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Degree-Based Topological Indices of Order Super Commuting Graphs of Finite Groups

Jayavel Pooja, Venkatesan Muthukumaran*, Suresh Elumalai, Selvaraj Balachandran

ABSTRACT: Consider D to be an equivalence relation on a group $\mathscr{H}$ and let Ψ be a graph type. Given the equivalence relation D, let $[y]$ denote the equivalence class of an element y. The vertex set $V(\Psi)$ of the D-super Ψ graph of $\mathscr{H}$ represents an undirected graph such that two distinct vertices $y, z \in \Psi$ are adjacent if $[y] = [z]$, or if there exist elements $y' \in [y]$ and $z' \in [z]$ such that y' and z' are adjacent in the Ψ graph of $\mathscr{H}$. In this study, we compute the degree-based topological indices of order super commuting graphs corresponding to various finite non-abelian groups, including dihedral, generalized quaternion, semidihedral, quasidihedral, $V_{8\vartheta}$, and $U_{6\vartheta}$ groups. This work lies in the systematic derivation of closed-form expressions for these indices over new classes of order super commuting graphs. The obtained results provide a deeper understanding of the structural properties and interrelations among these finite non-abelian groups through their topological descriptors.

Keywords: Chemical graphs, super commuting graphs, topological indices, finite groups, molecular structure.

Contents

1. Introduction

The properties of graphs have been exhaustively described on finite groups, along with their properties have been explored using group properties and vice versa. Additionally, finite groups are described if specific graph theoretic properties are attained. These graphs frequently produce instances and alternative examples for a range of graph theory conjectures and issues. Among the well-known graphs defined on groups are the generating graph, power graph, solvable graph, commuting graph, non-commuting graph, nilpotent graph, etc. Another class of graphs is formed on groups. Commuting, nilpotent, and solvable conjugacy class graphs are examples of such graphs.In 2022, Arunkumar et al. [4] introduced the concept of super graphs of groups by integrating two existing types of graphs defined on groups. The Laplacian Spectrum and Sombor Spectrum of super graphs for certain groups were further investigated in [[14], [18]].

Let D be an equivalence relation defined on a finite group $\mathscr{H}$. For any $y \in \mathscr{H}$, its D-equivalence class is denoted by $[y]$. If $[y] = [z]$ or if there exist $y' \in [y]$ and $z' \in [z]$ such that a commutes with b, then the K-super commuting graph on $\mathscr{H}$ is defined as a graph whose vertex set consists of such equivalence classes, and two distinct vertices y' and z' are adjacent. When $[y] = [z]$, the resulting graph is the K-commuting graph on $\mathscr{H}$, which has been discussed in [[9], [10], [17], [7]].

The order super commuting graph, denoted by $OSCom(\mathscr{H})$, is obtained by taking K as the relation of having the same order among elements of $\mathscr{H}$. Various forms of such graphs on finite groups were presented

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* Corresponding author.

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in [4], and their structural properties were analyzed in [[4], [5]]. Furthermore, several topological indices of super commuting graphs for specific groups were computed in [3], while their spectral properties were explored in [[2], [6]].

Compared to earlier studies on commuting graphs [[9], [10], [17], [7]], the investigations on super commuting graphs [[4], [3], [2], [6]] offer a broader framework that generalizes traditional commuting relations by incorporating equivalence-based partitions of group elements. This extension provides deeper insights into both the algebraic and spectral characteristics of group-based graphs. In this paper, we compute Distance-based topological indices of order super commuting graphs of finite non-abelian groups such as dihedral groups, dicyclic groups, semidihedral groups, quasidihedral groups, $V_{8\vartheta}$ groups, and $U_{6\vartheta}$ groups.

2. Preliminaries

Assume that $\Psi = (V(\Psi), E(\Psi))$ is a simple undirected graph. $|\Psi|$, the number of vertices, indicates the order of Ψ. The shortest path between v_1 and v_2 in Ψ is represented by the distance $dis(v_1, v_2)$. Two vertices, v_1 and v_2, are represented as v_1v_2 if they share an edge. The degree is specified by d_{v_1} of vertex v_1. $N(v_1)$ is the set of vertices in a graph, adjacent to v_1. A path of length $\text{dis}(v_1, v_2)$ from v_1 to v_2 is called a geodesic. $\text{ecc}(v_1)$ represents the eccentricity of a vertex $v_1 \in V$, which is the greatest distance between v_1 and any other vertex in Ψ. $diam(\Psi)$ represents the diameter of Ψ, which is the highest eccentricity of any vertex. Likewise, the vertex with the least eccentricity in Ψ corresponds to the radius of Ψ, which is represented by $\text{rad}(\Psi)$. If a graph's diameter and radius are the same, it is said to be self-centered.

The disjoint union of x copies of K_ϑ is denoted as xK_ϑ. $\vee$ is defined as the join of two graphs, $\cup$ is defined as union of two disjoint sets or graphs.

Let $\Psi_1(V(\Psi_1), E(\Psi_1))$ and $\Psi_2(V(\Psi_2), E(\Psi_2))$ be two connected graphs. $\Psi_1 \vee \Psi_2$ is their join graph, which is a graph with the vertex set $V(\Psi_1) \cup V(\Psi_2)$ and edge set

$$E(\Psi_1) \cup E(\Psi_2) \cup \Big\{yz : y \in V(\Psi_1), z \in V(\Psi_2)\Big\}.$$

Indices	Representations	Formulas
Randić index [12]	$R(\Psi)$	$\sum_{yz} \in E(\Psi) \frac{1}{\sqrt{d_y d_z}}$
Sombor index [11]	$SO(\Psi)$	$\sum_{yz \in E(\Psi)} \sqrt{d_y^2 + d_z^2}$
Harmonic index [15]	$Hr(\Psi)$	$\sum_{yz \in E(\Psi)} \frac{2}{d_y + d_z}$
Geometric Arithmetic index [16]	$GA(\Psi)$	$\sum_{yz \in E(\Psi)} \frac{2\sqrt{d_y d_z}}{d_y + d_z}$
Atom Bond Connectivity index [13]	$ABC(\Psi)$	$\sum_{yz \in E(\Psi)} \sqrt{\frac{d_y + d_z - 2}{d_y d_z}}$
Atom Bond Sum Connectivity index [1]	$ABS(\Psi)$	$\sum_{yz \in E(\Psi)} \sqrt{\frac{d_y + d_z - 2}{d_y + d_z}}$
Sum Connectivity index [20]	$\chi(\Psi)$	$\sum_{yz \in E(\Psi)} \frac{1}{\sqrt{d_y + d_z}}$
Symmetric Division Deg index [19]	$SDD(\Psi)$	$\sum_{yz \in E(\Psi)} \left(\frac{d_y}{d_z} + \frac{d_z}{d_y}\right)$

Table 1: Various types of degree-based topological indices considered.

The order super commuting graph of a group $\mathscr{H}$, represented as $OSCom(\mathscr{H})$, is defined as a graph where the vertex set is $V(OSCom(\mathscr{H})) = \mathscr{H}$. Two distinct vertices y and z are connected by an edge if either $[y] = [z]$, or there exist elements $a \in [y]$ and $b \in [z]$ such that $ab = ba$. Here, $[y]$, known as the order class of y in $\mathscr{H}$, consists of all elements in $\mathscr{H}$ that share the same order as y.

In this paper, we examine six specific types of groups: the dihedral groups $D_{2\vartheta}$, defined by the presentation $\langle \rho, \sigma \mid \rho^\vartheta = \sigma^2 = 1, \sigma\rho\sigma^{-1} = \rho^{-1} \rangle$ for $\vartheta \geq 3$ having order 2ϑ; the quaternion groups $Q_{4\vartheta}$, characterized

by the presentation $\langle \rho, \sigma \mid \rho^{2\vartheta} = 1,\, \rho^{\vartheta} = \sigma^2,\, \sigma\rho\sigma^{-1} = \rho^{-1} \rangle$ for $\vartheta \geq 2$ having order 4ϑ; the quasidihedral groups QD_{2^ϑ}, defined as $\langle \rho, \sigma \mid \rho^{2^{\vartheta-1}} = \sigma^2 = 1,\, \sigma\rho\sigma^{-1} = \rho^{2^{\vartheta-2}-1} \rangle$ for $\vartheta \geq 3$ having order 2^ϑ; the semidihedral groups $SD_{8\vartheta}$, which are defined by the presentation $\langle \rho, \sigma \mid \rho^{4\vartheta} = 1,\, \sigma^2 = 1,\, \sigma\rho\sigma^{-1} = \rho^{2\vartheta-1} \rangle$ for $\vartheta \geq 2$ having order 8ϑ; the groups $V_{8\vartheta}$, presented as $\langle \rho, \sigma \mid \rho^{2\vartheta} = 1,\, \sigma^4 = 1,\, \sigma\rho = \sigma^{-1}\rho^{-1},\, \sigma^{-1}\rho = \rho^{-1}\sigma \rangle$ having order 8ϑ, and $U_{6\vartheta}$; defined by $\langle \rho, \sigma \mid \rho^{2\vartheta} = 1,\, \sigma^3 = 1,\, \rho^{-1}\sigma\rho = \sigma^{-1} \rangle$ having order 6ϑ. The structure for these groups have been described in the S.Das [8].
The work in this article is structured as follows: In Chapter 3 provides an in-depth analysis of vertex and edge partitions within the order super commuting graphs associated with finite groups. In Chapter 4 4 explores various properties of these graphs, delving into their structural and mathematical characteristics. Finally, In Chapter 5 5 examines degree-based topological indices of the order super commuting graph $OSCom(\mathscr{H})$, presenting key insights and their implications.

3. Vertex and Edge Partitions

In this section, we explore the relationships between distances, edges, and the degree sequence within order super commuting graphs.
Edge partition of $OSCom(D_{2\vartheta})$ of Dihedral groups $D_{2\vartheta}$ for every $yz \in E(OSCom(D_{2\vartheta}))$:

(d_y, d_z) type edges	Distance (Φ)	Number of edges
$(\vartheta-1, 2\vartheta-1)$	$\Phi = 1$	$\vartheta - 1$
$(2\vartheta-1, \vartheta)$	$\Phi = 1$	ϑ
$(\vartheta-1, \vartheta-1)$	$\Phi = 1$	$\frac{(\vartheta-1)(\vartheta-2)}{2}$
(ϑ, ϑ)	$\Phi = 1$	$\frac{(\vartheta-1)\vartheta}{2}$
$(\vartheta, \vartheta-1)$	$\Phi = 2$	$(\vartheta-1)\vartheta$

Table 2: Values of distance and number of edges when ϑ is odd

(d_y, d_z) type edges	Distance (Φ)	Number of edges
$(2\vartheta-1, 2\vartheta-1)$	$\Phi = 1$	$(2\vartheta-1)\vartheta$

Table 3: Values of distance and number of edges when ϑ is even

Edge partition of $OSCom(Q_{4\vartheta})$ of Generalized Quaternion groups $Q_{4\vartheta}$ for every $yz \in E(OSCom(Q_{4\vartheta}))$:

(d_y, d_z) type edges	Distance (Φ)	Number of edges
$(4\vartheta-1, 4\vartheta-1)$	$\Phi = 1$	1
$(4\vartheta-1, 2\vartheta+1)$	$\Phi = 1$	4ϑ
$(4\vartheta-1, 2\vartheta-1)$	$\Phi = 1$	$4(\vartheta-1)$
$(2\vartheta+1, 2\vartheta+1)$	$\Phi = 1$	$(2\vartheta-1)\vartheta$
$(2\vartheta-1, 2\vartheta-1)$	$\Phi = 1$	$\frac{(2\vartheta-2)(2\vartheta-3)}{2}$
$(2\vartheta+1, 2\vartheta-1)$	$\Phi = 2$	$(\vartheta-1)4\vartheta$

Table 4: Values of distance and no.edges when ϑ is odd

(d_y, d_z) type edges	Distance (Φ)	Number of edges
$(4\vartheta-1, 4\vartheta-1)$	$\Phi = 1$	$(4\vartheta-1)2\vartheta$

Table 5: Values of distance and number of edges when ϑ is even

Table 6: Edge partition of $OSCom(QD_2^\vartheta)$ of Quasidihedral groups QD_2^ϑ for every $yz \in E(OSCom(QD_2^\vartheta))$

(d_y, d_z) type edges	Distance (Φ)	Number of edges
$(2^\vartheta-1, 2^\vartheta-1)$	$\Phi = 1$	$2^{2\vartheta-1} - 2^{\vartheta-1}$

Table 7: Edge partition of $OSCom(SD_{8\vartheta})$ of Semidihedral Groups $SD_{8\vartheta}$ for every $yz \in E(OSCom(SD_{8\vartheta}))$

(d_y, d_z) type edges	Distance (Φ)	Number of edges
$(8\vartheta-1, 8\vartheta-1)$	$\Phi = 1$	$4\vartheta(8\vartheta-1)$

Edge partition of $OSCom(V_{8\vartheta})$ of $V_{8\vartheta}$ for every $yz \in E(OSCom(V_{8\vartheta}))$:

(d_y, d_z) type edges	Distance (Φ)	Number of edges
$(8\vartheta-1, 8\vartheta-1)$	$\Phi = 1$	$\frac{(2\vartheta+4)(2\vartheta+3)}{2}$
$(8\vartheta-1, 4\vartheta+3)$	$\Phi = 1$	$4\vartheta(\vartheta+2)$
$(6\vartheta-1, 8\vartheta-1)$	$\Phi = 1$	$(4\vartheta-4)(2\vartheta+4)$
$(4\vartheta+3, 4\vartheta+3)$	$\Phi = 1$	$\vartheta(2\vartheta-1)$
$(6\vartheta-1, 6\vartheta-1)$	$\Phi = 1$	$\frac{(4\vartheta-4)(4\vartheta-5)}{2}$
$(4\vartheta+3, 6\vartheta-1)$	$\Phi = 2$	$8\vartheta(\vartheta-1)$

Table 8: Values of distance and number of edges when ϑ is odd

(d_y, d_z) type edges	Distance (Φ)	Number of edges
$(8\vartheta-1, 8\vartheta-1)$	$\Phi = 1$	$4\vartheta(8\vartheta-1)$

Table 9: Values of distance and number of edges when n is even

Table 10: Edge partition of $OSCom(U_{6\vartheta})$ of $U_{6\vartheta}$ for every $yz \in E(OSCom(U_{6\vartheta}))$

(d_y, d_z) type edges	Distance (Φ)	Number of edges
$(6\vartheta-1, 6\vartheta-1)$	$\Phi = 1$	$\frac{\vartheta(\vartheta-1)}{2}$
$(3\vartheta-1, 6\vartheta-1)$	$\Phi = 1$	$2\vartheta^2$
$(4\vartheta-1, 8\vartheta-1)$	$\Phi = 1$	$3\vartheta^2$
$(3\vartheta-1, 3\vartheta-1)$	$\Phi = 1$	$\vartheta(2\vartheta-1)$
$(4\vartheta-1, 4\vartheta-1)$	$\Phi = 1$	$\frac{3\vartheta(3\vartheta-1)}{2}$
$(3\vartheta-1, 4\vartheta-1)$	$\Phi = 2$	$6\vartheta^2$

4. Properties of Order Super Commuting Graphs of Finite Groups

This section examines various topological properties of order super commuting graphs associated with finite groups such as dihedral groups $D_{2\vartheta}$, generalized quaternion groups $Q_{4\vartheta}$, semidihedral groups QD_{2^ϑ}, quasidihedral groups $SD_{8\vartheta}$,$V_{8\vartheta}$ groups , and $U_{6\vartheta}$ groups.

Lemma 4.1. *[8] Consider $OSCom(\mathscr{H})$ be Order Super commuting graph of any finite non-abelian group $\mathscr{H}$.*

1. *If $\mathscr{H} = D_{2\vartheta}$, then* $OSCom(\mathscr{H}) \cong \begin{cases} K_1 \vee (K_{\vartheta-1} \cup K_\vartheta), & \textit{if } \vartheta \textit{ is odd,} \\ K_{2\vartheta}, & \textit{if } \vartheta \textit{ is even.} \end{cases}$
2. *If $\mathscr{H} = Q_{4\vartheta}$, then* $OSCom(\mathscr{H}) \cong \begin{cases} K_2 \vee (K_{2\vartheta-2} \cup K_{2\vartheta}), & \textit{if } \vartheta \textit{ is odd,} \\ K_{4\vartheta}, & \textit{if } \vartheta \textit{ is even.} \end{cases}$
3. *If $\mathscr{H} = QD_{2^\vartheta}$, then $OSCom(\mathscr{H}) \cong K_{2^\vartheta}$.*

4. *If* $\mathscr{H} = V_{8\vartheta}$, *then* $OSCom(\mathscr{H}) \cong \begin{cases} K_{2\vartheta+4} \vee (K_{2\vartheta} \cup K_{4\vartheta-4}), & \text{if } \vartheta \text{ is odd,} \\ K_{8\vartheta}, & \text{if } \vartheta \text{ is even.} \end{cases}$
5. *If* $\mathscr{H} = SD_{8\vartheta}$, *then* $OSCom(\mathscr{H}) \cong K_{8\vartheta}$.
6. *If* $\mathscr{H} = U_{6\vartheta}$, *then* $OSCom(\mathscr{H}) \cong K_{\vartheta} \vee (K_{2\vartheta} \cup K_{3\vartheta})$.

Consider an entity

$$C(SCom(\mathscr{H}), \Phi) = \{(j_1, j_2; j_1 j_2 \in V(P(\mathscr{H})))|\Phi(j_1, j_2)\}. \tag{4.1}$$

The following result shows the total distance of super commuting graphs given in Lemma 4.1.

Proposition 4.2. *The total possible distance of* $OSCom(\mathscr{H})$ *is given:*

1. *If* $D_{2\vartheta}$ *is a dihedral group of order* 2ϑ *where* ϑ *is odd, thus we have*

$$d(OSCom(D_{2\vartheta}), \Phi) = \begin{cases} \left.\begin{array}{ll} \vartheta^2, & \Phi = 1 \\ (\vartheta-1)2\vartheta, & \Phi = 2 \end{array}\right\} & \text{where } \vartheta \text{ is odd} \\ \\ \left.\begin{array}{ll} \vartheta(2\vartheta-1), & \Phi = 1 \end{array}\right\} & \text{where } \vartheta \text{ is even} \end{cases}$$

2. *If* $Q_{4\vartheta}$ *is a generalized quaternion group of order* 4ϑ *where* ϑ *is odd, thus we have .*

$$d(OSCom(Q_{4\vartheta}), \Phi) = \begin{cases} \left.\begin{array}{ll} (2\vartheta+1)2\vartheta, & \Phi = 1 \\ (\vartheta-1)8\vartheta, & \Phi = 2 \end{array}\right\} & \text{where } \vartheta \text{ is odd} \\ \\ \left.\begin{array}{ll} (4\vartheta-1)2\vartheta, & \Phi = 1 \end{array}\right\} & \text{where } \vartheta \text{ is even} \end{cases}$$

3. *If* QD_2^{ϑ} *is a quasidihedral group of order* 2^{ϑ}, *we have*

$$d(OSCom(QD_2^{\vartheta}), \Phi) = \begin{cases} 2^{2\vartheta-1} - 2^{\vartheta-1}, & \Phi = 1 \end{cases}$$

4. *If* $SD_{8\vartheta}$ *is a semidihedral group of order* 8ϑ *where* ϑ *is odd, thus we have*

$$d(OSCom(SD_{8\vartheta}), \Phi) = \begin{cases} 4\vartheta(8\vartheta-1), & \Phi = 1 \end{cases}$$

5. *If* $V_{8\vartheta}$ *is a group of order* 8ϑ *where* ϑ *is odd, thus we have*

$$d(OSCom(V_{8\vartheta}), \Phi) = \begin{cases} \left.\begin{array}{ll} 4\vartheta(6\vartheta+1), & \Phi = 1 \\ 16\vartheta(\vartheta-1), & \Phi = 2 \end{array}\right\} & \text{where } \vartheta \text{ is odd} \\ \\ \left.\begin{array}{ll} 4\vartheta(8\vartheta-1), & \Phi = 1 \end{array}\right\} & \text{where } \vartheta \text{ is even} \end{cases}$$

6. *If* $U_{6\vartheta}$ *is a group of order* 6ϑ, *we have*

$$d(OSCom(U_{6\vartheta}), \Phi) = \begin{cases} 3\vartheta(4\vartheta-1), & \Phi = 1 \\ 12\vartheta^2, & \Phi = 2. \end{cases}$$

Proof. The edge partition of $OSCom(D_{2\vartheta})$ of dihedral group $D_{2\vartheta}$ for every $yz \in V(OSCom(D_{2\vartheta}))$ is given in Table 2. For distance $\Phi = 1$, we sum all the number of edges having distance $\Phi = 1$ of (d_y, d_z) type edges and for distance $\Phi = 2$, we sum all the number of edges having distance $\Phi = 2$ of (d_y, d_z) type edges. Thus, adding all such entities, we get

$$d(OSCom(G), \Phi) = \begin{cases} \vartheta^2, & \Phi = 1 \\ 2\vartheta(\vartheta-1), & \Phi = 2. \end{cases}$$

By simplifying, we have

$$d(OSCom(G), \Phi) = \begin{cases} \vartheta^2, & \Phi = 1 \\ 2\vartheta^2 - 2\vartheta, & \Phi = 2. \end{cases}$$

The other parts can be similarly worked out. □

5. Degree-Based Topological Indices of $OSCom(\mathscr{H})$

Theorem 5.1. *The Randić index for $OSCom(\mathscr{H})$ of a finite group $\mathscr{H}$, the following holds.*

1. *For dihedral group $D_{2\vartheta}$, we have*

$$R(OSCom(D_{2\vartheta})) = \begin{cases} \frac{\sqrt{\vartheta}}{\sqrt{2\vartheta-1}} + \frac{\sqrt{\vartheta-1}}{\sqrt{2\vartheta-1}} + \frac{2\vartheta-3}{2}, & \textit{if } \vartheta \textit{ is odd,} \\ \vartheta, & \textit{if } \vartheta \textit{ is even.} \end{cases}$$

2. *For generalized quaternion group $Q_{4\vartheta}$, we have*

$$R(OSCom(Q_{4\vartheta})) = \begin{cases} \frac{4\vartheta}{\sqrt{(4\vartheta-1)(2\vartheta+1)}} + \frac{1}{4\vartheta-1} + \frac{4(\vartheta-1)}{\sqrt{(2\vartheta-1)(4\vartheta-1)}} \\ + \frac{(2\vartheta-1)\vartheta}{2\vartheta+1} + \frac{(2\vartheta-3)(\vartheta-1)}{2\vartheta-1}, & \textit{if } \vartheta \textit{ is odd,} \\ 2\vartheta, & \textit{if } \vartheta \textit{ is even.} \end{cases}$$

3. *For quasidihedral group QD_2^{ϑ}, we have*

$$R(OSCom(QD_2^{\vartheta})) = 2^{\vartheta-1}.$$

4. *For semidihedral group $SD_{8\vartheta}$, we have*

$$R(OSCom(SD_{8\vartheta})) = 4\vartheta.$$

5. *For $V_{8\vartheta}$ group, we have*

$$R(OSCom(V_{8\vartheta})) = \begin{cases} \frac{(2\vartheta+3)(\vartheta+2)}{8\vartheta-1} + \frac{4\vartheta(\vartheta+2)}{\sqrt{(4\vartheta+3)(8\vartheta-1)}} + \frac{\vartheta(2\vartheta-1)}{4\vartheta+3} \\ + \frac{8(\vartheta-1)(\vartheta+2)}{\sqrt{(6\vartheta-1)(8\vartheta-1)}} + \frac{2(4\vartheta-5)(\vartheta-1)}{6\vartheta-1}, & \textit{if } \vartheta \textit{ is odd,} \\ 4\vartheta, & \textit{if } \vartheta \textit{ is even.} \end{cases}$$

6. *For $U_{6\vartheta}$ group, we have*

$$R(OSCom(U_{6\vartheta})) = \frac{(\vartheta-1)\vartheta}{2(6\vartheta-1)} + \frac{2\vartheta^2}{\sqrt{(3\vartheta-1)(6\vartheta-1)}} + \frac{(2\vartheta-1)\vartheta}{3\vartheta-1} + \frac{3\vartheta(3\vartheta-1)}{2(4\vartheta-1)} + \frac{3\vartheta^2}{\sqrt{(4\vartheta-1)(6\vartheta-1)}}.$$

Proof. For every single group $\mathscr{H}$, the structure of $OSCom(\mathscr{H})$is described in lemma 4.1, and the Randić Index $R(OSCom(\mathscr{H}))$ is defined here.
For Randić index of the dihedral group having order 2ϑ, by using Lemma 4.1, Proposition 4.2, Tables 2 and 3, we have

$$R(\mathrm{OSCom}(D_{2\vartheta})) = \begin{cases} (\vartheta-1)\left(\frac{1}{\sqrt{(\vartheta-1)(2\vartheta-1)}}\right) + \vartheta\left(\frac{1}{\sqrt{\vartheta(2\vartheta-1)}}\right) \\ +\left(\frac{(\vartheta-1)(\vartheta-2)}{2}\right)\left(\frac{1}{\sqrt{(\vartheta-1)(\vartheta-1)}}\right) & \text{if } \vartheta \text{ is odd,} \\ +\left(\frac{\vartheta(\vartheta-1)}{2}\right)\left(\frac{1}{\sqrt{\vartheta^2}}\right), \\ (\vartheta(2\vartheta-1))\left(\frac{1}{\sqrt{(2\vartheta-1)^2}}\right) & \text{if } \vartheta \text{ is even.} \end{cases}$$

For the generalized quaternion group $Q_{4\vartheta}$, having order 4ϑ, by using Lemma 4.1, Proposition 4.2, Tables 4 and 5, we have

$$R(\mathrm{OSCom}(Q_{4\vartheta})) = \begin{cases} \left(\frac{1}{\sqrt{(4\vartheta-1)(4\vartheta-1)}}\right) + 4\vartheta\left(\frac{1}{\sqrt{(2\vartheta+1)(4\vartheta-1)}}\right) \\ +\frac{4(\vartheta-1)}{\sqrt{(2\vartheta-1)(4\vartheta-1)}} + \frac{\vartheta(2\vartheta-1)}{2\vartheta+1} & \text{if } \vartheta \text{ is odd,} \\ +\left(\frac{(2\vartheta-2)(2\vartheta-3)}{2}\right)\left(\frac{1}{\sqrt{(2\vartheta-1)^2}}\right), \\ \\ (2\vartheta(4\vartheta-1))\left(\frac{1}{\sqrt{(4\vartheta-1)(4\vartheta-1)}}\right), & \text{if } \vartheta \text{ is even.} \end{cases}$$

For the quasidihedral group QD_2^{ϑ} having order 2^{ϑ}, by using Lemma 4.1, Proposition 4.2, and Tables 6, we have

$$R(\mathrm{OSCom}(QD_2^{\vartheta})) = (2^{2\vartheta-1} - 2^{\vartheta-1})\left(\frac{1}{\sqrt{(2^{\vartheta}-1)^2}}\right).$$

For the semidihedral group $SD_{8\vartheta}$ having order 8ϑ, by using Lemma 4.1, Proposition 4.2, and Tables 7, we have

$$R(\mathrm{OSCom}(SD_{8\vartheta})) = (4\vartheta(8\vartheta-1))\left(\frac{1}{\sqrt{(8\vartheta-1)(8\vartheta-1)}}\right).$$

For the group $V_{8\vartheta}$, which has order 8ϑ, we have used Lemma 4.1, Proposition 4.2, and Tables 8 and 9 to obtain

$$R(\mathrm{OSCom}(V_{8\vartheta})) = \begin{cases} \left(\frac{(2\vartheta+4)(2\vartheta+3)}{2}\right)\left(\frac{1}{\sqrt{(8\vartheta-1)(8\vartheta-1)}}\right) \\ +4\vartheta(\vartheta+2)\left(\frac{1}{\sqrt{(4\vartheta+3)(8\vartheta-1)}}\right) \\ +(4\vartheta-4)(2\vartheta+4)\left(\frac{1}{\sqrt{(6\vartheta-1)(8\vartheta-1)}}\right) & \text{if } \vartheta \text{ is odd,} \\ +\vartheta(2\vartheta-1)\left(\frac{1}{\sqrt{(4\vartheta+3)(4\vartheta+3)}}\right) \\ +\left(\frac{(4\vartheta-4)(4\vartheta-5)}{2}\right)\left(\frac{1}{\sqrt{(6\vartheta-1)(6\vartheta-1)}}\right), \\ \\ (4\vartheta(8\vartheta-1))\left(\frac{1}{\sqrt{(8\vartheta-1)(8\vartheta-1)}}\right), & \text{if } \vartheta \text{ is even.} \end{cases}$$

For the group $U_{6\vartheta}$, which has order 6ϑ, we have used Lemma 4.1, Proposition 4.2, and Tables 10 to obtain

$$\begin{aligned} R(\mathrm{OSCom}(U_{6\vartheta})) = {} & \left(\frac{\vartheta(\vartheta-1)}{2}\right)\left(\frac{1}{\sqrt{(6\vartheta-1)(6\vartheta-1)}}\right) + 2\vartheta^2\left(\frac{1}{\sqrt{(3\vartheta-1)(6\vartheta-1)}}\right) \\ & + 3\vartheta^2\left(\frac{1}{\sqrt{(4\vartheta-1)(6\vartheta-1)}}\right) + (2\vartheta-1)\vartheta\left(\frac{1}{\sqrt{(3\vartheta-1)(3\vartheta-1)}}\right) \\ & + \left(\frac{3\vartheta(3\vartheta-1)}{2}\right)\left(\frac{1}{\sqrt{(4\vartheta-1)(4\vartheta-1)}}\right). \end{aligned}$$

□

Theorem 5.2. *The Sombor index for $OSCom(\mathscr{H})$ of a finite group $\mathscr{H}$, the following holds.*

1. *For dihedral group $D_{2\vartheta}$, we have*

$$SO(OSCom(D_{2\vartheta})) = \begin{cases} (\vartheta-1)\sqrt{5\vartheta^2-6\vartheta+2} + \vartheta\sqrt{5\vartheta^2-4\vartheta+1} \\ +\frac{(\vartheta-1)(2\vartheta^2-3\vartheta+2)}{\sqrt{2}}, & \textit{if } \vartheta \textit{ is odd,} \\ \\ (2\vartheta-1)^2\vartheta\sqrt{2}, & \textit{if } \vartheta \textit{ is even.} \end{cases}$$

2. *For generalized quaternion group $Q_{4\vartheta}$, we have*

$$SO(OSCom(Q_{4\vartheta})) = \begin{cases} \sqrt{2}(4\vartheta-1)+\sqrt{2}(\vartheta-1)(2\vartheta-3)(2\vartheta-1) & \\ +\vartheta\sqrt{2}(2\vartheta-1)(2\vartheta+1)+4(\vartheta-1)\sqrt{20\vartheta^2-12\vartheta+2} & \text{if } \vartheta \text{ is odd,} \\ +4\vartheta\sqrt{20\vartheta^2-4\vartheta+2}, & \\ & \\ 2\vartheta(4\vartheta-1)^2\sqrt{2}, & \text{if } \vartheta \text{ is even.} \end{cases}$$

3. *For quasidihedral group QD_2^{ϑ}, we have*

$$SO(OSCom(QD_2^{\vartheta})) = \sqrt{2}(2^{\vartheta-1})(2^{\vartheta}-1)^2$$

4. *For semidihedral group $SD_{8\vartheta}$, we have*

$$SO(OSCom(SD_{8\vartheta})) = 4\vartheta\sqrt{2}(8\vartheta-1)^2$$

5. *For $V_{8\vartheta}$ group, we have*

$$SO(OSCom(V_{8\vartheta})) = \begin{cases} \sqrt{2}\Big[(8\vartheta-1)(2\vartheta+3)(\vartheta+2)+\vartheta(4\vartheta+3)(2\vartheta-1) & \\ +2(\vartheta-1)(4\vartheta-5)(6\vartheta-1)+4\vartheta(\vartheta+2)\sqrt{40\vartheta^2+4\vartheta+5} & \text{if } \vartheta \text{ is odd,} \\ +8(\vartheta+2)(\vartheta-1)\sqrt{50\vartheta^2-14\vartheta+1}\Big], & \\ & \\ 4\vartheta\sqrt{2}(8\vartheta-1)^2, & \text{if } \vartheta \text{ is even.} \end{cases}$$

6. *For $U_{6\vartheta}$ group, we have*

$$SO(OSCom(U_{6\vartheta})) = \vartheta\sqrt{2}(27\vartheta^2-19\vartheta+3)+2\vartheta^2\sqrt{45\vartheta^2-18\vartheta+2}+3\vartheta^2\sqrt{52\vartheta^2-20\vartheta+2}$$

Proof. For every single group $\mathscr{H}$ the structure of $OSCom(\mathscr{H})$is described in lemma 4.1, and the Sombor Index $SO(OSCom(\mathscr{H}))$ is defined here.

For the dihedral group $D_{2\vartheta}$, using Lemma 4.1, Proposition 4.2, and Tables 2 and 3, we have

$$SO(\mathrm{OSCom}(D_{2\vartheta})) = \begin{cases} (\vartheta-1)\left(\sqrt{(2\vartheta-1)^2+(\vartheta-1)^2}\right)+\vartheta\left(\sqrt{\vartheta^2+(2\vartheta-1)^2}\right) & \\ +\left(\frac{(\vartheta-1)(\vartheta-2)}{2}\right)\left(\sqrt{(\vartheta-1)^2+(\vartheta-1)^2}\right) & \text{if } \vartheta \text{ is odd,} \\ +\left(\frac{\vartheta(\vartheta-1)}{2}\right)\left(\sqrt{\vartheta^2+\vartheta^2}\right), & \\ & \\ (\vartheta(2\vartheta-1))\left(\sqrt{2(2\vartheta-1)^2}\right), & \text{if } \vartheta \text{ is even.} \end{cases}$$

Using Lemma 4.1, Proposition 4.2, and Tables 4 and 5, we can analyze the generalized quaternion group $Q_{4\vartheta}$.

$$SO(\mathrm{OSCom}(Q_{4\vartheta})) = \begin{cases} \sqrt{2}(4\vartheta-1)+\sqrt{2}(\vartheta-1)(2\vartheta-3)(2\vartheta-1) & \\ +\vartheta\sqrt{2}(2\vartheta-1)(2\vartheta+1)+4\vartheta\left(\sqrt{(2\vartheta+1)^2+(4\vartheta-1)^2}\right) & \\ +4(\vartheta-1)\left(\sqrt{(2\vartheta-1)^2+(4\vartheta-1)^2}\right) & \text{if } \vartheta \text{ is odd,} \\ & \\ (2\vartheta(4\vartheta-1))\left(\sqrt{2(4\vartheta-1)^2}\right), & \text{if } \vartheta \text{ is even.} \end{cases}$$

For the quasidihedral group QD_2^ϑ, using Lemma 4.1, Proposition 4.2, and Tables 6, we have

$$SO(\mathrm{OSCom}(QD_2^\vartheta)) = (2^{2\vartheta-1} - 2^{\vartheta-1})\left(\sqrt{(2^\vartheta-1)^2+(2^\vartheta-1)^2}\right).$$

For the semidihedral group $SD_{8\vartheta}$, using Lemma 4.1, Proposition 4.2, and Tables 7, we have

$$SO(\mathrm{OSCom}(SD_{8\vartheta})) = (4\vartheta(8\vartheta-1))\left(\sqrt{(8\vartheta-1)^2+(8\vartheta-1)^2}\right).$$

For the $V_{8\vartheta}$ group, using Lemma 4.1, Proposition 4.2, Tables 8, and 9, we have

$$SO(\mathrm{OSCom}(V_{8\vartheta})) = \begin{cases} \left(\frac{(2\vartheta+4)(2\vartheta+3)}{2}\right)\left(\sqrt{(8\vartheta-1)^2+(8\vartheta-1)^2}\right) & \\ +4\vartheta(\vartheta+2)\left(\sqrt{(4\vartheta+3)^2+(8\vartheta-1)^2}\right) & \\ +(4\vartheta-4)(2\vartheta+4)\left(\sqrt{(6\vartheta-1)^2+(8\vartheta-1)^2}\right) & \text{if } \vartheta \text{ is odd,} \\ +\vartheta(2\vartheta-1)\left(\sqrt{(4\vartheta+3)^2+(4\vartheta+3)^2}\right) & \\ +\left(\frac{(4\vartheta-4)(4\vartheta-5)}{2}\right)\left(\sqrt{(6\vartheta-1)^2+(6\vartheta-1)^2}\right), & \\ & \\ (4\vartheta(8\vartheta-1))\left(\sqrt{(8\vartheta-1)^2+(8\vartheta-1)^2}\right), & \text{if } \vartheta \text{ is even.} \end{cases}$$

For the $U_{6\vartheta}$ group, using Lemma 4.1, Proposition 4.2, and Tables 10, we have

$$\begin{aligned} SO(\mathrm{OSCom}(U_{6\vartheta})) = {} & \left(\frac{\vartheta(\vartheta-1)}{2}\right)\left(\sqrt{(6\vartheta-1)^2+(6\vartheta-1)^2}\right) + 2\vartheta^2\left(\sqrt{(3\vartheta-1)^2+(6\vartheta-1)^2}\right) \\ & + 3\vartheta^2\left(\sqrt{(4\vartheta-1)^2+(6\vartheta-1)^2}\right) + \vartheta(2\vartheta-1)\left(\sqrt{(3\vartheta-1)^2+(3\vartheta-1)^2}\right) \\ & + \left(\frac{3\vartheta(3\vartheta-1)}{2}\right)\left(\sqrt{(4\vartheta-1)^2+(4\vartheta-1)^2}\right). \end{aligned}$$

□

Theorem 5.3. *The Harmonic index for* $OSCom(\mathscr{H})$ *of a finite group* $\mathscr{H}$*, the following holds.*

1. *For dihedral group* $D_{2\vartheta}$*, we have*

$$Hr(OSCom(D_{2\vartheta})) = \begin{cases} \frac{2(\vartheta-1)}{3\vartheta-2} + \frac{2\vartheta}{3\vartheta-1} + \frac{2\vartheta-3}{2}, & \text{if } \vartheta \text{ is odd,} \\ \vartheta, & \text{if } \vartheta \text{ is even.} \end{cases}$$

2. *For generalized quaternion group* $Q_{4\vartheta}$*, we have*

$$Hr(OSCom(Q_{4\vartheta})) = \begin{cases} \frac{1}{4\vartheta-1} + \frac{4}{3} + \frac{4(\vartheta-1)}{3\vartheta-1} + \frac{\vartheta(2\vartheta-1)}{2\vartheta+1} & \\ +\frac{(\vartheta-1)(2\vartheta-3)}{2\vartheta-1}, & \text{if } \vartheta \text{ is odd,} \\ 2\vartheta, & \text{if } \vartheta \text{ is even.} \end{cases}$$

3. *For quasidihedral group* QD_2^ϑ*, we have*

$$Hr(OSCom(QD_2^\vartheta)) = 2^{\vartheta-1}.$$

4. *For semidihedral group $SD_{8\vartheta}$, we have*
$$Hr(OSCom(SD_{8\vartheta})) = 4\vartheta.$$
5. *For $V_{8\vartheta}$ group, we have*
$$Hr(OSCom(V_{8\vartheta})) = \begin{cases} \frac{(\vartheta+2)(2\vartheta+3)}{8\vartheta-1} + \frac{4\vartheta(\vartheta+2)}{6\vartheta+1} + \frac{\vartheta(2\vartheta-1)}{4\vartheta+3} \\ + \frac{8(\vartheta-1)(\vartheta+2)}{7\vartheta-1} + \frac{2(\vartheta-1)(4\vartheta-5)}{6\vartheta-1}, & \text{if } \vartheta \text{ is odd,} \\ \\ 4\vartheta, & \text{if } \vartheta \text{ is even.} \end{cases}$$
6. *For $U_{6\vartheta}$ group, we have*
$$Hr(OSCom(U_{6\vartheta})) = \frac{\vartheta(\vartheta-1)}{2(6\vartheta-1)} + \frac{4\vartheta^2}{9\vartheta-2} + \frac{3\vartheta^2}{5\vartheta-1} + \frac{\vartheta(2\vartheta-1)}{3\vartheta-1} + \frac{3\vartheta(3\vartheta-1)}{2(4\vartheta-1)}.$$

Proof. For every single group $\mathscr{H}$, the structure of $OSCom(\mathscr{H})$is described in lemma 4.1and the Harmonic Index $Hr(OSCom(\mathscr{H}))$ is defined here.
To determine the harmonic index of the dihedral group with order 2ϑ, we use Lemma 4.1, Proposition 4.2, and Tables 2 and 3.

$$Hr(\text{OSCom}(D_{2\vartheta})) = \begin{cases} (\vartheta-1)\left(\frac{2}{(2\vartheta-1)+(\vartheta-1)}\right) + \vartheta\left(\frac{2}{\vartheta+(2\vartheta-1)}\right) \\ +\left(\frac{(\vartheta-2)(\vartheta-1)}{2}\right)\left(\frac{2}{2(\vartheta-1)}\right) & \text{if } \vartheta \text{ is odd,} \\ +\left(\frac{\vartheta(\vartheta-1)}{2}\right)\left(\frac{2}{2\vartheta}\right), \\ \\ (\vartheta(2\vartheta-1))\left(\frac{2}{2(2\vartheta-1)}\right) & \text{if } \vartheta \text{ is even.} \end{cases}$$

By using Lemma 4.1, Proposition 4.2, and Tables 4 and 5, we can determine the harmonic index of the generalized quaternion group of order 4ϑ.

$$Hr(\text{OSCom}(Q_{4\vartheta})) = \begin{cases} \left(\frac{2}{(4\vartheta-1)+(4\vartheta-1)}\right) + 4\vartheta\left(\frac{2}{(4\vartheta-1)+(2\vartheta+1)}\right) \\ +4(\vartheta-1)\left(\frac{2}{(2\vartheta-1)+(4\vartheta-1)}\right) & \text{if } \vartheta \text{ is odd,} \\ +\vartheta(2\vartheta-1)\left(\frac{2}{2(2\vartheta+1)}\right) \\ +\left(\frac{(2\vartheta-2)(2\vartheta-3)}{2}\right)\left(\frac{2}{2(2\vartheta-1)}\right), \\ \\ (2\vartheta(4\vartheta-1))\left(\frac{2}{(4\vartheta-1)+(4\vartheta-1)}\right), & \text{if } \vartheta \text{ is even.} \end{cases}$$

By using Lemma 4.1, Proposition 4.2, and Tables 6, we can determine the harmonic index of a quasidihedral group with order QD_2^{ϑ}.

$$Hr(\text{OSCom}(QD_2^{\vartheta})) = (2^{2\vartheta-1} - 2^{\vartheta-1})\left(\frac{2}{2(2^{\vartheta}-1)}\right).$$

By using Lemma 4.1, Proposition 4.2, and Tables 7, we can determine the harmonic index of the semidihedral group of order $SD_{8\vartheta}$.

$$Hr(\text{OSCom}(SD_{8\vartheta})) = (4\vartheta(8\vartheta-1))\left(\frac{2}{(8\vartheta-1)+(8\vartheta-1)}\right).$$

By using Lemma 4.1, Proposition 4.2, Tables 8 and 9, we can determine the harmonic index of the $V_{8\vartheta}$ group, which has order $V_{8\vartheta}$.

$$Hr(\text{OSCom}(V_{8\vartheta})) = \begin{cases} \left(\frac{(2\vartheta+4)(2\vartheta+3)}{2}\right)\left(\frac{2}{(8\vartheta-1)+(8\vartheta-1)}\right) \\ +4\vartheta(\vartheta+2)\left(\frac{2}{(4\vartheta+3)+(8\vartheta-1)}\right) \\ +(4\vartheta-4)(2\vartheta+4)\left(\frac{2}{(6\vartheta-1)+(8\vartheta-1)}\right) & \text{if } \vartheta \text{ is odd,} \\ +\vartheta(2\vartheta-1)\left(\frac{2}{(4\vartheta+3)+(4\vartheta+3)}\right) \\ +\left(\frac{(4\vartheta-4)(4\vartheta-5)}{2}\right)\left(\frac{2}{(6\vartheta-1)+(6\vartheta-1)}\right), \\ \\ (4\vartheta(8\vartheta-1))\left(\frac{2}{(8\vartheta-1)+(8\vartheta-1)}\right), & \text{if } \vartheta \text{ is even.} \end{cases}$$

By using Lemma 4.1, Proposition 4.2, and Tables 10, we can determine the harmonic index of the $U_{6\vartheta}$ group, which has order $U_{6\vartheta}$.

$$\begin{aligned} Hr(\text{OSCom}(U_{6\vartheta})) = &\left(\frac{\vartheta(\vartheta-1)}{2}\right)\left(\frac{2}{2(6\vartheta-1)}\right) + 2\vartheta^2\left(\frac{2}{(3\vartheta-1)+(6\vartheta-1)}\right) \\ &+ 3\vartheta^2\left(\frac{2}{(4\vartheta-1)+(6\vartheta-1)}\right) + \vartheta(2\vartheta-1)\left(\frac{2}{(3\vartheta-1)+(3\vartheta-1)}\right) \\ &+ \left(\frac{3\vartheta(3\vartheta-1)}{2}\right)\left(\frac{2}{(4\vartheta-1)+(4\vartheta-1)}\right). \end{aligned}$$

□

Theorem 5.4. *The Geometric Arithmetic index for $OSCom(\mathscr{H})$ of a finite group $\mathscr{H}$, the following holds.*

1. *For dihedral group $D_{2\vartheta}$, we have*

$$GA(OSCom(D_{2\vartheta})) = \begin{cases} \frac{2(\vartheta-1)^{\frac{3}{2}}\sqrt{2\vartheta-1}}{3\vartheta-2} + \frac{2\vartheta^{\frac{3}{2}}\sqrt{2\vartheta-1}}{3\vartheta-1} + (\vartheta-1)^2, & \text{if } \vartheta \text{ is odd,} \\ \vartheta(2\vartheta-1), & \text{if } \vartheta \text{ is even.} \end{cases}$$

2. *For generalized quaternion group $Q_{4\vartheta}$, we have*

$$GA(OSCom(Q_{4\vartheta})) = \begin{cases} 1 + \frac{4\sqrt{(4\vartheta-1)(2\vartheta+1)}}{3} + \vartheta(2\vartheta-1) \\ +\frac{4(\vartheta-1)\sqrt{(4\vartheta-1)(2\vartheta-1)}}{3\vartheta-1} + (\vartheta-1)(2\vartheta-3), & \text{if } \vartheta \text{ is odd,} \\ 2\vartheta(4\vartheta-1), & \text{if } \vartheta \text{ is even.} \end{cases}$$

3. *For quasidihedral group QD_2^{ϑ}, we have*

$$GA(OSCom(QD_2^{\vartheta})) = 2^{\vartheta-1}(2^{\vartheta}-1).$$

4. *For semidihedral group $SD_{8\vartheta}$, we have*

$$GA(OSCom(SD_{8\vartheta})) = 4\vartheta(8\vartheta-1).$$

5. *For $V_{8\vartheta}$ group, we have*

$$GA(OSCom(V_{8\vartheta})) = \begin{cases} (\vartheta+2)(2\vartheta+3)+\vartheta(2\vartheta-1)+2(\vartheta-1)(4\vartheta-5) \\ +\frac{4\vartheta(\vartheta+2)\sqrt{(4\vartheta+3)(8\vartheta-1)}}{6\vartheta+1}, & \textit{if } \vartheta \textit{ is odd,} \\ +\frac{8(\vartheta+2)(\vartheta-1)\sqrt{(6\vartheta-1)(8\vartheta-1)}}{7\vartheta-1} \\ \\ 4\vartheta(8\vartheta-1), & \textit{if } \vartheta \textit{ is even.} \end{cases}$$

6. *For $U_{6\vartheta}$ group, we have*

$$GA(OSCom(U_{6\vartheta})) = \vartheta(2\vartheta-1) + \frac{\vartheta(\vartheta-1)}{2} + \frac{3\vartheta(3\vartheta-1)}{2} + \frac{4\vartheta^2\sqrt{(3\vartheta-1)(6\vartheta-1)}}{9\vartheta-2} + \frac{3\vartheta^2\sqrt{(4\vartheta-1)(6\vartheta-1)}}{5\vartheta-1}.$$

Proof. For every single group $\mathscr{H}$, the structure of $OSCom(\mathscr{H})$ is described in lemma 4.1, and the Geometric Arithmetic Index $GA(OSCom(\mathscr{H}))$ is defined here.
Using Lemma 4.1, Proposition 4.2, and Tables 2 and 3, we can analyze the dihedral group $D_{2\vartheta}$.

$$GA(\mathrm{OSCom}(D_{2\vartheta})) = \begin{cases} (\vartheta-1)\left(\frac{2\sqrt{(\vartheta-1)(2\vartheta-1)}}{(\vartheta-1)+(2\vartheta-1)}\right) + \vartheta\left(\frac{2\sqrt{\vartheta(2\vartheta-1)}}{\vartheta+(2\vartheta-1)}\right) \\ +\left(\frac{(\vartheta-2)(\vartheta-1)}{2}\right)\left(\frac{2\sqrt{(\vartheta-1)^2}}{(\vartheta-1)+(\vartheta-1)}\right) & \text{if } \vartheta \text{ is odd,} \\ +\left(\frac{\vartheta(\vartheta-1)}{2}\right)\left(\frac{2\vartheta}{2\vartheta}\right), \\ \\ (\vartheta(2\vartheta-1))\left(\frac{2\sqrt{(2\vartheta-1)^2}}{2(2\vartheta-1)}\right) & \text{if } \vartheta \text{ is even.} \end{cases}$$

For the generalized quaternion group $Q_{4\vartheta}$, we have the following results using Lemma 4.1, Proposition 4.2, and Tables 4 and 5.

$$GA(\mathrm{OSCom}(Q_{4\vartheta})) = \begin{cases} \left(\frac{2\sqrt{(4\vartheta-1)(4\vartheta-1)}}{2(4\vartheta-1)}\right) + 4\vartheta\left(\frac{2\sqrt{(4\vartheta-1)(2\vartheta+1)}}{(4\vartheta-1)+(2\vartheta+1)}\right) \\ +4(\vartheta-1)\left(\frac{2\sqrt{(4\vartheta-1)(2\vartheta-1)}}{(4\vartheta-1)+(2\vartheta-1)}\right) & \text{if } \vartheta \text{ is odd,} \\ +\vartheta(2\vartheta-1)\left(\frac{2\sqrt{(2\vartheta+1)^2}}{(2\vartheta+1)+(2\vartheta+1)}\right) \\ +\left(\frac{(2\vartheta-2)(2\vartheta-3)}{2}\right)\left(\frac{2\sqrt{(2\vartheta-1)^2}}{2(2\vartheta-1)}\right), \\ \\ (2\vartheta(4\vartheta-1))\left(\frac{2\sqrt{(4\vartheta-1)(4\vartheta-1)}}{(4\vartheta-1)+(4\vartheta-1)}\right), & \text{if } \vartheta \text{ is even.} \end{cases}$$

For the quasidihedral group QD_2^{ϑ}, we have results using Lemma 4.1, Proposition 4.2, and Tables 6.

$$GA(\mathrm{OSCom}(QD_2^{\vartheta})) = (2^{2\vartheta-1} - 2^{\vartheta-1})\left(\frac{2\sqrt{(2^{\vartheta}-1)^2}}{2(2^{\vartheta}-1)}\right).$$

For semidihedral group $SD_{8\vartheta}$, using Lemma 4.1, Proposition 4.2 and Tables 7, we have

$$GA(\mathrm{OSCom}(SD_{8\vartheta})) = (4\vartheta(8\vartheta-1))\left(\frac{2\sqrt{(8\vartheta-1)(8\vartheta-1)}}{(8\vartheta-1)+(8\vartheta-1)}\right).$$

For the $V_{8\vartheta}$ group, using Lemma 4.1, Proposition 4.2, and Tables 8 and 9, we have

$$GA(\text{OSCom}(V_{8\vartheta})) = \begin{cases} \left(\frac{(2\vartheta+4)(2\vartheta+3)}{2}\right)\left(\frac{2\sqrt{(8\vartheta-1)(8\vartheta-1)}}{(8\vartheta-1)+(8\vartheta-1)}\right) & \\ +4\vartheta(\vartheta+2)\left(\frac{2\sqrt{(4\vartheta+3)(8\vartheta-1)}}{(4\vartheta+3)+(8\vartheta-1)}\right) & \\ +(4\vartheta-4)(2\vartheta+4)\left(\frac{2\sqrt{(6\vartheta-1)(8\vartheta-1)}}{(6\vartheta-1)+(8\vartheta-1)}\right) & \text{if } \vartheta \text{ is odd,} \\ +\vartheta(2\vartheta-1)\left(\frac{2\sqrt{(4\vartheta+3)(4\vartheta+3)}}{(4\vartheta+3)+(4\vartheta+3)}\right) & \\ +\left(\frac{(4\vartheta-4)(4\vartheta-5)}{2}\right)\left(\frac{2\sqrt{(6\vartheta-1)(6\vartheta-1)}}{(6\vartheta-1)+(6\vartheta-1)}\right), & \\ & \\ (4\vartheta(8\vartheta-1))\left(\frac{2\sqrt{(8\vartheta-1)(8\vartheta-1)}}{(8\vartheta-1)+(8\vartheta-1)}\right), & \text{if } \vartheta \text{ is even.} \end{cases}$$

For the $U_{6\vartheta}$ group, using Lemma 4.1, Proposition 4.2, and Tables 10, we have

$$\begin{aligned} GA(\text{OSCom}(U_{6\vartheta})) = {} & \left(\frac{\vartheta(\vartheta-1)}{2}\right)\left(\frac{2\sqrt{(6\vartheta-1)(6\vartheta-1)}}{(6\vartheta-1)+(6\vartheta-1)}\right) + 2\vartheta^2\left(\frac{2\sqrt{(3\vartheta-1)(6\vartheta-1)}}{(3\vartheta-1)+(6\vartheta-1)}\right) \\ & + 3\vartheta^2\left(\frac{2\sqrt{(4\vartheta-1)(6\vartheta-1)}}{(4\vartheta-1)+(6\vartheta-1)}\right) + \vartheta(2\vartheta-1)\left(\frac{2\sqrt{(3\vartheta-1)(3\vartheta-1)}}{(3\vartheta-1)+(3\vartheta-1)}\right) \\ & + \left(\frac{3\vartheta(3\vartheta-1)}{2}\right)\left(\frac{2\sqrt{(4\vartheta-1)(4\vartheta-1)}}{(4\vartheta-1)+(4\vartheta-1)}\right). \end{aligned}$$

□

Theorem 5.5. *The Atom Bond Sum Connectivity index for $OSCom(\mathscr{H})$ of a finite group $\mathscr{H}$, the following holds.*

1. *For dihedral group $D_{2\vartheta}$, we have*

$$ABS(OSCom(D_{2\vartheta})) = \begin{cases} (\vartheta-1)\sqrt{\frac{3\vartheta-4}{3\vartheta-2}} + \frac{(\vartheta-1)\sqrt{\vartheta(\vartheta-1)}}{2} & \\ +\vartheta\sqrt{\frac{3(\vartheta-1)}{3\vartheta-1}} + \frac{(\vartheta-2)\sqrt{(\vartheta-1)(\vartheta-2)}}{2}, & \text{if } \vartheta \text{ is odd,} \\ \vartheta\sqrt{2(\vartheta-1)(2\vartheta-1)}, & \text{if } \vartheta \text{ is even.} \end{cases}$$

2. *For generalized quaternion group $Q_{4\vartheta}$, we have*

$$ABS(OSCom(Q_{4\vartheta})) = \begin{cases} \sqrt{\frac{2(2\vartheta-1)}{4\vartheta-1}} + 4\vartheta\sqrt{\frac{3\vartheta-1}{3\vartheta}} + 4(\vartheta-1)\sqrt{\frac{3\vartheta-2}{3\vartheta-1}} & \\ +\vartheta(2\vartheta-1)\sqrt{\frac{2\vartheta}{2\vartheta+1}} + \frac{(2\vartheta-2)(2\vartheta-3)}{2}\sqrt{\frac{2(\vartheta-1)}{2\vartheta-1}}, & \text{if } \vartheta \text{ is odd,} \\ 2\vartheta\sqrt{(4\vartheta-2)(4\vartheta-1)}, & \text{if } \vartheta \text{ is even.} \end{cases}$$

3. *For quasidihedral group QD_2^{ϑ}, we have*

$$ABS(OSCom(QD_2^{\vartheta})) = 2^{\vartheta-1}\sqrt{(2^{\vartheta}-1)(2^{\vartheta}-2)}.$$

4. *For semidihedral group $SD_{8\vartheta}$, we have*

$$ABS(OSCom(SD_{8\vartheta})) = 4\vartheta\sqrt{(8\vartheta-2)(8\vartheta-1)}.$$

5. For $V_{8\vartheta}$ group, we have

$$ABS(OSCom(V_{8\vartheta})) = \begin{cases} (\vartheta+2)(2\vartheta+3)\sqrt{\frac{8\vartheta-2}{8\vartheta-1}} + 4\vartheta(\vartheta+2)\sqrt{\frac{6\vartheta}{6\vartheta+1}} & \\ +8(\vartheta-1)(\vartheta+2)\sqrt{\frac{7\vartheta-2}{7\vartheta-1}} + \vartheta(2\vartheta-1)\sqrt{\frac{4\vartheta+2}{4\vartheta+3}} & \text{if } \vartheta \text{ is odd,} \\ +2(\vartheta-1)(4\vartheta-5)\sqrt{\frac{6\vartheta-2}{6\vartheta-1}}, & \\ & \\ 4\vartheta\sqrt{(8\vartheta-2)(8\vartheta-1)}, \text{ if } \vartheta \text{ is even..} & \end{cases}$$

6. For $U_{6\vartheta}$ group, we have

$$ABS(OSCom(U_{6\vartheta})) = \frac{\vartheta(\vartheta-1)}{2}\sqrt{\frac{6\vartheta-2}{6\vartheta-1}} + 2\vartheta^2\sqrt{\frac{9\vartheta-4}{9\vartheta-2}} + 3\vartheta^2\sqrt{\frac{5\vartheta-2}{5\vartheta-1}}$$
$$+\vartheta(2\vartheta-1)\sqrt{\frac{3\vartheta-2}{3\vartheta-1}} + \frac{3\vartheta(3\vartheta-1)}{2}\sqrt{\frac{4\vartheta-2}{4\vartheta-1}}.$$

Proof. For every single group $\mathscr{H}$ the structure of $OSCom(\mathscr{H})$is as described in lemma 4.1, Atom Bond Sum Connectivity Index $ABS(OSCom(\mathscr{H}))$ is defined here.
By using Lemma 4.1, Proposition 4.2, Tables 2 and 3, and the Atom Bond Sum Connectivity index of the dihedral group with order 2ϑ, we obtain

$$ABS(\text{OSCom}(D_{2\vartheta})) = \begin{cases} (\vartheta-1)\left(\sqrt{\frac{(2\vartheta-1)+(\vartheta-1)-2}{(2\vartheta-1)+(\vartheta-1)}}\right) + \vartheta\left(\sqrt{\frac{(2\vartheta-1)+\vartheta-2}{(2\vartheta-1)+\vartheta}}\right) & \\ +\left(\frac{(\vartheta-2)(\vartheta-1)}{2}\right)\left(\sqrt{\frac{(\vartheta-1)+(\vartheta-1)-2}{(\vartheta-1)+(\vartheta-1)}}\right) & \text{if } \vartheta \text{ is odd,} \\ +\left(\frac{\vartheta(\vartheta-1)}{2}\right)\left(\sqrt{\frac{2\vartheta-2}{2\vartheta}}\right), & \\ & \\ (\vartheta(2\vartheta-1))\left(\sqrt{\frac{(2\vartheta-1)+(2\vartheta-1)-2}{(2\vartheta-1)+(2\vartheta-1)}}\right) & \text{if } \vartheta \text{ is even.} \end{cases}$$

By using Lemma 4.1, Proposition 4.2, Tables 4 and 5, and the Atom Bond Sum Connectivity index of the generalized quaternion group with order 4ϑ, we obtain

$$ABS(\text{OSCom}(Q_{4\vartheta})) = \begin{cases} \left(\sqrt{\frac{(4\vartheta-1)+(4\vartheta-1)-2}{2(4\vartheta-1)}}\right) + 4\vartheta\left(\sqrt{\frac{(4\vartheta-1)+(2\vartheta+1)-2}{(4\vartheta-1)+(2\vartheta+1)}}\right) & \\ +4(\vartheta-1)\left(\sqrt{\frac{(4\vartheta-1)+(2\vartheta-1)-2}{(4\vartheta-1)+(2\vartheta-1)}}\right) & \text{if } \vartheta \text{ is odd,} \\ +\vartheta(2\vartheta-1)\left(\sqrt{\frac{(2\vartheta+1)+(2\vartheta+1)-2}{(2\vartheta+1)+(2\vartheta+1)}}\right) & \\ +\left(\frac{(2\vartheta-3)(2\vartheta-2)}{2}\right)\left(\sqrt{\frac{(2\vartheta-1)+(2\vartheta-1)-2}{2(2\vartheta-1)}}\right), & \\ & \\ (2\vartheta(4\vartheta-1))\left(\sqrt{\frac{(4\vartheta-1)+(4\vartheta-1)-2}{(4\vartheta-1)+(4\vartheta-1)}}\right), & \text{if } \vartheta \text{ is even.} \end{cases}$$

By using Lemma 4.1, Proposition 4.2, and Table 6, we obtain the Atom Bond Sum Connectivity index of the quasidihedral group with order 2^{ϑ}, we obtain

$$ABS(\text{OSCom}(QD_2^{\vartheta})) = (2^{2\vartheta-1} - 2^{\vartheta-1})\left(\sqrt{\frac{(2^{\vartheta}-1)+(2^{\vartheta}-1)-2}{2(2^{\vartheta}-1)}}\right).$$

By using Lemma 4.1, Proposition 4.2, Table 7, and the Atom Bond Sum Connectivity index of the semidihedral group with order 8ϑ, we obtain

$$ABS(\mathrm{OSCom}(SD_{8\vartheta})) = (4\vartheta(8\vartheta-1))\left(\sqrt{\frac{(8\vartheta-1)+(8\vartheta-1)-2}{(8\vartheta-1)+(8\vartheta-1)}}\right).$$

By using Lemma 4.1, Proposition 4.2, Tables 8 and 9, and the Atom Bond Sum Connectivity index of the $V_{8\vartheta}$ group with order 8ϑ, we obtain

$$ABS(\mathrm{OSCom}(V_{8\vartheta})) = \begin{cases} \left(\frac{(2\vartheta+4)(2\vartheta+3)}{2}\right)\left(\sqrt{\frac{(8\vartheta-1)+(8\vartheta-1)-2}{(8\vartheta-1)+(8\vartheta-1)}}\right) \\ +4\vartheta(\vartheta+2)\left(\sqrt{\frac{(4\vartheta+3)+(8\vartheta-1)-2}{(4\vartheta+3)+(8\vartheta-1)}}\right) \\ +(4\vartheta-4)(2\vartheta+4)\left(\sqrt{\frac{(6\vartheta-1)+(8\vartheta-1)-2}{(6\vartheta-1)+(8\vartheta-1)}}\right) & \text{if } \vartheta \text{ is odd,} \\ +\vartheta(2\vartheta-1)\left(\sqrt{\frac{(4\vartheta+3)+(4\vartheta+3)-2}{(4\vartheta+3)+(4\vartheta+3)}}\right) \\ +\left(\frac{(4\vartheta-4)(4\vartheta-5)}{2}\right)\left(\sqrt{\frac{(6\vartheta-1)+(6\vartheta-1)-2}{(6\vartheta-1)+(6\vartheta-1)}}\right), \\ \\ (4\vartheta(8\vartheta-1))\left(\sqrt{\frac{(8\vartheta-1)+(8\vartheta-1)-2}{(8\vartheta-1)+(8\vartheta-1)}}\right), & \text{if } \vartheta \text{ is even.} \end{cases}$$

By using Lemma 4.1, Proposition 4.2, and Table 10, we obtain the Atom Bond Sum Connectivity index of the $U_{6\vartheta}$ group with order 6ϑ. Thus,

$$\begin{aligned} ABS(\mathrm{OSCom}(U_{6\vartheta})) = {} & \left(\frac{\vartheta(\vartheta-1)}{2}\right)\left(\sqrt{\frac{(6\vartheta-1)+(6\vartheta-1)-2}{(6\vartheta-1)+(6\vartheta-1)}}\right) \\ & + 2\vartheta^2\left(\sqrt{\frac{(3\vartheta-1)+(6\vartheta-1)-2}{(3\vartheta-1)+(6\vartheta-1)}}\right) \\ & + 3\vartheta^2\left(\sqrt{\frac{(4\vartheta-1)+(6\vartheta-1)-2}{(4\vartheta-1)+(6\vartheta-1)}}\right) \\ & + \vartheta(2\vartheta-1)\left(\sqrt{\frac{(3\vartheta-1)+(3\vartheta-1)-2}{(3\vartheta-1)+(3\vartheta-1)}}\right) \\ & + \left(\frac{3\vartheta(3\vartheta-1)}{2}\right)\left(\sqrt{\frac{(4\vartheta-1)+(4\vartheta-1)-2}{(4\vartheta-1)+(4\vartheta-1)}}\right). \end{aligned}$$

□

Theorem 5.6. *The Atom Bond Connectivity index for $OSCom(\mathscr{H})$ of a finite group $\mathscr{H}$, the following holds.*

1. *For dihedral group $D_{2\vartheta}$, we have*

$$ABC(OSCom(D_{2\vartheta})) = \begin{cases} \frac{\sqrt{\vartheta-1}\left(\sqrt{3\vartheta-4}+\sqrt{3\vartheta}\right)}{\sqrt{2\vartheta-1}} \\ +\frac{1}{\sqrt{2}}\left((\vartheta-1)^{\frac{3}{2}}+(\vartheta-2)^{\frac{3}{2}}\right), & \textit{if } \vartheta \textit{ is odd,} \\ \\ 2\vartheta\sqrt{\vartheta-1}, & \textit{if } \vartheta \textit{ is even.} \end{cases}$$

2. *For generalized quaternion group $Q_{4\vartheta}$, we have*

$$ABC(OSCom(Q_{4\vartheta})) = \begin{cases} \frac{2\sqrt{2\vartheta-1}}{4\vartheta-1} + \frac{(2\vartheta-2)(2\vartheta-3)\sqrt{\vartheta-1}}{2\vartheta-1} \\ + \frac{2\vartheta(2\vartheta-1)\sqrt{\vartheta}}{2\vartheta+1} + 4\vartheta\sqrt{\frac{(6\vartheta-2)}{(4\vartheta-1)(2\vartheta+1)}}, & \textit{if } \vartheta \textit{ is odd,} \\ +4(\vartheta-1)\sqrt{\frac{(6\vartheta-4)}{(2\vartheta-1)(4\vartheta-1)}} \\ \\ 4\vartheta\sqrt{2\vartheta-1}, & \textit{if } \vartheta \textit{ is even.} \end{cases}$$

3. *For quasidihedral group QD_2^{ϑ}, we have*

$$ABC(OSCom(QD_2^{\vartheta})) = 2^{\vartheta}\sqrt{2^{\vartheta-1}-1}.$$

4. *For semidihedral group $SD_{8\vartheta}$, we have*

$$ABC(OSCom(SD_{8\vartheta})) = 8\vartheta\sqrt{4\vartheta-1}.$$

5. *For $V_{8\vartheta}$ group, we have*

$$ABC(OSCom(V_{8\vartheta})) = \begin{cases} \frac{(2\vartheta+4)(2\vartheta+3)\sqrt{4\vartheta-1}}{8\vartheta-1} + \frac{(4\vartheta-4)(4\vartheta-5)\sqrt{3\vartheta-1}}{6\vartheta-1} \\ +8\vartheta(\vartheta+2)\sqrt{\frac{3\vartheta}{(4\vartheta+3)(8\vartheta-1)}} + \frac{2\vartheta(2\vartheta-1)\sqrt{2\vartheta+1}}{4\vartheta+3} & \textit{if } \vartheta \textit{ is odd,} \\ +(4\vartheta-4)(2\vartheta+4)\sqrt{\frac{(14\vartheta-4)}{(6\vartheta-1)(8\vartheta-1)}}, \\ \\ 8\vartheta\sqrt{4\vartheta-1}, & \textit{if } \vartheta \textit{ is even.} \end{cases}$$

6. *For $U_{6\vartheta}$ group, we have*

$$ABC(OSCom(U_{6\vartheta})) = \frac{\vartheta(\vartheta-1)\sqrt{3\vartheta-1}}{6\vartheta-1} + 2\vartheta^2\sqrt{\frac{9\vartheta-4}{(3\vartheta-1)(6\vartheta-1)}} + 3\vartheta^2\sqrt{\frac{10\vartheta-4}{(4\vartheta-1)(6\vartheta-1)}}$$
$$+ \frac{\vartheta(2\vartheta-1)\sqrt{6\vartheta-4}}{3\vartheta-1} + \frac{3\vartheta(3\vartheta-1)\sqrt{2\vartheta-1}}{4\vartheta-1}.$$

Proof. For every single group $\mathscr{H}$, the structure of $OSCom(\mathscr{H})$is described in lemma 4.1, and the Atom Bond Connectivity Index $ABC(OSCom(\mathscr{H}))$ is defined here.

For the dihedral group $D_{2\vartheta}$, we can apply Lemma 4.1, Proposition 4.2, refer to Tables 2 and 3, to derive the following results

$$ABC(\mathrm{OSCom}(D_{2\vartheta})) = \begin{cases} (\vartheta-1)\left(\sqrt{\frac{(2\vartheta-1)+(\vartheta-1)-2}{(2\vartheta-1)(\vartheta-1)}}\right) + \vartheta\left(\sqrt{\frac{\vartheta+(2\vartheta-1)-2}{\vartheta(2\vartheta-1)}}\right) \\ +\left(\frac{(\vartheta-1)(\vartheta-2)}{2}\right)\left(\sqrt{\frac{(\vartheta-1)+(\vartheta-1)-2}{(\vartheta-1)^2}}\right) & \text{if } \vartheta \text{ is odd,} \\ +\left(\frac{\vartheta(\vartheta-1)}{2}\right)\left(\sqrt{\frac{2\vartheta-2}{\vartheta^2}}\right), \\ \\ (\vartheta(2\vartheta-1))\left(\sqrt{\frac{(2\vartheta-1)+(2\vartheta-1)-2}{(2\vartheta-1)^2}}\right) & \text{if } \vartheta \text{ is even.} \end{cases}$$

For the generalized quaternion group $Q_{4\vartheta}$, we can apply Lemma4.1, Proposition 4.2, refer to Tables 4 and 5, to derive the following results

$$ABC(\mathrm{OSCom}(Q_{4\vartheta})) = \begin{cases} \left(\sqrt{\frac{(4\vartheta-1)+(4\vartheta-1)-2}{((4\vartheta-1)^2}}\right) + 4\vartheta\left(\sqrt{\frac{(4\vartheta-1)+(2\vartheta+1)-2}{(4\vartheta-1)(2\vartheta+1)}}\right) & \\ +4(\vartheta-1)\left(\sqrt{\frac{(4\vartheta-1)+(2\vartheta-1)-2}{(4\vartheta-1)(2\vartheta-1)}}\right) & \text{if } \vartheta \text{ is odd,} \\ +\vartheta(2\vartheta-1)\left(\sqrt{\frac{(2\vartheta+1)+(2\vartheta+1)-2}{(2\vartheta+1)^2}}\right) & \\ +\left(\frac{(2\vartheta-2)(2\vartheta-3)}{2}\right)\left(\sqrt{\frac{(2\vartheta-1)+(2\vartheta-1)-2}{(2\vartheta-1)^2}}\right), & \\ & \\ (2\vartheta(4\vartheta-1))\left(\sqrt{\frac{(4\vartheta-1)+(4\vartheta-1)-2}{(4\vartheta-1)(4\vartheta-1)}}\right), & \text{if } \vartheta \text{ is even.} \end{cases}$$

For the quasidihedral group QD_2^{ϑ}, using Lemma 4.1, Proposition 4.2, and Table 6, we have

$$ABC(\mathrm{OSCom}(QD_2^{\vartheta})) = (2^{2\vartheta-1} - 2^{\vartheta-1})\left(\sqrt{\frac{(2^{\vartheta}-1)+(2^{\vartheta}-1)-2}{(2^{\vartheta}-1)(2^{\vartheta}-1)}}\right).$$

For the semidihedral group $SD_{8\vartheta}$, using Lemma 4.1, Proposition 4.2, and Tables 7, we have

$$ABC(\mathrm{OSCom}(SD_{8\vartheta})) = (4\vartheta(8\vartheta-1))\left(\sqrt{\frac{(8\vartheta-1)+(8\vartheta-1)-2}{(8\vartheta-1)(8\vartheta-1)}}\right).$$

For the $V_{8\vartheta}$ group, using Lemma 4.1, Proposition 4.2, Tables 8 and 9, we have

$$ABC(\mathrm{OSCom}(V_{8\vartheta})) = \begin{cases} \left(\frac{(2\vartheta+4)(2\vartheta+3)}{2}\right)\left(\sqrt{\frac{(8\vartheta-1)+(8\vartheta-1)-2}{(8\vartheta-1)(8\vartheta-1)}}\right) & \\ +4\vartheta(\vartheta+2)\left(\sqrt{\frac{(4\vartheta+3)+(8\vartheta-1)-2}{(4\vartheta+3)(8\vartheta-1)}}\right) & \\ +(4\vartheta-4)(2\vartheta+4)\left(\sqrt{\frac{(6\vartheta-1)+(8\vartheta-1)-2}{(6\vartheta-1)(8\vartheta-1)}}\right) & \text{if } \vartheta \text{ is odd,} \\ +\vartheta(2\vartheta-1)\left(\sqrt{\frac{(4\vartheta+3)+(4\vartheta+3)-2}{(4\vartheta+3)(4\vartheta+3)}}\right) & \\ +\left(\frac{(4\vartheta-4)(4\vartheta-5)}{2}\right)\left(\sqrt{\frac{(6\vartheta-1)+(6\vartheta-1)-2}{(6\vartheta-1)(6\vartheta-1)}}\right), & \\ & \\ (4\vartheta(8\vartheta-1))\left(\sqrt{\frac{(8\vartheta-1)+(8\vartheta-1)-2}{(8\vartheta-1)(8\vartheta-1)}}\right), & \text{if } \vartheta \text{ is even.} \end{cases}$$

For the $U_{6\vartheta}$ group, using Lemma 4.1, Proposition 4.2, and Tables 10, we have

$$\begin{aligned} ABC(\mathrm{OSCom}(U_{6\vartheta})) = {} & \left(\frac{\vartheta(\vartheta-1)}{2}\right)\left(\sqrt{\frac{(6\vartheta-1)+(6\vartheta-1)-2}{(6\vartheta-1)(6\vartheta-1)}}\right) + 2\vartheta^2\left(\sqrt{\frac{(3\vartheta-1)+(6\vartheta-1)-2}{(3\vartheta-1)(6\vartheta-1)}}\right) \\ & + 3\vartheta^2\left(\sqrt{\frac{(4\vartheta-1)+(6\vartheta-1)-2}{(4\vartheta-1)(6\vartheta-1)}}\right) + \vartheta(2\vartheta-1)\left(\sqrt{\frac{(3\vartheta-1)+(3\vartheta-1)-2}{(3\vartheta-1)(3\vartheta-1)}}\right) \\ & + \left(\frac{3\vartheta(3\vartheta-1)}{2}\right)\left(\sqrt{\frac{(4\vartheta-1)+(4\vartheta-1)-2}{(4\vartheta-1)^2}}\right). \end{aligned}$$

□

Theorem 5.7. *The Sum Connectivity index for $OSCom(\mathscr{H})$ of a finite group $\mathscr{H}$, the following holds.*

1. *For dihedral group $D_{2\vartheta}$, we have*

$$\chi(OSCom(D_{2\vartheta})) = \begin{cases} \frac{\vartheta-1}{\sqrt{3\vartheta-2}} + \frac{\vartheta}{\sqrt{3\vartheta-1}} \\ +\frac{(\vartheta-2)\sqrt{\vartheta-1}+(\vartheta-1)\sqrt{\vartheta}}{2\sqrt{2}}, & \textit{if } \vartheta \textit{ is odd,} \\ \\ \frac{\vartheta\sqrt{2\vartheta-1}}{\sqrt{2}}, & \textit{if } \vartheta \textit{ is even.} \end{cases}$$

2. *For generalized quaternion group $Q_{4\vartheta}$, we have*

$$\chi(OSCom(Q_{4\vartheta})) = \begin{cases} \frac{1}{\sqrt{8\vartheta-2}} + \frac{4\vartheta}{\sqrt{6\vartheta}} + \frac{4(\vartheta-1)}{\sqrt{6\vartheta-2}} \\ +\frac{\vartheta(2\vartheta-1)}{\sqrt{4\vartheta+2}} + \frac{(\vartheta-1)(2\vartheta-3)}{\sqrt{4\vartheta-2}}, & \textit{if } \vartheta \textit{ is odd,} \\ \\ \vartheta\sqrt{8\vartheta-2}, & \textit{if } \vartheta \textit{ is even.} \end{cases}$$

3. *For quasidihedral group QD_2^{ϑ}, we have*

$$\chi(OSCom(QD_2^{\vartheta})) = 2^{\vartheta-2}\sqrt{2(2^{\vartheta}-1)}.$$

4. *For semidihedral group $SD_{8\vartheta}$, we have*

$$\chi(OSCom(SD_{8\vartheta})) = 2\vartheta\sqrt{16\vartheta-2}.$$

5. *For $V_{8\vartheta}$ group, we have*

$$\chi(OSCom(V_{8\vartheta})) = \begin{cases} \frac{(\vartheta+2)(2\vartheta+3)}{\sqrt{16\vartheta-2}} + \frac{4\vartheta(\vartheta+2)}{\sqrt{12\vartheta+2}} + \frac{\vartheta(2\vartheta-1)}{\sqrt{8\vartheta+6}} \\ +\frac{8(\vartheta+2)(\vartheta-1)}{\sqrt{14\vartheta-2}} + \frac{2(\vartheta-1)(4\vartheta-5)}{\sqrt{12\vartheta-2}}, & \textit{if } \vartheta \textit{ is odd,} \\ \\ 2\vartheta\sqrt{16\vartheta-2}, & \textit{if } \vartheta \textit{ is even.} \end{cases}$$

6. *For $U_{6\vartheta}$ group, we have*

$$\chi(OSCom(U_{6\vartheta})) = \frac{\vartheta(\vartheta-1)}{2\sqrt{12\vartheta-2}} + \frac{2\vartheta^2}{\sqrt{9\vartheta-2}} + \frac{3\vartheta^2}{\sqrt{10\vartheta-2}} + \frac{\vartheta(2\vartheta-1)}{\sqrt{6\vartheta-2}} + \frac{3\vartheta(3\vartheta-1)}{2\sqrt{8\vartheta-2}}.$$

Proof. For every single group $\mathscr{H}$ the structure of $OSCom(\mathscr{H})$is described in lemma 4.1, and the Sum Connectivity Index $\chi(OSCom(\mathscr{H}))$ is defined here.
By using Lemma 4.1, Proposition 4.2, Tables 2 and 3, and the Sum Connectivity index of the dihedral group with order 2ϑ, we obtain

$$\chi(\text{OSCom}(D_{2\vartheta})) = \begin{cases} (\vartheta-1)\left(\frac{1}{\sqrt{(\vartheta-1)+(2\vartheta-1)}}\right) + \vartheta\left(\frac{1}{\sqrt{\vartheta+(2\vartheta-1)}}\right) \\ +\left(\frac{(\vartheta-1)(\vartheta-2)}{2}\right)\left(\frac{1}{\sqrt{(\vartheta-1)+(\vartheta-1)}}\right) & \text{if } \vartheta \text{ is odd,} \\ +\left(\frac{\vartheta(\vartheta-1)}{2}\right)\left(\frac{1}{\sqrt{2\vartheta}}\right), \\ \\ (\vartheta(2\vartheta-1))\left(\frac{1}{\sqrt{(2\vartheta-1)+(2\vartheta-1)}}\right) & \text{if } \vartheta \text{ is even.} \end{cases}$$

By using Lemma 4.1, Proposition 4.2, Tables 4 and 5, and the Sum Connectivity index of the generalized quaternion group with order 4ϑ, we obtain

$$\chi(\mathrm{OSCom}(Q_{4\vartheta})) = \begin{cases} \left(\frac{1}{\sqrt{2(4\vartheta-1)}}\right) + 4\vartheta\left(\frac{1}{\sqrt{(4\vartheta-1)+(2\vartheta+1)}}\right) & \\ +4(\vartheta-1)\left(\frac{1}{\sqrt{(4\vartheta-1)+(2\vartheta-1)}}\right) & \text{if } \vartheta \text{ is odd,} \\ +\vartheta(2\vartheta-1)\left(\frac{1}{\sqrt{2(2\vartheta+1)}}\right) & \\ +\left(\frac{(2\vartheta-2)(2\vartheta-3)}{2}\right)\left(\frac{1}{\sqrt{2(2\vartheta-1)}}\right), & \\ & \\ (2\vartheta(4\vartheta-1))\left(\frac{1}{\sqrt{(4\vartheta-1)+(4\vartheta-1)}}\right), & \text{if } \vartheta \text{ is even.} \end{cases}$$

By using Lemma 4.1, Proposition 4.2, and Tables 6, and the Sum Connectivity index of the quasidihedral group with order 2^{ϑ}, we obtain

$$\chi(\mathrm{OSCom}(QD_2^{\vartheta})) = (2^{2\vartheta-1} - 2^{\vartheta-1})\left(\frac{1}{\sqrt{2(2^{\vartheta}-1)}}\right).$$

By using Lemma 4.1, Proposition 4.2, and Tables 7, and the Sum Connectivity index of the semidihedral group with order 8^{ϑ}, we obtain

$$\chi(\mathrm{OSCom}(SD_{8\vartheta})) = (4\vartheta(8\vartheta-1))\left(\frac{1}{\sqrt{(8\vartheta-1)+(8\vartheta-1)}}\right).$$

By using Lemma 4.1, Proposition 4.2, Tables 8 and 9, and the Sum Connectivity index of the $V_{8\vartheta}$ group with order 8^{ϑ}, we obtain

$$\chi(\mathrm{OSCom}(V_{8\vartheta})) = \begin{cases} \left(\frac{(2\vartheta+4)(2\vartheta+3)}{2}\right)\left(\frac{1}{\sqrt{(8\vartheta-1)+(8\vartheta-1)}}\right) & \\ +4\vartheta(\vartheta+2)\left(\frac{1}{\sqrt{(4\vartheta+3)+(8\vartheta-1)}}\right) & \\ +(4\vartheta-4)(2\vartheta+4)\left(\frac{1}{\sqrt{(6\vartheta-1)+(8\vartheta-1)}}\right) & \text{if } \vartheta \text{ is odd,} \\ +\vartheta(2\vartheta-1)\left(\frac{1}{\sqrt{(4\vartheta+3)+(4\vartheta+3)}}\right) & \\ +\left(\frac{(4\vartheta-4)(4\vartheta-5)}{2}\right)\left(\frac{1}{\sqrt{(6\vartheta-1)+(6\vartheta-1)}}\right), & \\ & \\ (4\vartheta(8\vartheta-1))\left(\frac{1}{\sqrt{(8\vartheta-1)+(8\vartheta-1)}}\right), & \text{if } \vartheta \text{ is even.} \end{cases}$$

By using Lemma 4.1, Proposition 4.2, and Tables 10, and the Sum Connectivity index of the $U_{6\vartheta}$ group with order 6^{ϑ}, we obtain

$$\begin{aligned} \chi(\mathrm{OSCom}(U_{6\vartheta})) = {} & \left(\frac{\vartheta(\vartheta-1)}{2}\right)\left(\frac{1}{\sqrt{(6\vartheta-1)+(6\vartheta-1)}}\right) + 2\vartheta^2\left(\frac{1}{\sqrt{(6\vartheta-1)+(3\vartheta-1)}}\right) \\ & + 3\vartheta^2\left(\frac{1}{\sqrt{(6\vartheta-1)+(4\vartheta-1)}}\right) + \vartheta(2\vartheta-1)\left(\frac{1}{\sqrt{(3\vartheta-1)+(3\vartheta-1)}}\right) \\ & + \left(\frac{3\vartheta(3\vartheta-1)}{2}\right)\left(\frac{1}{\sqrt{(4\vartheta-1)+(4\vartheta-1)}}\right). \end{aligned}$$

□

Theorem 5.8. *The Symmetric Division Deg index for $OSCom(\mathscr{H})$ of a finite group $\mathscr{H}$, the following holds.*

1. *For dihedral group $D_{2\vartheta}$, we have*

$$SDD(OSCom(D_{2\vartheta})) = \begin{cases} \frac{4\vartheta^3-2\vartheta+1}{2\vartheta-1}, & \text{if } \vartheta \text{ is odd,} \\ 2\vartheta(2\vartheta-1), & \text{if } \vartheta \text{ is even.} \end{cases}$$

2. *For generalized quaternion group $Q_{4\vartheta}$, we have*

$$SDD(OSCom(Q_{4\vartheta})) = \begin{cases} 2+\frac{4\vartheta(20\vartheta^2-4\vartheta+2)}{(2\vartheta+1)(4\vartheta-1)}+(8\vartheta^2-12\vartheta+6) & \\ +\frac{4(\vartheta-1)(20\vartheta^2-12\vartheta+2)}{(4\vartheta-1)(2\vartheta-1)}, & \text{if } \vartheta \text{ is odd,} \\ 4\vartheta(4\vartheta-1), & \text{if } \vartheta \text{ is even.} \end{cases}$$

3. *For quasidihedral group QD_2^{ϑ}, we have*

$$SDD(OSCom(QD_2^{\vartheta})) = 2^{\vartheta}(2^{\vartheta}-1).$$

4. *For semidihedral group $SD_{8\vartheta}$, we have*

$$SDD(OSCom(SD_{8\vartheta})) = 8\vartheta(8\vartheta-1).$$

5. *For $V_{8\vartheta}$ group, we have*

$$SDD(OSCom(V_{8\vartheta})) = \begin{cases} 24\vartheta^2-24\vartheta+32+8\vartheta(\vartheta+2)\left(\frac{40\vartheta^2+4\vartheta+5}{32\vartheta^2+20\vartheta-3}\right) & \\ +16(\vartheta-1)(\vartheta+2)\left(\frac{50\vartheta^2-14\vartheta+1}{48\vartheta^2-14\vartheta+1}\right), & \text{if } \vartheta \text{ is odd,} \\ 8\vartheta(8\vartheta-1), & \text{if } \vartheta \text{ is even.} \end{cases}$$

6. *For $U_{6\vartheta}$ group, we have*

$$SDD(OSCom(U_{6\vartheta})) = 14\vartheta^2-6\vartheta+2\vartheta^2\frac{45\vartheta^2-18\vartheta+2}{(3\vartheta-1)(6\vartheta-1)}+3\vartheta^2\frac{52\vartheta^2-20\vartheta+2}{(4\vartheta-1)(6\vartheta-1)}.$$

Proof. For every single group $\mathscr{H}$ the structure of $OSCom(\mathscr{H})$ is described in lemma 4.1, and the Symmetric Division Deg index $SDD(OSCom(\mathscr{H}))$ is defined here.

For the dihedral group $D_{2\vartheta}$, we can apply Lemma 4.1, Proposition 4.2, and refer to Tables 2 and 3, we have

$$SDD(\mathrm{OSCom}(D_{2\vartheta})) = \begin{cases} (\vartheta-1)\left(\frac{(2\vartheta-1)^2+(\vartheta-1)^2}{(\vartheta-1)(2\vartheta-1)}\right)+\vartheta\left(\frac{(2\vartheta-1)^2+\vartheta^2}{\vartheta(2\vartheta-1)}\right) & \\ +\left(\frac{(\vartheta-2)(\vartheta-1)}{2}\right)\left(\frac{(\vartheta-1)^2+(\vartheta-1)^2}{(\vartheta-1)(\vartheta-1)}\right) & \text{if } \vartheta \text{ is odd,} \\ +\left(\frac{\vartheta(\vartheta-1)}{2}\right)\left(\frac{2\vartheta^2}{\vartheta^2}\right), & \\ & \\ (\vartheta(2\vartheta-1))\left(\frac{(2\vartheta-1)^2+(2\vartheta-1)^2}{(2\vartheta-1)^2}\right) & \text{if } \vartheta \text{ is even.} \end{cases}$$

Using Lemma 4.1, Proposition 4.2, and Tables 4 and 5, we have results for the generalized quaternion group $Q_{4\vartheta}$,

$$SDD(\mathrm{OSCom}(Q_{4\vartheta})) = \begin{cases} \begin{aligned} &\left(\tfrac{(4\vartheta-1)^2+(4\vartheta-1)^2}{(4\vartheta-1)(4\vartheta-1)}\right) + 4\vartheta\left(\tfrac{(2\vartheta+1)^2+(4\vartheta-1)^2}{(2\vartheta+1)(4\vartheta-1)}\right) \\ &+4(\vartheta-1)\left(\tfrac{(2\vartheta-1)^2+(4\vartheta-1)^2}{(2\vartheta-1)(4\vartheta-1)}\right) \\ &+\vartheta(2\vartheta-1)\left(\tfrac{(2\vartheta+1)^2+(2\vartheta+1)^2}{(2\vartheta+1)(2\vartheta+1)}\right) \\ &+\left(\tfrac{(2\vartheta-2)(2\vartheta-3)}{2}\right)\left(\tfrac{(2\vartheta-1)^2+(2\vartheta-1)^2}{2(2\vartheta-1)}\right), \end{aligned} & \text{if } \vartheta \text{ is odd,} \\ \\ (2\vartheta(4\vartheta-1))\left(\tfrac{(4\vartheta-1)^2+(4\vartheta-1)^2}{(4\vartheta-1)(4\vartheta-1)}\right), & \text{if } \vartheta \text{ is even.} \end{cases}$$

Using Lemma 4.1, Proposition 4.2, and Tables 6, we have results for the quasidihedral group QD_2^{ϑ}; we have

$$SDD(\mathrm{OSCom}(QD_2^{\vartheta})) = (2^{2\vartheta-1} - 2^{\vartheta-1})\left(\frac{(2^{\vartheta}-1)^2+(2^{\vartheta}-1)^2}{(2^{\vartheta}-1)^2}\right).$$

Using Lemma 4.1, Proposition 4.2, and Tables 7, we present the results for the semidihedral group $SD_{8\vartheta}$.

$$SDD(\mathrm{OSCom}(SD_{8\vartheta})) = (4\vartheta(8\vartheta-1))\left(\frac{(8\vartheta-1)^2+(8\vartheta-1)^2}{(8\vartheta-1)(8\vartheta-1)}\right).$$

For the $V_{8\vartheta}$ group, using Lemma 4.1, Proposition 4.2, Tables 8 and 9, we have

$$SDD(\mathrm{OSCom}(V_{8\vartheta})) = \begin{cases} \begin{aligned} &\left(\tfrac{(2\vartheta+4)(2\vartheta+3)}{2}\right)\left(\tfrac{(8\vartheta-1)^2+(8\vartheta-1)^2}{(8\vartheta-1)(8\vartheta-1)}\right) \\ &+4\vartheta(\vartheta+2)\left(\tfrac{(4\vartheta+3)^2+(8\vartheta-1)^2}{(4\vartheta+3)(8\vartheta-1)}\right) \\ &+(4\vartheta-4)(2\vartheta+4)\left(\tfrac{(6\vartheta-1)^2+(8\vartheta-1)^2}{(6\vartheta-1)(8\vartheta-1)}\right) \\ &+\vartheta(2\vartheta-1)\left(\tfrac{(4\vartheta+3)^2+(4\vartheta+3)^2}{(4\vartheta+3)(4\vartheta+3)}\right) \\ &+\left(\tfrac{(4\vartheta-4)(4\vartheta-5)}{2}\right)\left(\tfrac{(6\vartheta-1)^2+(6\vartheta-1)^2}{(6\vartheta-1)(6\vartheta-1)}\right), \end{aligned} & \text{if } \vartheta \text{ is odd,} \\ \\ (4\vartheta(8\vartheta-1))\left(\tfrac{(8\vartheta-1)^2+(8\vartheta-1)^2}{(8\vartheta-1)(8\vartheta-1)}\right), & \text{if } \vartheta \text{ is even.} \end{cases}$$

For $U_{6\vartheta}$ group,we present the results using Lemma 4.1, Proposition 4.2, and Tables 10, we have

$$\begin{aligned} SDD(\mathrm{OSCom}(U_{6\vartheta})) = &\left(\frac{\vartheta(\vartheta-1)}{2}\right)\left(\frac{(6\vartheta-1)^2+(6\vartheta-1)^2}{(6\vartheta-1)(6\vartheta-1)}\right) + 2\vartheta^2\left(\frac{(3\vartheta-1)^2+(6\vartheta-1)^2}{(3\vartheta-1)(6\vartheta-1)}\right) \\ &+ 3\vartheta^2\left(\frac{(4\vartheta-1)^2+(6\vartheta-1)^2}{(4\vartheta-1)(6\vartheta-1)}\right) + \vartheta(2\vartheta-1)\left(\frac{(3\vartheta-1)^2+(3\vartheta-1)^2}{(3\vartheta-1)(3\vartheta-1)}\right) \\ &+ \left(\frac{3\vartheta(3\vartheta-1)}{2}\right)\left(\frac{(4\vartheta-1)^2+(4\vartheta-1)^2}{(4\vartheta-1)^2}\right). \end{aligned}$$

□

6. Conclusion

The article discusses many findings about the degree-based topological indices of specific kinds of super commuting graphs assigned to finite groups. The difficulty of determining topological indices for general super commuting graphs has yet to be addressed.

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Jayavel Pooja,
Department of Mathematics, College of Engineering and Technology,
SRM Institute of Science and Technology, Kattankulathur,
Tamil Nadu, India - 603203.
E-mail address: `pj5869@srmist.edu.in`

and

Venkatesan Muthukumaran,
Department of Mathematics, Faculty of Engineering and Technology,
SRM Institute of Science and Technology, Kattankulathur,
Tamil Nadu, India - 603203.

E-mail address: `muthukuv2@srmist.edu.in`

and

Suresh Elumalai,
Department of Mathematics, Faculty of Engineering and Technology,
SRM Institute of Science and Technology, Kattankulathur,
Tamil Nadu , India - 603203.
E-mail address: `sureshe1@srmist.edu.in, sureshkako@gmail.com`

and

Selvaraj Balachandran,
Department of Mathematics, School of Arts, Sciences, Humanities, and Education, SASTRA Deemed University,
Thanjavur 613 401, India.
E-mail address: `bala@maths.sastra.edu`