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A Green Inventory Model in Intuitionistic Fuzzy Environment With Remanufacturing of Defective Products Under Cap-and-Trade Policy *

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ABSTRACT: This research focuses on creating a multi-objective inventory management model for supply chains that addresses the difficulties of product deterioration and poor-quality production in an intuitive fuzzy environment. The purpose of this research is to lower both total operational costs and carbon emissions to improve supply chain profitability and prevent global warming. To minimize carbon emissions, the study considers carbon cap-and-trade policies and green technologies. Preservation technology is used to slow decreased product deterioration, while reworking abilities are used to repair imperfect items. Additionally, investments in quality improvements are considered to boost demand. In real-world scenarios, inventory management parameters are often uncertain, and thus, triangular intuitionistic fuzzy numbers are used to model these uncertainties. The study utilizes neutrosophic compromise programming to solve the resulting multi-objective model. The approach is demonstrated with a practical example, comparing crisp, fuzzy, and intuitionistic fuzzy models.

Keywords: Supply chain management, rework due to imperfect production, preservation and green technologies, enhancement of quality, carbon policies, Intuitionistic fuzzy set.

Contents

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1. Introduction

Reducing environmental pollution has become a central focus of global discussions today. Pollution leads to severe harm to the planet, contributing to issues like climate change and global warming. It also results in the rapid degradation of natural resources such as water, air, and soil quality. Over the past two decades, environmental pollution has escalated due to the excessive release of greenhouse gases (GHGs). More effective GHGs are carbon dioxide (CO_2), nitrous oxide (NO_2) and methane (CH_4) ([31]). The primary sources of CO_2 emissions are the transportation and industrial sectors. As a result, many companies are becoming more proactive in investing in green technology (GT) to reduce CO_2 emissions. The "Kyoto Protocol," established in 1997, aimed to identify solutions for tackling these issues by setting regulations to limit carbon emissions. [25] studied an inventory model for deteriorating products with GT investment. Several government-enforced policies, such as carbon tax (CT), carbon cap-and-trade (CCT), and carbon cap-and-offset (CCO), have been implemented to address this challenge. In this context, carbon emissions from supply chain inventory management (SCIM) are a significant concern, and various policies are being adopted by companies to reduce emissions and enhance SCIM sustainability. This study examines the impact of CCT policy and GT investments on the total carbon emissions and integrated costs within SCIM.

Consumers are increasingly concerned with the freshness of products, making the rate at which a product deteriorates a crucial factor in their purchasing decisions. This concern also introduces additional costs into the manufacturing process. To reduce deterioration, one solution is the use of Preservation Technology (PT), which helps reduce the rate of deterioration and minimizes product waste. However, fluctuations in storage conditions, such as temperature and humidity, cause the deterioration rate to vary ([46]). The goal of investing in PT is to stabilize these environmental factors, thereby decelerating the degradation process. As such, the impact of PT investments is a key focus of this study. Recently, Haldar et al., [15] and [16] developed two inventory models with deteriorating products under price dependent demand rate.

In inventory management, it is often assumed that all manufactured products are flawless and ready for sale. However, this assumption does not always reflect reality. Imperfect products are inevitable in the manufacturing process due to human errors and prolonged machine use. These defective products are typically inspected and reworked to make them marketable again. As a result, many researchers have explored inventory models that account for defective products. For example, [28] examined a sustainable production inventory model incorporating both synchronous and asynchronous reworking systems for defective products. The current study builds on this by exploring the effects of the synchronous reworking process within an SCIM framework, with an emphasis on quality improvement technology.

Parameters such as demand rate and various costs in SCIM models can experience minor fluctuations from their precise values, which may not follow any specific probability distribution. To address this, the fuzzy set (FS) theory, introduced by [48], is commonly applied to handle such uncertainties. FS theory has been successfully utilized in numerous fields, including SCIM. In FS theory, the membership function of an element is a value between 0 and 1, and the non-membership function is calculated by subtracting the membership degree from 1. Later, [5] expanded on FS by introducing the intuitionistic fuzzy set (IFS), where the sum of the membership degree and the non-membership degree of an element is constrained between 0 and 1. The hesitant degree, defined as the difference between 1 and the sum of membership and non-membership degrees, is zero in FS, but it can range between 0 and 1 in IFS. As a result, IFS offers a more flexible and practical approach for various applications, such as in algebraic structures, control systems, economics, and engineering. Many researchers have applied fuzzy set theories to model inventory and supply chain systems under uncertain conditions.

The triangular intuitionistic fuzzy number (TIFN) has been used to address decision-making problems in areas such as transportation, supply chain management, and inventory management. In SCIMs, uncertainties often arise in parameters like demand rate, deterioration rate, and variable costs per product. These uncertain parameters can be modeled as TIFNs. However, solving the model directly under an intuitionistic fuzzy (IF) environment is challenging due to the presence of both membership and non-membership functions. To overcome this, a ranking approach is employed to transform the IF parameters into crisp values using the (α, β) cut interval, allowing for the formulation of an equivalent deterministic model.

Based on the previous discussions, this study has three primary objectives. The first is to mathematically model an SCIM with a single manufacturer and a single retailer, addressing issues such as imperfect production and product deterioration. This model incorporates the inspection and reworking of defective products. The second objective is to reduce carbon emissions, which arise from production, transportation, and reworking processes. To achieve this, the study integrates a carbon cap-and-trade policy within the SCIM framework. The third goal is to optimize both operational costs and carbon emissions by developing a multi-objective SCIM model that includes the reworking process. Given the lack of precise information regarding SCIM parameters, these parameters are treated as uncertain. Therefore, the final innovation of this study is the use of TIFN to manage the uncertain parameters.

The rest of the paper contains 10 sections. Section 2 incorporates survey on related researches. Section 3 discusses the preliminaries about NS. Proposed model is defined in Section 4. Sections 5 consists of mathematical formulation of the SCIM model. Section 6 refers to the corresponding deterministic model. Solution procedure for solving the proposed model is discussed in Section 7. Section 8 contains one application example and discussion of obtained results. A sensitivity analysis and some important managerial insights are discussed in Section 9. Finally, conclusions of the proposed study and a few future research directions are presented in Section 10.

2. Research Overview

This paper provides a concise review of the literature in four key areas: (i) the SCIM problem related to deteriorating products, (ii) the reworking process for products with imperfect quality, (iii) methods for carbon emission reduction and management, and (iv) dealing with uncertain conditions.

2.1. SCIM problems for deteriorating products

For several decades, researchers have shown significant interest in SCIM issues related to deteriorating products. [45] explored an inventory model specifically for chilled foods, incorporating a price-dependent demand rate and a time-dependent deterioration rate. They considered price dependent demand rate and time dependent deterioration rate. Another study by [20] examined a supply chain inventory control problem involving a vendor and a retailer under a two-level trade credit policy. [6] suggested that carbon emissions in manufacturing processes could be mitigated through CCT policy and GT investments. Utilization of PT investment can significantly reduce deterioration rate. [34]investigated an EOQ model for deteriorating products, incorporating PT investment and stock-dependent demand.

2.2. Reworking process of imperfect quality products

Imperfect product quality is a regular occurrence in industrial systems. Numerous variables, including human mistake, improper handling, equipment failure, etc., can cause products to become poor quality. Certain defective goods are transported for reworking after being inspected. [3] investigated an EPQ model with synchronous and asynchronous reworking procedures in an imperfect production environment. They agreed with a constant demand rate. [23] explained a sustainable inventory problem including faulty products. They looked into the suggested model by taking into account the effects of a higher rate of deterioration and a discounted pricing for products of lower quality. They worked with CT policy and GT investment to maintain manageable carbon emissions. A synchronous reworking inventory model for deteriorating products in an imperfect manufacturing operation was evolved by [33]. In order to differentiate between perfect and flawed items in an EPQ model, [30] subsequently examined two different kinds of inspection procedures. Imperfect products are reworked after inspection. [9] has developed an EOQ model for both ideal and poor quality items, in which the latter are reworked after the manufacturing time is up. However, they haven't looked at how carbon emissions affect overall profit. A sustainable inventory model for defective and deteriorating products was examined in [23] in order to reduce carbon emissions from product storage and transit. Additionally, they have investigated how PT and GT investments affect the rates of deterioration and carbon emissions, respectively. [18] discussed an SCM for deteriorating products with imperfect manufacturing process under various carbon policies.

2.3. SCIM with different carbon policies

In recent years, various carbon policies have been implemented to mitigate carbon emissions generated during different activities within an SCIM. [21] proposed a retailer's inventory model incorporating CT regulation, assuming that carbon emissions result from product deterioration and storage. A two-echelon supply chain model under a CT policy was investigated by [38], who solved their model using a Stackelberg game-theoretical approach. [10] developed a low-carbon supply chain framework based on CT regulation and determined its optimal solution using the manufacturer-Stackelberg game theory. Additionally, [24] explored a sustainable supply chain model for deteriorating inventory, examining CT policy, GT investment, and PT investment to control carbon emissions associated with product deterioration and transportation. [1] examined how low-carbon performance impacts industrial sectors in developing countries. [4] discussed an SCIM with low carbon emissions under learning fuzzy theory.

2.4. SCIM under uncertain environment

The FS ([48]) is useful for handling imprecise or vague information in decision-making problems. Its fundamental element is a membership function, which ranges between 0 and 1. In inventory management, several parameters often exhibit imprecision. Numerous researchers have explored inventory models for deteriorating products under uncertain conditions. [32] examined an inventory model incorporating a fuzzy demand rate for deteriorating products. [29] introduced a green supply chain model that includes refurbishing activities, utilizing triangular fuzzy numbers to account for imprecise parameters such as demand rate, deterioration rate, and cannibalization parameters. Their study applied the sign distance method for defuzzification of fuzzy parameters.

Additionally, [12] investigated a sustainable traveling salesman problem in a type-2 fuzzy environment, incorporating carbon emissions. [13] addressed a multi-attribute decision-making problem for an EOQ model within an IF framework. [14] analyzed a multi-item inventory model with stock-dependent demand in an IF setting. During the COVID-19 pandemic, [37] explored a closed-loop supply chain network using the NS concept and solved their model through neutrosophic programming. [36] developed a back-ordering model under TIF environment.

2.5. Motivation

The literature review reveals that several researchers have explored various aspects, including SCIM for deteriorating products, reworking of imperfect-quality items, carbon policies for sustainable development, and handling uncertainty to manage imprecise parameters. However, the integration of these four topics has received limited attention. Additionally, only a few studies have specifically addressed the reworking process for imperfect-quality products. Hence, this study builds upon the work of [31] by introducing two types of reworking processes, incorporating PT investment to reduce deterioration rates, and implementing GT investment to mitigate carbon emissions. Given the lack of precise information, relevant data exhibit inherent uncertainty. To address this, TIFN is utilized to represent uncertain parameters, including initial demand rate, deterioration rate, and various variable costs. Furthermore, a CCT policy is examined for carbon emission reduction, and a comparative analysis of three different policies is conducted.

The key contributions of this study are as follows:

- A SCIM model with a synchronous reworking process for an imperfect production system of a deteriorating product is developed, extending the work of [31].
- PT investment is implemented to reduce the product's deterioration rate.
- CCT policy and GT investments are explored to mitigate carbon emissions and promote sustainability.

- TIFN is incorporated to mathematically handle uncertain parameters.
- The concept of NCP is utilized to solve deterministic multi-objective problems.

3. Preliminaries

Herein, a brief survey of relevant definitions and theorems about IFN is discussed subsequently. Those definitions and theorems are most important for this work.

Definition 3.1 Intuitionistic fuzzy set ([5]): An IFS $\grave{G}_I$ in a universal set P is of the form $\grave{G}_I = \{(p, \varrho_{\grave{G}_I}(p), \vartheta_{\grave{G}_I}(p)) : p \in P\}$ such that $0 \leq \varrho_{\grave{G}_I}(p) + \vartheta_{\grave{G}_I} \leq 1$. The functions

$$\varrho_{\grave{G}_I} : P \mapsto [0,1]; \text{i.e.}, p \in P \mapsto \varrho_{\grave{G}_I}(p) \in [0,1]$$
$$\text{and } \vartheta_{\grave{G}_I} : P \mapsto [0,1]; \text{i.e.}, p \in P \mapsto \vartheta_{\grave{G}_I}(p) \in [0,1]$$

define the membership degree and non-membership degree of an element $p \in P$. The acceptance degrees of $\varrho_{\grave{G}_I}(p)$ and $\vartheta_{\grave{G}_I}(p)$ can be arbitrary. The quantity $\Pi_{\grave{G}_I}(p) = 1 - \varrho_{\grave{G}_I}(p) - \vartheta_{\grave{G}_I}(p)$ is known as the measure of hesitant of the IFS $\grave{G}_I$.

Definition 3.2 (α, β)-cut: A (α, β)-cut set $(\grave{G}_I)_{\alpha,\beta}$ of an IFS is a crisp set and defined by $(\grave{G}_I)_{\alpha,\beta} = \{p \in P : \varrho_{\grave{G}_I}(p) \geq \alpha, \vartheta_{\grave{G}_I}(p) \leq \beta\}$. The fixed numbers α and β are taken from the closed interval $[0,1]$ satisfying the condition $\alpha + \beta \leq 1$.

Definition 3.3 Triangular intuitionistic fuzzy number (TIFN): As stated by [36], a TIFN is a specific form of an IFN, expressed as $\grave{G}_I^T = \langle p_1, p_2, p_3; p_1', p_2, p_3' \rangle$, where the condition $p_1' \leq p_1 \leq p_2 \leq p_3 \leq p_3'$ holds. The membership function (MF) and non-membership function (NMF) of the TIFN $\grave{G}_I^T$ are subsequently determined.

$$\varrho_{\grave{G}_I^T}(p) = \begin{cases} 0, & \text{if } p < p_1 \ \& \ p > p_3, \\ \frac{p-p_1}{p_2-p_1}, & \text{if } p_1 \leq p \leq p_2, \\ \frac{p_3-p}{p_3-p_2}, & \text{if } p_2 \leq p \leq p_3 \end{cases} \quad \& \quad \vartheta_{\grave{G}_I^T}(p) = \begin{cases} 1, & \text{if } p < p_1' \ \& \ p > p_3', \\ \frac{p_2-p}{p_2-p_1'}, & \text{if } p_1' \leq p \leq p_2, \\ \frac{p-p_2}{p_3'-p_2}, & \text{if } p_2 \leq p \leq p_3'. \end{cases} \tag{3.1}$$

If $p_1' = p_1 = p_2 = p_3 = p_3' = p$ then $\grave{G}_I^T = \langle p_1, p_2, p_3; p_1', p_2, p_3' \rangle$ refers to a real number p.

Definition 3.4 Expected value of a TIFN: Several defuzzification methods exist in literature review for defuzzifying a TIFN. In this study, the defuzzification procedure proposed by [42] is applied to defuzzify a TIFN.
Procedure:
The left and right α-cuts of $\grave{G}_I^T = \langle p_1, p_2, p_3; p_1', p_2, p_3' \rangle$ are as: $(\grave{G}_I^T)_\alpha^L = p_1 + (p_2 - p_1)\alpha$ and $(\grave{G}_I^T)_\alpha^R = p_3 - (p_3 - p_2)\alpha$, respectively. The right and left β-cuts of $\grave{G}_I^T$ are calculated as: $(\grave{G}_I^T)_\beta^R = p_2 + \beta(p_3' - p_2)$ and $(\grave{G}_I^T)_\beta^L = p_2 - \beta(p_2 - p_1')$, respectively. The expected value of $\grave{G}_I^T$ for the MF $\varrho_{\grave{G}_I^T}(p)$ is $\mathcal{R}_\varrho(\grave{G}_I^T) = \int_0^1 \frac{(\grave{G}_I^T)_\alpha^L + (\grave{G}_I^T)_\alpha^R}{2} d\alpha = \frac{p_1 + 2p_2 + p_3}{4}$. Similarly, the expected value of $\grave{G}_I^T$ for the NMF $\vartheta_{\grave{G}_I^T}$ is $\mathcal{R}_\vartheta(\grave{G}_I^T) = \int_0^1 \frac{(\grave{G}_I^T)_\beta^L + (\grave{G}_I^T)_\beta^R}{2} d\beta = \frac{p_1' + 2p_2 + p_3'}{4}$.

Now, the expected value of $\grave{G}_I^T$ is stated as: $\mathcal{R}(\grave{G}_I^T) = \kappa \mathcal{R}_\varrho(\grave{G}_I^T) + (1 - \kappa)\mathcal{R}_\vartheta(\grave{G}_I^T)$, where $0 \leq \kappa \leq 1$. When $\kappa = \frac{1}{2}$, then the expected value becomes

$$\mathcal{R}(\grave{G}_I^T) = \frac{1}{2}\mathcal{R}_\varrho(\grave{G}_I^T) + \frac{1}{2}\mathcal{R}_\vartheta(\grave{G}_I^T) = \frac{p_1 + p_1' + 4p_2 + p_3 + p_3'}{8}. \tag{3.2}$$

4. Model Establishment

In this section, the required notations, assumptions and problem description are presented subsequently.

4.1. Notations

The subsequent notations have been exploited for formulating the proposed SCIM:

Parameters

R_p :	A product's production rate per unit time.
R_r :	Rework rate per unit time for faulty products.
n_q :	Portion of good quality products manufactured ($0 < n_q \leq 1$).
r_q :	Portion of good quality products received from rework process ($0 < r_q \leq 1$).
d_{m0} :	Deterioration rate per unit time at the manufacturer's warehouse before investing in PT ($0 < d_{m0} < 1$).
d_m :	Deterioration rate per unit time at the manufacturer's warehouse after applying PT ($0 < d_m < 1$).
d_{r0} :	Deterioration rate per unit time at the retailer's storehouse before applying PT ($0 < d_{r0} < 1$).
d_r :	Deterioration rate per unit time at the retailer's storehouse after applying PT ($0 < d_r < 1$).
T :	Cycle time.
T_r :	Rework period in cycle time T.
$I_{mi}(t)$:	Level of acceptable inventory at time t for the manufacturer, $i = 1, 2, 3$.
$I_{mj}(t)$:	Level of reworkable inventory at time t for the manufacturer, $j = 4, 5$.
$I_r(t)$:	Level of inventory for the retailer at time t.
Q :	Quantity of products delivered by the manufacturer to the retailer per trip.
S_n :	Cost for setup of production process per cycle.
S_r :	Cost for setup of rework process per cycle.
c_p :	Cost for manufacturing each product ($).
O_r :	Ordering cost for the retailer per cycle.
c_r :	Cost for reworking of each reworkable product($).
p_w :	Purchasing cost for the retailer ($).
h_{mn} :	Cost for holding a good quality product at the manufacturer's warehouse ($/unit time).
h_{mr} :	Cost for holding a reworkable products at the manufacturer's warehouse ($/unit time).
h_r :	Cost for holding a product at the retailer's storehouse ($/unit time).
d_{cm} :	Deteriorating cost of each product for the manufacturer ($).
d_{cr} :	Deterioration cost of each product for the retailer ($).
s_c :	Discarding cost of each product ($).
i_c	Inspection cost of each product for the retailer ($).
f_{tc} :	Fixed transportation cost ($/delivery).
K :	Distance between the manufacturer and the retailer.
v_{tc} :	Variable transportation cost ($/unit product/km).
x_{tc} :	Transportation cost for empty truck ($/km).
Γ :	Investment for improving product's quality level.
η :	Fraction of PT investment which is paid by the retailer.
e_{sp} :	Carbon emission due to setup of production process (ton CO_2/setup).
e_{sr} :	Carbon emission due to setup of rework process (ton CO_2/setup).
e_p :	Carbon emission for manufacturing each product(ton CO_2).
e_r :	Carbon emission due to reworking process of each product (ton CO_2).
e_s :	Carbon emission due to discarding each non-reworked product (ton CO_2).
V :	Average energy consumption due to hold a product (kWh/unit time).
e_h :	Carbon emission due to holding products (ton CO_2/kWh).
e_d :	Carbon emission regarding deterioration of a product(ton CO_2).
e_v :	Carbon emission from transportation of products (ton CO_2/litre diesel fuel).
F_e :	Fuel consumption by an empty truck (litre/km).
F_l :	Fuel consumption by a loaded truck (litre/km).
u :	Permitted buying or selling price of carbon emission ($/ton CO_2).

Decision variables

N :	Number of deliveries from the manufacturer to the retailer per cycle.
G_I :	Investment for carbon emission reduction technology.
ξ :	PT investment ($/unit time).
a_s :	Selling price for each product ($).
T_p :	Production period in cycle time T.
q :	Product's quality level ($0 < q < 1$).

Other functions

Z_{IC} :	Integrated inventory cost of the manufacturer and the retailer per unit time ($).
T_{CE} :	Total carbon emissions per unit time (ton CO_2).
Z_1 :	Total of inventory cost and carbon emission cost per unit time ($).

4.2. Problem description and assumptions

This study examines an SCIM problem involving a single manufacturer and a single retailer operating over an infinite planning horizon. The manufacturer sources raw materials from a supplier to produce finished goods. Among the produced items, only perfect products are transported in N shipments to the retailer to meet consumer demand, with each shipment containing Q units. Product deterioration occurs at both the manufacturer's and retailer's warehouses. Additionally, over time, manufacturing machines may develop faults, leading to defective products. Some of these defective items undergo rework during new production cycles. However, not all defective products are reworkable; the non-reworkable ones are discarded immediately from the warehouse. The entire process—including setup, production, rework, deterioration, holding, discarding, and transportation—contributes to CO_2 emissions. To address this, the impact of carbon emissions on the total cost of SCIM is considered. The manufacturer also invests in GT to transition towards a more eco-friendly production system. A visual representation of the proposed SCIM model, incorporating the reworking of imperfect-quality products, is provided in Figure 1.

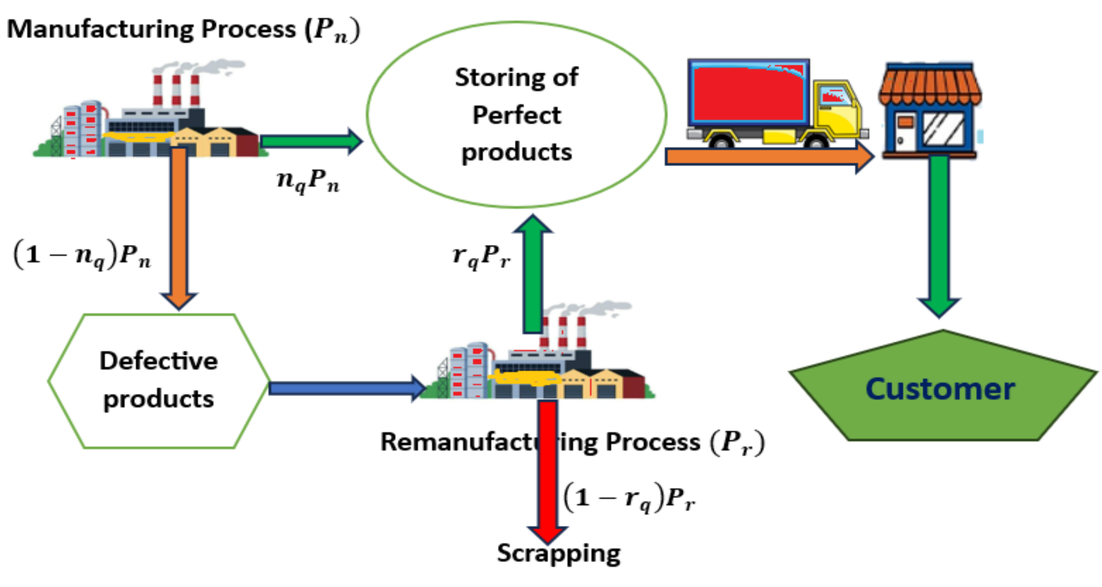

Figure 1: SCIM with rework (proposed by authors).

4.2.1. Assumptions. The required assumptions for developing proposed SCIM are as:

(i) An SCIM for a single manufacturer and a single retailer is incorporated.

(ii) Replenishment is done momentarily.

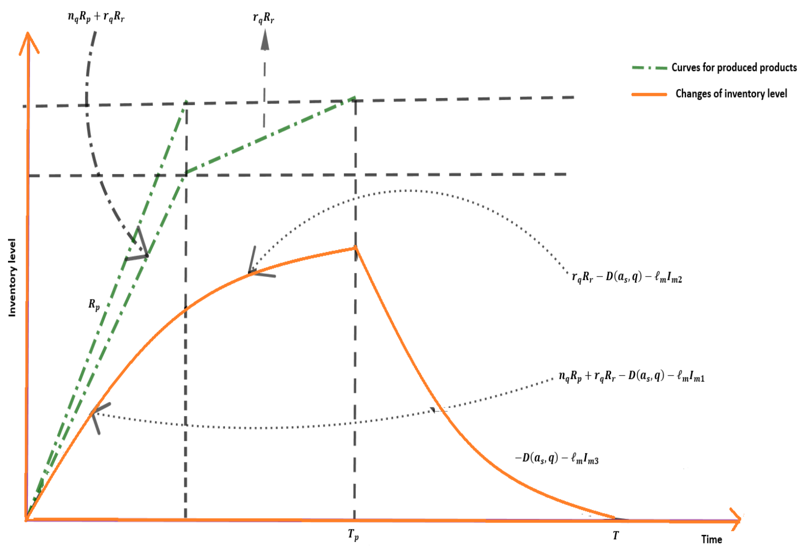

Figure 2: Inventory level for perfect goods of the manufacturer under synchronous rework process (proposed by authors).

(iii) Shortages are not deemed.

(iv) Product's demand depends on selling price, quality level of product.
The demand $D(a_s, q) = D_o + b_1 q - b_2 a_s$. Here, b_1 and b_2 are the sensitivity coefficients regarding q and a_s, respectively. Furthermore, D_o is the initial demand rate.

(v) PT is applied at the manufacturer's and retailer's warehouses as well as during transportation time of products. Therefore, after applying PT the deterioration rates for holding products at the manufacturer's warehouse and the retailer's store room are, respectively, $d_m = d_{m0}e^{-\epsilon\xi}$ and $d_r = d_{r0}e^{-\epsilon\xi}$. Here, ξ is the PT investment per unit time and ϵ is the sensitivity coefficient regarding reduction of deterioration rate that depends on ξ.

(vi) The retailer pays η portion of total PT cost and the manufacturer pays the remaining PT cost.

(vii) The manufacturer applies a GT during production process so that products become eco-friendly and carbon emission are reduced. The reduction rate θ $(0 < \theta < 1)$ of carbon emission is a function of GT investment G_I and it is defined as: $\theta = \theta_m(1 - e^{-\sigma G_I})$. Here, θ_m is the maximum reduction rate of carbon emission due to utilization of GT investment and σ is the efficiency level.

(viii) Initial demand rate, all deterioration rates and all variable costs per product are considered as uncertain in nature. These uncertain parameters are taken as TIFNs based on real situations.

5. Formulation of SCIM Model with Rework

This study develops an SCIM model incorporating a synchronous reworking process, where defective products are reworked simultaneously with regular production. This approach helps minimize defective goods while ensuring immediate fulfillment of demand.

A graphical representation of the proposed SCIM model with synchronous rework is provided in Figures 2 to 4. Specifically, Figure 2 illustrates the inventory level of perfect goods at the manufacturer's warehouse, while Figure 3 presents the inventory dynamics of defective goods. Additionally, Figure 4 depicts the inventory fluctuations at the retailer's storage facility.

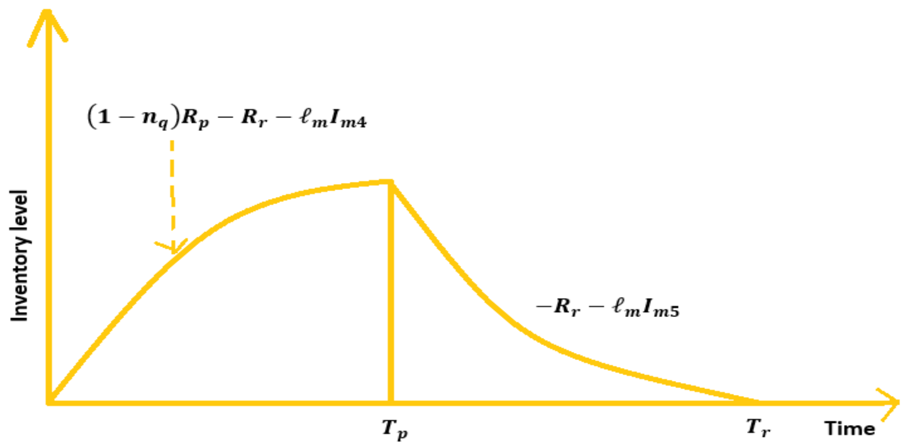

Figure 3: Inventory level of reworkable goods in synchronous rework system (proposed by authors).

5.1. Manufacturer's inventory level

The inventory level at the manufacturer's facility is influenced by factors such as production rate, reworking rate, deterioration, and demand rate. Based on Figure 2, the inventory dynamics of perfect goods can be described using the following differential equations, subject to specific boundary conditions:

$$\frac{dI_{m1}}{dt} = n_q R_p + r_q R_r - D(a_s, q) - d_m I_{m1}, \quad 0 \le t \le T_p, \ \ I_{m1}(0) = 0, \tag{5.1}$$

$$\frac{dI_{m2}}{dt} = r_q R_r - D(a_s, q) - d_m I_{m2}, \quad T_p \le t \le T_r, \ \ I_{m1}(T_p) = I_{m2}(T_p), \tag{5.2}$$

$$\frac{dI_{m3}}{dt} = -D(a_s, q) - d_m I_{m3}, \quad T_r \le t \le T, \ \ I_{m3}(T) = 0. \tag{5.3}$$

Similarly, the inventory level of re-workable goods is determined by solving the following differential equation, subject to the given equality constraints:

$$\frac{dI_{m4}}{dt} = (1-n_q)R_p - R_r - d_m I_{m4}, \quad 0 \le t \le T_p, \ \ I_{m4}(0) = 0, \tag{5.4}$$

$$\frac{dI_{m5}}{dt} = -R_r - d_m I_{m5}, \quad T_p \le t \le T_r, \ \ I_{m5}(T_r) = 0. \tag{5.5}$$

The corresponding inventory levels are obtained by solving the differential equations (5.1), (5.2), (5.3), (5.4) and (5.5) with the appropriate boundary conditions, as follows:

$$I_{m1}(t) = \frac{n_q R_p + r_q R_r - D(a_s, q)}{d_m}\left(1 - e^{-d_m t}\right), \quad 0 \le t \le T_p, \tag{5.6}$$

$$I_{m2}(t) = \frac{r_q R_r - D(a_s, q)}{d_m}\left(1 - e^{-d_m t}\right) + \frac{n_q R_p}{d_m}\left(e^{d_m T_p} - 1\right)e^{-d_m t}, \quad T_p \le t \le T_r, \tag{5.7}$$

$$I_{m3}(t) = \frac{D(a_s, q)}{d_m}\left(e^{d_m (T-t)} - 1\right), \quad t_r \le t \le T, \tag{5.8}$$

$$I_{m4}(t) = \frac{(1-n_q)R_p - R_r}{d_m}\left(1 - e^{-d_m t}\right), \quad 0 \le t \le T_p, \tag{5.9}$$

$$I_{m5}(t) = \frac{R_r}{d_m}\left(e^{d_m (T_r - t)} - 1\right), \quad T_p \le t \le T_r. \tag{5.10}$$

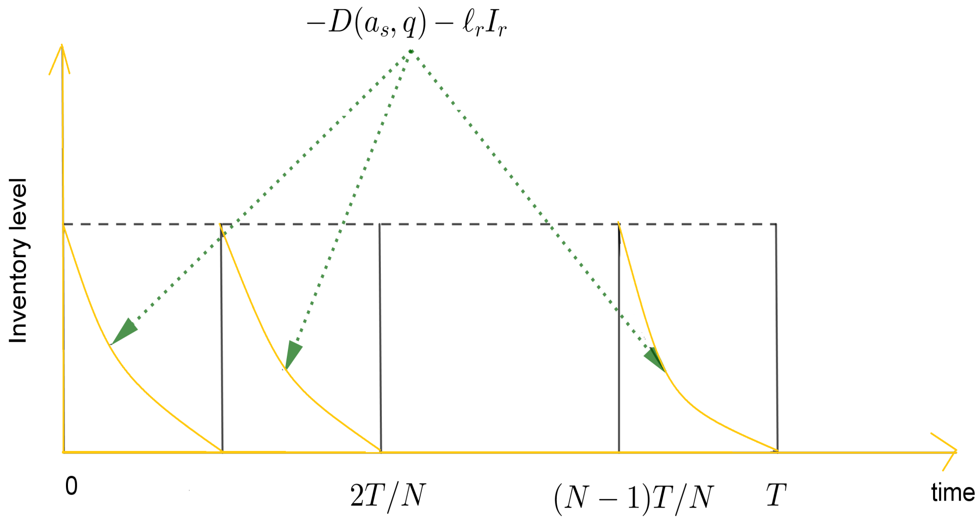

Figure 4: Retailer's inventory level (proposed by authors).

Figures 2 and 3 show that $I_{m2}(T_r) = I_{m3}(T_r)$ and $I_{m4}(T_p) = I_{m5}(T_p)$, respectively. These conditions provide the following:

$$T_r = \frac{1}{d_m} \ln \left[1 + \frac{(1-n_q)R_p}{R_r} \left(e^{d_m T_p} - 1 \right) \right], \tag{5.11}$$

$$T = \frac{1}{d_m} \ln \left[1 + \frac{r_q R_r}{D(a_s,q)} \left(e^{d_m T_r} - 1 \right) + \frac{n_q R_p}{D(a_s,q)} \left(e^{d_m T_p} - 1 \right) \right]. \tag{5.12}$$

5.2. Retailer's inventory level

Retailer's inventory level is effected by demand rate and deterioration rate of products. So, by observing Figure 4, the inventory level is determined by solving the differential equation (5.13) and the corresponding inventory level is presented in equation (5.14):

$$\frac{dI_r(t)}{dt} = -D(a_s,q) - d_r I_r(t), \quad 0 \le t \le \frac{T}{N}, \; I_r\left(\frac{T}{N}\right) = 0, \tag{5.13}$$

$$I_r(t) = \frac{D(a_s,q)}{d_r} \left(e^{d_r\left(\frac{T}{N}-t\right)} - 1 \right), \quad 0 \le t \le \frac{T}{N}. \tag{5.14}$$

5.3. Integrated inventory cost for synchronous rework

The total integrated inventory cost of the SCIM problem with synchronous reworking process is constructed by adding all costs of the manufacturer and all costs of the retailer. Costs related to the manufacturer are as follows:

1. Setup cost $SC = S_n + S_r$.

2. Production cost $PC = c_p R_p T_p$.

3. Reworking cost $RC = c_r R_r T_r$.

4. Holding cost at the manufacturer's warehouse is termed as HCM and defined by

$$\begin{aligned} HCM = \quad & h_{mn}\left[\int_0^{T_p} I_{m1}(t)dt + \int_{T_p}^{T_r} I_{m2}(t)dt + \int_{T_r}^{T} I_{m3}(t)dt - NQ\right] \\ & + h_{mr}\left[\int_0^{T_p} I_{m4}(t)dt + \int_{T_p}^{T_r} I_{m5}(t)dt\right]. \end{aligned}$$

5. Preservation investment of the manufacturer which is termed as PIM and $PIM = (1-\eta)\,\xi T$.

6. Deteriorating cost of the manufacturer, denoted by DCM and $DCM = d_{cm}\left[R_pT_p - D(a_s,q)T - R_rT_r\left(1-r_q\right)\right]$.

7. Discarding cost, termed as DIC and $DIC = s_cR_rT_r\left(1-r_q\right)$.

8. Quality improvement investment $QI = \Gamma q^2$.

9. Investment for GT during production time is termed as CMI and $CMI = G_I(R_pT_p + R_rT_r)$.

Now, costs related to the retailer are stated as:

1. Purchasing cost, represented by PUC and $PUC = p_wNQ$.

2. Ordering cost $OC = O_r$.

3. Deteriorating cost of the retailer, termed as DCR and $DCR = d_{cr}N\left(Q - \frac{D(a_s,q)T}{N}\right)$.

4. Preservation investment of the retailer which is denoted by PIR and calculated as: $PIR = \eta\xi T$.

5. Inspection cost $IC = i_cQN$.

6. Holding cost of the retailer which is denoted by HCR and $HCR = h_rN\int_0^{\frac{T}{N}} I_r(t)dt$.

7. Transportation cost, termed as TRC and $TRC = N\left[f_{tc} + v_{tc}QK + x_{tc}K\right]$.

Now, by adding all cost elements of the manufacturer and the retailer, the total integrated inventory cost per unit time is obtained and it is represented by Z_{IC}. Moreover, Z_{IC} is a function of T_p, delivery number N, product's selling price a_s, PT investment ξ and GT investment G_I. The function Z_{IC} is represented and evaluated as below:

$$\begin{aligned}
Z_{IC} = & \frac{1}{T}\left[SC+PC+RC+HCM+PIM+DCM+DIS+QI+PUC+OC\right] \\
& +\frac{1}{T}\left[DCR+CMI+PIR+IC+HCR+TRC\right] \\
= & \frac{1}{T}\left[S_n+S_r+c_pR_pT_p+c_rR_rT_r+h_{mn}\left(L_1+L_2+L_3\right)+h_{mr}\left(L_4+L_5\right)\right] \\
& +\frac{1}{T}\left[(1-\eta)\xi T+d_{cm}\left(R_pT_p-D(a_s,q)T-R_rT_r\left(1-r_q\right)\right)+s_cR_rT_r\left(1-r_q\right)\right] \\
& +\frac{1}{T}\left[\Gamma q^2+p_wQN+O_r+Nd_{cr}\left(Q-\frac{D(a_s,q)T}{N}\right)+\eta\xi T+i_cQN+Nh_rL_6\right] \\
& +\frac{1}{T}\left[N\left(f_{tc}+v_{tc}QK+x_{tc}K\right)+G_I(R_pT_p+R_rT_r)\right],
\end{aligned} \tag{5.15}$$

$$\text{where } L_1 = \int_0^{T_p} I_{m1}(t)dt = \frac{n_qR_p+r_qR_r-D(a_s,q)}{d_m^2}\left(e^{-d_mT_p}-1+d_mT_p\right), \tag{5.16}$$

$$\begin{aligned}
L_2 = & \int_{T_p}^{T_r} I_{m2}(t)dt = \int_{T_p}^{T_r}\frac{r_qR_r-D(a_s,q)}{d_m}\left(1-e^{-d_mt}\right)dt+\int_{T_p}^{T_r}\frac{n_qR_p}{d_m}\left(e^{d_mT_p}-1\right)e^{d_mt} \\
= & \frac{r_qR_r-D(a_s,q)}{d_m^2}\left(e^{-d_mT_r}-e^{-d_mT_p}+d_m\left(T_r-T_p\right)\right) \\
& -\frac{n_qR_p}{d_m^2}\left(e^{d_mT_p}-1\right)\left(e^{-d_mT_r}-e^{-d_mT_p}\right),
\end{aligned} \tag{5.17}$$

$$\begin{aligned}
L_3 = & \int_{T_r}^{T} I_{m3}(t)dt = \int_{T_r}^{T}\frac{D(a_s,q)}{d_m}\left(e^{d_m(T-t)}-1\right)dt \\
= & \frac{D(a_s,q)}{d_m^2}\left(e^{d_m(T-T_r)}-1-d_m\left(T-T_r\right)\right),
\end{aligned} \tag{5.18}$$

$$\begin{aligned}
L_4 = & \int_0^{T_p} I_{m4}(t)dt = \int_0^{T_p}\frac{\left(1-n_q\right)R_p-R_r}{d_m}\left(1-e^{-d_mt}\right)dt \\
= & \frac{\left(1-n_q\right)R_p-R_r}{d_m^2}\left(e^{-d_mT_p}-1+d_mT_p\right),
\end{aligned} \tag{5.19}$$

$$\begin{aligned}
L_5 = & \int_{T_p}^{T_r} I_{m5}(t)dt = \int_{T_p}^{T_r}\frac{R_r}{d_m}\left(e^{d_m(T_r-t)}-1\right)dt \\
= & \frac{R_r}{d_m^2}\left(e^{d_m(T_r-T_p)}-1+d_m\left(T_r-T_p\right)\right),
\end{aligned} \tag{5.20}$$

$$L_6 = \int_0^{\frac{T}{N}} I_r(t)dt = \int_0^{\frac{T}{N}}\frac{D}{d_r}\left(e^{d_r\left(\frac{T}{N}-t\right)}-1\right)dt = \frac{D}{d_r^2}\left(e^{d_r\frac{T}{N}}-1-d_r\frac{T}{n}\right). \tag{5.21}$$

5.3.1. *Integrated inventory cost under IF environment.* As per assumptions of the proposed model, initial demand rate D_o, deterioration rates l_{m0}, l_{r0} and all variable costs per product, i.e., c_p, O_r, c_r, p_w, h_{mn}, h_{mr}, h_r, d_{cm}, d_{cr}, s_c, i_c and v_{tc} are considered as uncertain in nature. These uncertain parameters are taken as TIFNs (by Definition 3.3) based on real situations. Therefore, the corresponding integrated inventory cost under neutrosophic environment is as follows:

$$
\begin{aligned}
\grave{Z_{IC}} = &\ \frac{1}{T}\left[S_n + S_r + \grave{c_p}R_pT_p + \grave{c_r}R_rT_r + \grave{h_{mn}}\left(\grave{L_1}+\grave{L_2}+\grave{L_3}\right) + \grave{h_{mr}}\left(\grave{L_4}+\grave{L_5}\right)\right] \\
&+\frac{1}{T}\left[(1-\eta)\xi T + \grave{d_{cm}}\left(R_pT_p - \grave{D}(a_s,q)T - R_rT_r\left(1-r_q\right)\right) + \grave{s_c}R_rT_r\left(1-r_q\right)\right] \\
&+\frac{1}{T}\left[\Gamma q^2 + \grave{p_w}QN + \grave{O_r} + N\grave{d_{cr}}\left(Q - \frac{\grave{D}(a_s,q)T}{N}\right) + \eta\xi T + \grave{i_c}QN + N\grave{h_r}\grave{L_6}\right] \\
&+\frac{1}{T}\left[N\left(f_{tc} + \grave{v_{tc}}QK + x_{tc}K\right) + G_I(R_pT_p + R_rT_r)\right], \qquad (5.22)
\end{aligned}
$$

$$
\text{where } \grave{L_1} = \frac{n_qR_p + r_qR_r - \grave{D}(a_s,q)}{\grave{d_m}^2}\left(e^{-\grave{d_m}T_p} - 1 + \grave{d_m}T_p\right), \qquad (5.23)
$$

$$
\begin{aligned}
\grave{L_2} = &\ \frac{r_qR_r - \grave{D}(a_s,q)}{\grave{d_m}^2}\left(e^{-\grave{d_m}T_r} - e^{-\grave{d_m}T_p} + \grave{d_m}\left(T_r - T_p\right)\right) \\
&-\frac{n_qR_p}{\grave{d_m}^2}\left(e^{\grave{d_m}T_p} - 1\right)\left(e^{-\grave{d_m}T_r} - e^{-\grave{d_m}T_p}\right), \qquad (5.24)
\end{aligned}
$$

$$
\grave{L_3} = \frac{\grave{D}(a_s,q)}{\grave{d_m}^2}\left(e^{\grave{d_m}(T-T_r)} - 1 - \grave{d_m}\left(T - T_r\right)\right), \qquad (5.25)
$$

$$
\grave{L_4} = \frac{\left(1-n_q\right)R_p - R_r}{\grave{d_m}^2}\left(e^{-\grave{d_m}T_p} - 1 + \grave{d_m}T_p\right), \qquad (5.26)
$$

$$
\grave{L_5} = \frac{R_r}{\grave{d_m}^2}\left(e^{\grave{d_m}(T_r-T_p)} - 1 + \grave{d_m}\left(T_r - T_p\right)\right), \qquad (5.27)
$$

$$
\grave{L_6} = \frac{\grave{D}}{\grave{d_r}^2}\left(e^{\grave{d_r}\frac{T}{N}} - 1 - \grave{d_r}\frac{T}{n}\right), \qquad (5.28)
$$

$$
\grave{D}(a_s,q) = \grave{D_o} + b_1q - b_2a_s, \quad \grave{d_m} = \grave{d_{m0}}e^{-\epsilon\xi}. \qquad (5.29)
$$

5.4. Total carbon emissions

Total carbon emissions from the manufacturer is calculated by adding the following components.

- CO_2 emission for setup of production and rework processes $= e_{S_p} + e_{S_r}$.
- CO_2 emission during production time $= e_pR_pT_p(1-\theta)$, where θ is the carbon emission reduction rate due to using of GT.
- CO_2 emission during reworking time $= e_rR_rT_r(1-\theta)$.
- CO_2 emission for scrapping the non-reworkable products $= e_s\left(1-r_q\right)R_rT_r$.
- CO_2 emission for holding products in the manufacturer's warehouse
  $= e_hV\left[\int_0^{T_p} I_{m1}(t)dt + \int_{T_p}^{T_r} I_{m2}(t)dt + \int_{T_r}^{T} I_{m3}(t)dt + \int_0^{T_p} I_{m4}(t)dt + \int_{T_p}^{T_r} I_{m5}(t)dt\right]$
  $= e_hV\left[L_1 + L_2 + L_3 + L_4 + L_5\right]$,
  where L_1, L_2, L_3, L_4 and L_5 are given in equations (5.16), (5.17), (5.18), (5.19) and (5.20), respectively.
- CO_2 emission due to deterioration $= e_d\left(R_pT_p - D(a_s,q)T - R_rT_r(1-r_q)\right)$.

Thus, total amount of carbon that is emitted by the manufacturer is obtained as:

$$
\begin{aligned}
CEM = &\ e_{S_p} + e_{S_r} + \left(e_pR_pT_p + e_rR_rT_r\right)(1-\theta) + e_s(1-r_q)R_rT_r \\
&+ e_hV\left[L_1 + L_2 + L_3 + L_4 + L_5\right] + e_d\left(R_pT_p - D(a_s,q)T - R_rT_r(1-r_q)\right).
\end{aligned}
$$

Total amount of carbon emissions by the retailer is calculated as follows:

$$\begin{aligned} CER = & \text{ Emission due to transportation} + \text{Emission due to deterioration} \\ & + \text{Emission from storing house} \\ = & \; N\left(KF_e e_v + KF_l e_v\right) + Ne_d\left(Q - \frac{D(a_s,q)T}{N}\right) + Ne_h V L_6, \end{aligned}$$

where $L_6 = \int_0^{\frac{T}{N}} I_r(t)dt$ and it is given in equation (5.21). Now, the total carbon emission that is emitted by the manufacturer and the retailer per unit time is formulated as:

$$\begin{aligned} T_{CE} = & \frac{(CEM + CER)}{T} \\ = & \frac{1}{T}\left[e_{S_p} + e_{S_r} + \left(e_p R_p T_p + e_r R_r T_r\right)(1-\theta) + e_s(1-r_q)R_r T_r\right] \\ & + \frac{1}{T}\left[e_h V\left(L_1 + L_2 + L_3 + L_4 + L_5\right) + e_d\left(R_p T_p - D(a_s,q)T - R_r T_r(1-r_q)\right)\right] \\ & + \frac{1}{T}\left[N\left(KF_e e_v + KF_l e_v\right) + Ne_d\left(Q - \frac{D(a_s,q)T}{N}\right) + Ne_h L_6\right]. \end{aligned} \tag{5.30}$$

5.4.1. Total carbon emissions under IF environment. In the similar way of calculating, the total integrated inventory cost under neutrosophic environment (discussed in Subsection 5.3.1), the total amount of carbon emissions under neotrosophic environment is determined by substituting the neutrosophic parameters in equation (5.30) and it is

$$\begin{aligned} \grave{T}_{CE} = & \frac{1}{T}\left[e_{S_p} + e_{S_r} + \left(e_p R_p T_p + e_r R_r T_r\right)(1-\theta) + e_s(1-r_q)R_r T_r\right] \\ & + \frac{1}{T}\left[e_h V\left(\grave{L}_1 + \grave{L}_2 + \grave{L}_3 + \grave{L}_4 + \grave{L}_5\right) + e_d\left(R_p T_p - D(a_s,q)T - R_r T_r(1-r_q)\right)\right] \\ & + \frac{1}{T}\left[N\left(KF_e e_v + KF_l e_v\right) + Ne_d\left(Q - \frac{\grave{D}(a_s,q)T}{N}\right) + Ne_h \grave{L}_6\right]. \end{aligned} \tag{5.31}$$

Total of integrated cost and carbon emission cost under CCT policy is calculated as: $\grave{Z}_1 = \grave{Z}_{IC} + u\left(\grave{T}_{CE} - C\right)^+ - u\left(C - \grave{T}_{CE}\right)$, where $\grave{Z}_{IC}$ and $\grave{T}_{CE}$ are presented in equations (5.22) and (5.31), respectively. Here, $\left(\grave{T}_{CE} - C\right)^+ = \max\left\{\left(\grave{T}_{CE} - C\right), 0\right\}$ and $\left(C - \grave{T}_{CE}\right)^+ = \max\left\{\left(C - \grave{T}_{CE}\right), 0\right\}$.

The corresponding MOSCIMP under CCT policy can be formulated as:

Model 1

$$\text{minimize} \quad \grave{Z}_1 = \grave{Z}_{IC} + u\left(\grave{T}_{CE} - C\right)^+ - u\left(C - \grave{T}_{CE}\right) \tag{5.32}$$

$$\text{minimize} \quad \grave{Z}_2 = \grave{T}_{CE} \tag{5.33}$$

$$\text{subject to} \quad T_r = \frac{1}{\grave{d}_m}\ln\left[1 + \frac{(1-n_q)R_p}{R_r}\left(e^{\grave{d}_m T_p} - 1\right)\right], \tag{5.34}$$

$$T = \frac{1}{\grave{d}_m}\ln\left[1 + \frac{r_q R_r}{\grave{D}(a_s,q)}\left(e^{\grave{d}_m T_r} - 1\right) + \frac{n_q R_p}{D}\left(e^{\grave{d}_m T_p} - 1\right)\right], \tag{5.35}$$

$$T_p > 0,\; a_s > c_p,\; 0 \le q \le 1,\; \xi > 0,\; G_I > 0,\; N \in I^+. \tag{5.36}$$

6. Deterministic Problem Formulation

As the above formulated MOSCIMP are studied by considering several inventory parameters as TIFNs so, the problems cannot be solved directly. Hence, a ranking index for defuzzification of an TIFN is utilized (one can see Definition 3.4) for transferring the proposed IFMOSCIMP (i.e., Model 1) into equivalent deterministic MOSCIMP (denoted as Model D1). Therefore, the deterministic MOSCIMP with synchronous rework under CCT policy is presented in Model D1.

Model D1

$$\begin{aligned}
\text{minimize} \quad & \mathcal{R}(\grave{Z}_1) = \mathcal{R}(\grave{Z}_{IC}) + u\left(\mathcal{R}(\grave{T}_{CE}) - C\right)^{+} \\
& \qquad\qquad - u\left(C - \mathcal{R}(\grave{T}_{CE})\right)^{+} && (6.1)\\
\text{minimize} \quad & \mathcal{R}(\grave{Z}_2) = \mathcal{R}(\grave{T}_{CE}) && (6.2)\\
\text{subject to} \quad & T_r = \frac{1}{\mathcal{R}(\grave{d}_m)} \ln\left[1 + \frac{(1-n_q)R_p}{R_r}\left(e^{\mathcal{R}(\grave{d}_m)T_p} - 1\right)\right], && (6.3)\\
& T = \frac{1}{\mathcal{R}(\grave{d}_m)} \ln\left[1 + \frac{r_q R_r}{\mathcal{R}(\grave{D}(a_s,q))}\left(e^{\mathcal{R}(\grave{d}_m)T_r} - 1\right)\right. \\
& \qquad \left. + \frac{n_q R_p}{\mathcal{R}(\grave{D}(a_s,q))}\left(e^{\mathcal{R}(\grave{d}_m)T_p} - 1\right)\right], && (6.4)\\
& T_p > 0,\ a_s > c_p,\ 0 \le q \le 1,\ \xi > 0,\ G_I > 0,\ N \in I^{+}.. && (6.5)
\end{aligned}$$

Here, the ranking indices of $\grave{Z}_{IC}$ and $\grave{T}_{CE}$ are of the following forms:

$$\begin{aligned}
\mathcal{R}(\grave{Z}_{IC}) = \ & \frac{1}{T}\left[S_n + S_r + \mathcal{R}(\grave{c}_p)R_pT_p + \mathcal{R}(\grave{c}_r)R_rT_r\right. \\
& + \mathcal{R}(\grave{h}_{mn})\left(\mathcal{R}(\grave{L}_1) + \mathcal{R}(\grave{L}_2) + \mathcal{R}(\grave{L}_3)\right) \\
& \left. + \mathcal{R}(\grave{h}_{mr})\left(\mathcal{R}(\grave{L}_4) + \mathcal{R}(\grave{L}_5)\right) + (1-\eta)\xi T\right] \\
& + \frac{1}{T}\left[\mathcal{R}(\grave{d}_{cm})\left(R_pT_p - \mathcal{R}(\grave{D}(a_s,q))T - R_rT_r(1-r_q)\right)\right. \\
& \left. + \mathcal{R}(\grave{s}_c)R_rT_r(1-r_q) + \Gamma q^2 + p_wQN + \mathcal{R}(\grave{O}_r)\right] \\
& + \frac{1}{T}\left[N\mathcal{R}(\grave{d}_{cr})\left(Q - \frac{\mathcal{R}(\grave{D}(a_s,q))T}{N}\right) + \eta\xi T\right] \\
& + \frac{1}{T}\left[\mathcal{R}(\grave{i}_c)QN + N\mathcal{R}(\grave{h}_r)\mathcal{R}(\grave{L}_6)\right. \\
& \left. + N\left(f_{tc} + \mathcal{R}(\grave{v}_{tc})QK + \mathcal{R}(\grave{x}_{tc})K\right) + G_I(R_pT_p + R_rT_r)\right], && (6.6)
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}(\grave{T}_{CE}) = & \frac{1}{T}\left[e_{S_p} + e_{S_r} + (e_p R_p T_p + e_r R_r T_r)(1-\theta) + e_s(1-r_q)R_r T_r\right] \\
& + \frac{1}{T}\left[e_h V\left(\mathcal{R}(\grave{L}_1) + \mathcal{R}(\grave{L}_2) + \mathcal{R}(\grave{L}_3)\right)\right] \\
& + \frac{1}{T}\left[e_h V\left(\mathcal{R}(\grave{L}_4) + \mathcal{R}(\grave{L}_5)\right)\right] \\
& + \frac{1}{T}\left[e_d\left(R_p T_p - \mathcal{R}(\grave{D}(a_s,q))T - R_r T_r(1-r_q)\right) + NKF_e e_v\right] \\
& + \frac{1}{T}\left[NKF_l e_v + Ne_d\left(Q - \frac{\mathcal{R}(\grave{D}(a_s,q))T}{N}\right)\right. \\
& \left. + Ne_h \mathcal{R}(\grave{L}_6)\right], \qquad (6.7)
\end{aligned}$$

$$\begin{aligned}
\text{where } \mathcal{R}(\grave{L}_1) = & \frac{n_q R_p + r_q R_r - \mathcal{R}(\grave{D}(a_s,q))}{\mathcal{R}(\grave{d}_m)^2}\left(e^{-\mathcal{R}(\grave{d}_m)T_p} - 1\right. \\
& \left. + \mathcal{R}(\grave{d}_m)T_p\right), \qquad (6.8)
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}(\grave{L}_2) = & \frac{r_q R_r - \mathcal{R}(\grave{D}(a_s,q))}{\mathcal{R}(\grave{d}_m)^2}\left(e^{-\mathcal{R}(\grave{d}_m)T_r} - e^{-\mathcal{R}(\grave{d}_m)T_p}\right. \\
& \left. + \mathcal{R}(\grave{d}_m)(T_r - T_p)\right) - \frac{n_q R_p}{\mathcal{R}(\grave{d}_m^2)}\left(e^{\mathcal{R}(\grave{d}_m)T_p} - 1\right) \\
& \left(e^{-\mathcal{R}(\grave{d}_m)T_r} - e^{-\mathcal{R}(\grave{d}_m)T_p}\right), \qquad (6.9)
\end{aligned}$$

$$\begin{aligned}
\mathcal{R}(\grave{L}_3) = & \frac{\mathcal{R}(\grave{D}(a_s,q))}{\mathcal{R}(\grave{d}_m)^2}\left(e^{\mathcal{R}(\grave{d}_m)(T-T_r)} - 1\right. \\
& \left. - \mathcal{R}(\grave{d}_m)(T - T_r)\right), \qquad (6.10)
\end{aligned}$$

$$\mathcal{R}(\grave{L}_4) = \frac{(1-n_q)R_p - R_r}{\mathcal{R}(\grave{d}_m)^2}\left(e^{-\mathcal{R}(\grave{d}_m)T_p} - 1 + \mathcal{R}(\grave{d}_m)T_p\right), \qquad (6.11)$$

$$\begin{aligned}
\mathcal{R}(\grave{L}_5) = & \frac{R_r}{\mathcal{R}(\grave{d}_m)^2}\left(e^{\mathcal{R}(\grave{d}_m)(T_r-T_p)} - 1\right. \\
& \left. + \mathcal{R}(\grave{d}_m)(T_r - T_p)\right), \qquad (6.12)
\end{aligned}$$

$$\mathcal{R}(\grave{L}_6) = \frac{\mathcal{R}(\grave{D}(a_s,q))}{\mathcal{R}(\grave{d}_r)^2}\left(e^{\mathcal{R}(\grave{d}_r)\frac{T}{N}} - 1 - \mathcal{R}(\grave{d}_r)\frac{T}{n}\right). \qquad (6.13)$$

7. Solution Procedure

This section introduces an NCP approach for addressing the deterministic MOSCIMPs under consideration. The NCP is initiated based on NS, which incorporates three objective functions: maximizing the degree of truth membership function (TMF), maximizing the degree of indeterminacy membership function (IMF), and minimizing the degree of falsity membership function (FMF). By employing this approach, decision-makers can gain clearer insights into real-world scenarios. The concept of a fuzzy decision set, denoted as F^D, was first proposed by [8] and is defined as: $F^D = F^G \cap F^C$, where F^G and F^C correspond to the fuzzy goal and fuzzy constraints, respectively. Over time, numerous researchers have leveraged this fuzzy decision set to tackle real-world problems involving uncertainty. Similarly, the neutrosophic decision set N^D, along with neutrosophic objectives and constraints, is interpreted as follows:

$$
\begin{aligned}
N^D &= \left(\cap_{m=1}^{M} F_m^G\right)\left(\cap_{n=1}^{N} F_n^C\right) = \{h, T_{N^D}(h), I_{N^D}(h), F_{N^D}(h)\},\\
\text{where } T_{N^D}(h) &= \min\left\{\begin{array}{l} T_{F_1^G}(h), T_{F_2^G}(h), \ldots, T_{F_M^G}(h) \\ T_{F_1^C}(h), T_{F_2^C}(h), \ldots, T_{F_M^C}(h) \end{array}\right\}, \ \forall\, h \in \mathbb{H},\\
I_{N^D}(h) &= \min\left\{\begin{array}{l} I_{F_1^G}(h), I_{F_2^G}(h), \ldots, I_{F_M^G}(h) \\ I_{F_1^C}(h), I_{F_2^C}(h), \ldots, I_{F_M^C}(h) \end{array}\right\}, \ \forall\, h \in \mathbb{H},\\
F_{N^D}(h) &= \min\left\{\begin{array}{l} F_{F_1^G}(h), F_{F_2^G}(h), \ldots, F_{F_M^G}(h) \\ F_{F_1^C}(h), F_{F_2^C}(h), \ldots, F_{F_M^C}(h) \end{array}\right\}, \ \forall\, h \in \mathbb{H}.
\end{aligned}
$$

The limits for individual goal functions have been computed for the purpose of defining the various membership functions of the deterministic MOSCIMP. The objective function $Z_j, j = 1, 2$ has upper and lower limits, represented by U_j and L_j, respectively, which are described below.

The solution sets $\mathbb{H}^j$, $j = 1, 2$ are produced by first solving each objective function Z_j, $j = 1, 2$ separately as a single objective function while taking into account the provided constraints of the MOSCIMP. The boundaries of each individual objective function are then obtained by substituting these solutions in each objective function, as shown below: $U_j = \max[Z_j(\mathbb{H}^j)]$ and $L_j = \min[Z_j(\mathbb{H}^j)]$ for all $j = 1, 2$. The j^{th} objective function boundaries for the neutrosophic environment are then calculated as follows:

$$U_j^T = U_j, \ \ L_j^T = L_j \qquad \text{for truth membership,} \tag{7.1}$$

$$U_j^I = L_j^T + \tau_J\left(U_j^T - L_j^T\right), \ \ L_j^I = L_j^T \qquad \text{for indeterminacy function,} \tag{7.2}$$

$$U_j^F = U_j^T, \ \ L_j^F = L_j^T + \upsilon_J\left(U_j^T - L_j^T\right) \qquad \text{for falsity membership.} \tag{7.3}$$

Here, $0 \leq \tau_j \leq 1$ and $0 \leq \upsilon_j \leq 1$ are predestined real numbers that decision makers assign for indeterminacy and falsity membership functions, respectively. Now, the linear membership functions depicted in equations (7.4), (7.5) and (7.6) under a neutrosophic environment are obtained by using the above calculated upper and lower bounds.

$$\text{The neutrosophic TMF is} \quad T_j(Z_j(h)) = \begin{cases} 1 & \text{for } Z_j(h) < L_j^T, \\ \frac{U_j^T - Z_j(h)}{U_j^T - L_j^T} & \text{for } L_j^T \leq Z_j(h) \leq U_j^T, \\ 0 & \text{for } Z_j(h) > U_j^T. \end{cases} \tag{7.4}$$

$$\text{The neutrosophic IMF is} \quad I_j(Z_j(h)) = \begin{cases} 1 & \text{for } Z_j(h) < L_j^I, \\ \frac{U_j^I - Z_j(h)}{U_j^I - L_j^I} & \text{for } L_j^I \leq Z_j(h) \leq U_j^I, \\ 0 & \text{for } Z_j(h) > U_j^I. \end{cases} \tag{7.5}$$

$$\text{The neutrosophic FMF is} \quad F_j(Z_j(h)) = \begin{cases} 1 & \text{for } Z_j(h) > U_j^F, \\ \frac{Z_j(h) - L_j^F}{U_j^F - L_j^F} & \text{for } L_j^F \leq Z_j(h) \leq U_j^F, \\ 0 & \text{for } Z_j(h) < L_j^F. \end{cases} \tag{7.6}$$

In the above formulating membership functions, $U_j^{(.)} \neq L_j^{(.)}$ for the objective function $Z_j, j = 1, 2$. If for any membership functions $U_j^{(.)} = L_j^{(.)}$, then the degree of this corresponding membership function becomes 1. Ultimately, the neutrosophic model to determine the solution of one deterministic MOSCIMP (i.e., Model D1) can be formulated by utilizing the discussed membership functions (7.4), (7.5) and (7.6) as follows:

Model N1

$$
\begin{aligned}
\text{maximize} \quad & (\delta + \theta - \nu)\\
\text{subject to} \quad & T_j(Z_j(h)) \geq \delta, \ \ I_j(Z_j(h)) \geq \theta, \ \ F_j(Z_j(h)) \leq \nu,\\
& \delta \geq \theta, \ \ \delta \geq \nu, \ \ \delta + \theta + \nu \leq 3, \ \ 0 \leq \tau_j \leq 1, \ \ 0 \leq \upsilon_j \leq 1,\\
& \text{constrains (6.3), (6.4) and (6.5).}
\end{aligned}
$$

Here, the objective functions $Z_1(h)$ and $Z_2(h)$ are stated in equations (6.1) and (6.2). In addition, the vector of decision variables $h = (T_p, a_s, q, \xi, G_I, N)$.

8. Numerical Example

A numerical example based on actual circumstances is examined to demonstrate the suggested model. The parameters are inherently ambiguous due to the lack of accurate information on their values. A company's organization must thus handle a number of challenging situations. A real MOSCIMP is taken into consideration where this kind of uncertainty is involved.

Example 1: (MOSCIMP with rework):
In this example, an MOSCIMP with rework is investigated under CCT policy. The values of the parameters are adopted from [31] and modified according to the proposed elaboration. The values of the parameters are as follows: $R_p = 2000000$ gallons/month, $R_r = 1000000$ gallons/month, $n_q = 0.95$, $r_q = 0.89$, $S_n = \$90000$, $S_r = \$70000$, $e_{S_p} = 25$ ton CO_2/gallon, $E_{S_r} = 20$ ton CO_2/gallon, $e_p = 2$ ton CO_2/gallon, $e_r = 1.5$ ton CO_2/gallon, $e_s = 0.3$ ton CO_2/gallon, $e_h = 0.0005$ ton CO_2/KWh, $e_v = 0.0026$ ton CO_2/litre diesel fuel, $e_d = 0.2$ ton CO_2/gallon, $f_{tc} = \$500$/trip, $K = 100$ Km, $C = 800000$ ton CO_2, $b_1 = 100$ gallons, $b_2 = 0.3$ gallons/\$. The TIFN parameters and their corresponding ranking indices are given in Table 1. Then, the problem is solved using the discussed NCP and the optimal solution is depicted in Table 1.

Table 1: TIFNs and their ranking indices.

Parameter	Value	Ranking index ($\mathcal{R}(.)$)
$\grave{D}_o$	$\langle 140229, 140958, 150124; 140200, 140958, 150160\rangle$	143068.125
$\grave{h}_{mn}$	$\langle 2, 2.8, 4; 1.8, 2.8, 4.8\rangle$	2.975
$\grave{h}_{mr}$	$\langle 2.14, 3, 3.75; 2, 3, 4\rangle$	2.98625
$\grave{h}_r$	$\langle 2.86, 4, 5; 2.5, 4, 5.75\rangle$	4.01375
$\grave{c}_p$	$\langle 7.43, 10.4, 13; 7, 10.4, 13.5\rangle$	10.31625
$\grave{c}_r$	$\langle 5.1, 7.2, 9; 4.98, 7.2, 9.3\rangle$	7.1475
$\grave{p}_w$	$\langle 8.5, 12, 15; 8, 12, 15.75\rangle$	11.90625
$\grave{O}_r$	$\langle 1142, 1600, 2000; 1100, 1600, 2050\rangle$	1586.5
$\grave{d}_{cm}$	$\langle 2.57, 3.6, 4.5; 2, 3.6, 5\rangle$	3.55875
$\grave{d}_{cr}$	$\langle 2.86, 4, 5; 2.5, 4, 5.5\rangle$	3.9825
$\grave{s}_c$	$\langle 2.28, 3.2, 4; 1.95, 3.2, 4.32\rangle$	3.16875
$\grave{v}_{tc}$	$\langle 0.006, 0.008, 0.01; 0.003, 0.008, 0.015\rangle$	0.00825

Table 2: Optimal solutions of Example 1.

Decision variable	Optimal value
Z_1	7081810
T_{CE}	490175
T_p	2.62
Q	3140150
a_s	23.5
N	5
G_I	30.6
ξ	15.14
q	0.99

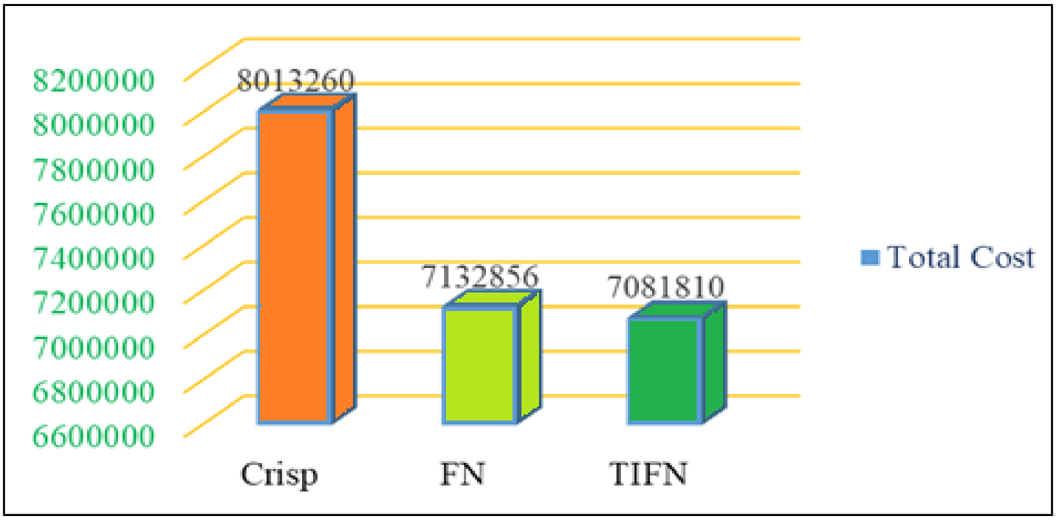

Figure 5: Comparison among crisp, FN and TIFN models (proposed by authors).

8.1. Discussion of results

From the optimal solutions depicted in Table 2, it is seen that in the synchronous reworking process, reworking activities are done during the production time. Hence, imperfect quality products are stored for less time. As a result, holding costs and carbon emissions as well as the carbon emission as a result of the storage of imperfect products are low in the synchronous reworking process. This fact reduces the total cost and the total amount of carbon emissions. Figure 5 shows the graphical comparison among crisp, FN and TIFN environments. It is noticed that the total cost of an SCIM is low when the TIFN is considered to handle uncertain parameters.

9. Sensitivity Analysis

This analysis looks at how changes in parameters affect the associated changes in cost and carbon emission values. Only one parameter at a time has its value modified from −50% to +50%; the values of the other parameters stay unchanged.

A graphical representation shows how the objective functions Z_1 and T_{CE} alter in response to changes in the values of different factors. The graphical illustrations are presented in Figures 6 and 7.

Depending on the obtained results as shown in Figures 6 and 7, the following observations can be stated:

- The generated total carbon emissions is increased due to increase the values of production rate and deterioration rate. In addition, an increase in rework rate R_r decreases the total generated carbon emission.

- The total cost is highly sensitive with respect to the change in production rate and deterioration rate, and it decreases with the increasing in rework rate.

- If rework cost and discarding cost increase, then the quantity of re-manufactured products decreases. Hence, to avoid shortages, more new products need to be manufactured that increases production run time as well as the production cost. Furthermore, the total cost as well as the total amount of carbon emissions increase.

- Due to an increase in cost for unit carbon emission, the total integrated cost increases and the total amount of carbon emissions decreases.

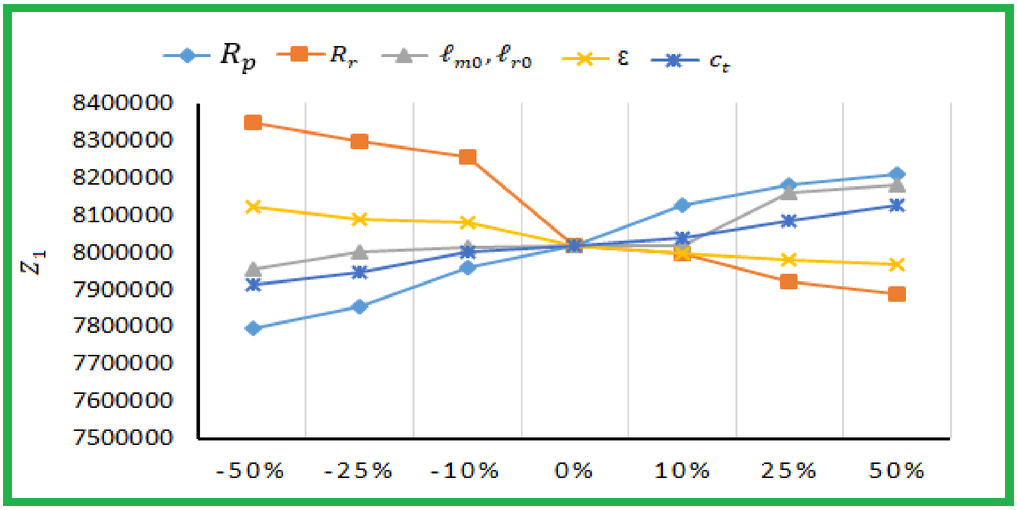

Figure 6: Change of Z_1 vs change in percentage of various parameters (proposed by authors).

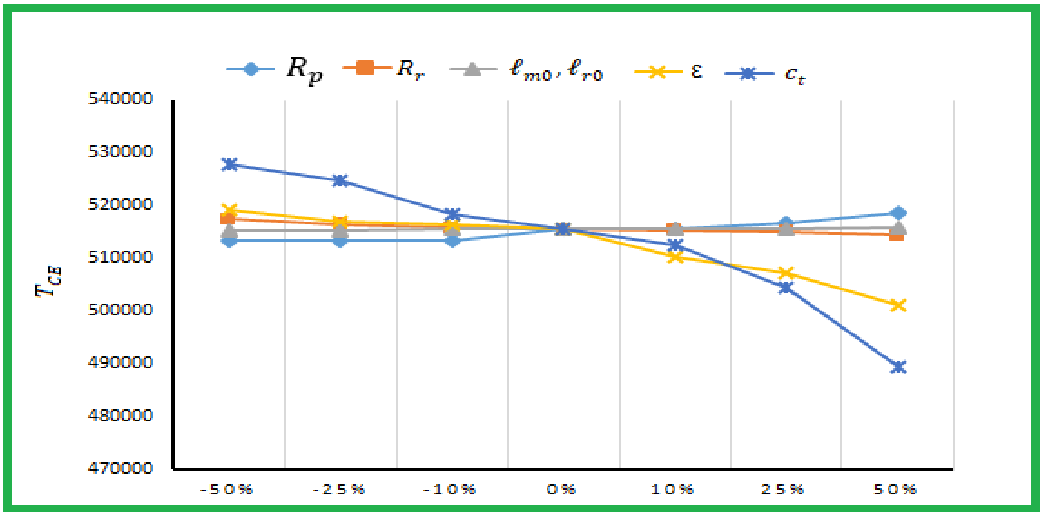

Figure 7: Change of T_{CE} vs change in percentage of various parameters (proposed by authors).

10. Conclusions and Outlook

An SCIM model with a manufacturer and a retailer has been analyzed by accounting for defective production under two different reworking methods in addition to product deterioration. A SCIM emits carbon through a variety of operational actions. As a result, the CCT policy was put into place to reduce carbon emissions and protect the planet from global warming. It has been examined how well synchronous reworking techniques reduce the amount of carbon emissions. The use of PT has been considered as an approach to lower the rate of deterioration. To reduce carbon emissions, the GT has been used. Additionally, methods for improving product quality have been included to raise product quality and lower the quantity of defective products. The expense associated with purchasing or selling permits under a CCT policy due to carbon emissions has been incorporated into the total integrated inventory cost. A multi-objective framework is developed to simultaneously minimize both the total integrated cost and carbon emissions. To better reflect real-world scenarios, uncertainty is considered in factors such as demand rate, deterioration rate at different stages, and all variable costs per product. These uncertainties are addressed using TIFN, which is transformed into a precise numerical value through a ranking index based on value and ambiguity measures. The proposed MOSCIMPs are effectively solved using the NCP approach. Additionally, a comprehensive real-world evaluation of the proposed models is conducted through one case study, along with a sensitivity analysis of key parameters. The study's results highlight that synchronizing the reworking process for defective products is a viable strategy for reducing both operational costs and environmental impact. Moreover, TIFN proves to be a practical tool for decision makers in SCIM, helping to manage complex uncertainties related to various critical parameters.

Future research can explore solving the MOSCIMP using the particle swarm optimization technique ([26]) or other metaheuristic algorithms ([41]). The study can be further extended by incorporating a multi-stage manufacturing system, as highlighted in previous research (one can see [11]). Furthermore, integrating multiple trade credit periods into the proposed framework could provide valuable insight. Researchers may also examine the impact of a double-level sustainable effort within the MOSCIMP model. To address parametric uncertainties, a robust optimization approach can be applied to enhance the reliability of decision making ([27]). Moreover, the Lp-metric method could be employed to solve the multi-objective optimization problem ([2]).

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