d \ldots)_d$, ($k$-times). This process is known as *deg-centrication process* (see [2]).

The *exact degree centric graph* or *exact deg-centric graph* of a graph G and denoted by G_{ed}, is the graph with $V(G_{ed}) = V(G)$ and $E(G_{ed}) = \{v_iv_j : d_G(v_i, v_j) = deg_G(v_i)\}$. This graph transformation is called exact deg-centrication (see [3]). The *upper degree centric graph* or *upper deg-centric graph* of a graph G and denoted by G_{ud}, is the graph with $V(G_{ud}) = V(G)$ and $E(G_{ud}) = \{v_iv_j : d_G(v_i, v_j) \geq deg_G(v_i)\}$. This graph transformation is called upper deg-centrication (see [5]). The *coarse degree centric graph* or *coarse deg-centric graph* of a graph G, denoted by G_{cd}, is the graph with $V(G_{cd}) = V(G)$ and $E(G_{cd}) = \{v_iv_j : d_G(v_i, v_j) > deg_G(v_i)\}$. This graph transformation is called coarse deg-centrication (see [7]). The *lower degree centric graph* or *lower deg-centric graph* of a graph G, denoted by G_{ld}, is the graph with $V(G_{ld}) = V(G)$ and $E(G_{ld}) = \{v_iv_j : d_G(v_i, v_j) < deg_G(v_i)\}$(see [6].

A set S is independent in a graph G if any pair of vertices $u, v \in S$ are nonadjacent in G. Maximum independent sets in G will also be called $\alpha-$sets in G. The independence number $\alpha(G)$ of a graph G is the cardinality of an $\alpha-$set in G. Thus, an independent set S in G is an $\alpha-$ set whenever $|S| = \alpha(G)$ (see [9]).

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Motivated by the aforementioned studies, we investigate the structure of maximum independent sets and determine the independence number of deg-centric graphs, a class of graphs characterised by degree-based centrality measures. Our goal is to explore how the structural properties of these graphs influence the size and distribution of independent sets.

The study of independent sets in transformed graphs often reveals how structural operations affect sparsity and vertex separation properties. In this context, deg-centric transformations provide a natural framework for examining how local degree constraints interact with global distance structure. Since adjacency in deg-centric graphs depends simultaneously on vertex degree and shortest-path distance, the resulting graphs frequently exhibit nontrivial structural behaviour that differs significantly from the original graph.

Understanding independence in such graphs is therefore important not only from a theoretical standpoint but also for identifying extremal configurations and structural bounds in degree–distance based graph models. In particular, investigating how the independence number evolves under deg-centrication may lead to new insights into the relationship between degree distribution, distance structure, and vertex separation properties.

Definition 1.1 [2] *The degree centric graph or deg-centric graph of a graph G, denoted by G_d, is a graph constructed from G such that, $V(G_d) = V(G)$ and $E(G_d) = \{v_iv_j : d_G(v_i, v_j) \leq deg_G(v_i)\}$. This graph transformation is called* deg-centrication *of the graph.*

Definition 1.2 [2] The *iterated deg-centric graph* of a graph G, denoted by G_{d^k}, is defined as the graph obtained by applying *deg-centrication* successively k-times. That is, $G_{d^k} = ((G_d)_{d\ldots})_d$, ($k$-times).

Theorem 1.1 [2] *The deg-centric graph of a non-star graph G with $\delta(G) \geq diam(G)$ is complete.*

Corollary 1.1 [2] *The deg-centric graph G_d of a non-star graph G with $deg_G(v_i) \geq e(v_i)$ is complete.*

2. Some Standard Results

This section recalls certain classical results on the independence number and its relations with other graph parameters, which serve as the foundation for further study in the context of deg-centric graphs [1].

These well-known results provide useful bounds and structural insights that help in estimating the independence number of derived graphs. They also offer important tools for comparing independence-related properties between a graph and its transformed versions. In the subsequent sections, these foundational results will be employed to analyse independence in various classes of deg-centric graphs.

Theorem 2.1 *For any simple connected deg-centric graph G_d of order n,*

$$1 \leq \alpha(G_d) \leq n.$$

Equality holds if and only if $G_d \cong K_n$ or $G_d \cong \overline{K}_n$ respectively.

Theorem 2.2 *For any G_d, the independence number $\alpha(G_d)$ and vertex cover number $\beta(G_d)$ satisfy*

$$\alpha(G_d) + \beta(G_d) = n.$$

Theorem 2.3 *Let $\delta(G_d)$ and $\Delta(G_d)$ denote the minimum and maximum degrees of G_d, respectively. Then*

$$\frac{n}{1 + \Delta(G_d)} \leq \alpha(G_d) \leq \frac{n(1 + \delta(G_d))}{\delta(G_d) + \Delta(G_d) + 1}.$$

Theorem 2.4 *If G_d has independence number $\alpha(G_d)$ and domination number $\gamma(G_d)$, then*

$$\alpha(G_d) \geq \gamma(G_d).$$

Equality holds if and only if every minimal dominating set is a maximum independent set.

Theorem 2.5 *For any deg-centric graph G_d without isolated vertices,*

$$\alpha(G_d) \leq n - \delta(G_d),$$

where $\delta(G_d)$ is the minimum degree.

Theorem 2.6 *If G_d is a regular graph of degree r, then*

$$\alpha(G_d) \geq \frac{n}{r+1}.$$

Equality holds for complete graphs and complete multipartite graphs with equal partite sets.

3. Independence Number of Deg-centric Graphs

In this section, we discuss the concept of independent sets and the independence number in the context of degree-centric graphs. Specifically, we examine how the structural properties influenced by vertex degrees affect the size and distribution of independent sets, and we explore methods for determining or estimating the independence number in such graphs.

Proposition 3.1 *For a connected graph G of order n, if $deg_G(v_i) \geq e_G(v_i)$, for all $v_i \in V(G)$, then, $\alpha(G_d) = 1$.*

Proof: Connected graph G of order n, if $deg_G(v_i) \geq e_G(v_i)$, then,in view of Theorem 1.1, $G_d \cong K_n$. The maximum independent set contains only one vertex. Therefore, the independence number is 1. Hence, $\alpha(G_d) = 1$. □

Proposition 3.2 *For any deg-centric graph G_d of order n, $G_d \cong \overline{K_n}$, $\alpha(G_d) = n$.*

Proof: The deg-centric graph of order n, $G_d \cong \overline{K_n}$, is an empty graph; no two vertices are connected by an edge, so any subset of vertices forms an independent set. Therefore, the maximum size of an independent set is simply the number of vertices in the graph. Hence, $\alpha(G_d) = n$. □

Proposition 3.3 *For a complete graph K_n,*

$$\alpha((K_n)_d) = 1.$$

Proof: For a complete graph K_n, $\delta(K_n) \geq e_G(v_i)$, the deg-centric graph of a complete graph K_n of order $n \geq 3$ is always isomorphic to the complete graph K_n. In view of Proposition 3.1, $\alpha((K_n)_d) = 1$. □

Proposition 3.4 *For $n \geq 1$,*

$$\alpha((P_n)_d) = \begin{cases} \lceil \frac{n}{2} \rceil; & \text{if } n \leq 3 \\ \lfloor \frac{n+2}{3} \rfloor; & \text{otherwise.} \end{cases}$$

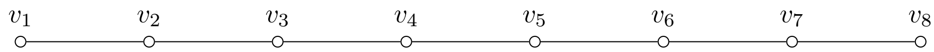
Proof: Consider a path graph P_n, on a horizontal line with the vertices labelled from left to right as v_1, v_2, v_3,..., v_n. If $n \leq 3$, clearly $(P_n)_d$ is isomorphic to P_n. Hence, $\alpha((P_n)_d) = \lceil \frac{n}{2} \rceil$.

For $n \geq 4$, by Definition 1.1, the pendant vertices of a path graph P_n have degree two, and the adjacent vertices of the pendant vertices have degree three in $(P_n)_d$. Vertices v_3,v_4,..., v_{n-2} have degree four in the deg-centric graph. In the $\alpha-$ set, it contains all the v_{i+3} vertices. Hence, $\alpha((P_n)_d) = \lfloor \frac{n+2}{3} \rfloor$. □

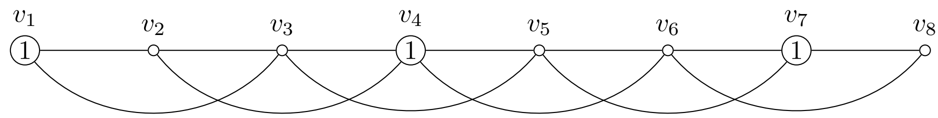
An illustration of proposition 3.4 is given in Figure 1.

Proposition 3.5 *For a cycle C_n, $n \geq 3$,*

$$\alpha((C_n)_d) = \begin{cases} 1; & \text{if } n \leq 5 \\ \lfloor \frac{n-3}{3} \rfloor + 1; & \text{otherwise.} \end{cases}$$



(a) P_8



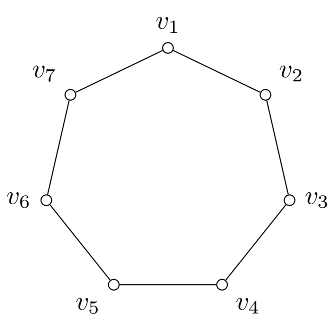
(b) $(P_8)_d$

Figure 1: $\alpha((P_8)_d) = 3$.

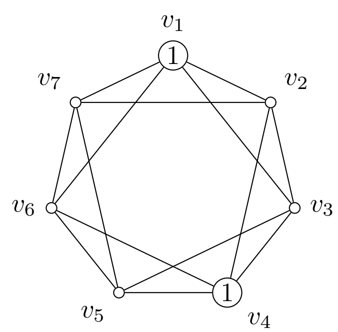
Proof: Consider that the deg-centric graph C_n, $n \leq 5$, clearly $(C_n)_d$ is complete graph. In view of Proposition 3.1, $\chi((C_n)_d) = 1$.

Now consider all other cases, In view of Definition 1.1, $\deg_{C_n}(v_i) = 2$, for all $v_i \in V(C_n)$, any vertex v_i in $(C_n)_d$ is adjacent to all vertices $d_{C_n}(v_i, v_j) \leq 2$. Then, the deg-centric graph, $(c_n)_d$ is always a 4-regular graph. In the α-set, it contains all the v_{i+3} vertices. Hence, $\alpha((C_n)_d) = \lfloor \frac{n-3}{3} \rfloor + 1$. □

An illustration of Proposition 3.5 is given in Figure 2.



(a) C_7



(b) $(C_7)_d$

Figure 2: $\alpha((C_7)_d) = 2$.

Proposition 3.6 *For $m, n \geq 2$,*

$$\alpha(K_{m,n}) = 1.$$

Proof: For $m, n \geq 2$, $K_{n,m}$ is a graph whose vertex set can be partitioned into two independent sets X, $|X| = n$ and Y, $|Y| = m$. Let $X = \{v_1, v_2, \ldots, v_n\}$ and $Y = \{u_{1,2}, \ldots, v_m\}$. In view of Definition 1.1, all the vertices of $k_{n,m}$ are adjacent in $(K_{n,m})_d$; that is, the deg-centric graph is complete. Hence, $\alpha(K_{m,n}) = 1$. □

Proposition 3.7 *For a star graph, $K_{1,n}$, $n \geq 0$,*

$$\alpha((K_{1,n})_d) = n.$$

Proof: Consider that the star graph, $k_{1,n}$, $n \geq 0$, is obtained by attaching n pendant vertices to a central vertex v_0. In view of Definition 1.1, the deg-centric graph of a star graph $k_{1,n}$, $n \geq 0$, is always isomorphic to the star graph. We know that the independence number of a star graph is always n. Hence, $\alpha((K_{1,n})_d) = n$. □

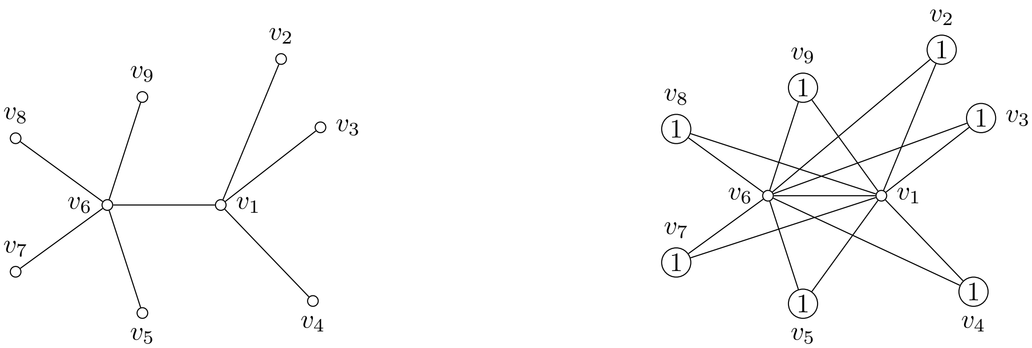
A non-trivial *bistar graph*, denoted by $S_{a,b}$, is a graph obtained by joining the centers of two non-trivial star graphs $K_{1,a}$, $a \geq 1$ and $K_{1,b}$, $b \geq 1$ with the edge v_0u_0.

Proposition 3.8 *For $a, b \geq 1$,*

$$\alpha((S_{a,b})_d) = a + b.$$

Proof: The bistar graph $S_{a,b}$, $a, b \geq 1$, let the pendant vertices of $K_{1,a}$ be the set $X = \{v_1, v_2, \ldots, v_a\}$ and let the pendant vertices of $K_{1,b}$ be the set $Y = \{u_1, u_2, \ldots, u_b\}$. Finally, let $W = \{v_0, u_0\}$ be center vertices. By Definition 1.1, it follows that both v_0, u_0 are adjacent with all other $a + b + 1$ vertices. Elements of sets X and Y are pendant vertices, then by Definition 1.1, no edges incident from these pendant vertices, which implies $a + b$ pendant vertices with degree two in $(S_{a,b})_d$. Therefore, all the vertices in the set X and Y are added to the α-set in $(S_{a,b})_d$. Finally, $\alpha((S_{a,b})_d) = a + b$. □

An illustration of Proposition 3.8 is given in Figure 3.



(a) $S_{4,3}$ (b) $(S_{4,3})_d$

Figure 3: : $\alpha((S_{4,3})_d) = 7$.

Proposition 3.9 *For $n \geq 3$,*

$$\alpha((W_{1,n})_d) = 1.$$

Proof: The wheel graph $W_{1,n}$, $n \geq 3$, note that, $deg(v_i) \geq e(v_i)$ in wheel graph, for all $v_i \in V(W_{1,n})$. Then, in view of the proposition 3.1, the α-set contains only one vertex in $(W_{1,n})_d$. Hence, $\alpha((W_{1,n})_d) = 1$. □

Proposition 3.10 *For $n \geq 3$,*

$$\alpha((DW_n)_d) = 1.$$

Proof: The double wheel graph DW_n, $n \geq 3$ is of the order $2n + 1$. Let $V(DW_n) = \{v_0, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$. In view of Proposition 3.1, the α-set contains only one vertex. Hence, $\alpha((DW_n)_d) = 1$. □

Proposition 3.11 *For $n \geq 3$,*

$$\alpha((H_{1,n})_d) = n.$$

Proof: The helm graph $H_{1,n,}$, $n \geq 3$, is of the order $2n+1$. Let $V(H_{1,n,}) = \{v_0, v_1, v_2, \ldots, v_n, \underbrace{u_1, u_2, \ldots, u_n}_{\text{pendant vertices}}\}$. In view of Definition 1.1, there are $2n$ edges incident in v_0 and v_i in $(H_{1,n})_d$. Then, we can add all the vertices u_i in the $\alpha-$ sets in $(H_{1,n})_d$. Also, the other $n+1$ vertices are adjacent to each other in $(H_{1,n})_d$. We cannot add these $n+1$ vertices in the $\alpha-$ sets. Hence, $\alpha((H_{1,n})_d) = n$. □

Proposition 3.12 *For $n \geq 3$,*

$$\alpha((CH_{1,n})_d) = \begin{cases} 1; & \text{if } n \leq 7 \\ \lfloor \frac{n-8}{4} \rfloor + 2; & \text{otherwise.} \end{cases}$$

Proof: Consider a closed helm graph $CH_{1,n}$ $n \geq 3$, is of the order $2n+1$. Let $V(CH_{1,n,}) = \{v_0, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$. For all $CH_{1,n}$, $n < 8$, $\delta(CH_{1,n}) = 3$. If $n = 3, 4, 5, 6, 7$ the diameter is bounded by $diam(CH_{1,n}) \leq 3$. Since $\delta(CH_{1,n}) \geq diam(CH_{1,n})$, $\delta(CH_{1,n}) = 3$. Then, the deg-centric graph of a closed helm graph $CH_{1,n}$ of order $n < 8$ is the complete graph, the result follows from Proposition 3.1.

For $n \geq 8$ we have $\delta(CH_{1,n}) = 3$ and $diam(CH_{1,n}) = 4$ in $CH_{1,n}$. The center vertex v_0, $deg(v_0) = n$ and $deg(v_n) = 4$ in $CH_{1,n}$. In view of Definition 1.1, $deg(v_0) = 2n$ and $deg(v_n) = 2n$ in deg-centric graph of the closed helm graph. We cannot add the v_0 and v_i vertices in the α- set. However, since $\deg_{CH_{1,n}}(u_i) = 3$, by Definition 1.1, distance less than or equal to three edges forms from u_i in $(CH_{1,n})_d$. In α- set we can add u_{4i+1}, where $i = 0, 1, 2, 3, \ldots$ vertices. Hence, $\alpha((CH_{1,n})_d) = \lfloor \frac{n-8}{4} \rfloor + 2$. □

Proposition 3.13 *For $n \geq 3$,*

$$\alpha((Wb_{1,n})_d) = n.$$

Proof: The web graph $Wb_{1,n}$, $n \geq 3$, is of the order $3n+1$. Let $V(Wb_{1,n,}) = \{v_0, v_1, v_2, \ldots, v_{n-1}, v_n, u_1, u_2, u_3, \ldots, u_n, \underbrace{w_1, w_2, w_3, \ldots, w_n}_{\text{pendant vertices}}\}$. If $n \geq 3$, the center vertex v_0, $deg(v_0) = n$ and $deg(v_n) = 4$ in $Wb_{1,n}$. In view of Definition 1.1, $deg(v_0) = 3n$ and $deg(v_n) = 3n$ in deg-centric graph of the web graph. We cannot add the v_0 and v_i vertices in the α- set. However, since $\deg_{Wb_{1,n}}(w_i) = 1$, by Definition 1.1, w_i are non adjacent each other in $(Wb_{1,n})_d$. Then, we can add these n vertices to α- set. In case of u_i vertices, we cannot add these vertices to α- set, these vertices are adjacent with w_i vertices. Hence, $\alpha((Wb_{1,n})_d) = n$. □

Proposition 3.14 *For $n \geq 3$,*

$$\alpha(F_{1,n}) = 1.$$

Proof: The flower graph $F_{1,n,}$, $n \geq 3$ is of the order $2n+1$. Let $V(F_{1,n,}) = \{v_0, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$. Since $\delta(F_{1,n,}) = 2 = diam(F_{1,n,}) = 2$, by Proposition3.1, $\alpha(F_{1,n}) = 1$. □

The *sunflower graph*, denoted by $SF_{1,n}$, $n \geq 3$ is obtained from the wheel $W_{1,n}$ by attaching n vertices u_i, $1 \leq i \leq n$ such that each u_i is adjacent to v_i and v_{i+1} and count the suffix is taken modulo n.

Proposition 3.15 *For $n \geq 3$,*

$$\alpha((SF_{1,n})_d) = \begin{cases} 1; & \text{if } n = 3 \\ \lfloor \frac{n-2}{2} \rfloor + 1; & \text{otherwise.} \end{cases}$$

Proof: The sunflower graph $SF_{1,n}$, $n \geq 3$, is of the order $2n+1$. Let $V(SF_{1,n,}) = \{v_0, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$. If $n = 3$, since $\delta(SF_{1,n}) = 2 = diam(SF_{1,n}) = 2$, by Proposition3.1, $\alpha((SF_{1,n})_d) = 1$. If $n \geq 4$, $\deg_{SF_{1,n}}(v_0) = n > e(v_0) = 2$ and $\deg_{SF_{1,n}}(v_i) = n+1 > e(v_i) = 2$ and $\deg_{SF_{1,n}}(v_i) = n+1 > e(v_i) = 2$, by the Definition1.1, $2n$ edges form from v_0 and v_i in deg-centric graph. We cannot add these vertices in α- set in $(SF_{1,n})_d$. Since $\deg_{SF_{1,n}}(u_i) = 2$, by the Definition1.1, distance one or two edges forms from u_i, that is $\deg_{(SF_{1,n})_d(u_i)} = n+3$. In α- set we can add u_{2i+1}, where $i = 0, 1, 2, 3, \ldots$ vertices. Hence, $\alpha((SF_{1,n})_d) = \lfloor \frac{n-2}{2} \rfloor + 1$. □

Proposition 3.16 *For $n \geq 3$, $\alpha(CSF_{1,n}) = 1$.*

Proof: The closed sunflower graph $CSF_{1,n,}$, is of the order $2n+1$. Let $V(CSF_{1,n,}) = \{v_0, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$. For $n \geq 3$, $\delta(CSF_{1,n,}) \geq diam(CSF_{1,n,})$, by Proposition 3.1, $\alpha(CSF_{1,n}) = 1$. □

Proposition 3.17 *For $n \geq 3$,*

$$\alpha((D_{1,n})_d) = 1.$$

Proof: The djembe graph $D_{1,n}$, $n \geq 3$, is of the order $2n+1$. Let $V(D_{1,n}) = \{v_0, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$. By Definition 1.1 and Theorem 1.1, the deg-centric graph of a djembe graph $D_{1,n}$, $n \geq 3$, is complete graph. That is, $(D_{1,n})_d \cong K_{2n+1}$. In view of Proposition 3.1, $\alpha((D_{1,n})_d) = 1$. □

Proposition 3.18 *For $n \geq 3$,*

$$\chi((Sl_n)_d) = n.$$

Proof: The sunlet graph Sl_n, $n \geq 3$ is of the order $2n$. Let $V(Sl_n) = \{v_1, v_2, \ldots, v_n, \underbrace{u_1, u_2, \ldots, u_n}_{\text{pendant vertices}}\}$. If $3 \leq n \leq 5$, in view of Definition 1.1, $\deg_{Sl_n}(v_i) = 3 > e(v_i) = 2$ then all v_i vertices are adjacent with other $2n-1$ vertices. Therefore, all the vertices u_i are added to the α-set in $(Sl_n)_d$. Finally, $\alpha((Sl_n)_d) = n$. If $n = 6$, by Definition 1.1, then all v_i vertices are adjacent to the other $2n-2$ vertices. However, since all u_i are pendant vertices, in view of Definition 1.1, no edge forms from a u_i in $(Sl_n)_d$. All the vertices u_i are added to the α-set in $(Sl_6)_d$. Hence, $\alpha((Sl_6)_d) = n$. If $n = 7$, by Definition 1.1, then all v_i vertices are adjacent to eleven vertices. However, since all u_i are pendant vertices, in view of Definition 1.1, no edge forms from a u_i in $(Sl_n)_d$. All the vertices u_i are added to the α-set in $(Sl_7)_d$. Hence, $\alpha((Sl_7)_d) = n$.

If $n \geq 8$, by Definition 1.1, then all v_i vertices are adjacent with eleven vertices. However, since all u_i are pendant vertices, no edge forms from a u_i in $(Sl_n)_d$. Then, all u_i have degree five in $(Sl_n)_d$. The deg-centric graph of (Sl_n), $n \geq 8$, with consecutively labeled rim vertices $\{v_1, v_2, v_3, \ldots, v_n\}$. The α-set contains all the u_i vertices in $(Sl_n)_d$. Hence, $\alpha((Sl_n)_d) = n$.

□

An illustration of a proposition 3.18 is given in Figure 4.

If the edge v_1v_3 joins vertices v_1 and v_3, then the *subdivision* of v_1v_3 replaces v_1v_3 by a new vertex v_2 and two new edges v_1v_2 and v_2v_3. A *gear graph*, denoted by G_n, $n \geq 3$, is a graph obtained by applying subdivision to each edge of the rim of a wheel graph $W_{1,n}$.

Proposition 3.19 *For $n \geq 3$,*

$$\alpha((G_n)_d) = \lfloor \frac{n}{2} \rfloor.$$

Proof: The gear graph, $n \geq 4$, is of the order $2n+1$. Let $V(G_n) = \{v_0, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$. Since $deg_G(v_0) = n > e(v_0) = 2$, by Definition1.1, $2n$ edges form from v_0 in $(G_n)_d$. Vertices v_i are adjacent to the center vertex v_0 However, since $\deg_{G_n}(v_i) = 3$, by Definition1.1, $2n$ edges form from a vertex v_i in $(G_n)_d$. We cannot add the v_0 and v_i vertices in the α-set. Since $\deg_{G_n}(u_i) = 2$,

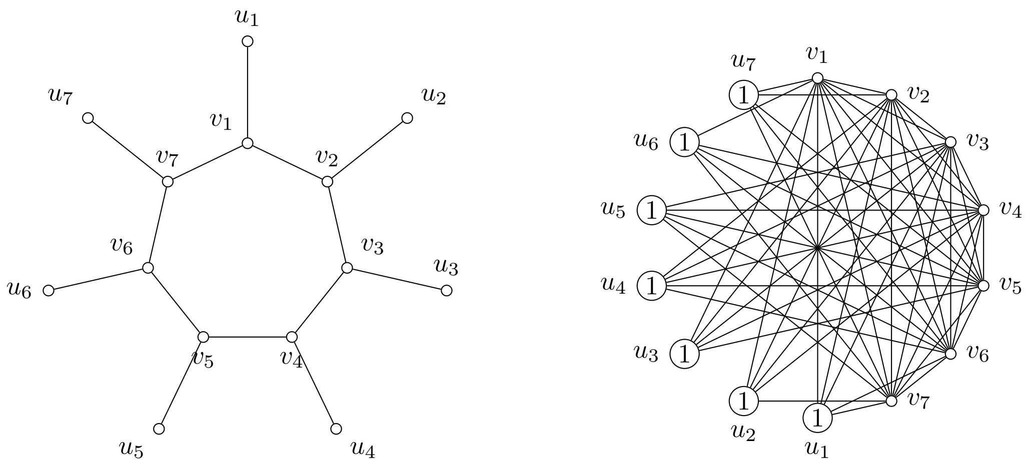


Figure 4: $\alpha(Sl_7)_d) = 7$.

then a distance less than or equal to two vertices forms edges from u_i, that is $\deg_{(G_n)_d(u_i)} = n+3$. We can add $\lfloor \frac{i}{2} \rfloor$ vertices to the maximum independent set. The α-set contains alternative u_i vertices in $(G_n)_d$. Hence, $\alpha((G_n)_d) = \lfloor \frac{n}{2} \rfloor$.

□

Proposition 3.20 *For $n \geq 3$,*

$$\alpha(Bl_{1,n}) = 1.$$

Proof: The blossom graph $Bl_{1,n,}$, $n \geq 3$ is of the order $2n+1$. Let $V(Bl_{1,n,}) = \{v_0, v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$. Since $\delta(Bl_{1,n,}) = 5 > diam(Bl_{1,n,}) = 2$, by Proposition3.1, $\alpha(Bl_{1,n}) = 1$. □

An *antiprism graph*, denoted by A_n, $n \geq 3$ is a graph obtained two cycles C_n and C'_n of order n with vertex sets $V = \{v_1, v_2, v_3, \ldots, v_n\}$ and $U = \{u_1, u_2, u_3, \ldots, u_n\}$ respectively. Join the vertices $u_i v_i$ and $u_i v_{i+1}$ to form the additional edges.

Proposition 3.21 *For $n \geq 3$,*

$$\alpha((A_n)_d) = \begin{cases} 1; & \textit{if } n \leq 8 \\ \lfloor \frac{n-9}{5} \rfloor + 2; & \textit{otherwise.} \end{cases}$$

Proof: For an antiprism graph A_n, $n \geq 3$, is of the order $2n$. Let $V(A_n) = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$.

1. (i) If $3 \leq n \leq 8$, $\deg_{A_n}(v_i) = deg_{A_n}(u_i) = 4 > e(v_i) = e(u_i)$, by Proposition 3.1, $\alpha((A_n)_d) = 1$.
2. (ii) If $n \geq 9$, $\deg_{A_n}(v_i) = \deg_{A_n}(u_i) = 4$, by Definition 1.1, $\deg_{(A_n)_d(v_i)} = \deg_{(A_n)_d(u_i)} = 16$. That implies $\varepsilon((A_n)_d) = 16n$. In the $\alpha-$ set, it contains v_{9i+5}, where $i = 0, 1, 2, 3 \ldots$ vertices, and u_{9i+1}, where $i = 0, 1, 2, 3 \ldots$ vertices. Hence, $\alpha((A_n)_d) = \lfloor \frac{n-9}{5} \rfloor + 2$.

□

Consider a complete graph K_n with the vertex set $V = \{v_1, v_2, v_3, \ldots, v_n\}$. Let $U = \{u_1, u_2, u_3, \ldots, u_n\}$ be a copy of $V(G)$ such that u_i corresponds to v_i. The *sun graph*, denoted by S_n, is a graph with vertex set $V \cup U$ and two vertices x and y are adjacent in S_n if $x \sim y$ in K_n and $x = u_i$, $y \in v_i, v_{i+1}$.

Table 1: Independence number of various deg-centric graphs

Graph G	$\alpha(G_d)$	**Condition / Remarks**
K_n	1	$n \geq 3$
P_n	$\left\lfloor \frac{n+2}{3} \right\rfloor$	$n \geq 4$
C_n	$\left\lfloor \frac{n-3}{3} \right\rfloor + 1$	$n \geq 6$
$K_{1,n}$	n	$n \geq 1$ (Star graph)
$K_{m,n}$	$\max(m, n)$	$m, n \geq 1$ (Complete bipartite graph)
$S_{a,b}$	$a + b$	$a, b \geq 1$ (Bistar graph)
$H_{1,n}$	n	$n \geq 3$ (Helm graph)
$CH_{1,n}$	$\left\lfloor \frac{n-8}{4} \right\rfloor + 2$	$n \geq 8$ (Closed helm graph)
$W_{1,n}$	1	$n \geq 3$ (Wheel graph)
DW_n	1	$n \geq 3$ (Double wheel graph)
$Wb_{1,n}$	n	$n \geq 3$ (Web graph)

Proposition 3.22 *For* $n \geq 3$,

$$\alpha((S_n)_d) = \begin{cases} 1; & \textit{if } n = 3 \\ \lfloor \frac{n-2}{2} \rfloor + 1; & \textit{otherwise.} \end{cases}$$

Proof: For a sun graph S_n, $n \geq 3$, is of the order $2n$. Let $V(S_n) = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$. For $n = 3$, $\delta(S_3) \geq diam(S_3)$, by Proposition 3.1, $\alpha((S_3)_d) = 1$.

If $n \geq 4$, $deg_{S_n}(v_i) = n + 1 > e(v_i) = 2$, by Definition 1.1, $(2n-1)$ edges form from v_i in $(S_n)_d$. We cannot add these vertices in α- set in $(S_n)_d$. However, since $\deg_{S_n}(u_i) = 2$, by Definition 1.1, distance one or two edges forms from u_i in $(S_n)_d$. In α- set we can add u_{2i+1}, where $i = 0, 1, 2, 3, \ldots$ vertices. Hence, $\alpha((S_n)_d) = \lfloor \frac{n-2}{2} \rfloor + 1$. □

Proposition 3.23 *For* $n \geq 3$,

$$\alpha(CS_n) = 1.$$

Proof: For a closed sun graph CS_n, $n \geq 3$, is of the order $2n$. Let $V(CS_n) = \{v_1, v_2, \ldots, v_n, u_1, u_2, \ldots, u_n\}$. For $n \geq 3$, $\delta(CS_n) \geq diam(CS_n)$, by Proposition 3.1, $\alpha(CS_n) = 1$. □

The *ladder graph*, L_n, $n \geq 1$ is obtained by taking two copies of a path P_n with respective vertices say, $v_1, v_2, v_3, \ldots, v_n$ and $u_1, u_2, u_3, \ldots, u_n$ and adding the edges $v_i u_i$, $1 \leq i \leq n$. Note that $L_n \cong P_n \Box K_2$ where $\Box$ denotes the Cartesian product.

Proposition 3.24 *For* $n \geq 1$,

$$\alpha((L_n)_d) = \begin{cases} 1; & \textit{if } n \leq 2 \\ \lfloor \frac{n}{3} \rfloor + 1; & \textit{otherwise.} \end{cases}$$

Proof: By applying Definition 1.1, it easily follows that $\varepsilon(L_{1_d}) = 1$, $\varepsilon(L_{2_d}) = 6$, $\varepsilon(L_{3_d}) = 13$, $\varepsilon(L_{4_d}) = 24$, $\varepsilon(L_{5_d}) = 37$ and $\varepsilon(G_d) = \varepsilon(H_d) + 11$ where $H = L_{n-1}$ and $n \geq 6$ [8]. In the least α- set of $(L_n)_d$, we can add the vertices u_{3i}, where $i = 1, 3, 5 \ldots$, the vertex v_1 and the vertices v_{6i}, where $i = 1, 2, 3 \ldots$ to the α- set, all other vertices are adjacent to these vertices. Hence, $\alpha((L_n)_d) = \lfloor \frac{n}{3} \rfloor + 1$. □

4. Conclusion

The concept of independent sets in deg-centric graphs has been explored, and the independence number for the deg-centrication of various graph classes has been investigated. Several preliminary results have been presented to lay the groundwork for future studies. As a potential direction for further research, graph-theoretical parameters can be examined in the context of deg-centric graphs across different graph classes to yield valuable insights. Additionally, new researchers may consider exploring various forms of graph independent sets within the framework of deg-centric graphs.

Furthermore, the structural behaviour of deg-centric transformations suggests that several classical graph invariants, including domination parameters, connectivity measures, and chromatic properties, may exhibit interesting variations under deg-centric operations. Establishing relationships between these parameters in a graph and its deg-centric counterpart could lead to new bounds and structural characterisations.

Another promising direction lies in the study of algorithmic aspects of deg-centric graphs, particularly the computational complexity of determining independence-related parameters. Extensions of this work to weighted graphs, dynamic graphs, or probabilistic network models may also provide useful insights for applications in network analysis. Overall, the results obtained in this study form a foundation for the systematic development of theory related to deg-centric graph transformations and open several avenues for further research.

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