



On Generalization of Generalized Star Generalized Closed Sets in Topological Spaces

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ABSTRACT: In this paper, we introduce a new class of closed sets called the generalization of generalized star generalized closed sets (briefly $(gg)^*g$ -closed sets) in topological spaces. And, we scrutinized some of their characterization and its relationship along other closed sets.

Keywords: $(gg)^*g$ -closed set, $(gg)^*$ -open sets, g -closure.

Contents

1 Introduction	1
2 Preliminaries	2
3 Generalization of Generalized Star Generalized Closed Sets	6
4 Independency of $(gg)^*g$-Closed Sets with Other Closed Sets	9
5 Characteristics of $(gg)^*g$-Closed Sets	10
6 Conclusion	14

1. Introduction

In the field of topology, finding exact answers to many problems is challenging. Consider problems involving closure space, proximity space, and compactifications. As a result, topologists have developed several tools for us, including nets, ideals, filters, and grills. Researchers have become quite interested in primal set theory lately. One of the most essential things to look at is how the operators described by the primal set behave in a qualitative way. The tools listed above worked quite well for this purpose. Topologists gave us related topological spaces, which were seen as three variables that acted differently. Using these concepts and methods, researchers looked into a primal generalized topological space. In another study, topologists came up with distinct kinds of open sets that had some different properties. This sort of work is becoming more common in the subject of topology. This way of doing research has an effect on the ideas in topological theory that go along with it. The reason for this technique pays a lot of attention to the issue of continuity. This principle has led to several ways of breaking down continuous functions. General topology is very important in many fields of science, including data mining, computational topology for geometric design and molecular design, computer-aided design, computer-aided geometric design, engineering design, digital topology, information systems, quantum physics, high energy physics, and superstring theory. Chang [42] was the first to talk about the topological structures of set theories that deal with uncertainty. By extending rough set theory, Lashin et al. [47] looked at topological spaces. Shabir and Naz [53] came up with the idea of soft topological spaces based on an initial universe with a fixed set of parameters. Senel and Cagman [44] then expanded the idea of bitopological spaces to soft bitopological spaces and found some connections between soft topology and soft bitopology. In [43], the authors of this paper defined and looked into the ideas of soft closed sets, soft α -closed sets, soft semi-closed sets, soft pre-closed sets, regular soft closed sets, soft g -closed sets, and soft sg -closed sets in soft bi-topological spaces. When studying generic topology, the ideas of closed sets and open sets are very important. Levine [49] came up with the idea of generalised closed sets in topological spaces in 1970. He also defined a $T(1/2)$ -space as one where the closed sets and the generalised closed sets are the same. Dunham and Levine [45] looked into other features of generalised closed sets. We have

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changed and looked at the idea of generalised closed sets by employing weaker types of open sets, such as α -open sets [51], semi-open sets [48], preopen sets [50], and semi-preopen sets [40]. Abd El-Monsef et al. [46] came up with a weak variant of open sets called β -open sets in 1983. The idea of β -open sets is the same as that of semi-preopen sets [40]. Noiri and Hatir [52] came up with the ideas of Λ sp-sets, Λ sp-closed sets, and spg-closed sets in 2004 and looked into what these sets are like. The author of [41] spoke about and looked into the ideas of (Λ, sp) -open sets and (Λ, sp) -closed sets, which are based on the ideas of Λ sp-sets and β -closed sets. Recent research scholars have proposed a new concept of closed sets in topological spaces, which is called "On Generalization of Generalized Star Generalized Closed Sets in Topological Spaces." The concept of generalization of generalized closed sets (briefly gg -closed) was introduced by Basavaraj, M. Ittanagi and H.G. Govardhana Reddy in 2017. The concept of generalization of generalized star closed sets (briefly $(gg)^*$ -closed) was introduced by I. Christal Bai and T. Shyla Isac Mary in 2018. In this paper, a new class of closed sets called $(gg)^*$ g-closed sets is introduced, and its characteristics are analyzed.

2. Preliminaries

Throughout this paper, (X, τ) represents the topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological space (X, τ) , the closure of A and the interior of A are denoted by $cl(A)$ and $int(A)$ respectively. $X \setminus A$ denotes the complement of A in X .

Definition 1. A subset A of a topological space (X, τ) is called a

1. **pre-open** if $A \subseteq int\ cl(A)$.
pre-closed if $cl\ int(A) \subseteq A$ [11].
2. **semi-open** if $A \subseteq cl\ int(A)$.
semi-closed if $int\ cl(A) \subseteq A$ [8].
3. **semi-pre-open** if $A \subseteq cl\ int\ cl(A)$.
semi-pre-closed if $int\ cl\ int(A) \subseteq A$ [13].
4. **α -open** if $A \subseteq int\ cl\ int(A)$.
 α -closed if $cl\ int\ cl(A) \subseteq A$ [20].
5. **regular open** if $A = int(cl(A))$.
regular closed if $cl\ int(A) = A$ [7].
6. **π -open** if A is the union of regular open sets.
 π -closed if A is the intersection of regular closed sets [12].
7. **b -open** if $A \subseteq cl\ int(A) \cup int\ cl(A)$.
 b -closed if $cl\ int(A) \cap int\ cl(A) \subseteq A$ [34].
8. **β -open** if $A \subseteq cl\ int\ cl(A)$.
 β -closed if $int\ cl\ int(A) \subseteq A$ [19].

The closure of a subset A of topological space (X, τ) is the smallest closed set containing A , is denoted by $cl(A)$.

The generalized closure (briefly g -closure) of a subset $A \subseteq X$ is the smallest g -closed set containing A is denoted by $gcl(A)$.

The regular closure of a subset $A \subseteq X$ is the intersection of all regular closed sets containing A is denoted by $rcl(A)$.

The α -closure of a subset $A \subseteq X$ is the intersection of all α -closed sets containing A is denoted by $\alpha cl(A)$.

The b -closure of a subset $A \subseteq X$ is the intersection of all b -closed sets containing A is denoted by $bcl(A)$.

Definition 2. A subset of a topological space (X, τ) is called a

1. Generalized closed set (briefly g -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X . [25]
2. Regular semi-open if there exists a regular open set U such that $U \subseteq A \subseteq cl(A)$. [12]
3. Generalization of generalized closed set (briefly gg -closed) if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open in X . [3]
4. Generalization of generalized star closed set (briefly $(gg)^*$ -closed) if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is gg -open in X . [9]
5. Generalized b -closed set (briefly gb -closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X . [34].
6. Generalized star closed set (briefly g^* -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in X . [27]
7. r^*g^* -closed set if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in X [23].
8. gb^* -closed set if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^* -open in X [34].
9. Generalized semi-pre closed set (briefly gsp -closed) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X [28]
10. Generalized semi-pre star closed set (briefly $(gsp)^*$ -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is gsp -open in X [28].
11. $g^\#$ -closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is αg -open in X [4].
12. g^{**} -closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^* -open in X [30].

13. \hat{g}^* -s-closed set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is \hat{g} -open in X [1].
14. Regular generalized closed set (briefly rg-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X [26].
15. w or \hat{g} -closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X [2].
16. Generalized semi closed set (briefly gs-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X [20].
17. Generalized star semi-regular closed set (briefly g^* sr-closed) if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is gs-open in X [37].
18. gs^* -closed set or $(gs)^*$ -closed if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is gs-open in X [1].
19. sg^* -closed set or $(sg)^*$ -closed or g^*s^* -closed set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^* -open in X [1].
20. gs^{**} -closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ whenever U is gs^* -open in X [1].
21. Semi generalized closed set (briefly sg-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X [20].
22. Generalized star semi-closed set (briefly g^*s -closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is gs-open in X [37].
23. Generalized pre-closed set (briefly gp-closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X [11].
24. Generalized pre-star closed set (briefly gp^* -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is gp-open in X [16].
25. Generalized semi-pre regular closed set (briefly $gspr$ -closed) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X [13].
26. Generalized pre-regular closed set (briefly gpr -closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X [21].
27. Generalized regular star closed set (briefly gr^* -closed) if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in X [15].

28. Regular beta closed set (briefly $r\beta$ -closed) if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is β -open in X [19].
29. Generalized star pre closed set (briefly g^*p -closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g -open in X [32].
30. $(g^*p)^*$ -closed set if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^*p -open in X [29].
31. Generalized alpha regular closed set (briefly $g\alpha r$ -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X [33].
32. Generalized alpha regular star closed set (briefly $(g\alpha r)^*$ -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $g\alpha r$ -open in X [35].
33. Generalized alpha regular star-star closed set (briefly $(g\alpha r)^{**}$ -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $g\alpha r^*$ -open in X [35].
34. Weakly generalized closed set (briefly wg -closed) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is open in X [14].
35. Weakly semi closed set (briefly ws -closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is w -open in X [10].
36. Alpha generalized regular closed set (briefly αgr -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X [36].
37. Regular generalized b-closed set (briefly rgb -closed) if $bcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X [34].
38. Alpha generalized closed set (briefly αg -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X [20].
39. Alpha generalized star closed set (briefly αg^* -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in X [20].
40. Generalized alpha closed set (briefly $g\alpha$ -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in X [18].
41. Generalized regular closed set (briefly gr -closed) if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X [7].

42. Regular weakly generalized closed set (briefly rwg-closed) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X [22].
43. Regular generalized weakly closed set (briefly rgw-closed) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open in X [2].
44. R^* -closed set if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open in X [4].
45. $R^\#$ -closed set if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is R^* -open in X [4].
46. $r^\wedge g$ -closed set if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X [31].
47. Pre-generalized closed set (briefly pg-closed) if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is pre-open in X [16].
48. Strongly generalized star closed set (briefly strongly g^* -closed) if $cl(int(A)) \subseteq U$ whenever $A \subseteq U$ and U is g -open in X [27].
49. Regular weakly closed set (briefly rw-closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi-open in X [5].

The complements of the above mentioned closed sets are their respective open sets.

3. Generalization of Generalized Star Generalized Closed Sets

Definition 3. A subset A of a topological space (X, τ) is called a generalization of generalized star generalized closed set (briefly $(gg)^*g$ -closed) if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is $(gg)^*$ -open in (X, τ) . The set of all $(gg)^*g$ -closed sets in (X, τ) is denoted by $(GG)^*GC(X)$.

Example 1. : Let $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$,
 $\tau_c = \{\emptyset, \{a, c, d\}, \{a, b, d\}, \{a, d\}, \{a\}, X\}$.
 $(gg)^*$ -open = $\{\emptyset, \{c, d\}, \{b, d\}, \{b, c\}, \{d\}, \{c\}, \{b\}, X\}$,
 $(gg)^*g$ -closed = $\{\emptyset, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}, X\}$.

Proposition 3.1. Every closed set is $(gg)^*g$ -closed.

Let A be a closed set in X , such that $A \subseteq U$, and U is a $(gg)^*$ -open set in X . Since A is closed, so $cl(A) = A \subseteq U$ [36]. But we have, $gcl(A) \subseteq cl(A) \subseteq U$ [20]. Therefore, $gcl(A) \subseteq U$ and hence A is $(gg)^*g$ -closed.

Remark 1. : The converse part of the above proposition need not be true as shown in the following example.

Example 2. : Let $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$,
 $\tau_c = \{\emptyset, \{a, c, d\}, \{a, b, d\}, \{a, d\}, \{a\}, X\}$. Then the set $\{b, c, d\}$ is $(gg)^*g$ -closed but not closed.

Proposition 3.2. :

- (i) Every regular closed set is $(gg)^*g$ -closed.
- (ii) Every π -closed set is $(gg)^*g$ -closed.

- (iii) Every g^* -closed set is $(gg)^*g$ -closed.
- (iv) Every $(g\alpha r)^{**}$ -closed set is $(gg)^*g$ -closed.
- (v) Every gr^* -closed set is $(gg)^*g$ -closed.
- (vi) Every $(gg)^*$ -closed set is $(gg)^*g$ -closed.
- (vii) Every $r\beta$ -closed set is $(gg)^*g$ -closed.
- (viii) Every gp^* -closed set is $(gg)^*g$ -closed.
- (ix) Every $(gsp)^*$ -closed set is $(gg)^*g$ -closed.
- (x) Every $g^\#$ -closed set is $(gg)^*g$ -closed.
- (xi) Every $(gs)^*$ -closed set is $(gg)^*g$ -closed.
- (xii) Every g^*sr -closed set is $(gg)^*g$ -closed.
- (xiii) Every r^*g^* -closed set is $(gg)^*g$ -closed.
- (xiv) Every $(g^*p)^*$ -closed set is $(gg)^*g$ -closed.

Proof. (i) Let A be a regular closed set in X , such that $A \subseteq U$ and U is $(gg)^*$ -open set in X . Since A is regular closed, so $rcl(A) = A \subseteq U$ [35]. But we have $gcl(A) \subseteq rcl(A) \subseteq U$ [19]. Therefore $gcl(A) \subseteq U$ and hence A is $(gg)^*g$ -closed.

- (ii) Let A be a π -closed set in X . Since every π -closed set is closed [12] and from Proposition 1 every closed set is $(gg)^*g$ -closed, and hence A is $(gg)^*g$ -closed set in X .
- (iii) Let A be a g^* -closed set in X , such that $A \subseteq U$ and U is $(gg)^*$ -open in X . Since every $(gg)^*$ -open set is g -open [9], therefore U is g -open in X . Since A is g^* -closed, $cl(A) \subseteq U$. But we have $gcl(A) \subseteq cl(A) \subseteq U$ [20]. Therefore $gcl(A) \subseteq U$ and hence A is $(gg)^*g$ -closed.
- (iv) Let A be a $(g\alpha r)^{**}$ -closed set. Since every $(g\alpha r)^{**}$ -closed set is g^* -closed [35] and from Proposition 1(iii) every g^* -closed set is $(gg)^*g$ -closed, hence every $(g\alpha r)^{**}$ -closed set is $(gg)^*g$ -closed.
- (v) Let A be a gr^* -closed set in X , such that $A \subseteq U$ and U is $(gg)^*$ -open set in X . Since every $(gg)^*$ -open set is g -open in X [9], therefore U is g -open in X . Since A is gr^* -closed, $rcl(A) \subseteq U$. But we have $gcl(A) \subseteq rcl(A) \subseteq U$ [19]. Therefore $gcl(A) \subseteq U$ and hence A is $(gg)^*g$ -closed.
- (vi) Let A be a $(gg)^*$ -closed set in X . Since every $(gg)^*$ -closed set is gr^* -closed [9] and from Proposition 1(v) every gr^* -closed set is $(gg)^*g$ -closed, hence every $(gg)^*$ -closed set is $(gg)^*g$ -closed.
- (vii) Let A be a $r\beta$ -closed set in X . Since every $r\beta$ -closed set is gr^* -closed [19] and from Proposition 1(v) every gr^* -closed set is $(gg)^*g$ -closed, hence every $r\beta$ -closed set is $(gg)^*g$ -closed.
- (viii) Let A be a gp^* -closed set in X , such that $A \subseteq U$ and U is $(gg)^*$ -open set in X . Since every $(gg)^*$ -open set is gp -open [9], therefore U is gp -open in X . Since A is gp^* -closed, $cl(A) \subseteq U$. But we have $gcl(A) \subseteq cl(A) \subseteq U$ [20]. Therefore $gcl(A) \subseteq U$ and hence A is $(gg)^*g$ -closed.
- (ix) Let A be a $(gsp)^*$ -closed set in X , such that $A \subseteq U$ and U is $(gg)^*$ -open in X . Since every $(gg)^*$ -open set is gsp -open [9], therefore U is gsp -open in X . Since A is $(gsp)^*$ -closed, $cl(A) \subseteq U$. But we have $gcl(A) \subseteq cl(A) \subseteq U$ [20]. Therefore $gcl(A) \subseteq U$ and hence A is $(gg)^*g$ -closed.

- (x) Let A be a $g^\#$ -closed set in X , such that $A \subseteq U$ and U is $(gg)^*$ -open set in X . Since every $(gg)^*$ -open set is αg -open [9], therefore U is αg -open in X . Since A is $g^\#$ -closed, $cl(A) \subseteq U$. But we have $gcl(A) \subseteq cl(A) \subseteq U$ [20]. Therefore $gcl(A) \subseteq U$ and hence A is $(gg)^*g$ -closed.
- (xi) Let A be a $(gs)^*$ -closed set in X , such that $A \subseteq U$ and U is $(gg)^*$ -open set in X . Since every $(gg)^*$ -open set is gs -open [9], therefore U is gs -open and since A is $(gs)^*$ -closed, $cl(A) \subseteq U$. But we have $gcl(A) \subseteq cl(A) \subseteq U$ [20]. Therefore $gcl(A) \subseteq U$ and hence A is $(gg)^*g$ -closed.
- (xii) Let A be a g^*sr -closed set in X . Let U be a $(gg)^*$ -open set in X , such that $A \subseteq U$. Since every $(gg)^*$ -open set is gs -open [9], therefore U is gs -open and since A is g^*sr -closed, $rcl(A) \subseteq U$. But we have $gcl(A) \subseteq rcl(A) \subseteq U$ [19]. Therefore $gcl(A) \subseteq U$ and hence A is $(gg)^*g$ -closed.
- (xiii) Let A be a r^*g^* -closed set in X , such that $A \subseteq U$ and U is $(gg)^*$ -open set in X . Since every $(gg)^*$ -open set is g -open [9], therefore U is g -open in X . Since A is r^*g^* -closed, $rcl(A) \subseteq U$. But we have $gcl(A) \subseteq rcl(A) \subseteq U$ [19]. Therefore $gcl(A) \subseteq U$ and hence A is $(gg)^*g$ -closed.
- (xiv) Let A be a $(g^*p)^*$ -closed set in X , such that $A \subseteq U$ and U is $(gg)^*$ -open set in X . Since every $(gg)^*$ -open set is g^*p -open [9], therefore U is g^*p -open. Since A is $(g^*p)^*$ -closed, $cl(A) \subseteq U$. But we have $gcl(A) \subseteq cl(A) \subseteq U$ [20]. Therefore $gcl(A) \subseteq U$ and hence A is $(gg)^*g$ -closed. \square

Remark 2. : *The converse of the above preposition need not be true as shown in the following examples.*

Example 3. : *Let $X = \{a, b, c, d\}$ with $\tau = \{\emptyset, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$ and $\tau_c = \{\emptyset, \{a, c, d\}, \{a, b, d\}, \{a, d\}, \{a\}, X\}$.*

Then the following statements hold:

1. $\{a, b, c\}$ is $(gg)^*g$ -closed but not regular closed.
2. $\{a\}$ is $(gg)^*g$ -closed but not π -closed.
3. $\{b, c, d\}$ is $(gg)^*g$ -closed but not g^* -closed.
4. $\{b, c, d\}$ is $(gg)^*g$ -closed but not $(gar)^{**}$ -closed.
5. $\{b, c, d\}$ is $(gg)^*g$ -closed but not gr^* -closed.
6. $\{a\}$ is $(gg)^*g$ -closed but not $(gg)^*$ -closed.
7. $\{a\}$ is $(gg)^*g$ -closed but not $r\beta$ -closed.
8. $\{a, b\}$ is $(gg)^*g$ -closed but not gp^* -closed.
9. $\{a, c\}$ is $(gg)^*g$ -closed but not $(gsp)^*$ -closed.
10. $\{a, b, c\}$ is $(gg)^*g$ -closed but not $g^\#$ -closed.
11. $\{a, c\}$ is $(gg)^*g$ -closed but not $(gs)^*$ -closed.
12. $\{a, b\}$ is $(gg)^*g$ -closed but not g^*sr -closed.
13. $\{b, c, d\}$ is $(gg)^*g$ -closed but not r^*g^* -closed.
14. $\{a, b, c\}$ is $(gg)^*g$ -closed but not $(g^*p)^*$ -closed.

Proposition 3.3. :

- (i) *Every $(gg)^*g$ -closed set is $r^\wedge g$ -closed.*
- (ii) *Every $(gg)^*g$ -closed set is rwg -closed.*

Proof. :

- (i) Let A be a $(gg)^*g$ -closed set in X , such that $A \subseteq U$ and U be a regular open set in X . Since every regular open set is $(gg)^*$ -open [36], therefore U is $(gg)^*$ -open. Since A is $(gg)^*$ -closed, $gcl(A) \subseteq U$ and hence A is $r^\wedge g$ -closed.
- (ii) Let A be a $(gg)^*g$ -closed set in X , such that $A \subseteq U$, U be a regular open set in X . Since every regular open set is $(gg)^*$ -open [36], therefore U is $(gg)^*$ -open. Since A is $(gg)^*g$ -closed, $gcl(A) \subseteq U$. But we have $cl(int(A)) \subseteq gcl(A) \subseteq U$. Therefore $cl(int(A)) \subseteq U$ and hence A is rwg -closed.

□

Remark 3. : *The converse of the above proposition need not be true as shown in the following examples.*

Example 4. : *Let $X = \{a, b, c, d\}$ with $\tau = \{\phi, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$ and $\tau_c = \{\phi, \{a, c, d\}, \{a, b, d\}, \{a, d\}, \{a\}, X\}$. Then the set*

1. $\{d\}$ is $r^\wedge g$ -closed but not $(gg)^*g$ -closed.
2. $\{b, c\}$ is rwg -closed but not $(gg)^*g$ -closed.

4. Independency of $(gg)^*g$ -Closed Sets with Other Closed Sets

The following example shows that the concept of $(gg)^*g$ -closed set is independent from R^* -closed, g^*p -closed, g^*s^* -closed, strongly g^* -closed, g^*s -closed, wg -closed, pg -closed, ws -closed, gp -closed, αg -closed, gs -closed, gsp -closed, sg -closed, $g\alpha$ -closed, α -closed, b -closed, semi-closed, gb -closed, gb^* -closed, $g^\wedge s$ -closed sets.

Example 5. : *Let $X = \{a, b, c, d\}$ with $\tau = \{\phi, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$ and $\tau_c = \{\phi, \{a, c, d\}, \{a, b, d\}, \{a, d\}, \{a\}, X\}$. Then the set*

1. $\{a\}$ is $(gg)^*g$ -closed but not R^* -closed, and $\{b, c\}$ is R^* -closed but not $(gg)^*g$ -closed.
2. $\{b, c, d\}$ is $(gg)^*g$ -closed but not g^*p -closed, and $\{b, d\}$ is g^*p -closed but not $(gg)^*g$ -closed.
3. $\{b, c, d\}$ is $(gg)^*g$ -closed but not g^*s^* -closed, and $\{c, d\}$ is g^*s^* -closed but not $(gg)^*g$ -closed.
4. $\{b, c, d\}$ is $(gg)^*g$ -closed but not strongly g^* -closed, and $\{d\}$ is strongly g^* -closed but not $(gg)^*g$ -closed.
5. $\{a, b, c\}$ is $(gg)^*g$ -closed but not g^*s -closed, and $\{b, d\}$ is g^*s -closed but not $(gg)^*g$ -closed.
6. $\{d\}$ is wg -closed but not $(gg)^*g$ -closed, and $\{b, c, d\}$ is $(gg)^*g$ -closed but not wg -closed.
7. $\{a, b\}$ is $(gg)^*g$ -closed but not pg -closed, and $\{b, d\}$ is pg -closed but not $(gg)^*g$ -closed.
8. $\{b, c, d\}$ is $(gg)^*g$ -closed but not ws -closed, and $\{c, d\}$ is ws -closed but not $(gg)^*g$ -closed.
9. $\{b, c, d\}$ is $(gg)^*g$ -closed but not gp -closed, and $\{d\}$ is gp -closed but not $(gg)^*g$ -closed.
10. $\{b, c, d\}$ is $(gg)^*g$ -closed but not αg -closed, and $\{d\}$ is αg -closed but not $(gg)^*g$ -closed.
11. $\{a, b, d\}$ is $(gg)^*g$ -closed but not gs -closed, and $\{d\}$ is gs -closed but not $(gg)^*g$ -closed.
12. $\{b, c, d\}$ is $(gg)^*g$ -closed but not gsp -closed, and $\{b\}$ is gsp -closed but not $(gg)^*g$ -closed.
13. $\{a, b, c\}$ is $(gg)^*g$ -closed but not sg -closed, and $\{c\}$ is sg -closed but not $(gg)^*g$ -closed.
14. $\{a, b\}$ is $(gg)^*g$ -closed but not $g\alpha$ -closed, and $\{b, d\}$ is $g\alpha$ -closed but not $(gg)^*g$ -closed.
15. $\{a, b, c\}$ is $(gg)^*g$ -closed but not α -closed, and $\{d\}$ is α -closed but not $(gg)^*g$ -closed.

16. $\{c\}$ is b -closed but not $(gg)^*g$ -closed, and $\{a, b, c\}$ is $(gg)^*g$ -closed but not b -closed.
17. $\{a, b, c\}$ is $(gg)^*g$ -closed but not semi-closed, and $\{b, d\}$ is semi-closed but not $(gg)^*g$ -closed.
18. $\{b, c, d\}$ is $(gg)^*g$ -closed but not gb -closed, and $\{b\}$ is gb -closed but not $(gg)^*g$ -closed.
19. $\{b, c, d\}$ is $(gg)^*g$ -closed but not gb^* -closed, and $\{b\}$ is gb^* -closed but not $(gg)^*g$ -closed.
20. $\{b, c, d\}$ is $(gg)^*g$ -closed but not $g^\wedge s$ -closed, and $\{c\}$ is $g^\wedge s$ -closed but not $(gg)^*g$ -closed.

Remark 4. : From the above discussion and known results, the relationship between $(gg)^*g$ -closed sets and other existing generalizations of closed sets are established in Figure 1.

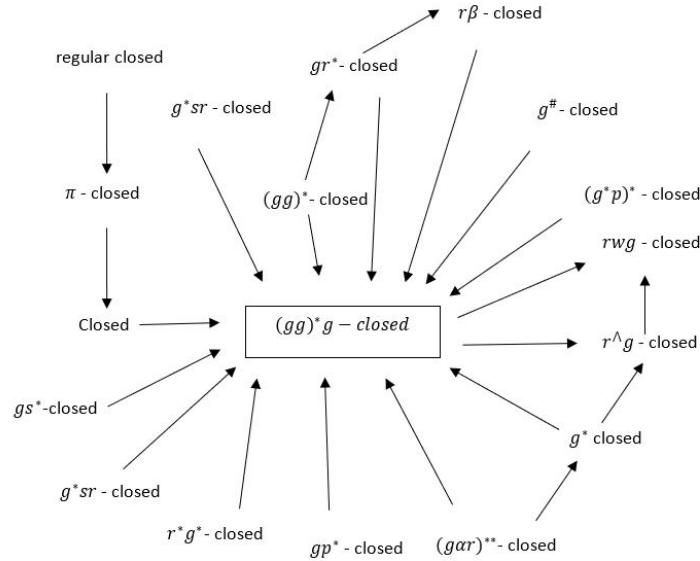


Figure 1: $A \rightarrow B$ means that set A implies set B , but not conversely

5. Characteristics of $(gg)^*g$ -Closed Sets

Theorem 5.1. : The finite union of any two $(gg)^*g$ -closed sets of X is $(gg)^*g$ -closed.

Proof. Let A and B be $(gg)^*g$ -closed sets in X . Let U be a $(gg)^*g$ -open set in X , such that $A \cup B \subseteq U$. Then $A \subseteq U$ and $B \subseteq U$. Since A and B are $(gg)^*g$ -closed sets in X , therefore $gcl(A) \subseteq U$ and $gcl(B) \subseteq U$. We have

$$gcl(A \cup B) = gcl(A) \cup gcl(B) \subseteq U \quad [20].$$

Thus, $gcl(A \cup B) \subseteq U$. Therefore, $A \cup B$ is a $(gg)^*g$ -closed set. \square

Remark 5. : The intersection of two $(gg)^*g$ -closed sets in X need not be a $(gg)^*g$ -closed set in X , as seen in the following example.

Example 6. : Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$ and the collection of $(gg)^*g$ -closed sets is given by $\{\emptyset, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}, X\}$. If $A = \{a, b\}$ and $B = \{b, c, d\}$, then A and B are $(gg)^*g$ -closed sets. But $A \cap B = \{b\}$ is not a $(gg)^*g$ -closed set in X .

Theorem 5.2. : A subset A of X is $(gg)^*g$ -closed in X and $A \subseteq B \subseteq gcl(A)$. Then B is also $(gg)^*g$ -closed in X .

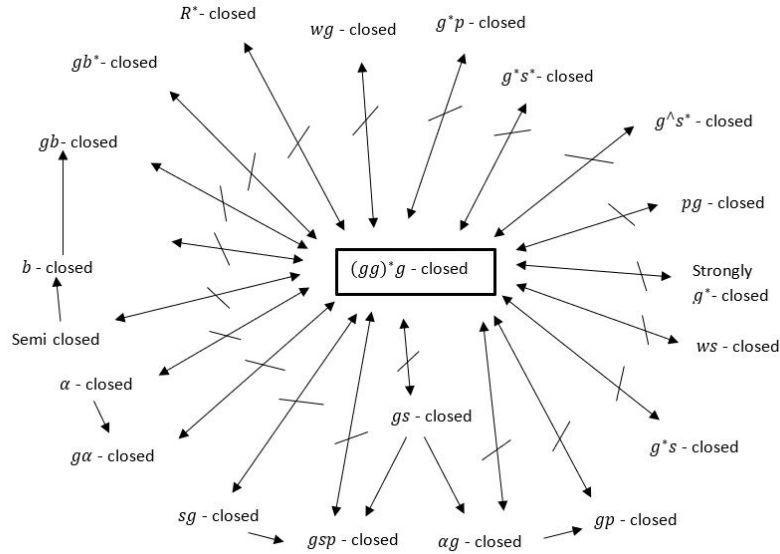


Figure 2: $A \not\rightarrow B$ means set A and set B are independent of each other.

Proof. : Let $B \subseteq U$, where U is a $(gg)^*$ -open set in X . Then $A \subseteq B$ implies $A \subseteq U$. Since A is $(gg)^*$ -closed, $gcl(A) \subseteq U$. By hypothesis, $gcl(B) \subseteq gcl(gcl(A)) = gcl(A) \subseteq U$ [20]. Therefore, $gcl(B) \subseteq U$, and hence B is $(gg)^*$ -closed in X . \square

Remark 6. : *The converse part of Theorem 5 is not true as seen from the example given below.*

Example 7. : *Let $X = \{a, b, c, d\}$, $\tau = \{\phi, \{b\}, \{c\}, \{b, c\}, \{b, c, d\}, X\}$ be a topology on X . Then $A = \{c\}$ and $B = \{a, b\}$ are $(gg)^*$ -closed sets in X . But $A \subseteq B$ is not a subset of $gcl(A)$, since $gcl(A) = \{a, c\}$.*

Theorem 5.3. : *A subset A of X is $(gg)^*$ -closed in X if and only if $gcl(A) \setminus A$ does not contain any non-empty $(gg)^*$ -closed set in X .*

Proof. : Suppose that A is a $(gg)^*$ -closed set in X . Let us assume that B is a non-empty $(gg)^*$ -closed subset of $gcl(A) \setminus A$ in X , such that $B \subseteq gcl(A) \setminus A$. That implies

$$B \subseteq gcl(A) \cap (X \setminus A) \quad \text{and} \quad B \neq \phi,$$

which implies

$$B \subseteq gcl(A) \tag{1}$$

and

$$B \subseteq X \setminus A \Rightarrow A \subseteq X \setminus B.$$

Since A is $(gg)^*$ -closed and $X \setminus B$ is $(gg)^*$ -open in X , [definition 3] and so

$$gcl(A) \subseteq X \setminus B \Rightarrow B \subseteq X \setminus gcl(A). \tag{2}$$

Therefore, from (1) and (2), we get

$$B \subseteq gcl(A) \cap (X \setminus gcl(A)) = \phi,$$

which implies $B = \phi$, a contradiction. Hence, $gcl(A) \setminus A$ does not contain any non-empty $(gg)^*$ -closed set in X .

Conversely, assume that $gcl(A) \setminus A$ does not contain any non-empty $(gg)^*$ -closed set in X . Let $A \subseteq U$, where U is a $(gg)^*$ -open set in X . Suppose $gcl(A) \not\subseteq U$. Then

$$gcl(A) \cap (X \setminus U)$$

is a non-empty $(gg)^*$ -closed set and is contained in $gcl(A) \setminus A$, which contradicts the assumption. Therefore,

$$gcl(A) \subseteq U,$$

and hence A is $(gg)^*$ - g -closed in X . \square

Lemma 5.4. : *A subset A of X is $(gg)^*$ - g -closed in X . Then $gcl(A) \setminus A$ does not contain any non-empty regular closed set in X .*

Proof. : Suppose that A is a $(gg)^*$ - g -closed set in X . To prove: $gcl(A) \setminus A$ does not contain any non-empty regular closed set in X . The proof follows from the known fact that every regular closed set is $(gg)^*$ -closed (see [9]), and by theorem 6, for a $(gg)^*$ - g -closed set A , $gcl(A) \setminus A$ does not contain any non-empty $(gg)^*$ -closed set in X . Therefore, $gcl(A) \setminus A$ cannot contain any non-empty regular closed set in X . \square

Theorem 5.5. : *Let $A \subseteq X$. Then A is g -closed if and only if $gcl(A) \setminus A$ is $(gg)^*$ -closed and $(gg)^*$ - g -closed in X .*

Proof. : Suppose A is a g -closed set in X . Then $gcl(A) = A$, which implies $gcl(A) \setminus A = \emptyset$. Thus, $gcl(A) \setminus A$ is a $(gg)^*$ -closed set in X [theorem 6].

Conversely, suppose A is $(gg)^*$ - g -closed in X and $gcl(A) \setminus A$ is a $(gg)^*$ -closed set in X . By theorem 6, this implies $gcl(A) \setminus A = \emptyset$, then $gcl(A) = A$. Therefore, A is g -closed in X . \square

Theorem 5.6. : *A topological space (X, τ) , for every $x \in X$, the set $X \setminus \{x\}$ is either $(gg)^*$ - g -closed or $(gg)^*$ -open set in X .*

Proof. : Let $x \in X$. Suppose that $X \setminus \{x\}$ is not $(gg)^*$ -open in X . Then X is the only $(gg)^*$ -open set containing $X \setminus \{x\}$. That implies $X \setminus \{x\} \subseteq X$, and $gcl(X \setminus \{x\}) \subseteq gcl(x) \subseteq X$, which implies $gcl(X \setminus \{x\}) \subseteq X$. Hence, $X \setminus \{x\}$ is $(gg)^*$ - g -closed. \square

Theorem 5.7. : *Let X and Y be two topological spaces and $A \subseteq Y \subseteq X$. If A is $(gg)^*$ - g -closed in X , then A is $(gg)^*$ - g -closed relative to Y .*

Proof. : Let U be a $(gg)^*$ -open set in X and suppose $A \subseteq Y \cap U$. Since A is $(gg)^*$ - g -closed in X , we have $A \subseteq U$ and hence $gcl(A) \subseteq U$. This implies $Y \cap gcl(A) \subseteq Y \cap U$, where $Y \cap gcl(A)$ is the g -closure of A in Y and $Y \cap U$ is open in Y . Hence A is $(gg)^*$ - g -closed relative to Y . \square

Theorem 5.8. : *A subset $A \subseteq X$ is $(gg)^*$ - g -closed if and only if $gcl(A) \cap A^c$ contain any non-empty closed set in X .*

Proof. : Let A be a $(gg)^*$ - g -closed subset of X . Also, if possible, let R be a non-empty closed subset of X , such that $R \subseteq gcl(A) \cap A^c$ that implies $R \subseteq gcl(A)$ and $R \subseteq A^c$. Since R is a closed subset of X , R^c is an open subset of X , and $R^c \supseteq A$. Since every open set is $(gg)^*$ -open [9] and since A is $(gg)^*$ - g -closed, $gcl(A) \subseteq R$. But $R \subseteq gcl(A)$, so we get a contradiction, which leads to the conclusion that $R = \emptyset$. So $gcl(A) \cap A^c$ contain any non-empty closed set in X . Conversely, let S be an open set in X such that $A \subseteq S$. Then $S^c \subseteq A^c$, and S^c is a closed subset of X . Since every open set is $(gg)^*$ -open [9], S is a $(gg)^*$ -open set. Suppose, if possible, $gcl(A) \not\subseteq S$. Now $S^c \subseteq A^c$ implies $gcl(A) \cap S^c \subseteq gcl(A) \cap A^c$. Then $gcl(A) \cap S^c$ is a non-empty closed subset of $gcl(A) \cap A^c$, which is a contradiction. Therefore, $gcl(A) \subseteq S$. Hence A is $(gg)^*$ - g -closed in X . \square

Theorem 5.9. : *If A is a $(gg)^*$ -open subset of X and $(gg)^*$ - g -closed in X , then A is g -closed in X .*

Proof. : Let A be $(gg)^*$ -open in X and $(gg)^*$ - g -closed, $gcl(A) \subseteq A$, but we know that $A \subseteq gcl(A)$ therefore $A = gcl(A)$. Hence A is g -closed in X . □

Theorem 5.10. : *If A is regular open and rg -closed in (X, τ) , then A is $(gg)^*$ - g -closed in (X, τ) .*

Proof. : Let A be a regular open and rg -closed set in X . Let U be any $(gg)^*$ -open set in X such that $A \subseteq U$. Since A is regular open and rg -closed, we have $cl(A) \subseteq A \subseteq U$. But $gcl(A) \subseteq cl(A) \subseteq A$ [20], therefore $gcl(A) \subseteq A \subseteq U$, Thus $gcl(A) \subseteq U$ Hence, A is $(gg)^*$ - g -closed in X . □

Theorem 5.11. : *If A is open and gr -closed in (X, τ) , then A is $(gg)^*$ - g -closed in (X, τ) .*

Proof. : Let A be an open and gr -closed set in X . Let U be any $(gg)^*$ -open set in X such that $A \subseteq U$. Since A is open and gr -closed, $rcl(A) \subseteq A \subseteq U$. But $gcl(A) \subseteq rcl(A) \subseteq A$ [19], therefore $gcl(A) \subseteq A \subseteq U$, Thus $gcl(A) \subseteq U$ hence A is $(gg)^*$ - g -closed in X . □

Theorem 5.12. : *If A is open, rw -closed, and regular semi-open in (X, τ) , then A is $(gg)^*$ - g -closed in (X, τ) .*

Proof. : Let A be regular semi-open and rw -closed in X . Let U be any $(gg)^*$ -open set in X such that $A \subseteq U$. Since A is regular semi-open and rw -closed [51], $cl(A) \subseteq A \subseteq U$. Also, $gcl(A) \subseteq cl(A) \subseteq A$ [20]. Therefore $gcl(A) \subseteq A \subseteq U$, Thus $gcl(A) \subseteq U$ hence A is $(gg)^*$ - g -closed in X . □

Theorem 5.13. : *If A is regular semi-open and gg -closed in (X, τ) , then A is $(gg)^*$ - g -closed in (X, τ) .*

Proof. : Let A be regular semi-open and gg -closed in X . Let U be any $(gg)^*$ -open set in X such that $A \subseteq U$. Since A is regular semi-open and gg -closed [3], $gcl(A) \subseteq A \subseteq U$. Therefore $gcl(A) \subseteq U$, hence A is $(gg)^*$ - g -closed. □

Theorem 5.14. : *Let (X, τ) be an extremely disconnected space. If A is open and g -closed in (X, τ) , then A is $(gg)^*$ - g -closed in (X, τ) .*

Proof. : Let A be open and g -closed in (X, τ) . Let U be any $(gg)^*$ -open set in X such that $A \subseteq U$. Since X is extremely disconnected, $cl(A) = A$ [10]. Also, since A is open and g -closed, $gcl(A) = A$ [3]. We know that $gcl(A) \subseteq cl(A) = A \subseteq U$ [20]. Thus $gcl(A) \subseteq U$, and hence A is $(gg)^*$ - g -closed in (X, τ) . □

Theorem 5.15. : *If A is regular open and gpr -closed in (X, τ) , then A is $(gg)^*$ - g -closed in (X, τ) .*

Proof. : Let A be regular open and gpr -closed in (X, τ) . Let U be any $(gg)^*$ -open set in X such that $A \subseteq U$. Since A is regular open and gpr -closed, $pcl(A) \subseteq U$. Also, $gcl(A) \subseteq pcl(A) \subseteq A$ [19]. therefore $gcl(A) \subseteq A \subseteq U$, Thus $gcl(A) \subseteq U$ hence A is $(gg)^*$ - g -closed in X . □

Theorem 5.16. : *Let $X \subseteq Y \subseteq Z$, where Y is $(gg)^*$ -open and $(gg)^*$ - g -closed in Z . If X is $(gg)^*$ - g -closed in Y , then X is $(gg)^*$ - g -closed in Z .*

Proof. : Let U be a $(gg)^*$ -open set in Z such that $X \subseteq U$. Then $X \subseteq U \cap Y$, where $U \cap Y$ is a $(gg)^*$ -open set in Y .

Since X is $(gg)^*$ - g -closed in Y ,

$$gcl(X) \subseteq U \cap Y.$$

That is,

$$gcl(X) \cap Y \subseteq U \cap Y.$$

Since $X \subseteq Y$, we have:

$$gcl(X) \subseteq gcl(Y).$$

Since Y is both $(gg)^*$ -open and $(gg)^*$ - g -closed in Z . Then Y is g -closed [by theorem 11] Therefore,

$$gcl(Y) = Y.$$

Thus,

$$gcl(X) \subseteq Y \Rightarrow gcl(X) = gcl(X) \cap Y.$$

Hence,

$$gcl(X) = gcl(X) \cap Y \subseteq U \cap Y \subseteq U.$$

Therefore, $gcl(X) \subseteq U$, and so X is $(gg)^*$ - g -closed in Z . \square

Definition 4. : [9] Let (X, τ) be a topological space and $A \subseteq X$. Then

$$(gg)^* - ker(A) = \bigcap \{G \in (gg)^* - O(X) \mid A \subseteq G\}.$$

Theorem 5.17. : A subset A of a topological space (X, τ) is $(gg)^*$ - g -closed if and only if $gcl(A) \subseteq (gg)^* - ker(A)$.

Proof. : **Necessity:** Assume that A is a $(gg)^*$ - g -closed set in X . Suppose $x \in gcl(A)$ but $x \notin (gg)^* - ker(A)$. Then there exists a $(gg)^*$ -open set U such that $A \subseteq U$ and $x \notin U$. Since A is $(gg)^*$ - g -closed and U is a $(gg)^*$ -open set containing A , we have $gcl(A) \subseteq U$ (by definition 1). Therefore, $x \notin gcl(A)$, which contradicts our assumption that $x \in gcl(A)$. Hence, $x \in (gg)^* - ker(A)$. This shows that $gcl(A) \subseteq (gg)^* - ker(A)$.

Sufficiency: Let $gcl(A) \subseteq (gg)^* - ker(A)$ and $A \subseteq U$, where U is any $(gg)^*$ -open set. Since $(gg)^* - ker(A) \subseteq U$, it follows that $gcl(A) \subseteq U$. Hence, A is a $(gg)^*$ - g -closed set in X . \square

6. Conclusion

In this paper, we deliberated about $(gg)^*$ - g -closed sets in topological spaces acquired from $(gg)^*$ -open sets and g -closure. In the future, we can continue to a more advanced level with nano topological spaces and other topological spaces.

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