



## Searching the Optimal Hamiltonian Cycle That is the Solution to the Traveling Salesman Problem

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**ABSTRACT:** The traveling salesman problem is a major challenge for researchers in mathematics and computer science. The challenge lies in the nature of its solution, which requires finding the shortest route to visit several cities only once and return to the starting point. In fact, the difficulty of solving it increases significantly as the number of cities increases. This is because the number of possible solutions grows rapidly with increasing numbers of cities. Consequently, reaching the final solution requires a longer time, which makes it belongs to the category an NP-hard problem. Therefore, this paper studies the traveling salesman problem to attempt to develop a suitable approach to reach its final solution, given its significant importance in many areas of life. In this regard, this study resulted in the design of a new algorithm to solve the linear traveling salesman problem. While there is a slight similarity between the traveling salesman problem and assignment problems in terms of the nature of the solution, this played a significant role in the design of the new algorithm. Obviously, the methodology of the new algorithm is based on the Hungarian method used to find the optimal solution to assignment problems. This, in turn, contributed to the harvesting of the most feasible solutions to all issues of the traveling salesman problem when using the new algorithm. Despite the difficulty and complexity of solving this problem, the new algorithm was characterized by its efficiency in solving the traveling salesman problem in record time and effort.

**Key Words:** Optimization theory, operations research, graph applications, the first approach to traveling Salesman problem (1<sup>st</sup> ATSP), Hungarian method, optimal control.

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### 1. Introduction

The term "optimal" refers to the most effective or appropriate outcome or state in a given situation, taking into account specific constraints relates to a specific criterion or goal. In decision-making, the optimal solution is the one that yields the best feasible results, taking into account specific constraints. It is worth noting that "optimal" is relevant in the context of various scientific applications, most notably mathematics, computing, engineering, design, networks, health, economics, industry, etc. [19]. In particular, the traveling salesman problem is one of the most prominent real-world problems that requires continuous optimization to achieve the best feasible solutions. Clearly, optimization often involves algorithms that provide feasible solutions to the traveling salesman problem, aiming to achieve a solution that is better than the previous one. Certainly, the resulting solution cannot be deemed the optimal solution to a specific issue within the traveling salesman problem if the issue is large. In fact, the reason behind this is the lack of an algorithm that guarantees the optimal solution within a specific time frame for issues in this problem, especially when the issue is large, given the complexity of the problem. This

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has made it an NP-hard problem [5]. In this regard, the traveling salesman problem is among the most widely known problems in mathematics and computer programming and is considered a classic example of a combinatorial optimization problem. Its solution involves determining the shortest closed path that passes through a set of locations (such as cities or similar) only once, without excluding any of locations, and then returning to the starting point. Based on this, the resulting solution constitutes what is known as a Hamiltonian cycle, which is a spanning subgraph of the Hamiltonian graph of a given network [18]. It is worth noting that interest in studying this problem and searching for the best solution has arisen in view of its widespread applications in various fields, including vehicle routing, planning, scheduling, and others. Hence, one of the most prominent research questions related to this topic is: Can the condition of the traveling salesman problem be met for all issues of this problem when solving it and with the optimal numerical value? As there is another question revolves around the extent to which algorithms can be used which it is provide the optimal solution for problems with a structure somewhat similar to the traveling salesman problem? Indeed, within the framework of the last question, the assignment problem is one of problems closest to the traveling salesman problem in terms of their general structure. In this regard, this study explored the possibility of employing the Hungarian algorithm to achieve the most effective solution to the traveling salesman problem, as this algorithm, in turn, ensures the optimal solution to all issues of the assignment problem. On the other hand, the literature on the traveling salesman problem is numerous, but the most important among them, in terms of methodology and results, have been identified [17,15,22,24,9,11,13,8,14,3,20]. There are also many studies related to problems that are related to the traveling salesman problem and the nature of what has been presented in this paper [23,10,6,25,7,1,12,21,2,4]. In general, the role of literature is absolutely essential when embarking on a study on a research problem, and one of the most prominent of these roles is building an idea about the nature of the problem, which helps in addressing it and achieving the most effective solutions for it. There is no doubt that studies mentioned and others represent efforts made by their authors in the right place and are worthy of appreciation and respect. However, this does not mean that some of them are not without some gaps and weaknesses that require further research to limit their impact on achieving the desired results.

In this paper, a new approach, the first approach to traveling salesman problem ( $1^{st}$ ATSP), is developed. This approach aims to find the optimal Hamiltonian cycle that represents the optimal solution to the traveling salesman problem when the objective function is to obtain the lowest feasible value among the other solutions. Clearly, the reason for this name is that another research study was conducted on this problem, but with a different objective function than the one in this study, and both studies form a chain with each other. Returning to the achievement presented in this paper, work was done to design a mathematical algorithm for  $1^{st}$ ATSP supported by computer code running in the MATLAB program mentioned below in the appendices section. Specifically, the  $1^{st}$ ATSP algorithm is based on the Hungarian method concerned with finding the optimal solution to the assignment problem. In fact, the reason for this lies in the similarity between the traveling salesman problem and the assignment problem in terms of structure and the nature of the solution. Although this similarity is limited, it has proven to be significant, embodied by the creation of the  $1^{st}$ ATSP algorithm, which has achieved impressive results. Furthermore, the reason for choosing the Hungarian method to develop the  $1^{st}$ ATSP algorithm is evident in its efficiency in achieving the optimal solution for all issues of the assignment problem, regardless of their size [19]. In fact, after solving many cases by using the  $1^{st}$ ATSP algorithm, its outstanding performance and flexibility of its steps were demonstrated. When comparing the results, it was found that the  $1^{st}$ ATSP algorithm's results were superior to those of the nearest neighbor and cheapest link algorithms. Furthermore, the  $1^{st}$ ATSP algorithm overcame some of the shortcomings faced by other algorithms. For example, it ensured that the traveling salesman problem condition was met when solving issues of this problem, compared to modern algorithms [16]. In conclusion, the study presented in this paper has yielded several contributions, the most prominent of which is achieving the optimal solution for issues of the traveling salesman problem, regardless of the number of vertices in the issue graph, provided that the number is even. In addition, the possibility of solving these cases by using the MATLAB code for the  $1^{st}$ ATSP algorithm, which was presented in this study. Finally, authors of this paper express that what is presented in it is characterized by originality and that it has not been noticed that it has been published in any other journal within limits of reviewing previous studies related to the problem presented in this

paper.

## 2. Methodology

### 2.1. About 1<sup>st</sup> ATSP

1<sup>st</sup> ATSP is an abbreviation for the first approach to traveling salesman problem, which aims to find the most feasible solution to the traveling salesman problem, which represents what is known as an optimal Hamiltonian cycle. In this context, the need to use 1<sup>st</sup> ATSP arises when decision makers seek to find the minimum feasible solution among the other solutions of issues. In fact, the 1<sup>st</sup> ATSP algorithm's methodology is based on the Hungarian method used to find the optimal solution to linear assignment problems. Based on the above, it is worth noting that the network graph of the salesperson's travel between the locations to be visited is transformed into a mathematical matrix. This matrix contains numerical values that represent weights of edges between vertices of the network graph. What's noteworthy is the 1<sup>st</sup> ATSP algorithm's ability to solve any issue, whether its weight matrix is symmetric or asymmetric. This is an important feature of the 1<sup>st</sup> ATSP algorithm. In this framework, it is important to note assumptions underlying the 1<sup>st</sup> ATSP algorithm's operation, which are as follows:

- I. The issue to be solved must be within the linear problems.
- II. The simple complete graph ( $K_n$ ) of the network of cities and routes through which the salesman travels.
- III. The total number of vertices in the issue network graph must be an even number of vertices.
- IV. Weights of edges connecting vertices of the graph that are non-zero numerical values.

### 2.2. Algorithm of 1<sup>st</sup> ATSP

This subsection will demonstrate how to use the 1<sup>st</sup> ATSP algorithm to solve the traveling salesman problem, detailing its steps, along with its inputs and outputs, as follows.

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#### Algorithm 1 1<sup>st</sup> ATSP Algorithm

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**Input:**

Edges weight matrix ( $WM$ ) of the graph ( $K_n$ ).

- 1: Apply steps of the Hungarian method algorithm [19].
- 2: Draw the graph of the issue network but without edges, i.e. draw the null graph  $\overline{K_n}$ .
- 3: Draw on  $\overline{K_n}$  only edges between vertices that were assigned in step 1.
- 4: If the Hamiltonian cycle is obtained from step 3, the total weight is calculated and thus the solution steps are complete. Otherwise, proceed to the next step.
- 5: If the Hamiltonian cycle is not obtained from step 3, steps of the Hungarian method are reapplied, excluding cells allocated in step 1 when performing the new application of the Hungarian method. Then, the following cases are considered:
  - a. Draw on  $\overline{K_n}$  only edges between vertices that were assigned in step 5 only. If the Hamiltonian cycle is obtained after the second application, the total weight is calculated and thus the solution steps are completed.
  - b. Draw on  $\overline{K_n}$  only edges between vertices assigned in both steps 1 and 5. If the Hamiltonian cycle is obtained by combining assignments resulting from steps 1 and 5, the total weight is calculated and thus the solution steps are complete.
  - c. If a Hamiltonian cycle is not obtained in the previous two cases,  $a$  and  $b$ , steps of the Hungarian method are repeated. Then, draw on  $\overline{K_n}$  only edges between vertices assigned to obtain a Hamiltonian cycle. Then, the total weight is calculated, thus completing the solution steps.

**Output:**

Hamiltonian cycle with total weight that satisfies the objective function

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### 3. Examples of Practical Application

Since presenting the algorithm alone without addressing its application to practical cases is insufficient to demonstrate its performance and the quality of its results, this section has been dedicated to that purpose.” To avoid prolixity, it has been sufficient to adopt the presentation of two cases of the traveling salesman problem, whose objective is to obtain the optimal Hamiltonian cycle with the lowest total weight of edges that comprise that cycle.

**Example 3.1** *There is an issue related to the traveling salesman problem with a network whose data are based on the graph  $K_6$  shown in Figure 1, which requires finding a Hamiltonian cycle has a total weight that is as smallest as possible.*

**Solution**

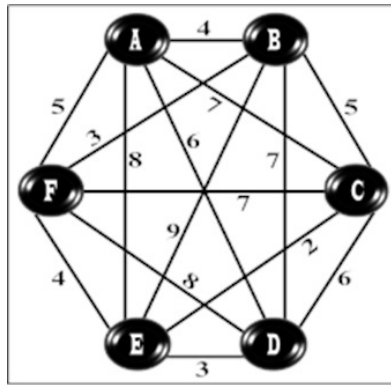


Figure 1: Graph of Example 3.1.

The starting point is to provide inputs for the 1<sup>st</sup> ATSP algorithm in order to start it, and all of this is shown in the following tables.

Table 1: Weight matrix of  $K_6$

	A	B	C	D	E	F
A	NF	4	7	6	8	5
B	4	NF	5	7	9	3
C	7	5	NF	6	2	7
D	6	7	6	NF	3	8
E	8	9	2	3	NF	4
F	5	3	7	8	4	NF

Table 2: After implementing the Hungarian method

	A	B	C	D	E	F
A	NF	0	2	0	5	0
B	0	NF	2	3	8	0
C	2	2	NF	1	0	3
D	0	3	1	NF	0	3
E	5	8	0	0	NF	2
F	0	0	3	3	2	NF

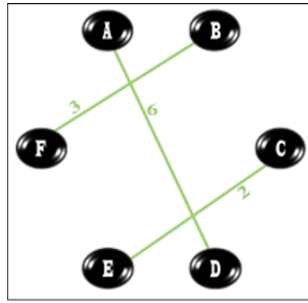


Figure 2: Graph by allocation in Table 2

Table 3: After implementing the Hungarian method again

	A	B	C	D	E	F
A	NF	0	1	NF	5	0
B	0	NF	0	3	7	NF
C	1	0	NF	0	NF	1
D	NF	3	0	NF	0	3
E	5	7	NF	0	NF	1
F	0	NF	1	3	1	NF

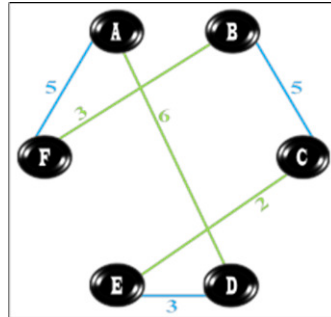


Figure 3: Graph by allocation in Tables 2 and 3

**Outputs:**

The Hamiltonian cycle as in Figure 3 has the total weight of 24 units.

**Example 3.2** *There is an issue related to the traveling salesman problem with a network whose data are based on the graph  $K_8$  and whose edge weight matrix is given in Table 4. Which requires finding a Hamiltonian cycle has a total weight that is as smallest as possible.*

Table 4: Weight matrix of  $K_8$

	A	B	C	D	E	F	G	H
A	NF	15	8	13	6	18	7	9
B	14	NF	4	11	8	5	9	6
C	5	9	NF	6	16	12	14	7
D	12	7	5	NF	10	13	9	8
E	7	12	17	8	NF	9	11	15
F	9	6	8	5	12	NF	10	13
G	10	8	14	10	9	6	NF	4
H	4	11	6	9	13	5	7	NF

### Solution

The starting point is to provide inputs for the 1<sup>st</sup> ATSP algorithm in order to start it, and all of this is shown in the following tables.

Table 5: After implementing the Hungarian method

	A	B	C	D	E	F	G	H
A	NF	9	4	8	0	13	0	5
B	10	NF	0	6	2	0	2	2
C	0	2	NF	0	9	6	6	2
D	7	0	0	NF	3	7	1	3
E	0	3	10	0	NF	1	1	8
F	5	0	4	0	6	NF	3	9
G	6	2	10	5	3	1	NF	0
H	0	5	2	4	7	0	0	NF

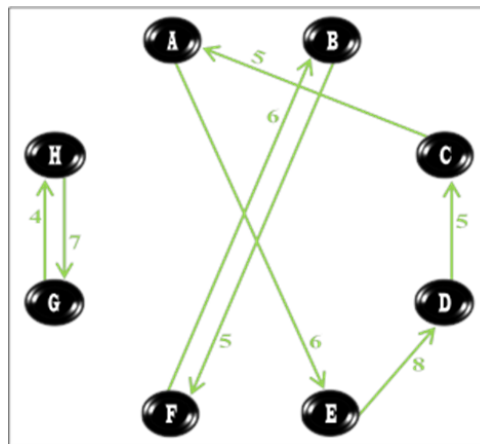


Figure 4: Graph by allocation in Table 5

Table 6: After implementing the Hungarian method again

	A	B	C	D	E	F	G	H
A	NF	9	4	9	NF	13	0	3
B	10	NF	0	5	0	NF	2	0
C	NF	2	NF	0	7	6	6	0
D	7	0	NF	NF	1	7	1	1
E	0	2	9	NF	NF	0	0	5
F	4	NF	3	0	3	NF	2	6
G	5	1	9	5	0	0	NF	NF
H	0	5	2	5	5	0	NF	NF

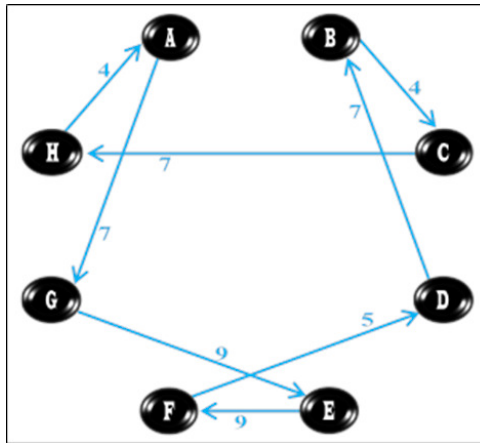


Figure 5: Graph by allocation in Table 6

**Outputs:**

The Hamiltonian cycle as in Figure 5 has the total weight of 52 units.

#### 4. Analysis of Results

This study addresses the traveling salesman problem, one of the most prominent combinatorial optimization problems. This is due to the occurrence of a combinatorial explosion in the feasible solution region when the problem size increases slightly. In fact, this study presents a new approach, the 1<sup>st</sup> ATSP algorithm, which relies on applying the Hungarian method more than once to ensure obtaining a cycle containing all vertices of the problem network. Another reason for applying the Hungarian method more than once in the 1<sup>st</sup> ATSP algorithm is that a feasible solution to the traveling salesman problem, represented by a Hamiltonian cycle, is not obtained when applying the Hungarian method only once, as mentioned in examples in the previous section. The best evidence for this is a comparison with [25], which shows that applying the Hungarian method only once does not guarantee obtaining a Hamiltonian cycle.

#### 5. Conclusion

With the completion of the study presented in this paper, it is worth noting that its most prominent outputs are the design of the 1<sup>st</sup> ATSP algorithm. Indeed, its importance is highlighted by its ability to achieve the optimal solution to the linear traveling salesman problem in record time. In this regard, the construction of the 1<sup>st</sup> ATSP algorithm relied on the Hungarian method used to solve assignment problems characterized by a minimization objective function. This is due to the Hungarian method's efficiency in achieving the optimal solution to assignment problems. Although the Hungarian method is unable to solve all issues of the traveling salesman problem due to the structure and nature of the problem, this study contributed to employing the Hungarian method to develop the 1<sup>st</sup> ATSP algorithm, which is specifically designed to achieve the optimal solution to the traveling salesman problem. What is noteworthy is that the 1<sup>st</sup> ATSP algorithm distinguishes itself from other algorithms in terms of its efficiency in solving all issues, regardless of whether the weight matrix is symmetric or asymmetric. Overall, 1<sup>st</sup> ATSP results were efficient and trustworthy for decision makers in solving the traveling salesman problem, provided the total number of vertices in the network graph for any given issue was an even number. In light of this, researchers are recommended to benefit from this study for future research, which could lead to more surprising results. Furthermore, decision makers can apply outputs of this study to real-world situations and observe the extent to which they fit their objectives.

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