Bol. Soc. Paran. Mat. (3s.) **v. 2026 (44) 6** : 1–8.
©SPM – E-ISSN-2175-1188 ISSN-0037-8712
SPM: www.spm.uem.br/bspm doi:10.5269/bspm.79585

α-Symmetric and α-Door Space

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ABSTRACT: The aim of this research is to apply a new topological spaces are named α-symmetric and α-door. we show that no relation between them. Also we show that α-door is hereditary property. Definitions, properties, examples and remarks of symmetric and door space via α-open set.

Keywords: α-symmetric, $S\alpha$-symmetric, $P\alpha$-symmetric, α^*-symmetric, α-door space, $S\alpha$-door, $P\alpha$-door, α^*-door space

Contents

1. Introduction

In the mathematical paper, N. Levine [8], defined Q-set. N. Levine in [9] introduced s-open set. O.Njasted defined α and $\alpha*$-open set in [11]. In (1969) J. Dugundji in [5] investigated door space. In (1982) A. S. Mashhour gave pre-open set. N.K. Ahmed [2] defined s-closure of set. In (1994) M. Caldas presented s-symmetric [3]. α-Exterior, α-Fronter and α-boundary studied by M. Caldas in 2003, see [4]. N.H. AL-Tabatabai [13] gave α-closure and α-interior in (2004) . pre-closure investigated by N.J. Kaisam [6] in 2005. H.M. Salih [12], defined s-door space . In (2011) G.A. Khtan [7], presented pre-door space. And in 2021 he introduced α^*-irresolute continuous function in [1].

In this work we presented the concepts α-symmetric and α-door space and some properties about them . We have α-symmetric is need not to be α-door and α-door is not α-symmetric . Also we have the subspace of α-door space is α-door. we introduced theorems, examples and remarks about them . We will denote (Semi, Pre-, α-, α^*-, Semiα-, pre α- and α^*-)open set by the symbols (S-O., P-O., α-O., α^*-O. , $S\alpha$-O., $P\alpha$-O.-) also denote Semi symmetric, pre symmetric, α-symmetric , Semi α-symmetric , Pre α-symmetric and α^*- symmetric by the symbols (S-sym., P-sym., α-sym., $S\alpha$-sym., $P\alpha$-sym. and α^*- sym.)

2. Preliminaries

Definition 2.1 *If (Ω, μ) be a topological space and $W \subseteq \Omega$ is said to be*

1. *Semi-open (S-O.)[9], if $W \subseteq \overline{W^\circ}$. Semi-closed (S-C.) if $\overline{W}^\circ \subseteq W$.*
2. *Pre-open (P-O.) [10], if $W \subseteq \overline{W}^\circ$. Pre-closed (P-C.) if $\overline{W^\circ} \subseteq W$.*
3. *α-open (α-O.) [11], if $W \subseteq \overline{W^\circ}^\circ$. α-closed (α-C.) if $\overline{\overline{W}^\circ} \subseteq W$*
4. *Q-set [8], if $\overline{W^\circ} = \overline{W}^\circ$*

Definition 2.2 *If (Ω, μ) be a topological space and W sub set of Ω is called*

1. *Semi-closure of W [2], defined by $\overline{W}^S = \cap\{H : H \text{ is S-C.}; W \subseteq H\}$.*

2020 *Mathematics Subject Classification:* 54A40.
Submitted October 16, 2025. Published February 17, 2026

2. *Pre-closure of W* [6], *defined by* $\overline{W}^{P} = \cap\{H : H \text{ is P-C.}; W \subseteq H\}$.
3. *α-closure of W* [13], *defined by* $\overline{W}^{\alpha} = \cap\{H : H \text{ is } \alpha\text{-C.}; W \subseteq H\}$.
4. *α-kernel of W defined by* $\alpha\text{-}Ker(W) = \cap\{H : H \text{ is } \alpha\text{-O.}; W \subseteq H\}$.
5. *α-interior of W* [13], *defined by* $W^{\circ^{\alpha}} = \cup\{K : K \text{ is } \alpha\text{-O.}; K \subseteq W\}$.
6. *α-exterior of W* [4], *defined by* $\alpha\text{-}Ext(W) = \cup\{V : V \text{ is } \alpha\text{-O.}; V \subseteq W^{c}\}$.
7. *α-frontier of W* [4], *defined by* $\alpha\text{-}Fr(W) = \overline{W}^{\alpha} \cap \overline{W^{c}}^{\alpha}$.
8. *α-border of W* [4], *defined by* $\alpha\text{-}b(W) = W \cap \overline{W^{c}}^{\alpha}$.

Definition 2.3 *If* (Ω, μ) *be a space and W sub set of* Ω *is called*

1. *α^*-open set (α^*-O.)* [11], *if* $W \subseteq \overline{W^{\circ\alpha}}^{\alpha\circ\alpha}$*.$\alpha^*$-closed ($\alpha^*$-C.) if* $\overline{\overline{W}^{\alpha\circ\alpha}}^{\alpha} \subseteq W$.
2. *Semi α-open set (Sα-O.) if* $W \subseteq \overline{W^{\circ\alpha}}^{\alpha}$. *Semi α-closed (Sα-O.) if* $\overline{W}^{\alpha\circ\alpha} \subseteq W$.
3. *Pre α-open set (Pα-O.) if* $W \subseteq \overline{W}^{\alpha\circ\alpha}$. *Preα-closed (Pα-C.) if* $\overline{W^{\circ\alpha}}^{\alpha} \subseteq W$.
4. *α-Q-set if* $\overline{W^{\circ\alpha}}^{\alpha} = \overline{W}^{\alpha\circ\alpha}$.

Definition 2.4 *Let* (Ω, μ) *be a space and* $W \subseteq \Omega$ *is called*

1. *α^*-closure of W given by* $\overline{W}^{\alpha *} = \cap\{H : H \text{ is } \alpha^*\text{-C.}; W \subseteq H\}$.
2. *Sα-closure of W given by* $\overline{W}^{S\alpha} = \cap\{H : H \text{ is } S\alpha\text{-C.}; W \subseteq H\}$.
3. *Pα-closure of W given by* $\overline{W}^{P\alpha} = \cap\{H : H \text{ is } P\alpha\text{-C.}; W \subseteq H\}$.

Remark 2.1

1. Every α-O. is S-O..
2. Sα-closure of W given by $\overline{W}^{S\alpha} = \cap\{H : H \text{ is } S\alpha\text{-}C.; W \subseteq H\}$.
3. Every α-O. is α^*-O. [11].

But S-O. need not to be α-O. for the following example

Example 2.1 Suppose $\Omega = \{f, g, h, j\}, \tau = \{\emptyset, \Omega, \{f\}, \{h\}, \{f, h\}\}$ is topology on Ω. If $A = \{f, g\}$ it is clear that A is S-O. but not α-O.

Theorem 2.1 *Every Sα-O. is α^*-O.*

Proof: Let W is Sα-O. thus $W \subseteq \overline{W^{\circ\alpha}}^{\alpha} \subseteq \overline{W^{\circ\alpha}}^{\alpha\circ\alpha}$. Then $W \subseteq \overline{W^{\circ\alpha}}^{\alpha\circ\alpha}$ is α^*-O. □

Theorem 2.2

1. *W is P-O. if W is α-O. and Q-set*
2. *W is Pα-O. if W is α^*-O. and α Q-set then*

Proof:

1. Suppose W is α-O. thus $W \subseteq \overline{W^{\circ}}^{\circ}$. Since W is Q-set hence $W \subseteq \overline{W}^{\circ\circ}$. Hence $W \subseteq \overline{W}^{\circ}$ then W is P-O.

2. Suppose W is α^*-O. thus $W \subseteq \overline{W^{\circ\alpha}}^{\alpha\circ\alpha}$. Since W is α Q-set hence $W \subseteq \overline{W}^{\alpha\circ\alpha\circ\alpha}$. Then $W \subseteq \overline{W}^{\alpha\circ\alpha}$ thus W is $P\alpha$-O.

□

Remark 2.2 If W is α Q-set in Ω thus W is Sα-O. if and only if Pα-O.

Theorem 2.3 *The product of two α-O. is α-O.*

Proof: Let M and N are α-O. then $M \times N \subseteq \overline{M^\circ}^\circ \times \overline{N^\circ}^\circ = (\overline{M^\circ} \times \overline{N^\circ})^\circ = (\overline{M^\circ \times N^\circ})^\circ = \overline{(M \times N)^\circ}^\circ$. We have $M \times N \subseteq \overline{(M \times N)^\circ}^\circ$. Therefore $M \times N$ is α-O. □

Theorem 2.4 *If Ω is a space and $c \in W$ then $c \in \overline{W}^\alpha$ if and only if for all E is α-O. containing c such that $E \cap W \neq \emptyset$.*

Proof: Clear. □

Theorem 2.5 *If $W \subseteq \Omega$ we have $\overline{W^c}^\alpha = W^{\circ\alpha c}$ and $\left(\overline{W}^\alpha\right)^c = (W^c)^{\circ\alpha}$*

Proof: Clear. □

Theorem 2.6 *If W sub set of Ω then*

1. $\overline{W}^S \subseteq \overline{W}^\alpha$
2. $\overline{W}^p \subseteq \overline{W}^\alpha$ *if every α-O. is Q-set.*
3. $\overline{W}^{\alpha*} \subseteq \overline{W}^\alpha$.
4. $\overline{W}^{P\alpha} \subseteq \overline{W}^{\alpha*}$ *if every α^*-O. is αQ-set .*

Proof:

1. By Remark 2.1 and Theorem 2.4
2. Let $c \in \overline{W}^p$. Suppose V is α-O. such that c contained in V. Since V is Q-set thus it is Pα- O. by Theorem 2.2 (1) hence $V \cap W \neq \emptyset$. Therefore $c \in \overline{W}^\alpha$.
3. Clear.
4. Let $c \in \overline{W}^{P\alpha}$. Suppose V is α^*-O. such that c contained in V. Since V is αQ -set thus it is Pα-O. By Theorem 2.2(2) hence V∩W $\neq \emptyset$. Therefore $c \in \overline{W}^{\alpha*}$.

□

Theorem 2.7 *For Ω is space and i, j are points in Ω then $i \in \alpha$-$Ker(\{j\})$ iff $j \in \overline{\{i\}}^\alpha$.*

Proof: Let $i \notin \alpha$-$Ker(\{n\})$. Thus $\exists\alpha$-O. set G containing j not i. Then $G \cap \overline{\{i\}}^\alpha = \emptyset$. we have $j \notin \overline{\{i\}}^\alpha$. Suppose $j \notin \overline{\{i\}}^\alpha$. Hence $\exists\alpha$-O. set H where $j \in H$ and $i \notin H$. Therefore $H \cap \{i\} = \emptyset$. Then $i \notin \alpha$-$Ker(\{j\})$. □

3. α-Symmetric Space

Definition 3.1 *For all $r, t \in \Omega$ then the topological space (Ω, μ) is said to be*

1. *Semi-symmetric (S-sym.) [3], if $t \in \overline{\{r\}}^{s}$ implies that $r \in \overline{\{t\}}^{s}$.*
2. *Pre-symmetric (P-sym.) if $t \in \overline{\{r\}}^{P}$ means that $r \in \overline{\{t\}}^{P}$.*
3. *α-symmetric (α-sym.) if $t \in \overline{\{r\}}^{\alpha}$ shows that $r \in \overline{\{t\}}^{\alpha}$.*
4. *α^*-symmetric (α^*-sym.) if $t \in \overline{\{r\}}^{\alpha *}$ infers that $r \in \overline{\{t\}}^{\alpha *}$.*
5. *Semi α-symmetric ($S\alpha$-sym.) if $t \in \overline{\{r\}}^{S\alpha}$ implies that $r \in \overline{\{t\}}^{S\alpha}$.*
6. *Pre α-symmetric ($P\alpha$-sym.) if $t \in \overline{\{r\}}^{P\alpha}$ shows that $r \in \overline{\{t\}}^{P\alpha}$.*

Theorem 3.1

1. *Every S-sym. is α-sym.*
2. *Every α^*-sym. is $S\alpha$-sym.*

Proof:

1. Let $m, n \in \Omega$. S-sym. then $n \in \overline{\{m\}}^{S}$ implies that m containing in $\overline{\{n\}}^{S}$. Since $\overline{\{m\}}^{S}$ subset of $\overline{\{m\}}^{\alpha}$, $\overline{\{n\}}^{S}$ subset of $\overline{\{n\}}^{\alpha}$ thus n containing in $\overline{\{m\}}^{\alpha}$ implies that m containing in $\overline{\{n\}}^{\alpha}$ therefore Ω α-sym.
2. By Theorem 2.2 (2).

□

Theorem 3.2

1. *If Ω is P-sym. and each subset of Ω is Q-set thus Ω is α-sym.*
2. *If Ω is $P\alpha$-sym. and each subset of Ω is αQ-set thus Ω is α^*-sym.*

Proof:

1. Let $m, n \in \Omega$. Let E and K are α-O. such that $m \in E$ and $n \in K$. Since E and K are Q-sets hence E and k are P-O. by Theorem 2.2 (1) we have $E \cap n \neq \emptyset$ and $K \cap m \neq \emptyset$. Hence n containing in $\overline{\{m\}}^{P}$ implies that m contained in $\overline{\{n\}}^{P}$ therefore $\overline{\{m\}}^{P}$ subset of $\overline{\{m\}}^{\alpha}$ and $\overline{\{n\}}^{P}$ subset of $\overline{\{n\}}^{\alpha}$ by Theorem 2.6 (2) hence Ω is α-sym.
2. Let $m, n \in \Omega$. Let E and K are α^*-O. such that $m \in E$ and $n \in K$. Since E and K are Q-sets hence E and k are $P\alpha$-O. by Theorem 2.2 (2) we have $E \cap n \neq \emptyset$ and $K \cap m \neq \emptyset$. Hence n contained in $\overline{\{m\}}^{P\alpha}$ implies that m contained in $\overline{\{n\}}^{P\alpha}$ therefore $\overline{\{m\}}^{P\alpha}$ subset of $\overline{\{m\}}^{\alpha *}$ and $\overline{\{n\}}^{P\alpha}$ subset of $\overline{\{n\}}^{\alpha *}$ by Theorem 2.6 (4) hence Ω is α^*-sym.

□

Theorem 3.3 *If Ω is α-sym. and $m \in \Omega$ then*

1. *$\overline{\{m\}}^{\alpha} \subseteq \alpha\text{-}Ker(\{m\})$ for all m containing in Ω.*
2. *$(\alpha_K er\,(\{m\})^{c} \subseteq (\{m\}^{c})^{o\alpha}$ for all m contained in Ω.*

3. Ω has not singletons α-C.

Proof:

1. Suppose m contain in Ω. Let n contain in $\overline{m}^{\alpha}$. Since Ω is α-sym. so that m contain in $\overline{n}^{\alpha}$. Hence $n \in \alpha\text{-}Ker(\{m\})$ by Theorem 2.4. We have $\overline{m}^{\alpha} \subseteq \alpha\text{-}Ker(\{m\})$

2. Clear.

3. Suppose $\{m\}$ is α-C. in Ω then $\{m\} = \overline{\{m\}}^{\alpha}$. Since $m \neq n$ thus $\overline{\{m\}}^{\alpha}$ not containing n and $\overline{\{n\}}^{\alpha}$ not containing m this contradiction since Ω is α-sym.

□

Definition 3.2 *For $W \subseteq \Omega$ is said to be α-dense if $\overline{W}^{\alpha} = \Omega$.*

Theorem 3.4 *If Ω is α-sym. and $\{m\}$ is α-dense then $\alpha\text{-}Ker(\{m\}) = \Omega$ for all m in Ω.*

Proof: Suppose m containing in Ω. Since $\{m\}$ α-dense therefore $\overline{\{m\}}^{\alpha} = \Omega$, Since Ω is α-sym. thus α-Ker$(\{m\}) = \Omega$ for all m in Ω. □

Theorem 3.5 *If b, d are contained in Ω is α-sym. then $\overline{\{b\}}^{\alpha} \cap \overline{\{d\}}^{\alpha} \neq \emptyset$*

Proof: Let $\overline{\{b\}}^{\alpha} \cap \overline{\{d\}}^{\alpha} = \emptyset$. Since $b \in \overline{\{b\}}^{\alpha}, d \in \overline{\{d\}}^{\alpha}$ hence $b \notin \overline{\{d\}}^{\alpha}$ and $d \notin \overline{\{b\}}^{\alpha}$ therefore Ω not α-sym. this contradiction we have $\overline{\{b\}}^{\alpha} \cap \overline{\{d\}}^{\alpha} \neq \emptyset$ □

Theorem 3.6 *If (Ω, μ) is α-sym. then*

1. $b \in \alpha\text{-}Fr(\{d\})$ and $d \in \alpha\text{-}Fr(\{b\})$ $\forall b, d \in \Omega$

2. $b \notin \alpha\text{-}b(\{d\})$ and $d \notin \alpha\text{-}b(\{b\}) \forall\ b, d \in \Omega$

Proof:

i. Since Ω is α-sym. then $d \in \overline{\{b\}}^{\alpha}$. implies that $d \in \overline{\{b\}}^{\alpha}$. Since $\alpha\text{-}Fr(\{d\}) = \overline{\{d\}}^{\alpha} \cap \overline{\{d\}^{c}}^{\alpha}$ and $b \in \overline{\{d\}}^{\alpha}, b \in \Omega\text{-}d = \{d\}^{c}$ subset of $\overline{\{d\}^{c}}^{\alpha}$. we have $d \in \alpha\text{-}Fr(b)$.

ii. Clear.

□

Theorem 3.7 *A space (Ω, μ) is α-sym. iff $l_1 \notin \alpha\text{-}Ext(\{l_2\})$ means that $l_2 \notin \alpha\text{-}Ext(\{l_1\})$ $\forall$ l_1, l_2 contained in Ω.*

Proof: Let l_1, l_2 contained in Ω. Since Ω α-sym. hence $l_1 \in \overline{\{l_2\}}^{\alpha}$ implies that $l_2 \in \overline{\{l_1\}}^{\alpha}$ thus $l_1 \notin \overline{l_2}^{\alpha c}$. Therefore $l_1 \notin \{l_2\}^{c \circ \alpha}$. Then $l_1 \notin \alpha\text{-}Ext(\{l_2\})$ implies that $l_2 \in \alpha\text{-}Ext(\{l_1\})$.

Suppose $l_1 \notin \alpha\text{-}Ext(\{l_2\})$ means that $l_2 \notin \alpha\text{-}Ext(\{l_1\})$ then $l_2 \notin \{l_1\}^{c \circ \alpha}$ and $l_1 \notin \{l_2\}^{c \circ \alpha}$ for every $l_1, l_2 \in \Omega$ We have $l_2 \notin \overline{\{l_1\}}^{\alpha c}, l_1 \notin \overline{\{l_2\}}^{\alpha c}$. Therefore $l_2 \in \overline{\{l_1\}}^{\alpha}$ implies that $l_1 \in \overline{\{l_2\}}^{\alpha}$, therefore Ω α-sym. □

Theorem 3.8 *If Ω_1 and Ω_2 are α-sym. The product of them is α-sym.*

Proof: Let (u, v) and$(p, q) \in \Omega_1 \times \Omega_2$ thus $u, p \in \Omega_1$ and $v, q \in \Omega_2$. Since Ω_1 and Ω_2 are α-sym. thus $u \in \overline{\{p\}}^{\alpha}$ and $v \in \overline{\{q\}}^{\alpha}$ implies that $P \in \overline{\{u\}}^{\alpha}, q \in \overline{\{v\}}^{\alpha}$ hence $(p, q) \in \overline{\{u\}}^{\alpha} \times \overline{\{v\}}^{\alpha} \subseteq \overline{\{u\} \times \{v\}}^{\alpha}$ thus $(p, q) \in \overline{\{u, v\}}^{\alpha}$ implies that $(u, v) \in \overline{\{p, q\}}^{\alpha}$ therefore $\Omega_1 \times \Omega_2$ is α-sym. □

4. α-door Spac

Definition 4.1 *A space Ω is named.*

1. *Door [5], if for each W contained in Ω is open or closed or both.*
2. *Semi-door (S-door) [12], if for all W contained in Ω is S-O. or S-C. or both.*
3. *Pre-door (P-door) [7], if for any W contained in Ω is P-O. or P-C. both.*
4. *α-door if for each W contained in Ω is α-O. or α-C. or both.*
5. *α-door if for each W contained in Ω is α-O. or α-C. or both.*
6. *α^*-door if for each W contained in Ω is α^*-O. or α^*-C. or both.*
7. *Semi α-door (Sα-door) if for all W contained in Ω is Sα-O. or Sα-C. or both.*
8. *Pre α -door (Pα-door) if for each W contained in Ω is Pα-O. or Pα-C. both.*

Remark 4.1 Every α-door is S-door.

The following example shows the converse of Remark 4.1 is not true

Example 4.1 Suppose $\Omega = \{e_1, e_2, e_3, e_4\}, \mu = \{\emptyset, \Omega, \{e_1\}, \{e_3\}, \{e_1, e_3\}\}$ is topology on Ω. It is easy to know that Ω is S-door but not α-door.

Theorem 4.1 *Every Sα-door is α^*-door.*

Proof: By Theorem 2.1 □

Remark 4.2

i. α-door is not necessary α-sym.
ii. α-sym. is not necessary α-door.

Example 4.2

i. If τ is discreet on X therefore (X, τ) α-door is not α-sym.
ii. If τ is indiscreet on X therefore (X, τ) α-sym. is not α-door.

Theorem 4.2

i. *If Ω is α-door and each subset of Ω is Q-set therefore Ω is P-door.*
ii. *If Ω is α^*-door and each subset of Ω is αQ-set therefore Ω is Pα-door.*

Proof: Clear. □

Definition 4.2 *[1] A mapping $k : (\Omega_1, \mu) \to (\Omega_2, \mu')$ is said to be α^*-irresolute continuous [1] if $k^{-1}(G)$ is α^*-O. in Ω_1 for all α^*-O. in Ω_2.*

Theorem 4.3 *If $k : (\Omega_1, \mu) \to (\Omega_2, \mu')$ one to one and α^*-irresolute continuous such that Ω_1 is α^*-door thus Ω_2 is α^*-door*

Proof: Let Ω_2 is α^*-door and $G \subseteq \Omega_1$ then $k(G) \subseteq \Omega_2$. Since Ω_2 is α^*-door hence $k(G)$ is α^*-O. or α^*-C. in Ω_2. Since k is α^*-irresolute continuous therefore inverse image of $k(G)$ is α^*-O. or α^*-C. in Ω_1. Since k is one to one hence $\left(k^{-1}(k(G)\right) = G$. We have Ω_1 is α^*-door. □

Theorem 4.4 *α-door is hereditary property*

Proof: Suppose (Ω, μ) α-door space. And H contained in Ω, let K contained in H. Thus K subset in Ω hence U is α-O. or α-C. set in Ω. We have $K \cap H = K$. Hence K is α-O. or α-C. in H. Then (H, μ_H) α-door. □

Theorem 4.5 *If Ω is α-door then*

1. *Either α-$Fr(V) = \overline{V^{\circ\alpha}}^{\alpha} \cap V^C$ or α-$Fr(V) = \alpha - b(V) = V \cap \overline{V}^{\alpha\circ\alpha C}$ for all V subset of Ω.*
2. *Either α-$Fr(V) = \overline{V}^{\alpha} \cap \alpha$-$Ext(V)$ or α-$Fr(V) = V \cap \overline{\alpha\text{-}Ext(V)}^{\alpha}$ for all V subset of Ω.*

Proof: Suppose V subset of Ω where Ω α-door therefor V is α-O. or α-C. or both

1. If V α-O. thus $\Omega - V$ is α-C. then α-$Fr(V) = \overline{V^{\circ\alpha}}^{\alpha} \cap \Omega - V$.
   If V is α-C. then α-$Fr(V) = V \cap \overline{V^c}^{\alpha} = \alpha$-$b(V)$.
   Since $\Omega - V$ is α-O. We have α-$Fr(V) = V \cap \overline{V^{C\circ\alpha}}^{\alpha} = V \cap \overline{\overline{V}^{\alpha C}}^{\alpha} = V \cap \overline{V}^{\alpha\circ\alpha C}$
2. If W is α-O. thus $\Omega - V$ is α-C. then α-$Fr(V) = \overline{V}^{\alpha} \cap \Omega - V = V^{\alpha} \cap V^{\circ\alpha C} = \overline{V}^{\alpha} \cap \alpha$-$Ext(\Omega - V)$.
   If W is α-C. then $\Omega - V$ is α-O. hence α-$Fr(V) = V \cap \overline{V^{c\circ\alpha}}^{\alpha} = W \cap \overline{\alpha\text{-}Ext(V)}^{\alpha}$.

□

Theorem 4.6 *If Ω α-door space thus There is no $G \subseteq \Omega$ where G and G^c are α-dense.*

Proof: Suppose $G \subseteq \Omega$ where G and G^c are α-dense. Thus $\overline{G}^{\alpha} = \Omega$ and $\overline{G^c}^{\alpha} = \Omega$. Therefore G not α-C. Since $\overline{G^c}^{\alpha} = G^{\circ\alpha c} = \Omega$. We have $G^{\circ\alpha} = \emptyset \neq G$, thus G not α-O. so as Ω not α-door this contradiction we have Ω α-door space. □

Remark 4.3 If Ω_1 and Ω_2 are α-door space but $\Omega_1 \times \Omega_2$ is need not be α-door

Example 4.3 Suppose $\Omega_1 = \{i, j\}$ and $\Omega_2 = \{p, q\}$ such that $\mu_1 = \{\emptyset, \Omega_1, \{i\}\}$ is topology on Ω_1, $\mu_2 = \{\emptyset, \Omega_2, \{p\}\}$ is topology on Ω_1. It is clear that Ω_1 and Ω_2 are α-door and $\{i\} \times \{j\}$ is not α-O. and not α-C. in $\Omega_1 \times \Omega_2$ thus the product of Ω_1 and Ω_2 is not α-door.

Theorem 4.7 *If every subset of Ω_1 and Ω_2 are α-clopen then $\Omega_1 \times \Omega_2$ is α-door .*

Proof: Clear. □

Figure 1 explain the relation among new types symmetric and door spaces.

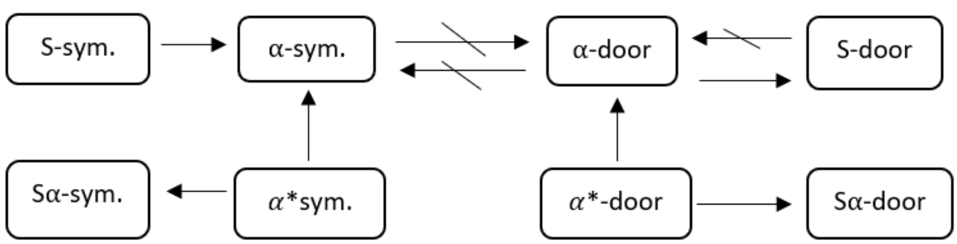

Figure 1: Relationships of types of symmetric and door spaces.

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