



## Some New Identities Involving Generalized the Multivariable $H$ –Functions

Priyanka Gupta, Bhupendra Tripathi \* and Roshani Singh

ABSTRACT: This study establishes some new identities involving the multivariable H-functions. The symbolic manipulation of equations through the use of multivariable H-functions is crucial and useful for both applied and pure mathematics. The findings obtained here are fundamental in nature and comprise both fresh and well-known findings as specific examples.

Keywords: H-function of two variables, multivariable H-functions, identities, hyper geometric function.

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### 1. Introduction

The multivariable H-function, which was introduced by Srivastava (1978) and investigated by Srivastava et al.(1982) [4,5] in term of a multiple Mellin -Bernes type contour integral as

$$\begin{aligned}
 H[z_1, \dots, z_r] &= H_{p,q;p_1,q_1:\dots:p_r,q_r}^{0,n;m_1,n_1:\dots:m_r,n_r} \left[ \begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \middle| \begin{matrix} (a_j; \alpha_j^1, \dots, \alpha_j^r)_{1,p}; (c_j^1, \gamma_j^1)_{1,p_1}; \dots; (c_j^r, \gamma_j^r)_{1,p_r} \\ (b_j; \beta_j^1, \dots, \beta_j^r)_{1,q}; (d_j^1, \delta_j^1)_{1,q_1}; \dots; (d_j^r, \delta_j^r)_{1,q_r} \end{matrix} \right] \\
 &= \frac{1}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \psi(\xi_1, \dots, \xi_r) \left\{ \prod_{i=1}^r \phi_i(\xi_i) z_i^{\xi_i} \right\} d\xi_1 \dots d\xi_r.
 \end{aligned} \tag{1.1}$$

where  $\omega = \sqrt{-1}$  and

$$\psi(\xi_1, \dots, \xi_r) = \frac{\prod_{j=1}^n \Gamma(1 - a_j + \sum_{i=1}^r \alpha_j^{(i)} \xi_i)}{\prod_{j=n+1}^p \Gamma(a_j - \sum_{i=1}^r \alpha_j^{(i)} \xi_i) \prod_{j=1}^q \Gamma(1 - b_j + \sum_{i=1}^r \beta_j^{(i)} \xi_i)} \tag{1.2}$$

$$\phi_i(\xi_i) = \frac{\prod_{j=n_i+1}^{m_i} \Gamma(1 - c_j^{(i)} + \gamma_j^{(i)} \xi_i) \prod_{j=1}^{m_i} \Gamma(d_j^{(i)} + \delta_j^{(i)} \xi_i)}{\prod_{j=1}^{p_i} \Gamma(c_j^{(i)} + \gamma_j^{(i)} \xi_i) \prod_{j=m_i+1}^{q_i} \Gamma(1 - d_j^{(i)} + \delta_j^{(i)} \xi_i)} \quad (i = 1, \dots, r) \tag{1.3}$$

And  $L_j = L_{\omega\tau_j\infty}$  represents the contours which start at the point  $\tau_j - \omega\infty$  and terminate at the points  $\tau_j - \omega\infty$  with  $\tau_j \in \Re = (-\infty, \infty) (j = 1, \dots, r)$ .

In eqn 1.1,  $i$  in the superscript  $(i)$  stands for the number of primes e.g.  $b^{(1)} = b', b^{(2)} = b''$  and so on; and an empty product is interpreted as unity. Suppose, as use that the parameters

$$a_j, j = 1, \dots, p; c_j^{(i)}, j = 1, \dots, p_i$$

\* Corresponding author

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$$b_j, j = 1, \dots, q; d_j^{(i)}, j = 1, \dots, q_i; \hat{I}(1, \dots, r)$$

are complex numbers and the associated coefficients

$$\alpha_j, j = 1, \dots, p; \gamma_j^{(i)}, j = 1, \dots, p_i; \quad (1.4)$$

$$\beta_j, j = 1, \dots, q; \delta_j^{(i)}, j = 1, \dots, q_i; \forall i \in \{1, \dots, r\} \quad (1.5)$$

positive real numbers such that the left of the contour . Also

$$V_i = \sum_{j=1}^p \alpha_j^{(i)} + \sum_{j=1}^{p_i} \gamma_j^{(i)} - \sum_{j=1}^q \beta_j^{(i)} - \gamma \sum_{j=1}^{q_i} \delta_j^{(i)} < 0 \quad (1.6)$$

$$\Omega_i = \sum_{j=n+1}^p \alpha_j^{(i)} - \sum_{j=1}^q \beta_j^{(i)} + \sum_{j=1}^{m_i} \delta_j^{(i)} - \sum_{j=1}^{q_i} \delta_j^{(i)} + \sum_{j=1}^{n_i} \gamma_j^{(i)} - \sum_{j=n+1}^{p_i} \gamma_j^{(i)} > 0 \quad (1.7)$$

Where the integral  $n, p, q, m_i, n_i, p_i$  and  $q_i$  are constrained by the inequalities  $p \geq n \geq 0, q \geq 0, q_i \geq m_i \geq 1$  and  $p_i \geq n_i \geq 1, \forall i \in \{1, 2, \dots, r\}$  and the inequalities hold for suitably restricted values of the complex variables  $z_1, \dots, z_r$ . The sequence of parameters in eqn 1.1 are such that none of poles of the integrand coincide, that is, the poles of the integrand coincide, that is, the poles of the integrand in eqn 1.1 are simple. The contour  $L_i$  in the complex  $\xi_i$ - plane is of the Mellin - Barnes type which runs from  $-\omega_\infty$  to  $\omega_\infty$  with indentations, if necessary to ensure that all the poles of  $\Gamma(d_j^{(i)} + \delta_j^{(i)} \xi_i)$ , where  $j = 1, \dots, m_i$  are separated from those of  $\Gamma(1 - c_j^{(i)} + \gamma_j^{(i)} \xi_i)$ , where  $j = 1, \dots, n_i$ .

## 2. Result Required in the Sequel

From Rainville(1971) [3], my investigation requires the following formula:

$$z\Gamma z = \Gamma(n+1) \quad (2.1)$$

$$\Gamma(\alpha-r)\Gamma(1-\alpha+r) = (-1)^r \Gamma(1-\alpha)\Gamma\alpha \quad (2.2)$$

## 3. Identities

In this paper, we will to establish some new identities involving generalized the multivariable H-functions defined by Mittal and Gupta(1972) [2]. Provided that where  $|arg(z_\rho)| < \frac{1}{2}V_\rho\pi, \forall \rho \in \{1, \dots, r\}$  is given in eqn 1.6.

$$\begin{aligned} & H_{p,q;p_1+2,q_1+2;\dots;p_r,q_r}^{0,n;m_1,n_1;\dots;m_r,n_r} \left[ \begin{array}{c} z_1 \\ \vdots \\ z_r \end{array} \left| \begin{array}{c} (a_j, \alpha_j^1, \dots, \alpha_j^r)_{1,p}; (c_j^1, \gamma_j^1)_{1,p_1}, (\sigma+r, \nu)(\rho+r, \mu); \dots; (c_j^r, \gamma_j^r)_{1,p_r} \\ (b_j; \beta_j^1, \dots, \beta_j^r)_{1,q}; (d_j^1, \delta_j^1)_{1,q_1}, (\sigma+r, \nu)(\rho+r, \mu); \dots; (d_j^r, \delta_j^r)_{1,q_r} \end{array} \right. \right] \\ &= (-1)^{2r} H_{p,q;p_1+2,q_1+2;\dots;p_r,q_r}^{0,n;m_1,n_1;\dots;m_r,n_r} \left[ \begin{array}{c} z_1 \\ \vdots \\ z_r \end{array} \left| \begin{array}{c} (a_j, \alpha_j^1, \dots, \alpha_j^r)_{1,p}; (c_j^1, \gamma_j^1)_{1,p_1}, (\sigma, \nu)(\rho, \mu); \dots; (c_j^r, \gamma_j^r)_{1,p_r} \\ (b_j; \beta_j^1, \dots, \beta_j^r)_{1,q}; (d_j^1, \delta_j^1)_{1,q_1}, (\sigma, \nu)(\rho, \mu); \dots; (d_j^r, \delta_j^r)_{1,q_r} \end{array} \right. \right]. \end{aligned} \quad (3.1)$$

$$\begin{aligned} & H_{p,q;p_1+2,q_1+2;\dots;p_r,q_r}^{0,n;m_1+2,n_1+2;\dots;m_r,n_r} \left[ \begin{array}{c} z_1 \\ \vdots \\ z_r \end{array} \left| \begin{array}{c} (a_j, \alpha_j^1, \dots, \alpha_j^r)_{1,p}; (c_j^1, \gamma_j^1)_{1,p_1}, (\sigma+r, \nu)(\rho+r, \mu); \dots; (c_j^r, \gamma_j^r)_{1,p_r} \\ (b_j; \beta_j^1, \dots, \beta_j^r)_{1,q}; (d_j^1, \delta_j^1)_{1,q_1}, (\sigma+r, \nu)(\rho+r, \mu); \dots; (d_j^r, \delta_j^r)_{1,q_r} \end{array} \right. \right] \\ &= (-1)^{2r} H_{p,q;p_1+2,q_1+2;\dots;p_r,q_r}^{0,n;m_1+2,n_1+2;\dots;m_r,n_r} \left[ \begin{array}{c} z_1 \\ \vdots \\ z_r \end{array} \left| \begin{array}{c} (a_j, \alpha_j^1, \dots, \alpha_j^r)_{1,p}; (c_j^1, \gamma_j^1)_{1,p_1}, (\sigma, \nu)(\rho, \mu); \dots; (c_j^r, \gamma_j^r)_{1,p_r} \\ (b_j; \beta_j^1, \dots, \beta_j^r)_{1,q}; (d_j^1, \delta_j^1)_{1,q_1}, (\sigma, \nu)(\rho, \mu); \dots; (d_j^r, \delta_j^r)_{1,q_r} \end{array} \right. \right]. \end{aligned} \quad (3.2)$$

$$\begin{aligned}
 & H_{p,q;p_1+1,q_1+2;\dots;p_r,q_r}^{0,n;m_1+2,n_1;\dots;m_r,n_r} \left[ \begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \left| \begin{matrix} (a_j, \alpha_j^1, \dots, \alpha_j^r)_{1,p} : (c_j^1, \gamma_j^1)_{1,p_1}, (1-\rho, \mu); \dots; (c_j^r, \gamma_j^r)_{1,p_r} \\ (b_j; \beta_j^1, \dots, \beta_j^r)_{1,q} : (1, \sigma), (2-\rho, \mu), (d_j^1, \delta_j^1)_{1,q_1}; \dots; (d_j^r, \delta_j^r)_{1,q_r} \end{matrix} \right. \right] \\
 &= (1-\rho) H_{p,q;p_1+1,q_1+2;\dots;p_r,q_r}^{0,n;m_1+1,n_1;\dots;m_r,n_r} \left[ \begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \left| \begin{matrix} (a_j, \alpha_j^1, \dots, \alpha_j^r)_{1,p} : (c_j^1, \gamma_j^1)_{1,p_1}; \dots; (c_j^r, \gamma_j^r)_{1,p_r} \\ (b_j; \beta_j^1, \dots, \beta_j^r)_{1,q} : (1, \sigma), (d_j^1, \delta_j^1)_{1,q_1}; \dots; (d_j^r, \delta_j^r)_{1,q_r} \end{matrix} \right. \right] \\
 &\quad - H_{p,q;p_1+1,q_1+2;\dots;p_r,q_r}^{0,n;m_1+1,n_1+1;\dots;m_r,n_r} \left[ \begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \left| \begin{matrix} (a_j, \alpha_j^1, \dots, \alpha_j^r)_{1,p} : (0, \mu), (c_j^1, \gamma_j^1)_{1,p_1}; \dots; (c_j^r, \gamma_j^r)_{1,p_r} \\ (b_j; \beta_j^1, \dots, \beta_j^r)_{1,q} : (1, \sigma), (d_j^1, \delta_j^1)_{1,q_1}, (1, \mu); \dots; (d_j^r, \delta_j^r)_{1,q_r} \end{matrix} \right. \right].
 \end{aligned} \tag{3.3}$$

$$\begin{aligned}
 & H_{p,q;p_1+1,q_1+2;\dots;p_r,q_r}^{0,n;m_1+1,n_1+1;\dots;m_r,n_r} \left[ \begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \left| \begin{matrix} (a_j, \alpha_j^1, \dots, \alpha_j^r)_{1,p} : (1-\rho, \mu), (c_j^1, \gamma_j^1)_{1,p_1}; \dots; (c_j^r, \gamma_j^r)_{1,p_r} \\ (b_j; \beta_j^1, \dots, \beta_j^r)_{1,q} : (1, \sigma), (d_j^1, \delta_j^1)_{1,q_1}, (\rho, \mu); \dots; (d_j^r, \delta_j^r)_{1,q_r} \end{matrix} \right. \right] \\
 &= (1-\rho) H_{p,q;p_1+1,q_1+2;\dots;p_r,q_r}^{0,n;m_1+1,n_1;\dots;m_r,n_r} \left[ \begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \left| \begin{matrix} (a_j, \alpha_j^1, \dots, \alpha_j^r)_{1,p} : (c_j^1, \gamma_j^1)_{1,p_1}; \dots; (c_j^r, \gamma_j^r)_{1,p_r} \\ (b_j; \beta_j^1, \dots, \beta_j^r)_{1,q} : (1, \sigma), (d_j^1, \delta_j^1)_{1,q_1}; \dots; (d_j^r, \delta_j^r)_{1,q_r} \end{matrix} \right. \right] \\
 &\quad + H_{p,q;p_1+1,q_1+2;\dots;p_r,q_r}^{0,n;m_1+1,n_1+1;\dots;m_r,n_r} \left[ \begin{matrix} z_1 \\ \vdots \\ z_r \end{matrix} \left| \begin{matrix} (a_j, \alpha_j^1, \dots, \alpha_j^r)_{1,p} : (0, \mu), (c_j^1, \gamma_j^1)_{1,p_1}; \dots; (c_j^r, \gamma_j^r)_{1,p_r} \\ (b_j; \beta_j^1, \dots, \beta_j^r)_{1,q} : (1, \sigma), (d_j^1, \delta_j^1)_{1,q_1}, (1, \mu); \dots; (d_j^r, \delta_j^r)_{1,q_r} \end{matrix} \right. \right].
 \end{aligned} \tag{3.4}$$

**Proof:** To prove above eqn 3.1 we use 1.1 and Fox (1961) [1]. We get following integral

$$\begin{aligned}
 &= \frac{1}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \frac{1}{\Gamma(1-\sigma+v\xi) - r\Gamma(1-\sigma+v\xi) + r\Gamma(1-\rho+\mu\xi) + r\Gamma(1-(1-\sigma+\mu\xi)+r)} \\
 &\quad \psi(\xi_1, \dots, \xi_r) \{ \prod_{i=1}^r \phi_i(\xi_i) z_i^{\xi_i} \} d\xi_1 \dots d\xi_r \\
 &= \frac{(-1)^{2r}}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \frac{1}{\Gamma(1-(1-\sigma+v\xi)\Gamma(1-\sigma+v\xi)\Gamma(1-(1-\rho+\mu\xi)\Gamma(1-\sigma+\mu\xi)} \\
 &\quad \psi(\xi_1, \dots, \xi_r) \{ \prod_{i=1}^r \phi_i(\xi_i) z_i^{\xi_i} \} d\xi_1 \dots d\xi_r
 \end{aligned}$$

[On using eqn 2.2]

$$\begin{aligned}
 &= \frac{(-1)^{2r}}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \frac{1}{\Gamma\sigma + v\xi\Gamma(1-\sigma+v\xi)\Gamma\rho - \mu\xi\Gamma(1-\rho+\mu\xi)} \\
 &\quad \psi(\xi_1, \dots, \xi_r) \{ \prod_{i=1}^r \phi_i(\xi_i) z_i^{\xi_i} \} d\xi_1 \dots d\xi_r
 \end{aligned}$$

which in the light of eqn 1.1 provides right hand side of 3.1. Similarly, the result 3.2 can be established.

Again we have to prove equation 3.3, consider left hand side of 3.3 after using 1.1 to obtain

$$\begin{aligned}
 &= \frac{1}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \psi(\xi_1, \dots, \xi_r) \frac{\Gamma(1-\sigma\xi)\Gamma(2-\rho-\mu\xi)}{\Gamma(1-\rho-\mu\xi)} \{ \prod_{i=1}^r \phi_i(\xi_i) z_i^{\xi_i} \} d\xi_1 \dots d\xi_r \\
 &= \frac{1}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \psi(\xi_1, \dots, \xi_r) \Gamma(1-\sigma\xi)(1-\rho-\mu\xi) \{ \prod_{i=1}^r \phi_i(\xi_i) z_i^{\xi_i} \} d\xi_1 \dots d\xi_r
 \end{aligned}$$

[On using eqn 2.1]

$$\begin{aligned}
 &= \frac{1-\rho}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \psi(\xi_1, \dots, \xi_r) \Gamma(1-\sigma\xi) \{ \prod_{i=1}^r \phi_i(\xi_i) z_i^{\xi_i} \} d\xi_1 \dots d\xi_r \\
 &= \frac{1}{(2\pi\omega)^r} \int_{L_1} \dots \int_{L_r} \psi(\xi_1, \dots, \xi_r) \frac{\Gamma(1-\sigma\xi)\Gamma(1+v\xi)}{\Gamma(v\xi)} \{ \prod_{i=1}^r \phi_i(\xi_i) z_i^{\xi_i} \} d\xi_1 \dots d\xi_r
 \end{aligned}$$

Which in the light of eqn 1.1 provides right hand side of eqn 3.3. Similarly, the result can be established.

#### 4. Particular Cases

Putting  $r = 2$ , in Identities 3.1, 3.2, 3.3 also the new results may be realized to the H-function of two variables:

$$\begin{aligned}
\text{(a)} \quad & H_{p,q;p_1+1,q_1+2;p_2,q_2}^{0,n;m_1+2,n_1;m_2,n_2} \left[ \begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \end{matrix} \left| \begin{matrix} (a_j, \alpha_j^1, \alpha_j^2)_{1,p}; (c_j^1, \gamma_j^1)_{1,p_1} (1-\rho, \mu); (c_j^2, \gamma_j^2)_{1,p_2} \\ (b_j, \beta_j^1, \beta_j^2)_{1,q}; (1, \sigma), (2-\rho, \mu) (d_j^1, \delta_j^1)_{1,q_1}; (d_j^2, \delta_j^2)_{1,q_2} \end{matrix} \right. \right] \\
&= (1-\rho) H_{p,q;p_1,q_1+2;p_2,q_2}^{0,n;m_1+1,n_1;m_2,n_2} \left[ \begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \end{matrix} \left| \begin{matrix} (a_j, \alpha_j^1, \alpha_j^2)_{1,p}; (\rho-1, \mu), (c_j^1, \gamma_j^1)_{1,p_1}; (c_j^2, \gamma_j^2)_{1,p_2} \\ (b_j, \beta_j^1, \beta_j^2)_{1,q}; (1, \sigma) (d_j^1, \delta_j^1)_{1,q_1}; (d_j^2, \delta_j^2)_{1,q_2} \end{matrix} \right. \right] \\
&\quad - H_{p,q;p_1+1,q_1+2;p_2,q_2}^{0,n;m_1+1,n_1+1;m_2,n_2} \left[ \begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \end{matrix} \left| \begin{matrix} (a_j, \alpha_j^1, \alpha_j^2)_{1,p}; (0, \mu), (c_j^1, \gamma_j^1)_{1,p_1}; (c_j^2, \gamma_j^2)_{1,p_2} \\ (b_j, \beta_j^1, \beta_j^2)_{1,q}; (1, \sigma) (d_j^1, \delta_j^1)_{1,q_1}; (1, \mu); (d_j^2, \delta_j^2)_{1,q_2} \end{matrix} \right. \right] \tag{4.1}
\end{aligned}$$

$$\begin{aligned}
\text{(b)} \quad & H_{p,q;p_1+1,q_1+2;p_2,q_2}^{0,n;m_1+1,n_1+1;m_2,n_2} \left[ \begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \end{matrix} \left| \begin{matrix} (a_j, \alpha_j^1, \alpha_j^2)_{1,p}; (1-\rho, \mu), (c_j^1, \gamma_j^1)_{1,p_1}; (c_j^2, \gamma_j^2)_{1,p_2} \\ (b_j, \beta_j^1, \beta_j^2)_{1,q}; (1, \sigma), (d_j^1, \delta_j^1)_{1,q_1}; (\rho, \mu); (d_j^2, \delta_j^2)_{1,q_2} \end{matrix} \right. \right] \\
&= (1-\rho) H_{p,q;p_1,q_1+1+2;p_2,q_2}^{0,n;m_1+1,n_1;m_2,n_2} \left[ \begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \end{matrix} \left| \begin{matrix} (a_j, \alpha_j^1, \alpha_j^2)_{1,p}; (c_j^1, \gamma_j^1)_{1,p_1}; (c_j^2, \gamma_j^2)_{1,p_2} \\ (b_j, \beta_j^1, \beta_j^2)_{1,q}; (1, \sigma) (d_j^1, \delta_j^1)_{1,q_1}; (d_j^2, \delta_j^2)_{1,q_2} \end{matrix} \right. \right] \\
&\quad + H_{p,q;p_1+1,q_1+2;p_2,q_2}^{0,n;m_1+1,n_1+1;m_2,n_2} \left[ \begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \end{matrix} \left| \begin{matrix} (a_j, \alpha_j^1, \alpha_j^2)_{1,p}; (0, \mu), (c_j^1, \gamma_j^1)_{1,p_1}; (c_j^2, \gamma_j^2)_{1,p_2} \\ (b_j, \beta_j^1, \beta_j^2)_{1,q}; (1, \sigma) (d_j^1, \delta_j^1)_{1,q_1}; (1, \mu); (d_j^2, \delta_j^2)_{1,q_2} \end{matrix} \right. \right] \tag{4.2}
\end{aligned}$$

$$\begin{aligned}
\text{(c)} \quad & H_{p,q;p_1+2,q_1+2;p_2,q_2}^{0,n;m_1,n_1;m_2,n_2} \left[ \begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \end{matrix} \left| \begin{matrix} (a_j, \alpha_j^1, \alpha_j^2)_{1,p}; (c_j^1, \gamma_j^1)_{1,p_1} (\sigma+r, \nu) (\sigma+r, \mu); (c_j^2, \gamma_j^2)_{1,p_2} \\ (b_j, \beta_j^1, \beta_j^2)_{1,q}; (d_j^1, \delta_j^1)_{1,q_1}; (\sigma+r, \nu) (\sigma+r, \mu); (d_j^2, \delta_j^2)_{1,q_2} \end{matrix} \right. \right] \\
&= (-1)^{2r} H_{p,q;p_1+2,q_1+2;p_2,q_2}^{0,n;m_1,n_1;m_2,n_2} \left[ \begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \end{matrix} \left| \begin{matrix} (a_j, \alpha_j^1, \alpha_j^2)_{1,p}; (c_j^1, \gamma_j^1)_{1,p_1} (\sigma, \nu) (\sigma, \mu); (c_j^2, \gamma_j^2)_{1,p_2} \\ (b_j, \beta_j^1, \beta_j^2)_{1,q}; (d_j^1, \delta_j^1)_{1,q_1}; (\sigma, \nu) (\sigma, \mu); (d_j^2, \delta_j^2)_{1,q_2} \end{matrix} \right. \right] \tag{4.3}
\end{aligned}$$

$$\begin{aligned}
\text{(d)} \quad & H_{p,q;p_1+2,q_1+2;p_2,q_2}^{0,n;m_1+2,n_1+2;m_2,n_2} \left[ \begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \end{matrix} \left| \begin{matrix} (a_j, \alpha_j^1, \alpha_j^2)_{1,p}; (\sigma+r, \nu) (\sigma+r, \mu) (c_j^1, \gamma_j^1)_{1,p_1}; (c_j^2, \gamma_j^2)_{1,p_2} \\ (b_j, \beta_j^1, \beta_j^2)_{1,q}; (\sigma+r, \nu) (\sigma+r, \mu), (d_j^1, \delta_j^1)_{1,q_1}; (d_j^2, \delta_j^2)_{1,q_2} \end{matrix} \right. \right] \\
&= (-1)^{2r} H_{p,q;p_1+2,q_1+2;p_2,q_2}^{0,n;m_1+2,n_1+2;m_2,n_2} \left[ \begin{matrix} z_1 \\ \cdot \\ \cdot \\ \cdot \end{matrix} \left| \begin{matrix} (a_j, \alpha_j^1, \alpha_j^2)_{1,p}; (\sigma, \nu) (\sigma, \mu) (c_j^1, \gamma_j^1)_{1,p_1}; (c_j^2, \gamma_j^2)_{1,p_2} \\ (b_j, \beta_j^1, \beta_j^2)_{1,q}; (\sigma, \nu) (\sigma, \mu), (d_j^1, \delta_j^1)_{1,q_1}; (d_j^2, \delta_j^2)_{1,q_2} \end{matrix} \right. \right] \tag{4.4}
\end{aligned}$$

Again, putting  $n = p = q = 0, r = 1$  in Identities 3.1, 3.2, 3.3, 3.4 also ; the new results may be realized to the H-function of one variable

$$\begin{aligned}
\text{(e)} \quad & H_{p_1+1,q_1+2}^{m_1+2,n_1} \left[ z \left| \begin{matrix} (a_j, \alpha_j)_{1,p}; (1-\rho, \mu) \\ (1, \sigma), (2-\rho, \mu), (b_j, \beta_j)_{1,q}, (\rho, \mu) \end{matrix} \right. \right] \\
&= (1-\rho) H_{p_1,q_1+1}^{m_1+1,n_1} \left[ z \left| \begin{matrix} (a_j, \alpha_j)_{1,p} \\ (1, \sigma), (b_j, \beta_j)_{1,q} \end{matrix} \right. \right] - H_{p_1+1,q_1+2}^{m_1+1,n_1+1} \left[ z \left| \begin{matrix} (0, \mu), (a_j, \alpha_j)_{1,p} \\ (1, \sigma), (b_j, \beta_j)_{1,q}, (1, \mu) \end{matrix} \right. \right] \tag{4.5}
\end{aligned}$$

$$\begin{aligned}
\text{(f)} \quad & H_{p_1+1,q_1+2}^{m_1+1,n_1+1} \left[ z \left| \begin{matrix} (1-\rho, \mu), (a_j, \alpha_j)_{1,p} \\ (1, \sigma), (b_j, \beta_j)_{1,q}, (\rho, \mu) \end{matrix} \right. \right] \\
&= (1-\rho) H_{p_1,q_1+1}^{m_1+1,n_1} \left[ z \left| \begin{matrix} (a_j, \alpha_j)_{1,p} \\ (1, \sigma), (b_j, \beta_j)_{1,q} \end{matrix} \right. \right] - H_{p_1+1,q_1+2}^{m_1+1,n_1+1} \left[ z \left| \begin{matrix} (0, \mu), (a_j, \alpha_j)_{1,p} \\ (1, \sigma), (b_j, \beta_j)_{1,q}, (1, \mu) \end{matrix} \right. \right] \tag{4.6}
\end{aligned}$$

$$\begin{aligned}
\text{(g)} \quad & H_{p_1+2,q_1+2}^{m_1,n_1} \left[ z \left| \begin{matrix} (a_j, \alpha_j)_{1,p}; (\sigma+r, \nu) (\rho+r, \mu) \\ (b_j, \beta_j)_{1,q}, (\rho+r, \nu) (\rho+r, \mu) \end{matrix} \right. \right] \\
&= (-1)^{2r} H_{p_1+2,q_1+2}^{m_1,n_1} \left[ z \left| \begin{matrix} (a_j, \alpha_j)_{1,p}; (\sigma, \nu) (\rho, \mu) \\ (b_j, \beta_j)_{1,q}, (\rho, \mu) (\rho, \mu) \end{matrix} \right. \right] \tag{4.7}
\end{aligned}$$

$$\begin{aligned}
\text{(h)} \quad & H_{p_1+2,q_1+2}^{m_1+2,n_1+2} \left[ z \left| \begin{matrix} (\sigma+r, \nu) (\rho+r, \mu), (a_j, \alpha_j)_{1,p} \\ (\rho+r, \nu) (\rho+r, \mu), (b_j, \beta_j)_{1,q} \end{matrix} \right. \right] \\
&= (-1)^{2r} H_{p_1+2,q_1+2}^{m_1,n_1} \left[ z \left| \begin{matrix} (\sigma, \nu) (\rho, \mu), (a_j, \alpha_j)_{1,p} \\ (\rho, \mu) (\rho, \mu), (b_j, \beta_j)_{1,q} \end{matrix} \right. \right] \tag{4.8}
\end{aligned}$$

## 5. Conclusion

The Multivariable H-function, presented in this paper, is quite basic in nature. Therefore, on specializing the parameters of this function, we may obtain various other special functions such as Fox's H-function, Meijer's G-function, Wright's generalized Bessel function, Wright's generalized hypergeometric function, Mac-Robert's E-function, generalized hypergeometric function, Bessel function of first kind, modified Bessel function, Whittaker function, exponential function, binomial function etc. as its special cases, and therefore, various unified integral presentations can be obtained as special cases of our results.

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## References

1. Fox, C. (1961). The G and H-functions as symmetrical Fourier kernels. *American Mathematical Society*, **98**, 395–429.
2. Mittal, P.K. and Gupta, K.C. (1972). An integral involving generalized function of two variables. *Proceedings of the Indian Academy of Sciences*, **75A**, 117–123.
3. Rainville, E.D. (1971). *Special Functions*. Macmillan, New York.
4. Srivastava, H.S.P. (1978). H-functions of two variables I. *Indore University Research Journal of Science*, **5**(1–2), 87–93.
5. Srivastava, H.M., Gupta, K.C. and Goyal, S.P. (1982). *The H-Functions of One and Two Variables with Applications*. South Asian Publishers, New Delhi and Madras.

*Priyanka Gupta*

*Department of Mathematics,*

*Government (PG) college,*

*Amarpatan-485775, Madhya Pradesh, India.*

*E-mail address: priyanka103gupta@gmail.com*

*and*

*Bhupendra Tripathi*

*Department of Mathematics,*

*Lakshmi Narain College of Technology and Science,*

*Bhopal-462021, Madhya Pradesh, India.*

*E-mail address: btripathirewa@yahoo.co.in*

*and*

*Roshani Singh*

*Department of Mathematics,*

*Lakshmi Narain College of Technology,*

*Bhopal-462021, Madhya Pradesh, India.*

*E-mail address: roshanisingh11011101@gmail.com*