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Economic Order Quantity Under Constant Demand, Preservation, Trade Credit and Shortages of Non-Instantaneous Deteriorating Items

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ABSTRACT: In this paper, an Economic order quantity (EOQ) is proposed for an inventory model , integrating constant demand for non-instantaneous deteriorating items with dual trade credit policy, preservation techniques and shortages . Preservation investment reduces the rate of deterioration of products and maximizes the retailer's profit. Dual trade credit policy allows the retailer to receive a permissible and interest free period of credit from the supplier and offers the similar benefit to the customer. Shortages are allowed and assumed to be completely backlogged. Theorems are proved analytically to maximize the total profit , supported with numerical illustrations and sensitivity analysis. The findings highlight that optimal balancing of preservation cost with credit policy enhances profit considerably.

Keywords: Economic order quantity, dual trade credit, completely backlogged shortages, preservation, constant demand.

Contents

1. Introduction

To maintain supply and demand of a product, inventory maintenance is crucial. Holding excessive inventory can also lead to significant loss and inefficient inventory management. Inventory maintenance is indeed a complex process. In EOQ models, products are assumed to be non- perishable and hence have an infinite life shell. However, in practical life situations, inventory products face many risks. Deterioration in inventory is a significant concern for business, as it can lead to financial losses .The model assumes that items do not start to deteriorate immediately. Certain items remain in good condition for some time and start to deteriorate after that. Alongside demand considerations, deterioration of inventory items plays a critical role in determining optimal inventory policies, especially for perishable goods such as fruits, vegetables, dairy products, and volatile chemicals. Thus, the concept of non-instantaneous deterioration items cannot be overlooked . While deterioration is inevitable, it can be controlled to some extent through strategic investments in preservation technologies, such as cold storage or humidity control systems. By reducing the deterioration rate, firms can minimize losses and lower overall inventory costs.Here, dual credit policy is allowed as a part of the strategy to transform possible customers to potential buyers. Suppliers give retailers a credit policy and retailers allow customers to avail the same policy. Apart from maximizing his profit, they also have to minimize their loss. Consumer's demand is influenced by stock

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availability, particularly in retail environments such as supermarkets, where larger stock levels tend to attract more customers by offering a wider range of choices.In the case of unavailability of a product, shortages occur and some customers are willing to wait until the next replenishment . Others may not wait , which results in goodwill cost. This demand which is delayed due to the shortage of the product is called backlogging.This study proposes an inventory model that incorporates stock-dependent demand and constant deterioration. To mitigate the effects of deterioration, the model introduces preservation technology investment as a decision variable. In addition, due to high product demand and ongoing deterioration, shortages may arise. Since stockouts lead to potential customer dissatisfaction and penalty costs, the model permits partial backordering, a common real-world practice where only a portion of unmet demand is backordered based on customer willingness to wait.The degree of partial backordering is assumed to be a function of the waiting time, with longer waiting periods leading to higher backorder rates. This approach reflects more realistic consumer behavior during shortages and enhances the practical relevance of the model. The proposed framework aims to assist retailers in making informed decisions regarding inventory control, preservation investment, and customer service strategies to optimize overall profitability.

2. Literature Review

Vijayashree.M [29] derived economic order quantity, taking exponential penalty cost for deteriorating items with shortages completely backlogged. Khanra.S [18] developed EOQ model for deteriorating items with permissible delay in payment with demand rate taken as a quadratic function with respect to time. Yogendra kumar [30] used Weibull distribution to study the effect of deterioration items when shortages were partially backlogged. Madhukar Nagare [20] determined economic order quantity for perishable items when the number of replenishments were considered to be discrete variable, while other parameters were continuous. Chung - Yuan Dye [9] used fractional algorithm for partial backlogging deteriorating items under the effect of preservation technology to prove the uniqueness of solution as a global maximum. Chuanxu Wang [10] derived optimal price for deteriorating items for seasonal deteriorating products like mango during the storage time using ramp type function. Chih – Te Yang [12] proposed model for joint price and preservation technology for deteriorating products with dynamic pricing. Sudarshan bardhan [27] formulated an inventory model with preservation technology for non -instantaneous deteriorating items and found maximum total profit. K.V.Geetha [19] provided optimal solution for non-instantaneous deteriorating items with shortages allowed and completely backlogged. They considered demand to be constant and allowed a permissible trade credit, preservation cost was not included. S.W.Setiawan [28] explored model for deteriorating goods mathematically with shortages, without preservation and tradecredit. Aref Gholami [4] developed algorithm to control lead time using normal-log distribution and gamma distribution with shortages. Jie Min [17] discussed permissible delay in payment when the demand is constant to maximize the retailers profit for deteriorating items with two level trade credit policy. Chung – Yuan Dye [11] examined credit period with preservation technology for deteriorating items. Indrajitsingha S.K [16] developed a fuzzy model for seasonal deteriorating products by using signed distance method with shortages partially backlogged. Abu Hashan Md Mashud [2] proved that preservation cost and trade credit helps in minimizing the deterioration cost when shortages were partially backlogged and used MATLAB to prove numerical results. Biman Kanti Nath [7] found an optimal ordering policy for non-instantaneous deteriorating items for a two-warehouse problem with backlogged shortages and preservation technology. Here, the cost parameters were assumed as interval numbers. Aashish [1] systemically studied about carbon emission for sustainable environment using preservation technology with stochastic demand supply and constructed a model [29] when shortages were allowed and fully backlogged. Ali Akbar Sheikh [3] described an inventory model for time dependent demand. Shortages were allowed and partially backlogged. Preservation technology was incorporated with permissible delay in payment. Anthony [5] developed sustainable inventory model for quadratic demand of non instantaneous deteriorating items with shortages for a three warehouse problem. B.Babangida [6] determined ideal strategy for a retailer for non-instantaneous decaying goods with two phase demand rate, shortages, permissible delay in payment, two storage facilities, constant deterioration rate. Biswajit Sarkar [8] studied deteriorating products of ramp type demand with preservation technology and backlogged shortages without credit policy. Dharmendar [13] classified the model with constant deterioration rate, price-dependent demand, partially backlogged shortages and preservation technology to find the total inventory cost. G.Santhi [14] analysed inventory model in two different warehouse for price and time dependent demand with shortages completely backlogged with preservation. Ihsaan [15] analysed EOQ model to integrate improved preservation technology for non-instantaneous deteriorating items to enhance the durability of inventory .Two different channels to increase profit were discovered using boundary condition. Neeraj Kumar [21] implemented the shelf life for deteriorating productions of a two warehouse problem with partially backlogged shortages where one warehouse is owned and the other one is rented. Preservation technology in rented warehouse was considered comparatively good than the one owned. Pankaj Bhatnagar [22] adapted waste management system, reworking and preservation technology for deteriorating products with shortages partially backlogged. Priyanka Singh [23] developed inventory model for deteriorating items for price stock dependent demand, shortages completely backlogged and preservation. Ravendra Kumar [24] and Rupali [25] formulated mathematical model under inflation and trade credit for deteriorating items when shortages were partially backlogged. Shehu [26] considered shortages and partially backlogged shortages of deteriorating items in pharmaceutical industry with three parameter weibull distribution as deterioration rate and price dependent demand. Based on the analysis of the reviewed literature, a significant research gap has been identified. While many studies have considered complete backlogging of shortages for non-instantaneous deteriorating items with preservation techniques, the incorporation of a dual credit policy remains largely unexplored. This study aims to bridge this gap by integrating all these key features into a unified inventory model from retailer's point of view.

3. Assumptions and Notations

Assumptions

1. Only one item is considered.
2. Preservation cost is incorporated. $m(0) = 0$, as no investment in preservation leads to reduction in deterioration.$\lim_{\mu \to \infty} m(\mu) = 1$, as investment increases, maximum reduction approaches. Also, $m'(\mu) > 0$ and $m''(\mu) < 0$.
3. Shortages are allowed.
4. Shortages are completely backlogged and is specified by $(1 + r(T-t))^{-1}$.
5. Dual trade credit policy is allowed where the suppliers give retailers a trade credit period P. Retailers pay a partial sum of P at the time of ordering goods and has to pay the remaining within the credit period. Similiarly, retailers also give customers a partial trade credit period Q where the customers pay a partial sum at the time of buying and has to pay the remaining within the credit period.
6. Uniform demand rate is a known constant.
7. Replacement repair does not occur during the period.
8. Deterioration rate is a constant and is considered only after its life time.
9. Infinite replenishment rate and lead time is zero.
10. Sales revenue is generated from an interest bearing account, when the amount is not returned.

Notations

1.	T	Inventory cycle
2.	W(t)	Inventory at any time t
3.	D	Demand
4.	m(μ)	deterioration rate in shortage phase
5.	r	parameter for backlogging , $0 \leq \ r \leq 1$.
6.	c_h	holding cost in \$ per unit
7.	n	shortage cost in \$ per unit
8.	a	purchasing cost in \$ per order
9.	p	selling price in \$ per order
10.	μ	preservation cost in \$ per unit
11.	M_B	maximum shortage that can be backlogged per cycle
12.	M_I	maximum inventory
13.	t_0	time when inventory level is zero, $W(t_0) = 0$
14.	O_{cost}	ordering cost
15.	i_e	Interest earned by the retailer in \$ per unit
16.	i_p	Interest paid by the retailer in \$ per unit
		Decision variables - T, t_0, μ

4. Mathematical Formulation

Consider the interval $[0, T]$ to be the length of the inventory period.
Inventory dynamics is described by two differential equations as follows :

1. Inventory level in stock phase ($0 \ < t \ < \ t_0$)

$$\frac{d\,(W\,(t))}{dt} + m\,(\mu\,)\,W(t) \;=\; -\;D \tag{4.1}$$

Here inventory decreases due to both demand and deterioration

2. Inventory level in the shortage phase ($t_0 < t < T$)

$$\frac{d\,(W\,(t))}{dt} \;= -rD \tag{4.2}$$

Here, demand exceeds supply and shortages are backlogged.

General solution of (4.1) is

$$W\,(t) \;=\; -\;\frac{D}{m\,(\mu\,)} + C_1 e^{-m(\mu)t} \tag{4.3}$$

At $t = t_0$, $W\,(t) = 0$

$$C_1 = \frac{D}{m\,(\mu\,)} e^{m(\mu)t_0}$$

$$W\,(t) = \;\frac{D}{m\,(\mu\,)}\;(e^{\;m(\mu\;)(t_0 -\;t)} - 1) \tag{4.4}$$

is the solution of (4.1)
Maximum Inventory at $t = t_0$

$$M_I = \;\frac{D}{m\,(\mu\,)}\;\left(e^{\;m(\mu\;)t_0} \;- 1\right) \tag{4.5}$$

Solution of (4.2) is as follows :

$$W\,(t) = -rDt + C_2 \tag{4.6}$$

C_2 can be found by applying the boundary condition $W\,(T) = -M_B$

$$C_2 = -M_B + rDT$$

$$W\,(t) = rD\,(T - t\,) - M_B \tag{4.7}$$

is the solution of (4.2).
Hence the solution of differential equations are

W(t)= $\begin{cases} \frac{D}{m\mu}(e^{m\mu(t_0-t} - 1, (0 < t < t_0) \\ rD(T-t) - M_B \quad\quad , (t_0 < t < T) \end{cases}$

Maximum backlog

At $t = t_0$, $W\,(t) = 0$

$$M_B = rD(T - t_0) \tag{4.8}$$

Total order per cycle is

$$Tot_{ord} = M_I + M_B$$

$$Tot_{ord} = \frac{D}{m\,(\mu\,)}\ \left(e^{\,m(\mu\,)t_0} - 1\right) +\ rD(T - t_0) \tag{4.9}$$

The various costs associated with the required Inventory model are listed below:

1. Ordering cost for the inventory is

$$O_{cost} = a\ (\ Tot_{ord})$$

$$O_c =\ a\ D \left\{ \frac{(e^{\ m(\mu\ )t_0}\ - 1)}{m\,(\mu\ )}\ +\ \ r(T - t_0\ ) \right\} \tag{4.10}$$

2. Holding cost is given by

$$H_{cost} =\ \ c_h \int_0^{t_0} W(t)dt\ = c_h \int_0^{t_0} \frac{D}{m\,(\mu\ )}\ (e^{m(\mu\ )(\ t_0\ -\ t)} - 1)dt$$

$$H_{cost} = \frac{c_h D}{m\,(\mu\ )}\ \left[\frac{e^{\ m(\mu\ )t_0}\ \ - 1}{m\,(\mu\ )} -\ t_0 \right] \tag{4.11}$$

3. Sales revenue for the inventory is

$$SR = p\int_0^{t_0} Ddt + pM_B = pDt_0\ + pM_B$$

$$SR = pD(t_0 + r(T - t_0)) \tag{4.12}$$

4. Shortage cost is given by

$$S_c = -n\int_{t_0}^{T} W\,(t)\,dt\ =\ -\,n\int_{t_0}^{T} (rD\,(T - t\,) - M_B)\ dt$$

$$S_c =\ nrD \left[\frac{T^2}{2} -\ Tt_0\ +\ \ \frac{{t_0}^{\,2}}{2} \right] \tag{4.13}$$

5. Preservation cost

$$PC = \mu T \tag{4.14}$$

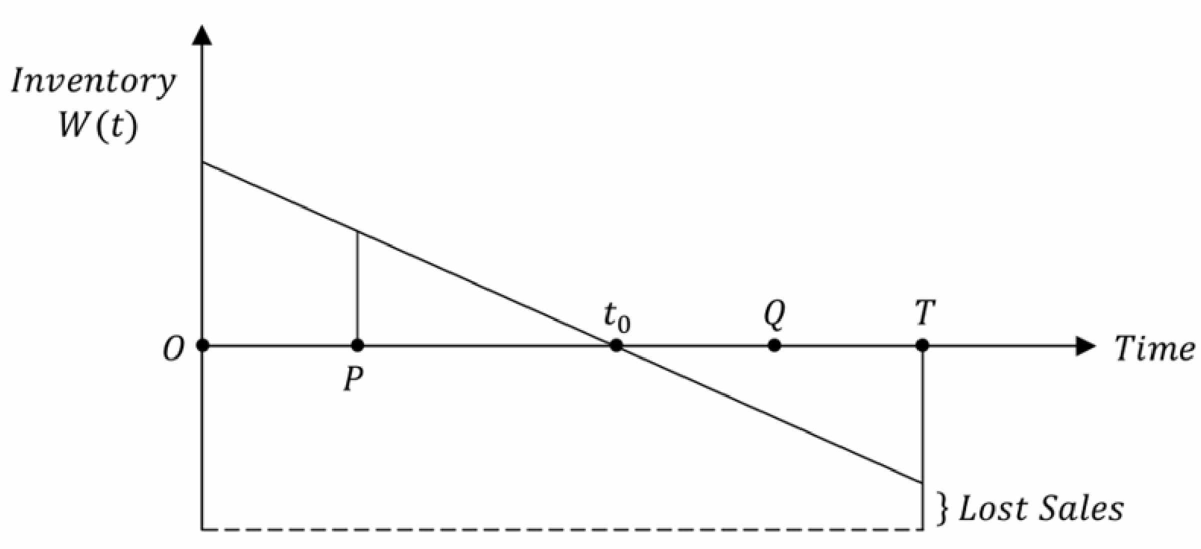

Figure 1: Inventory when $P < t_0 < Q$

6. Deterioration cost

$$d_c = \delta \int_0^{t_0} W(t)dt = \delta \int_0^{t_0} \frac{D}{m(\mu)} (e^{m(\mu)(t_0 - t)} - 1)dt$$

$$d_c = \frac{\delta D}{m(\mu)} \left[\frac{e^{m(\mu)t_0} - 1}{m(\mu)} - t_0 \right] \tag{4.15}$$

7. Interest earned and Interest paid
   This is evaluated for two different cases for the credit period P.
   i) $P < t_0 < Q$ and ii)$P > t_0$
   Case 1 :$P < t_0 < Q$
   In this case, we consider t_0 to be more than the credit period received by the retailer.
   Interest Earned by the retailers

$$I_{e1} = pi_e \left[\int_0^{t_0} \int_0^{t} Ddvdt + M_B t_0 \right] = pi_e \left[D\frac{t_0^2}{2} + M_B t_0 \right]$$

$$I_{e1} = pi_e D \left[\frac{t_0^2}{2} + r(T - t_0)\, t_0 \right] \tag{4.16}$$

Interest paid by Retailer

$$I_{p1} = ai_p \int_P^{t_0} W(t)dt = ai_p \int_P^{t_0} \frac{D}{m(\mu)} (e^{m(\mu)(t_0 - t)} - 1)dt$$

$$I_{p1} = \frac{ai_p D}{m(\mu)} \left[\frac{e^{m(\mu)(t_0 - P)} - 1}{m(\mu)} - (t_0 - P) \right] \tag{4.17}$$

Total Profit

$$TP_1(\mu, T, t_0) = \frac{1}{T}[sales\ revenue + interest\ earned - (preservation\ cost + deterioration\ cost$$
$$+ ordering\ cost + holding\ cost + shortage\ cost + interest\ paid)]$$

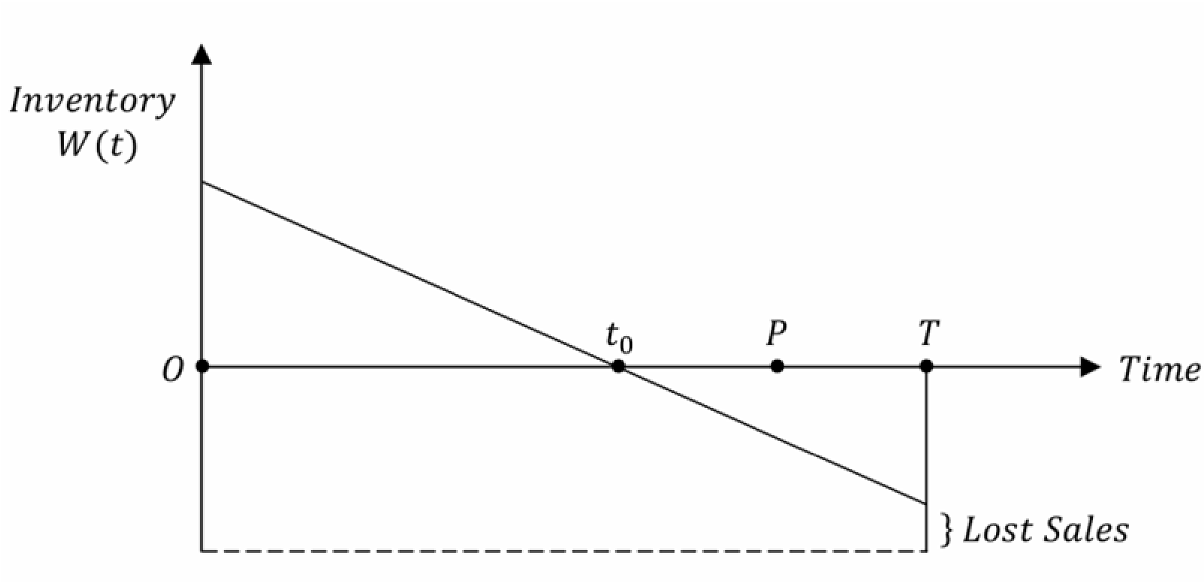

Figure 2: Inventory when $P > t_0$

$$TP_1(\mu,T,t_0) = \frac{1}{T}[pD(t_0 + r(T - t_0)) + pi_e[D\frac{t_0^2}{2} + rD(T - t_0)t_0] - [\mu T + \frac{\delta D}{m(\mu)}[\frac{e^{m(\mu)t_0} - 1}{m(\mu)} - t_0] \tag{4.18}$$

$$+ a[\frac{D}{m(\mu)}(e^{m(\mu)t_0} - 1) + rD(T - t_0)] + \frac{c_h D}{m(\mu)}(\frac{e^{m(\mu)t_0} - 1}{m(\mu)} - t_0)$$

$$+ nrD[\frac{T^2}{2} - Tt_0 + \frac{{t_0}^2}{2}] + \frac{ai_p D}{m(\mu)}[\frac{e^{m(\mu)(t_0 - P)} - 1}{m(\mu)} - (t_0 - P)]]]$$

Case 2 : $P > t_0$
Here, no interest is paid on the outstanding amount by the retailer in this period as he is within the stipulated credit period. This allows the retailer to sell the product and generate his revenue, before he pays the supplier.

Interest Earned by the retailers

$$I_{e2} = pi_e \left[\int_0^{t_0} \int_0^t D dv dt + (P - t_0)D + M_B P\right]$$

$$I_{e2} = pi_e D \left[\frac{t_0^2}{2} + (P - t_0) + r(T - t_0) P\right] \tag{4.19}$$

$$I_{p2} = 0 \tag{4.20}$$

Total Profit

$$TP_2(\mu,T,t_0) = \frac{1}{T}[\textit{sales revenue} + \textit{interest earned} - (\textit{preservation cost} + \textit{deterioration cost}$$
$$+ \textit{ordering cost} + \textit{holding cost} + \textit{shortage cost})]$$

$$= \frac{1}{T}[pD(t_0 + r(T - t_0)) + pi_e[\frac{t_0^2}{2} + (P - t_0) + r(T - t_0)p] - [\mu T + \frac{\delta D}{m(\mu)}[\frac{e^{m(\mu)t_0} - 1}{m(\mu)} - t_0]$$

$$+ a[\frac{D}{m(\mu)}(e^{m(\mu)t_0} - 1) + rD(T - t_0)] + \frac{c_h D}{m(\mu)}(\frac{e^{m(\mu)t_0} - 1}{m(\mu)} - t_0) + nrD[\frac{T^2}{2} - Tt_0 + \frac{t_0^2}{2}]]] \tag{4.21}$$

Theorem 4.1 *i) The total profit function $TP_1(\mu\ ,\ T\ ,t_0\ )$ is a concave function with respect to μ for any given value of T and t_0.*
ii) The total profit function $TP_1(\mu\ ,\ T\ ,\ t_0)$ is a concave function with respect to t_0 for any given value of μ and T.
iii) If $P < t_0 < Q$, there exist a unique maximum solution T_1^* for $TP_1(\mu\ ,\ T\ ,t_0\ )$ and $TP_1(\mu\ ,\ T\ ,\ \ t_0)$ is concave with respect to T for any given value of μ and t_0,

Proof: The necessary and sufficient condition for a function to be concave and have a maximum unique solution is obtained by showing the first partial derivative to zero and second partial derivative to be less than zero.
Expanding (4.18) by Taylor's expansion and approximating till second order we have

$$\begin{aligned} TP_1(\mu,T,t_0) = &\frac{1}{T}[pD(t_0+r(T-t_0))+pi_e[D\frac{t_0^2}{2}+rD(T-t_0)t_0] \\ &-[\mu T+\frac{\delta D}{m(\mu)}[\frac{1+\ m(\mu)t_0+(m(\mu)t_0)^2-1}{m(\mu)}-t_0]+a(\frac{D}{m(\mu)}(1+m(\mu)t_0+(m(\mu)t_0)^2-1) \\ &+rD(T-t_0))+\frac{c_hD}{m(\mu)}[\frac{1+\ m(\mu)t_0+(m(\mu)t_0)^2-1}{m(\mu)}-t_0] \\ &+nrD[\frac{T^2}{2}-Tt_0+\frac{{t_0}^2}{2}]+\frac{ai_pD}{m(\mu)}[\frac{1+\ m(\mu)(t_0-P)+(m(\mu)(t_0-P))^2-1}{m(\mu)}-(t_0-P)]]] \end{aligned} \tag{4.22}$$

On simplification

$$\begin{aligned} TP_1(\mu,T,t_0) = &\frac{1}{T}[pD(t_0+r(T-t_0))+pDi_e[\frac{t_0^2}{2}+r(T-t_0)t_0]-[\mu T+\delta Dt_0^2 \\ &+aD(t_0(1+m(\mu)t_0)+r(T-t_0r))+c_hDt_0^2+nrD[\frac{T^2}{2}-Tt_0+\frac{{t_0}^2}{2}]+ai_pD(t_0-P)^2]] \end{aligned} \tag{4.23}$$

i) Differentiating (4.23) with respect to μ, we have

$$\frac{\partial TP_1(\mu\ ,\ T\ ,\ t_0\ )}{\partial\mu} = \frac{-1}{T}\left[T\ +aDm'(\mu)t_0^2\right]=0$$

$$\frac{\partial^2 TP_1(\mu\ ,\ T\ ,\ t_0\ )}{\partial\mu^2} = \frac{2}{T^{\ 2}}\ aDm''(\mu)t_0<0 \text{ as } m''(\mu)<0$$

Hence, total profit function $TP_1(\mu\ ,\ T\ ,t_0\ )$ is a concave function with respect to μ.

ii) Taking first and second order partial derivative of (4. 23) with respect to t_0, we have

$$\begin{aligned} \frac{\partial TP_1(\mu,T,t_0)}{\partial t_0} = &\frac{1}{T}[pD(1-r)+pDi_e[t_0+r(T-2t_0)]-[2\delta Dt_0 \\ &+aD((1+2m(\mu)t_0)-r)+2Dc_ht_0+nrD[-T+t_0]+2ai_pD(t_0-P)]]=0 \end{aligned}$$

$$\frac{\partial^2 TP_1(\mu\ ,\ T\ ,\ t_0\ )}{\partial {t_0}^{\ 2}} = -\frac{D}{T}\left[pi_e\quad[2r-1]+\ [2\delta+2am\left(\mu\right)+2c_h+nr+2ai_p)]\right]<0$$

Thus, total profit function $TP_1(\mu\ ,\ T\ ,t_0\ )$ is a concave function with respect to t_0.

iii) The necessary condition for the function $TP_1(\mu, T, t_0)$ to be maximum is $\frac{\partial TP_1(\mu, T, t_0)}{\partial T} = 0$. Taking first and second order partial derivative of (4.23) with respect to T, we have

$$\begin{aligned}\frac{\partial TP_1(\mu, Tt_0)}{\partial T} =& \frac{-1}{T^2}[[pD(t_0 + r(T - t_0)) + pDi_e[\frac{t_0^2}{2} + r(T - t_0)t_0] \\ &- [\mu T + \delta D t_0^2 + aD(t_0(1 + m(\mu)t_0) + r(T - t_0) + c_h D t_0^2 + nrD[\frac{T^2}{2} - Tt_0 \\ &+ \frac{{t_0}^2}{2}] + ai_p D(t_0 - P)^2]]] + \frac{1}{T}[pDr(1 + i_e t_0) - [\mu + arD + nrD(T - t_0)]] = 0.\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 TP_1(\mu, T, t_0)}{\partial T^2} =& \frac{-2}{T^3}[pDt_0 + pDrt_0 + pDi_e\frac{t_0^2}{2} + pDi_e r t_0^2 + \mu T + \delta D t_0^2 + aDt_0 + aDm(\mu)t_0^2 \\ &+ rT + rt_0 + c_h D t_0^2 + nrD\frac{{t_0}^2}{2} + ai_p D t_0^2 + 2ai_p D t_0 P + ai_p D P^2 + nrDT^2] < 0\end{aligned}$$

$$\frac{\partial^2 TP_1(\mu, T, t_0)}{\partial T^2} < 0 \text{ when } P < t_0 < Q.$$

Hence, $TP_1(\mu, T, t_0)$ is concave with respect to T.

Unique maximum solution T_1^* for $TP_1(\mu, T, t_0)$ is obtained from (4.24). □

Theorem 4.2 *i) The total profit function $TP_2(\mu, T, t_0)$ is a concave function with respect to μ for any given value of T and t_0.*
ii) The total profit function $TP_2(\mu, T, t_0)$ is a concave function with respect to t_0 for any given value of μ and T.
iii) If $P > t_0$, there exist a unique maximum solution T_2^* for $TP_2(\mu, T, t_0)$ and $TP_2(\mu, T, t_0)$ is concave with respect to T for any given value of μ and t_0,

Proof: The proof is similar to Theorem (4.1)
Expanding (4.21) by Taylor's expansion and approximating till second order, we have

$$\begin{aligned}TP_2(\mu, T, t_0) =& \frac{1}{T}[pD(t_0 + r(T - t_0)) + pi_e D[\frac{t_0^2}{2} + (P - t_0) + r(T - t_0)p] \\ &- [\mu T + \frac{\delta D}{m(\mu)}[\frac{1 + m(\mu)t_0 + (m(\mu)t_0)^2 - 1}{m(\mu)} - t_0] + a(\frac{D}{m(\mu)}(1 + m(\mu)t_0 + (m(\mu)t_0)^2 - 1) \\ &+ rD(T - t_0)) + \frac{c_h D}{m(\mu)}[\frac{1 + m(\mu)t_0 + (m(\mu)t_0)^2 - 1}{m(\mu)} - t_0] + nrD[\frac{T^2}{2} - Tt_0 + \frac{t_0^2}{2}]]]\end{aligned} \tag{4.24}$$

On simplification

$$\begin{aligned}TP_2(\mu, T, t_0) =& \frac{1}{T}[pD(t_0 + r(T - t_0)) + pDi_e[\frac{t_0^2}{2} + (P - t_0) + r(T - t_0)p] - [\mu T + \delta D t_0^2 \\ &+ aD(t_0(1 + m(\mu)t_0) + r(T - t_0)) + c_h D t_0^2 + nrD[\frac{T^2}{2} - Tt_0 + \frac{{t_0}^2}{2}]]]\end{aligned} \tag{4.25}$$

i) Differentiating (4.26) with respect to μ, we have

$$\frac{\partial TP_2(\mu, T, t_0)}{\partial \mu} = \frac{-1}{T}\left[T + aDm'(\mu)t_0^2\right] = 0$$

$$\frac{\partial^2 TP_2(\mu, T, t_0)}{\partial \mu^2} = \frac{2}{T^2} aDm''(\mu)t_0 < 0 \quad as \quad m''(\mu) < 0$$

Hence, total profit function $TP_2(\mu, T, t_0)$ is a concave function with respect to μ.

ii) Taking first and second order partial derivative of (4.26) with respect to t_0, we have

$$\frac{\partial TP_2(\mu,T,t_0)}{\partial t_0} = \frac{1}{T}[pD(1-r)+pDi_e[t_0-1-Pr] - [2\delta Dt_0+aD)(1+2m(\mu)t_0)-r)+2Dc_ht_0+nrD[-T+t_0]]]=0$$

$$\frac{\partial^2 TP_2(\mu\ ,\ T\ ,\ t_0\ )}{\partial {t_0}^{\ 2}} = -\frac{D}{T}\ [2\delta+2am\,(\mu)\ +2c_h+nr+pi_e\ ]<0$$

Thus, total profit function $TP_2\,(\mu\ ,\ T\ ,t_0\ )$ is a concave function with respect to t_0.

iii) The necessary condition for the function $TP_1\,(\mu\ ,\ T\ ,\ t_0\ )$ to be maximum is $\frac{\partial TP_2(\mu\ ,\ T\ ,t_0\ )}{\partial T}=0$.
Taking first and second order partial derivative of (4.26) with respect to T, we have

$$\frac{\partial TP_2(\mu,T,t_0)}{\partial T} = \frac{-1}{T^2}[pD(t_0+r(T-t_0))+pDi_e[\frac{t_0^2}{2}+(P-t_0)+r(T-t_0)P] - [\mu T+\delta Dt_0^2+aD(t_0(1+m(\mu)t_0)+r(T-t_0))+c_hDt_0^2 + nrD[\frac{T^2}{2}-Tt_0+\frac{{t_0}^2}{2}]]]+\frac{1}{T}[pDr(1+i_eP)-[\mu+r+nrD(T-t_0)]]=0.$$

$$\frac{\partial^2 TP_2(\mu,T,t_0)}{\partial T^2} = \frac{-2}{T^3}[pDt_0+pDrt_0+pDi_e\frac{t_0^2}{2}+PpDi_e+pDi_et_0+Prt_0+\delta Dt_0^2 + aDt_0+aDm(\mu)t_0^2+rt_0+c_hDt_0^2+nrDTt_0+nrD\frac{t_0^2}{2} + Ppi_eDrT+2ai_pDt_0P+nrD\frac{T^2}{2}]<0$$

$$\frac{\partial^2 TP_2(\mu\ ,\ T\ ,\ t_0\ )}{\partial T^2}<0 \text{ when } P>t_0.$$

Unique maximum solution T_2^* for $TP_2\,(\mu\ ,\ T\ ,t_0\ )$ is obtained from (4.2).
Hence, $TP_2\,(\mu\ ,\ T\ ,\ \ t_0)$ is concave with respect to T. □

5. Numerical Examples

The following two examples are illustrated for the cases $P<t_0$ and $P>t_0$ when the credit period P received by the retailer is less than the credit period Q, given by the retailer to the customer.

Example 5.1 *Let T = 11 months, P = 2 months, Q = 7 months , D = 200 units/month , r= 0.5,* $a=\$50/unit$, $p=\$100/unit$, $\mu=\$5/unit$, $i_e=0.2$, $i_p=0.3$, $c_h=\$4/unit$, $n=\$10/unit$, $\delta=\$5$, *optimum* $t_0=3.5$ *months, Optimum quantity ordered is 240.4 units. Total Optimum Maximum profit in this case is* $TP_1\,(\mu\ ,\ T\ ,\ \ t_0)=\$12,470$.

Example 5.2 *Taking values as in example 1, except , P = 2 months, Q = 5 months, optimum* $t_0=1.8$ *months, Optimum quantity ordered is 195.6 units*
Total Optimum Maximum profit in this case is $TP_2\,(\mu\ ,\ T\ ,\ \ t_0)=\$12,250$.

6. Sensitivity Analysis

Based on the above two examples, sensitivity analysis is performed and total profit is calculated by changing only one parameter at a time and others fixed from -20% to 20%. Total profits in both the cases are calculated by varying one of the decision variables μ, T, t_0 and keeping the other two to be fixed values as in the examples. While calculating the sensitivity analysis of $TP_2\,(\mu,T,t_0)$ in table 2, by varying t_0, the values are truncated at 10%. Beyond this, the condition $P=2<t_0$ is violated . However, it does not affect the optimum value of the analysis.

Table 1: Sensitivity Analysis of Total Profit $TP_1(\mu, T, t_0)$ when $P = 2 < t_0$

Change in %	μ	**Total Profit (\$)(by varying μ)**	**T**	**Total Profit (\$) (by varying T)**	t_0	**Total Profit (\$) (by varying t_0)**
−20%	4.00	12,300	8.80	12,120	2.80	12,190
−15%	4.25	12,360	9.35	12,260	2.98	12,280
−10%	4.50	12,410	9.90	12,370	3.15	12,360
−5%	4.75	12,450	10.45	12,440	3.32	12,420
0	**5.00**	**12,470**	**11.00**	**12,470**	**3.50**	**12,470**
5%	5.25	12,460	11.55	12,460	3.67	12,460
10%	5.50	12,430	12.10	12,420	3.85	12,420
15%	5.75	12,380	12.65	12,360	4.02	12,350
20%	6.00	12,310	13.20	12,280	4.20	12,270

Table 2: Sensitivity Analysis of Total Profit $TP_2(\mu, T, t_0)$ when $P = 2 > t_0$

Change in %	μ	**Total Profit (\$)(by varying μ)**	**T**	**Total Profit (\$) (by varying T)**	t_0	**Total Profit (\$) (by varying t_0)**
-20%	3.84	11,870	8.40	11,900	1.44	12,000
-15%	4.08	12,010	8.93	12,050	1.53	12,080
-10%	4.32	12,120	9.45	12,140	1.62	12,140
-5%	4.56	12,200	9.98	12,200	1.71	12,200
0	**4.80**	**12,250**	**10.50**	**12,250**	**1.80**	**12,250**
5%	5.04	12,230	11.03	12,230	1.89	12,230
10%	5.28	12,180	11.55	12,180	1.98	12,180
15%	5.52	12,110	12.08	12,110	2.07	12,110
20%	5.76	12,020	12.60	12,020	2.16	12,020

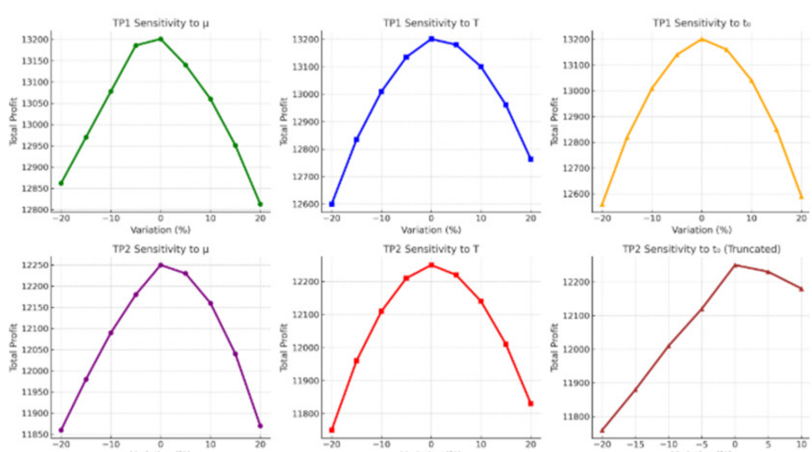

Figure 3: Graphical Representation of Total Profits TP_1 and TP_2 vs (μ, T, t_0)

7. Insights

i) Preservation (μ) : In case 1, inventory is held for a longer time. To protect against increased deterioration, the retailer invests in preservation. In case 2, there is less need for preservation as deterioration time is low. In both the cases, total profit increases as μ increases . Beyond the optimum value, profits starts to decline. This indicates that the diminishing returns of the total profit is due to higher investment in preservation. Investing moderately in preservation may yield higher returns without risk.

ii) Inventory cycle Time (T): In case 1, longer cycle means fewer orders per year. In case 2, since the cycles are shorter, revenue is comparatively less. In both the cases, profit increases steadily from -20% upto the optimum cycle time and gradually decreases. This creates a sharp concave curve and implies that small deviations from optimal value of T significantly affects profit. Longer cycle increases profit, but too longer cycle reduces profit due to higher holding cost and the interests.

iii) Inventory depletion time (t_0): Total profit increases towards the optimum point and then declines . But in case 2, the sensitivity of t_0 has to be truncated at 10% to maintain the condition $P > t_0$.

iv) Credit Period(P): When the credit period is shorter than the inventory depletion time as in case 1, interest paid rises. Case 2 benefits from interest earned , but requires control over depletion timing and backlogging of shortages.

v) Comparison of two cases : Under the given parameters, we see that case 1 yields slightly higher profit than case 2 . In case 1, credit period ends before inventory runs out, retailer pays interest to the supplier, requires longer cycle and preservation and yields higher profit. But, in case 2, credit period ends after inventory is depleted, retailer earns interest on revenue, requires quicker depletion smaller cycles and gets slightly lower profits.

8. Conclusion

This paper presents the importance of jointly optimizing preservation investment, inventory cycle and shortage management under trade credit constraints .The model highlights insights for retailers handling non- instantaneous deteriorated goods, suggesting that credit policies with optimal preservation can improve profitability. Profits in stock phase is higher than in the case when shortages are backlogged, under specific parameters due to strategic use of preservation and inventory cycle. Importance of adhering to trade credits was visible when variations in t_0 were limited in case 2. In conclusion, retailers can enhance their overall profit, by strategically balancing investment in preservation and inventory timing in alignment with credit policy when shortages are completely backlogged.

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