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A New Approach to Abel Statistical Convergence in Metric Spaces

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ABSTRACT: In this study, we first consider the sequences in the sense of Abel statistical together with the functions preserving the convergence of this kind of sequences called Abel statistical continuous functions in a metric space X. Then we relate this kind of continuity with some others. A function f is Abel statistically continuous on a subset E of a metric space X, if it preserves Abel statistical convergent sequences, i.e. $(f(p_k))$ is Abel statistically convergent whenever (p_k) is an Abel statistical convergent sequence of points in E, where a sequence (p_k) of points in X is called Abel statistically convergent to a point L in X if $\lim_{x\to 1^-}(1-x)\sum_{k\in\mathbf{N}:d(p_k,L)\geq\varepsilon} x^k = 0$ for every $\varepsilon > 0$. Some other types of continuities are also studied and interesting results are obtained.

Keywords: Statistical convergence, Abel series method, continuity compactness.

Contents

1. Introduction

The continuity concept plays a very important role in computer science, combinatorics, geographic information systems, information theory, economics, biological science, population modelling, and motion modelling in robotics. $\mathbf{N}$, $\mathbf{R}$, and X denote the set of positive integers and the set of real numbers, a metric space, respectively. The boldface letters $\mathbf{p}$, $\mathbf{q}$, $\mathbf{r}$, and $\mathbf{w}$ will be used for sequences $\mathbf{p} = (p_k)$, $\mathbf{q} = (q_k)$, $\mathbf{r} = (r_k)$, $\mathbf{w} = (w_k)$, ... of points in X. The continuity of a function f with the domain a subset E in a metric space X is equivalent to the preserving of the function the convergences of the sequences of the points in E. Motivated by this fact about the continuity of a function concerning the convergent sequences, many types of continuities in such as real and metric spaces have been developed by introducing and investigating the convergences associated with them. The most common of these continuities can be recalled with their citations as follows. By the abuse of the language we miss the word "continuity" for the continuities and just rephrase the names of the methods associated with: slowly oscillating continuity ([13]), Δ slowly oscillating continuity ([8]), ward continuity ([16], [2],[20]), statistical continuity, ([17], [5]), λ-statistical continuity ([26]), rho statistical continuity, ([32]), lacunary statistical continuity ([21] , [33]), ideal sequential continuity ([7,19]), N_θ-sequential continuity ([4]. delta Abel statistical continuity ([41]). Parallel to the usual theory for the convergences of the sequences with real terms, a generalization of the usual convergence idea of real-valued sequences for statistical convergence the reader is referred to the references [28], [29], [15], [9], [23], and [10]. A sequence (p_k) of points in X is called statistically convergent to an $L \in X$ if $\lim_{n\to\infty}\frac{1}{n}|\{k \leq n : d(p_k, L) \geq \varepsilon\}| = 0$ for every $\varepsilon > 0$, and this is denoted by $st - \lim p_k = L$. A sequence (p_k) is called lacunary statistically convergent ([31] ,[43]) to an $L \in X$ if $\lim_{r\Rightarrow\infty}\frac{1}{h_r}|\{k \in I_r : d(p_k, L) \geq \varepsilon\}| = 0$ for every $\varepsilon > 0$, where $I_r = (k_{r-1}, k_r]$, and $k_0 = 0$, $h_r : k_r - k_{r-1} \Rightarrow \infty$ as $r \Rightarrow \infty$ and $\theta = (k_r)$ is an increasing sequence of positive integers, and this is denoted by $S_\theta - \lim p_n = L$ (see [12], [38], and [39]). Throughout this paper, we assume that $\liminf_r \frac{k_r}{k_{r-1}} > 1$. A sequence (p_k) of real numbers is called Abel convergent (or Abel summable) to L if the series $\Sigma_{k=0}^{\infty} p_k x^k$ is convergent for $0 < x < 1$ and $\lim_{x\to 1^-}(1-x)\sum_{k=0}^{\infty} p_k x^k = L$ ([1], [35], [30],

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[40], [42]). In this case, we write $Abel - \lim p_k = L$. A will denote the set of Abel convergent sequences. A subset E of $\mathbf{R}$ is called Abel sequentially compact if whenever $\mathbf{p} = (p_k)$ is a sequence of point in E, there is an Abel convergent subsequence $\mathbf{r} = (r_k) = (p_{n_k})$ of $\mathbf{p}$, i.e. $\lim_{x\to 1^-}(1-x)\sum_{k=0}^{\infty} r_k x^k$ exists and belongs to E. ([6]).

A method of sequential convergence is a linear function G defined on a linear subspace of s, denoted by c_G, into X where s denotes the space of all sequences. A sequence $\mathbf{p}=(p_n)$ is said to be G-convergent to L if $\mathbf{p}\in c_G$, and $G(\mathbf{p}) = L$ ([11]). A method G is called regular if every convergent sequence $\mathbf{p}=(p_n)$ is G-convergent with G($\mathbf{p}$)=lim $\mathbf{p}$. A method G is called subsequential if whenever $\mathbf{p}$ is G-convergent with G($\mathbf{p}$)=L, then there is a subsequence (p_{n_k}) of the sequence $\mathbf{p}$ with $\lim_k p_{n_k} = L$. A function f is called G-continuous if G(f($\mathbf{p}$))=f(G($\mathbf{p}$)) for any G-convergent sequence $\mathbf{p}$ ([14], [3], [36]). Any matrix summability method on a subspace of s is a method of sequential convergence. Abel summability method is a regular method of sequential convergence in this manner.

The purpose of this paper is to extend the concept of Abel statistically convergence to metric spaces, and investigate the concept of Abel statistical continuity in metric spaces presenting interesting results.

2. Abel statistical compactness in metric spaces

Recently the concept of Abel statistical convergence of a sequence is introduced and investigated in [42] (see also [37]). Although the definitions and the most of the results are also valid in a topological Hausdorff group, which allows countable local base at the origin, we investigate the notion in the metric space setting:

Definition 2.1 A sequence $\mathbf{p} = (p_k)$ is called Abel statistically convergent to a point L if Abel density of the set $\{k \in \mathbf{N} : d(p_k, L) \geq \varepsilon\}$ is 0, i.e. $\lim_{x\to 1^-}(1-x)\sum_{k\in\mathbf{N}:d(p_k,L)\geq\varepsilon} x^k = 0$ for every $\varepsilon > 0$, and denoted by $Abel_{st} - lim\ p_k = L$ ([37]).

We note that Abel statistical limit of an Abel statistical convergent sequence is unique.

Theorem 2.1 *If a sequence $\mathbf{p} = (p_k)$ is Abel statistically convergent to L_1 and L_2, then $L_1 = L_2$.*

Proof: Write $d(L_1, L_2) = \alpha$. Then $\lim_{x\to 1^-}(1-x)\sum_{k\in\mathbf{N}:d(L_1,L_2)\geq\alpha} x^k = 1$. On the other hand, we have $\sum_{k\in\mathbf{N}:d(L_1,L_2)\geq\alpha} x^k \leq \sum_{k\in\mathbf{N}:d(L_1,p_k)\geq\frac{\alpha}{3}} x^k + \sum_{k\in\mathbf{N}:d(p_k,L_2)\geq\frac{\alpha}{3}} x^k$ for every $0 < x < 1$. Now it follow that $\lim_{x\to 1^-}(1-x)\sum_{k\in\mathbf{N}:d(L_1,L_2)\geq\alpha} x^k \leq \lim_{x\to 1^-}(1-x)\sum_{k\in\mathbf{N}:d(L_1,p_k)\geq\frac{\alpha}{3}} x^k$
$+ \lim_{x\to 1^-}(1-x)\sum_{k\in\mathbf{N}:d(p_k,L_2)\geq\frac{\alpha}{3}} x^k = 0 + 0 = 0$.
This implies that $\lim_{x\to 1^-}(1-x)\sum_{k\in\mathbf{N}:d(L_1,L_2)\geq\alpha} x^k = 0$. This is a contradiction. □

Theorem 2.2 *Any convergent sequence is Abel statistically convergent, i.e. the Abel statistical method is regular.*

Proof: let (p_k) be a convergent sequence with $lim\ p_k = L$. Then for any $\varepsilon > 0$ there exists a $k_0 \in \mathbf{N}$ such that $d(p_k, L) < \varepsilon$ for $k \geq k_0$. Thus the number of indices is less than k_0, i.e. $\{k \in \mathbf{N} : d(p_k, L) \geq \varepsilon\} \subseteq \{1, 2, 3, ..., k_0\}$. Hence $(1-x)\sum_{k\in\mathbf{N}:d(p_k,L)\geq\varepsilon} x^k \leq (1-x)\sum_{k=0}^{k_0} x^k = 1 - x^{k_0+1}$. Now it follow that $\lim_{x\to 1^-}(1-x)\sum_{k\in\mathbf{N}:d(p_k,L)\geq\varepsilon} x^k = 0$. So the Abel statistical sequential method is regular. □

On the other hand, Abel statistical sequential method is a sequential method in the manner of [14], [3], [18], and [36].

Now we give a characterization of Abel statistical convergence in the following:

Theorem 2.3 . *A sequence (x_n) is Abel statistically convergent if and only if the following condition is satisfied.*

$(A_{st}C)$ For each $\varepsilon > 0$ there exists a subsequence of $(x_{k'(r)})$ of (x_n) such that $\lim_{r\Rightarrow\infty} x_{k'(r)} = \ell$ and $\lim_{x\to 1^-}(1-x)\sum_{k\in\mathbf{N}:d(p_k,p_{k'(r)})\geq\varepsilon} x^k = 0$

Proof: Take any sequence with $Abel_st-\lim_{n\Rightarrow\infty} p_n=\ell$. Write $K^{(j)}=\{k\in\mathbf{N}: k\leq n \ and \ d(p_k,\ell)\leq 1/j\}$ for any positive integer j. Thus for each j, $K(j+1)\subset K(j)$ and $\lim_{x\to 1^-}(1-x)\sum_{K(j)}x^k=1$.

Choose $m(1)$ such that $n>m(1)$ implies that $(1-x)\sum_{K(j)}x^k>0$, i.e. $K^{(1)}\mathbf{N}eq\emptyset$. Then for each positive integer r such that $m(1)\leq r<m(2)$, choose $k(r)\in K^p$, i.e. $p(x_{k'(r)},\ell)\leq 1$. In general, choose $m(p+1)>m(p)$ such that $r>m(p+1)$ implies that $K^{p+1}\mathbf{N}eq\emptyset$. Then for all r satisfying $m(p)\leq r<m(p+1)$, choose $k'(r)\in K^p$, i.e. $d(x_{k'(r)},\ell)<1/p$. Then

$(1-x)\sum_{\{k\leq n: d(x_k,x_{k'(r)})<1/2} x^k \leq (1-x)\sum_{\{k\leq n: d(x_k,\ell)<1/2\}} + (1-x)\sum_{\{k\leq n: d(\ell,x_{k'(r)})<1/2\}}$

Since $Abel_{st}-\lim_{n\Rightarrow\infty} x_n=\ell$, and $\lim_{r\Rightarrow\infty} x_{k'(r)}=\ell$, $(1-x)\sum_{k\in\mathbf{N}: d(p_k,\ell)\geq\varepsilon}x^k\leq (1-x)\sum_{k\in\mathbf{N}: d(p_k,p_{k'(r)})\geq\varepsilon}x^k + (1-x)\sum_{k\in\mathbf{N}: d(p_{k'(r)},\ell)\geq\varepsilon}x^k$, hence the condition $(A_{st}C)$ is satisfied. This completes the proof of the theorem.

□

Corollary 2.1 *Any Abel statistically convergent sequence has a convergent subsequence.*

Proof: The proof follows from the preceding theorem, so is omitted. □

The preceding theorem ensures that Abel statistical limit method is subsequential, where a sequential method G is called subsequential if a sequence is G-summable to a point in X, then there is convergent subsequence of the sequence with G-limit is equal to the limit of the subsequence.

Definition 2.2 A subset E of X is called Abel statistically compact if any sequence of points in E has an Abel statistical convergent subsequence whose Abel statistical limit is in E, i.e. whenever $\mathbf{p}=(p_n)$ is a sequence of points in E, there is an Abel statistical convergent subsequence $\mathbf{r}=(r_k)=(r_{n_k})$ of the sequence $\mathbf{p}$ satisfying $Abel_{st}-lim\,\mathbf{r}\in E$.

Theorem 2.4 *A subset of X is Abel statistically compact if and only if it is compact in the ordinary sense.*

Proof: The proof of this theorem follows from Lemma 2 in [3], and Corollary 2.1 so is omitted. □

Definition 2.3 A point L in X is said to be in the Abel statistical sequential closure of a subset E of X, denoted by $\overline{E}^{Abel_{st}}$ if there is a sequence $\mathbf{p}=(p_k)$ of points in E such that $Abel_{st}-limp_k=L$, and it is called Abel statistically sequentially closed if $\overline{E}^{Abel_{st}}=E$.

In the following we see that ordinary closure of a subset of X coincides with the Abel statistically sequential closure.

Theorem 2.5 $\overline{E}^{Abel_{st}}=\overline{E}$

3. Abel statistical continuity in metric spaces

We now introduce a new type of continuity defined via Abel statistical convergent sequences.

Definition 3.1 A function f is called Abel statistically continuous if it preserves Abel statistical convergent sequences, i.e. $(f(p_k))$ is Abel statistical convergent to $f(L)$ whenever (p_k) is Abel statistically convergent to L.

In connection with Abel statistical convergent sequences and convergent sequences the problem arises to investigate the following types of continuity of functions on X.

(A_{st}) $(p_n)\in\mathbf{A}_{st}\Rightarrow(f(p_n))\in\mathbf{A}_{st}$

$(A_{st}c)$ $(p_n)\in\mathbf{A}_{st}\Rightarrow(f(p_n))\in c$

(c) $(p_n)\in c\Rightarrow(f(p_n))\in c$

(cA_{st}) $(p_n)\in c\Rightarrow(f(p_n))\in\mathbf{A}_{st}$

We see that A_{st} is Abel statistical continuity of f, and (c) states the ordinary continuity of f. We easily see that (c) implies (cA_{st}), (A_{st}) implies (cA_{st}), and (A_{st}c) implies (A_{st}). The converses are not always true as the identity function could be taken as a counter example for all the cases.

Now we give the implication (A_{st}) implies (c), i.e. any Abel statistical continuous function is continuous in the ordinary sense.

Theorem 3.1 *If a function f is Abel statistically continuous on a subset E of X, then it is continuous on E in the ordinary sense.*

Proof: The proof of this theorem follows from the preceding theorem and Lemma 2 in [3], so is omitted. □

Corollary 3.1 *Any Abel statistical continuous function on an Abel statistical compact subset of X is uniformly continuous.*

It is well known that uniform limit of a sequence of continuous functions is continuous. This is also true for Abel statistical continuity, i.e. uniform limit of a sequence of Abel statistical continuous functions is Abel statistical continuous.

Theorem 3.2 *If (f_n) is a sequence of Abel statistical continuous functions defined on a subset E of X and (f_n) is uniformly convergent to a function f, then f is Abel statistically continuous on E.*

Proof: Let (p_n) be an Abel statistical convergent sequence of points in E. Write $Abel_{st}-\lim p_n = L$. Take any $\varepsilon > 0$. Since (f_n) is uniformly convergent to f, there exists an $N \in \mathbf{N}$ such that $d(f_k(t), f(t)) < \varepsilon/3$ for all $t \in E$ whenever $k \geq N$. Thus $\lim_{x\to 1^-}(1-x)\sum_{d(f(p_k),f_N(p_k))\geq\frac{\varepsilon}{3}} x^k = 0$. Since f_N is Abel statistically continuous, we have
$\lim_{x\to 1^-}(1-x)\sum_{d(f_N(p_k)),f_N(L)|)\geq\frac{\varepsilon}{3}} x^k = 0$.
On the other hand,
$\sum_{d(f(p_k),f(L))\geq\varepsilon} x^k \leq \sum_{d(f(p_k),f_N(p_k))\geq\frac{\varepsilon}{3}} x^k + \sum_{d(f_N(p_k)),f_N(L))\geq\frac{\varepsilon}{3}} x^k + \sum_{d(f_N(L),f(L))\geq\frac{\varepsilon}{3}} x^k$
for every x satisfying $0 < x < 1$. Hence
$\lim_{x\to 1^-}(1-x)\sum_{d(f(p_k),f(L))\geq\varepsilon} x^k \leq$
$\lim_{x\to 1^-}(1-x)\sum_{d(f(p_k),f_N(p_k))\geq\frac{\varepsilon}{3}} x^k + \lim_{x\to 1^-}(1-x)\sum_{d(f_N(p_k)),f_N(L))\geq\frac{\varepsilon}{3}} x^k +$
$\lim_{x\to 1^-}(1-x)\sum_{d(f_N(L),f(L))\geq\frac{\varepsilon}{3}} x^k = 0+0+0 = 0$.
This completes the proof of the theorem. □

In the following theorem we prove that the set of Abel statistical continuous functions is a closed subset of the space of continuous functions.

Theorem 3.3 *The set of Abel statistical continuous functions on a subset E of X is a closed subset of the set of all continuous functions on E, i.e. $\overline{\boldsymbol{A}_{st}C(E)} = \boldsymbol{A}_{st}C(E)$, where $\boldsymbol{A}_{st}C(E)$ is the set of all Abel statistical continuous functions on E, $\overline{\boldsymbol{A}_{st}C(E)}$ denotes the set of all cluster points of $\boldsymbol{A}_{st}C(E)$.*

Proof: Let f be any element in the closure of $\mathbf{AC}(E)$. Then there exists a sequence (f_n) of points in $\mathbf{AC}$(E) such that $\lim_{k\to\infty} f_k = f$. To show that f is Abel statistical continuous, take any Abel statistical convergent sequence (p_k) of points E with Abel statistical limit L. Let $\varepsilon > 0$. Since (f_k) is convergent to f, there exists a positive integer N such that $d(f_k(t), f(t)) < \varepsilon/3$ for all $t \in E$ whenever $k \geq N$. Thus $\lim_{x\to 1^-}(1-x)\sum_{d(f(p_k),f_N(p_k))\geq\frac{\varepsilon}{3}} x^k = 0$. Since f_N is Abel statistically continuous, $\lim_{x\to 1^-}(1-x)\sum_{d(f_N(p_k)),f_N(L))\geq\frac{\varepsilon}{3}} x^k = 0$. On the other hand,
$\sum_{d(f(p_k),f(L))\geq\varepsilon} x^k \leq$
$\sum_{d(f(p_k),f_N(p_k))\geq\frac{\varepsilon}{3}} x^k + \sum_{d(f_N(p_k)),f_N(L))\geq\frac{\varepsilon}{3}} x^k + \sum_{d(f_N(L),f(L))\geq\frac{\varepsilon}{3}} x^k$
for every x satisfying $0 < x < 1$. Hence
$\lim_{x\to 1^-}(1-x)\sum_{d(f(p_k),f(L))\geq\varepsilon} x^k \leq$
$\lim_{x\to 1^-}(1-x)\sum_{d(f(p_k),f_N(p_k))\geq\frac{\varepsilon}{3}} x^k + \lim_{x\to 1^-}(1-x)\sum_{d(f_N(p_k)),f_N(L))\geq\frac{\varepsilon}{3}} x^k +$

$\lim_{x\to 1^-}(1-x)\sum_{d(f_N(L),f(L))\geq\frac{\varepsilon}{3}} x^k = 0+0+0 = 0.$
This completes the proof of the theorem. □

Corollary 3.2 *The set of all Abel statistical continuous functions on a subset E of X is a complete subspace of the space of all continuous functions on E.*

Proof: The proof follows from the preceding theorem, and the fact that the set of all continuous functions on E is complete. □

Theorem 3.4 *Abel statistical continuous image of any Abel statistical compact subset of X is Abel statistically compact.*

Proof: The proof follws easily, so is omitted. □

For $G := Abel - lim$, we have the following:

Theorem 3.5 *If a function f is Abel statistical continuous on a subset E of X, then*

$$f(\overline{B}^{Abel_{st}}) \subset \overline{(f(B))}^{Abel_{st}}$$

for every subset B of E.

Proof: The proof follows from the regularity and subsequenelity of Abel statistical sequential method, and Theorem 8 on page 316 of [3]. □

4. Conclusion

In this paper we introduce a concept of Abel statistical continuity in a metric space X, and present theorems related to this kind of continuity, and some other kinds of continuities. The concept of Abel statistical compactness is also introduced and investigated. One may expect this investigation to be a useful tool in the field of analysis in modeling various problems occurring in many areas of science, dynamical systems, computer science, information theory, and biological science.

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