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# Metro Domination Number of Certain Graphs

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ABSTRACT: We study the metro domination number $\gamma_\beta(G)$, defined as the minimum size of a dominating set that also resolves a connected, simple, finite graph $G$. We present explicit formulas for $\gamma_\beta(G)$ for four classical graph families—Fan graphs, Firecracker graphs, Banana trees, and Coconut trees—together with concise proofs and examples. The presentation unifies domination and metric-dimension arguments, producing statements that are consistent with recent developments in the literature.

Key Words: Dominating set, domination number, metric dimension, fan graph, banana tree, coconut tree, Firecracker graph.

## Contents



## 1. Introduction

All graphs considered are connected, simple, and finite. Let $G = (V, E)$ and let $d_G(u, v)$ denote the distance between vertices $u$ and $v$ in $G$ in 1960. Hedetneimi and Laskar(1991) [1], The study of domination in graphs has evolved since early foundational works introducing domination and related parameters.The term dominating set and domination number were used by ore [3] in 1962. A resolving set is a vertex subset whose distance vectors distinguish all vertices; the minimum size of such a set is the metric dimension $\beta(G)$. A dominating set that is also resolving is called a *metro dominating set*, and its minimum size is the metro domination number $\gamma_\beta(G)$. Sooryanarayan B and Raghunath introduced the metro domination number [4]. On the k-metrodomination number of paths, cycles studied by Lakshminarayana S and M Vishukumar[5].

## 2. Definitions

**Definition 2.1 (Dominating set)** *A set $D \subseteq V(G)$ is a dominating set if every vertex in $V(G) \setminus D$ has a neighbor in $D$. The minimum cardinality is the domination number $\gamma(G)$.*

**Definition 2.2 (Resolving set)** *A set $S \subseteq V(G)$ is a resolving set if for every $u, v \in V(G)$ there exists $w \in V(G)$ with $d(u, w) \neq d(v, w)$. The minimum cardinality is the metric dimension $\beta(G)$.*

**Definition 2.3 (Metro dominating set)** *A dominating set $D$ that is also a resolving set is a metro dominating set. Its minimum cardinality is the metro domination number $\gamma_\beta(G)$.*

**Definition 2.4 (Fan Graph)** *The Fan graph $F_{m,n}$ is the join $\overline{K_m} + P_n$, where $\overline{K_m}$ is an empty graph on $m$ vertices and $P_n$ is a path on $n$ vertices.*

**Definition 2.5 (Firecracker Graph)** *The Firecracker graph $F(m, n)$ is obtained by linking one leaf from each of $m$ copies of an $n$-star in series.*

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**Definition 2.6 (Banana Tree)** *The Banana tree $B(m,n)$ is obtained by attaching one leaf of each of $m$ copies of an $n$-star to a new root vertex $v$.*

**Definition 2.7 (Coconut Tree)** *The Coconut tree $CT(m,n)$ is constructed from a path $P_m$ by appending $n$ pendant edges at one end vertex.*

## 3. Main Results

**Theorem 3.1** *For the Fan graph $F_{m,2}$, the metro domination number equals $m$, for $m \geq 1$.*

**Proof:** Let $G \cong F(m,2) = \overline{K_m} + P_2$ with vertex set consisting of $m$ isolated vertices and two vertices in the path. Any resolving set must contain all but at most one of the $m$ isolated vertices to distinguish them, and any dominating set must include a vertex adjacent to the $P_2$ vertices. Selecting $m$ vertices—one from the $P_2$ part and $m-1$ among the isolated vertices—yields a metro dominating set of size $m$. Conversely, fewer than $m$ vertices cannot resolve all $m$ isolated vertices simultaneously.
Hence $\gamma_\beta(F_{m,2}) = m$. □

**Example 3.1** *the metro domination number of fan graph $F_{3,2}$ is shown in the Figure 1.*

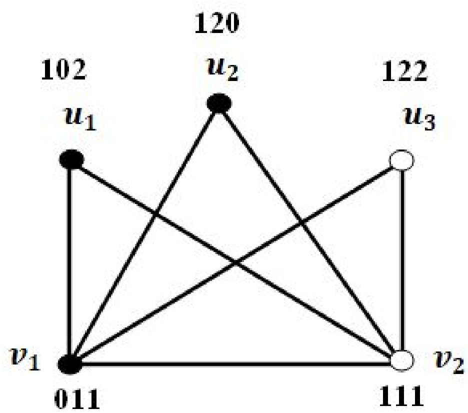


Figure 1: Fan Graph $\gamma_\beta(F_{3,2}) = 3$.

**Theorem 3.2** *For the Firecracker graph $F(m,n)$, $\gamma_\beta(F(m,n)) = (n-2)m$ for $n \geq 4$.*

**Proof:** View $F(m,n)$ as $m$ copies of an $n$-star linked through one leaf of each star. Each star contributes $n-2$ leaves that must be represented in any resolving set to distinguish symmetric leaves, and domination is concurrently enforced through the star centers or selected leaves. Choosing $n-2$ vertices per star gives a metro dominating set of size $(n-2)m$, and any smaller set fails either to dominate all leaves or to resolve them across the $m$ copies.

**Case 1:** Let $n = 2$ and $n = 3$:
the metrodomination number of firecracker graph $F(m,2)$ and $F(m,3)$ is
$\gamma_\beta F(m,2) = 2$ and $\gamma_\beta F(m,3) = 2$.

**Case 2:** Let $n \geq 4$:
since in a firecrack graph $F(m,n)$ chossing $n-2$ nodes from n the metro domination number for $F(m,n)$ is $(n-2)m$
thus $\gamma_\beta(F(m,n)) = (n-2)m$ for $n \geq 4$.

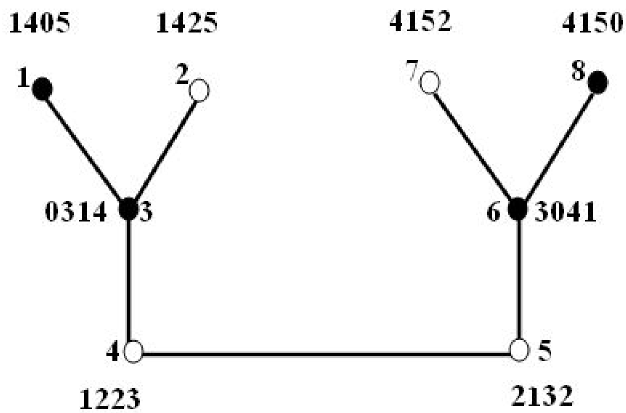


Figure 2: Firecracker Graph $\gamma_\beta(F(2,4)) = 4$

□

**Example 3.2** *the metro domination number of firecracker graph $F(2,4)$ is shown in the Figure 2.*

**Theorem 3.3** *For the Banana tree $B(m,n)$, $\gamma_\beta(B(m,n)) = (n-2)m + 1$ for $m \geq 1$ and $n \geq 4$.*

**Proof:** Let $B(m,n)$ be formed by attaching one leaf from each $n$-star to a new root vertex $v$. The root requires one additional representative to dominate and resolve the attachment point, while each of the $m$ stars contributes $n-2$ vertices to resolve symmetric leaves and ensure domination. The construction yields a metro dominating set of size $(n-2)m+1$, and minimality follows from symmetry arguments on the stars combined with the necessity to resolve vertices adjacent to the root.

**Case 1:** Let $n = 1$
The metro domination number of banana tree graph $B(m,1)$
Metro domination number $\gamma_\beta(G)$ is m.

**Case 2:** Let $n = 2$
The metro domination number of banana tree graph $B(m,2)$
Metro domination number $\gamma_\beta(B(1,2)) = 2$ and $\gamma_\beta(B(2,2)) = 2$
the metro domination number $\gamma_\beta(G) = (m-1)$ where $m \geq 3$.

**Case 3:** Let $n = 3$
The metro domination number of banana tree graph $B(m,3)$
Metro domination number $\gamma_\beta(B(1,3)) = 3$ and
the metro domination number $\gamma_\beta(G) = (n-1)m$ where $m \geq 2$.

**Case 4:** Let $n \geq 4$.
The metro domination number of banana tree graph G is $\gamma_\beta(B(m,n)) = (n-2)m + 1$.

□

**Example 3.3** *the metro domination number of banana tree graph $B(2,4)$ is shown in the Figure 3.*

**Theorem 3.4** *For the Coconut tree $CT(m,n)$, $\gamma_\beta(CT(m,n)) = \lceil \frac{m-2}{3} \rceil + n$, where $m \geq 1$, $n \geq 1$.*

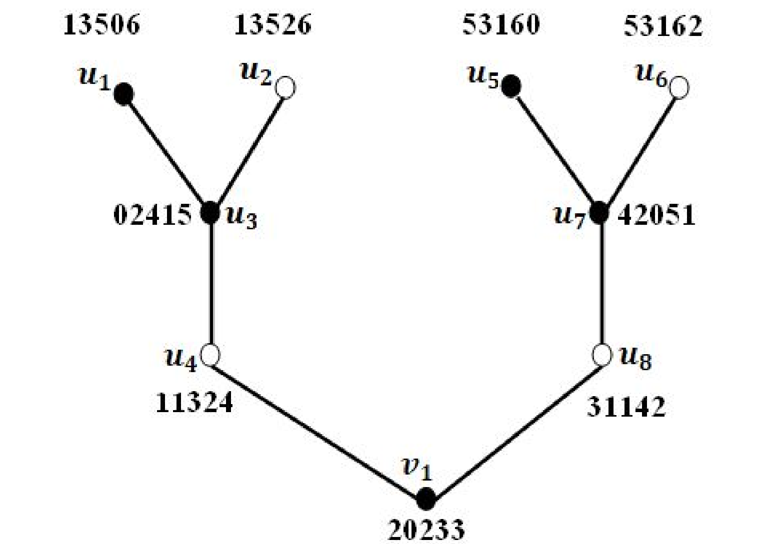


Figure 3: Banana tree Graph $\gamma_\beta(B(2,4)) = 5$

**Proof:** Let $CT(m,n)$ be obtained from the path $P_m$ by appending $n$ pendant edges at one end. The term $\lceil \frac{m-2}{3} \rceil$ is inherited from the domination of $P_m$ while preserving resolution along the path; the $n$ pendants require $n$ additional vertices to dominate and to break symmetries at the appended end. Combining these contributions yields the formula above. □

**Example 3.4** *the metro domination number of coconut tree graph $CT(4,5)$ is shown in the Figure 4.*

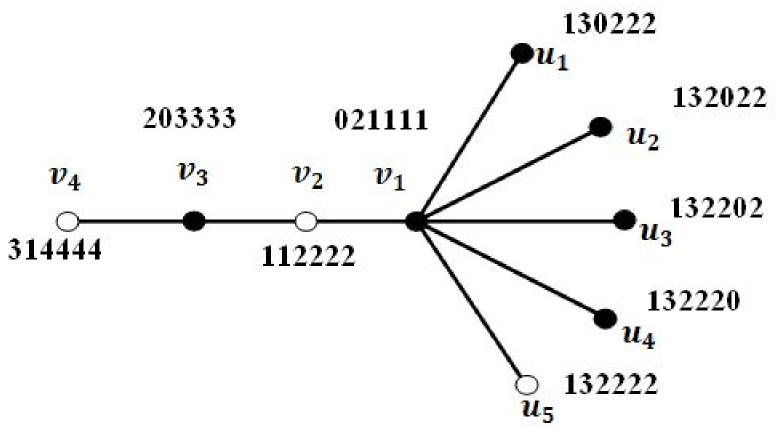


Figure 4: coconut tree Graph $\gamma_\beta(CT(4,5)) = 6$

## 4. Conclusion

We derived explicit formulas for the metro domination number of Fan graphs, Firecracker graphs, Banana trees, and Coconut trees using unified domination–resolution arguments. These cases illustrate how structural symmetry drives lower bounds and sharp constructions.

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