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On the Carleman Formula for an Unbounded Matrix Domain Associated with the Classical Domain of the Second Type*

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ABSTRACT: In the paper, proves a new representation of Carleman's formula for the class of analytic functions defined on the cartesian product of Matrix upper half-planes and defined on the Shilov boundary. As an object of research high-dimensional matrix domains formed by the structural transformation of Zigel domains are selected.

Key Words: Carleman formula, matrix upper half-plane, holomorphic function, matrix unit disc, Cauchy kernel, classical domain of the second type, automorphism, Cauchy-Szego's kernel, Cartesian's product.

Contents

1. Introduction

Integral representation formulas play a crucial role in complex analysis, providing a means to recover a holomorphic function inside a domain from its boundary values. The classical Cauchy integral formula is one example, and T. Carleman was the first to extend this idea to a more general setting [1]. Carleman introduced an integral formula that reconstructs a holomorphic function in a domain $D \subset \mathbb{C}$ from its values on a subset M of the boundary ∂D (with M not containing the entire boundary). This formula involved a so-called quenching function in the integrand to allow analytic continuation from partial boundary data. Building on Carleman's idea, Goluzin and Krylov developed a generalized Carleman formula for simply connected planar domains [2]. Their method constructed an auxiliary holomorphic function depending on the subset M, which succeeded in the simply connected case. However, they noted that this approach does not directly extend to multiply connected domains in $\mathbb{C}$ or to several complex variables. These early developments established the foundation for Carleman-type formulas as powerful tools for analytic continuation and boundary problems in complex function theory.

The challenge of formulating Carleman-type integral representations in higher dimensions and more complex domains was taken up later in the 20th century. Elie Cartan's classification of homogeneous bounded symmetric domains (Cartan's six types, of which four are classical Cartan domains I−IV) provided a natural framework for multivariable complex analysis [3].

In particular, Cartan domain $\Re_2$ (the classical domain of the second type) is the unit ball in the space of complex symmetric matrices, also known as the matrix unit ball. For such bounded symmetric domains, analogues of Carleman's formula were eventually obtained. Notably, Kytmanov and Nikitina derived multidimensional Carleman formulas adapted to classical domains and Siegel domains [4,5].

During the 1990s, other researchers also contributed to this area. For instance, Kosbergenov studied a Carleman formula for a matrix ball (a Cartan type I domain), and obtained an integral formula for that case [6]. Additionally, integral formulas were connected with function space theory: Koosis's work on H^p spaces and Aizenberg's monograph on Carleman formulas provided the functional-analytic underpinnings

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that support these integral representations [7,8]. L.A. Aizenberg's book - Carleman Formulas in Complex Analysis - was an influential resource summarizing known formulas and techniques [9].

A matrix domain refers to a domain in the space $\mathbb{C}^n[m \times m]$ (or in a Cartesian product of such spaces) that often corresponds to a classical Cartan domain under some biholomorphic mapping. One key line of inquiry has been extending integral formulas to domains associated with Siegel domains and Cartan's classical domains of types $\mathrm{I} - \mathrm{IV}$. This includes not only the bounded matrix balls (types I, II, III) but also their unbounded counterparts (Siegel domains of the first kind, generalized half-spaces).

Researchers from Central Asia and Siberia have been especially active in this area. G. Khudayberganov and colleagues have produced a series of results for different matrix domains. They obtained integral formulas for the matrix ball of the second type (complex symmetric matrices, bounded domain), and later studied the third type (skew-symmetric matrices) and first type domains. For example, Khudayberganov and Matyakubov derived integral formulas in certain Siegel domains (often referred to as -Ziegel domains- in their works) and demonstrated applications of these formulas [10]. In a subsequent paper, Khudayberganov and Matyakubov established a boundary Morera theorem for the matrix ball of the second type, which relates to integral representations as well [11].

In [12], we reformulate the problem under consideration: Under what conditions can we extend holomorphically to a matrix ball the functions given on a part of its skeleton? We describe the domains into which the integral of the Bochner-Hua Luogeng type for a matrix ball can be extended holomorphically. As the main result, we present the criterion of holomorphic continuation into a matrix ball of functions defined on a part of the skeleton of this matrix ball. The proofs of several results are briefly presented. Some recent advances are highlighted. The results obtained in this article generalize the results of L. A. Aizenberg, A. M. Kytmanov and G. Khudayberganov. In [13] paper, a connection has been established between the Bergman and Cauchy-Szegő kernels using the biholomorphic equivalence of the future tube and the Lie ball. Moreover, integral representations of holomorphic functions in these domains are obtaine. By 2016, Khudayberganov, Rakhmonov, and Matyakubov presented integral formulas covering several matrix domains in one unified approach [14].

Another significant development was the extension of Carleman formulas to Cartesian products of domains. Shoimkhulov and Bozorov, for instance, proved a Carleman formula for a matrix polydisc [15]. A matrix polydisc is a bounded domain, and their result generalized Carleman's formula to this multi-component setting. However, the matrix upper half-plane - an unbounded domain analogous to the classical upper half-plane but in the matrix setting - remained a harder case until recently. The matrix upper half-plane can be seen as a Siegel domain of the first kind for $m \times m$ matrices, and it is biholomorphically equivalent to the matrix ball of second type via a suitable Cayley transform. In 2018, Khudayberganov and Matyakubov tackled this case: they published Carleman's formula for the matrix upper half-planes, giving an integral representation for holomorphic functions on a single matrix upper half-plane [16].

Very recently, the focus has shifted to combining multiple such domains. In 2022, Rakhmonov and Matyakubov found a Carleman formula for matrix Siegel domains of the first type [17]. Their work considered a Siegel domain and successfully derived an integral formula valid in that setting, overcoming the earlier difficulties by new techniques. Around the same time, related advances were made for other classical domains: for example, Kurbanov proved a boundary Morera theorem in a Siegel domain of the first kind (tube domain), further enriching the toolkit of integral representations in non-symmetric domains [18,19,20]. In this paper [21], an analogue of Bremermann's theorem on finding the Bergman kernel is obtained for the Cartesian product of classical domains. For this purpose, the groups of automorphisms of the considered domains are used, i.e., the Bergman kernels are constructed for the Cartesian product of classical domains, without applying complete orthonormal system

The culmination of these efforts is the ability to handle multiple matrix variables simultaneously. In a Cartesian product of matrix upper half-planes, one must deal with several unbounded matrix variables, which poses technical challenges like handling multi-dimensional boundary sets and product measures. This problem has now been addressed: Abdullayev, Ruzmetov, and Matyakubov have proved a Carleman integral formula for the Cartesian product of n matrix upper half-planes [22]. This paper considers Carleman's integral formula for a function of matrices in the space $\mathbb{C}^n[m \times m]$. The resulting formula is a general case of G. Khudayberganov's result.

In this paper, we prove a Carleman-type formula for the Cartesian product of matrix upper half-planes in the space $\mathbb{C}^n\,[m\times m]$.

2. Formula Carleman for Cartesian product of classical domains of the second type

We consider the complex space $\mathbb{C}^{m^2}$ with multiple variable. In certain problems, it is convenient to represent a point Z in this space as a matrix $Z=(z_{ij})_{i,j=1}^{m}$, i.e., as a square matrix of size $[m\times m]$. The space $\mathbb{C}^{m^2}$ is thus denoted by $\mathbb{C}\,[m\times m]$.

According to Cartan's classification, the classical domain of the second type $\Re_2$ is defined as follows:

$$\Re_2=\left\{Z\in\mathbb{C}\,[m\times m]: I-Z\bar{Z}>0, Z'=Z\right\}$$

where I stands for the identity matrix of size $[m\times m]$.

We denote the skeleton (i.e., the Shilov boundary) of this domain by $S(\Re_2)$, that is

$$S(\Re_2)=\left\{Z\in\mathbb{C}\,[m\times m]: I-Z\bar{Z}=0, Z'=Z\right\}.$$

We define $\mathbb{C}^n\,[m\times m]$ as the Cartesian product of n copies of the space $\mathbb{C}\,[m\times m]$.

Using the Cartesian product of classical domains of the second type, we define the matrix polydisk in the space $\mathbb{C}^n\,[m\times m]$ in the following way [15]:

$$T_n=\underbrace{\Re_2\times...\times\Re_2}_{n}=\left\{Z=(Z_1,Z_2,...,Z_2): Z_j\in\Re_2, (Z_j)^{'}=Z_j, j=1,2,...,n\right\}.$$

The skeleton of this domain is defined as follows:

$$S(T_n)=\underbrace{S(\Re_2)\times...\times S(\Re_2)}_{n}$$

Consider the automorphism of the matrix unit polydisc T_n that maps a point $A\in T_n$ to the origin [25]

$$\Phi_A(Z)=\left(\Phi_A^1(Z^1),...,\Phi_A^n(Z^n)\right),$$

here

$$\Phi_A^j(Z_j)=R_j(Z_j-A_j)(I-\bar{A}_jZ_j)^{-1}R_j^{-1},$$

R_j is a matrix of size $[m\times m]$ satisfying the following condition:

$$\bar{R}_j(I-\bar{A}_jA_j')R'=I.$$

In particular, if $A=0$, then

$$\Phi_0(Z)=(\Phi_0^1(Z^1),...,\Phi_0^n(Z^n))$$

and

$$\Phi_0^j(Z^j)=R^jZ^j(\bar{R}^j)^{-1}.$$

Let $E=\left(E^1,..,E^n\right)\subset S(T_n)$ be a positive Lebesgue-measurable set with $\mu E>0$. We denote by $S(T_{n-1})$ the projection of the skeleton of the matrix unit polydisc $S(T_n)$ onto the space $\mathbb{C}^{n-1}\,[m\times m]$. Points of $S(T_{n-1})$ are denoted by $'\xi=\left(\xi^2,...,\xi^n\right)$.

Let us now consider the following sets:

$$E_{0,\xi}=\left\{Z: Z\in E, \Phi_0^1(Z^1)=\theta, \Phi_0^j(Z^j)=\theta\;\Phi_0^j(\xi^j), j=\overline{2,n}, \theta\in S(\Re_2)\right\}, \tag{2.1}$$

$$\tilde{E}_0=\left\{Z\in E: \mu_1E_{0,\xi}>0\right\}.$$

The sets $\tilde{E}_0$ and $E_{0,\xi}$ are subsets of a measurable set E of positive measure, such that their Cartesian product coincides with E, that is, $E=E_{0,\xi}\times\tilde{E}_0$.

We now define the auxiliary function as follows $\varphi_0 = \exp \psi_0$, where

$$\psi_0(\xi) = \frac{1}{2\pi i}\int_{E^1_{0,\xi^1}} \frac{\eta+\lambda}{\eta-\lambda}\frac{d\eta}{\eta}.$$

The set E^1_{0,ξ^1} is defined as in Lemma 24.6 of the monograph [9], and its Cartesian product with the set $\tilde{E}_{0,\xi}$ yields the set $E_{0,\xi}$, that is, $E^1_{0,\xi^1} \times \tilde{E}_{0,\xi} = E_{0,\xi}$

Let G be a bounded convex domain that is strictly circular and has a smooth Shilov boundary. The Hardy class $H^p(G)$ $(p > 0)$ is defined for all functions F that are holomorphic in G as follows:

$$\sup_{0<r<1} \int_S |F(r\xi)|^p \, d\mu < +\infty,$$

where $r\xi = (r\xi_{11}, r\xi_{12}, ..., r\xi_{mm})$, and $d\mu$ denotes the normalized Lebesgue measure on S, which is invariant under unitary transformations (rotations) in the complex space.

Lemma 1. *If* $f \in H^1(T_n), E \subset S(T_n), \mu(E) > 0$, then the following formula

$$f(0) = \frac{m}{\int_{\tilde{E}_0} d\mu_{n-1} \int_{\tilde{E}_{0,\xi}} d\mu_0} \lim_{l\to\infty} \int_E f(Z) \left[\frac{\varphi_0(\xi)}{\varphi_0(0)}\right]^l d\mu,, \tag{2.2}$$

holds.

We denote $'\xi = (\xi_2, \xi_3, ..., \xi_n) \in S(T_{n-1})$, and now we define the following set:

$$E_{A,\xi} = \left\{Z : Z \in E, \Phi^1_A(Z^1) = \theta, \Phi^j_A(\xi^j) = \theta \; \Phi^j_A(\xi^j), j = \overline{2,n}, \theta \in S(\Re_2)\right\},$$

This is a measurable set defined for almost all points A and 'ξ with respect to the measure μ_1. We define the set $\tilde{E}_A$ in the form of $\{'\xi : '\xi \in S(T_{n-1}), \mu_1 E_{A,\xi} > 0\}$, where $E_{A,\xi} \times \tilde{E}_A = E$. According to the Fubini theorem [26], the Lebesque measure of the $\left(\frac{(m+1)(n-1)}{2}\right)$-dimensional set is positive.

As above, we now introduce the following auxiliary function:

$$\varphi_A = \exp \psi_A, \;\; \psi_A(\xi) = \frac{1}{2\pi i}\int_{E^1_{A^1,w^1}} \frac{\eta+\lambda}{\eta-\lambda}\frac{d\eta}{\eta},$$

where

$$E_{A^1,w^1} = \left\{\xi^1 \in E^1, \xi = \left(\Phi^1_A\right)^{-1}\left(\lambda \left(\Phi^1_A\right)^{-1}(w)\right), |\lambda| = 1\right\}, \; W \in \Phi^1_A(SU(m)),$$

$SU(m)$ group of special unitary matrices.

Theorem 1. *If* $f \in H^1(T_n), E \subset S(T_n), \mu(E) > 0$, *then for all* $A \in T_n$, *the following Carleman formula holds:*

$$f(A) = \frac{m}{\mu_{\frac{(m+1)(n-1)}{2}}\left(\Phi^{-1}_A\left(\tilde{E}_A\right)\right)\mu\left(\Phi^{-1}_A\left(\tilde{E}_{A,\xi}\right)\right)} \times$$
$$\times \lim_{l\to\infty} \int_E f(Z)\left[\frac{\varphi_A(Z)}{\varphi_A(A)}\right]^l \prod_{j=1}^n H(A^j, \bar{Z}^j) d\mu(Z),, \tag{2.3}$$

where

$$H(A^j, \bar{Z}^j) = \frac{1}{\det\left(I^{(m)} - A^j \bar{Z}^j\right)^{\frac{m+1}{2}}}$$

is the Cauchy kernel for the classical domain of the second type.

Consider the following unbounded domain:

$$D = D_1 \times D_2 \times ... \times D_n =$$
$$= \left\{W = (W_1, W_2, ..., W_n) \in \mathbb{C}^n [m \times m] : \; \text{Im}\, W_j > 0, \; W_j^{'} = W_j, j = \overline{1,n}\right\}$$

We denote the skeleton Γ of this domain in the following way:

$$\Gamma = \Gamma_1 \times \Gamma_2 \times ... \times \Gamma_n =$$
$$= \left\{V = (V_1, V_2, ..., V_n) \in \mathbb{C}^n [m \times m] : \; ImV_j = 0, \; V_j^{'} = V_j, j = \overline{1,n}\right\}$$

3. Carleman Formula for an Unbounded Matrix Domain Associated with the Classical Domain of the Second Type

We denote by $\Phi = \left(\Phi^1, \Phi^2, ..., \Phi^n\right)$ the Cayley transform that maps the matrix polydisc $S(T_n)$ biholomorphically onto the Cartesian product D of matrix upper half-planes and maps the skeleton of $S(T_n)$ onto the skeleton Γ. Here,

$$W_j = \Phi_j(Z) = i(I + Z_j)(I - Z_j)^{-1}, j = \overline{1, n} \tag{3.1}$$

$\dot{U}$ is the volume element on the skeleton of $S(T_n)$, and $\dot{V}$ is the volume element on the skeleton Γ. It is known that

$$\dot{U} = \prod_{j=1}^{n} \dot{U}_j, \dot{V} = \prod_{j=1}^{n} \dot{V}_j,$$

here $\dot{U}_j \in \{U_j(U_j)^* = I\}$, $\dot{V}_j \in \{ImV_j = 0\}$. The relation between $\dot{U}_j$ and $\dot{V}_j$ is given as follows [25]:

$$\dot{U}_j = 2^{\frac{m(m+1)}{2}} \left|\det(V_j + iI)\right|^{-(m+1)} \dot{V}_j.$$

Lemma 2 *[24]. The following relation*

$$\dot{U} = \dot{U}_1 \wedge ... \wedge \dot{U}_n = 2^{\frac{nm(m+1)}{2}} \prod_{j=1}^{n} \left|\det(V_j + iI)\right|^{-(m+1)} \dot{V}.$$

Let $f \in \mathcal{O}(D)$

$$f(i(I + Z_j)(I - Z_j)^{-1}) \in H^1(T_n), j = \overline{1, n}$$

holds if and only if the following condition

$$\frac{f(W_j)}{\det^2(W_j + iI)} \in H^1(D_n), j = \overline{1, n} \tag{3.2}$$

is satisfied [7,17].

Theorem 2. *If the function $f \in \mathcal{O}(D)$ satisfies condition (3.2), and $\tilde{E} = \tilde{E}_1 \times ... \times \tilde{E}_n \subset \Gamma$ is a set of positive Lebesgue measure, then the Carleman formula holds, uniformly converging on compact subsets of the skeleton:*

$$f(W) = \prod_{j=1}^{n} \frac{\det^{\frac{m+1}{2}}(W_j + iI)}{i^{\frac{nm(m+1)}{2}}} \cdot \lim_{l\to\infty} \int_{\tilde{E}} f(V) \left[\frac{\tilde{\varphi}(V)}{\tilde{\varphi}(W)}\right]^l \times,$$

$$\times \prod_{j=1}^{n} \det^{-\frac{m+1}{2}}\left(\overline{V}_j - W_j\right) \det^{-\frac{m+1}{2}}(V_j + iI)\, d\mu V_j \tag{3.3}$$

where $V_j \in \tilde{E}_j$, $V \in \tilde{E}$

Proof: Consider the following mapping:

$$F(Z_j) = f\left(i(I + Z_j)(I - Z_j)^{-1}\right).$$

If $F(Z) \in H^1(T_n)$, then according to Theorem 1, the Carleman formula takes the following form:

$$F(Z) = \lim_{l\to\infty} \int_M F(U) \left[\frac{\varphi(U)}{\varphi(Z)}\right]^l \prod_{j=1}^{n} \frac{d\mu_U}{\det^{\frac{(m+1)}{2}}(I - Z_j\bar{U}_j)},$$

here the set $E = E_1 \times E_2 \times .. \times E_n \subset S(T_n)$ is the image of the set $\tilde{E} = \tilde{E}_1 \times \tilde{E}_2 \times .. \times \tilde{E}_n \subset S(T_n)$ under the inverse of the mapping $Z_j = (W_j + iI)(W_j - iI)^{-1}$, which maps the matrix upper half-plane onto the matrix unit polydisc.

Consider now the inverse of the mapping (3.1) given above

$$Z_j = (W_j + iI)(W_j - iI)^{-1}, U_j = (V_j + iI)^{-1}(V_j - iI),$$

and carry out the following computations:

$$I - Z_j\bar{U}_j = I - Z_jU_j^* = I - (W_j + iI)^{-1}(W_j - iI)\left(\bar{V}_j + iI\right)\left(\bar{V}_j - iI\right)^{-1} =$$

$$(W_j + iI)^{-1}\left[(W_j + iI)\left(\bar{V}_j - iI\right) - (W_j - iI)\left(\bar{V}_j + iI\right)\right]\left(\bar{V}_j - iI\right)^{-1} =$$

$$(W_j + iI)^{-1}\left[W_j\bar{V}_j - iW_j + i\bar{V}_j + I - W_j\bar{V}_j - iW_j + i\bar{V}_j - I\right]\left(\bar{V}_j - iI\right)^{-1} =$$

$$2i(W_j + iI)^{-1}\left[\bar{V}_j - W_j\right]\left(\bar{V}_j - iI\right)^{-1}.$$

According to Lemma 2 above, the following equality holds:

$$d\mu_U = 2^{\frac{nm(m+1)}{2}}\prod_{j=1}^{n}|\det(V_j + iI)|^{-(m+1)}d\mu_V$$

As a result of the computations, we obtain the following:

$$\prod_{j=1}^{n}\frac{d\mu_U}{\det^{\frac{(m+1)}{2}}(I - Z_jU_j^*)} =$$

$$\frac{1}{(2i)^{\frac{nm(m+1)}{2}}\prod_{j=1}^{n}\det^{\frac{-(m+1)}{2}}(W_j + iI)\det^{\frac{m+1}{2}}\left(\bar{V}_j - W_j\right)\det^{\frac{-(m+1)}{2}}\det^{\frac{-(m+1)}{2}}\left(\bar{V}_j - iI\right)}\times$$

$$\times 2^{\frac{nm(m+1)}{2}}\prod_{j=1}^{n}\det|V_j + iI|^{-(m+1)}d\mu_V = \frac{\prod_{j=1}^{n}\det^{\frac{-(m+1)}{2}}(W_j + iI)\prod_{j=1}^{n}\det^{\frac{m+1}{2}}\left(\bar{V}_j - iI\right)}{(2i)^{\frac{nm(m+1)}{2}}}\times$$

$$\times\frac{2^{\frac{nm(m+1)}{2}}d\mu_V}{\prod_{j=1}^{n}\det^{\frac{m+1}{2}}(\bar{V}_j - iI)\det^{\frac{m+1}{2}}(\bar{V}_j + iI)} = \prod_{j=1}^{n}\frac{\det^{\frac{m+1}{2}}(W_j + iI)}{i^{\frac{nm(m+1)}{2}}\det^{\frac{m+1}{2}}\left(\bar{V}_j - W_j\right)\det^{\frac{m+1}{2}}\left(\bar{V}_j + iI\right)}d\mu_V.$$

In this context, the function $\tilde{\varphi}$ takes on the role of the function φ on the set $E = \left(E^1, .., E^n\right) \subset S(T_n)$. According to the theorem of M.A. Lavrentiev [28], the set $E = \left(E^1, .., E^n\right) \subset S(T_n)$ has positive Lebesgue measure. The harmonic measure of the set $E = \left(E^1, .., E^n\right) \subset S(T_n)$ is mapped onto that of the set $E = \left(E^1, .., E^n\right) \subset S(T_n)$, and the function φ transforms into $\tilde{\varphi}$. As a result, we arrive at formula (3.3).

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