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Secure Fuzzy Information Networks with Applications to Social Media Platforms

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ABSTRACT: In this paper, we introduce the concept of a fuzzy information network, where nodes retain a portion of their information internally to ensure self-protection against potential threats. To quantify the degree of security, we propose a fuzzy information number(FIN), calculated from the retained information within the network. Based on the threshold values of this information number, we define secure fuzzy information networks, along with the families of α-secure and β-secure fuzzy information networks. To demonstrate the effectiveness of the proposed framework, we present an application of secure fuzzy information networks in the context of social media platforms.

Key Words: Fuzzy information index number, α-secure and β-secure fuzzy information networks.

Contents

1. Introduction

Fuzzy networks are increasingly recognized as powerful tools in computer science, with applications spanning data mining, image segmentation, clustering, image analysis, and network design. Their ability to manage vagueness and uncertainty makes them particularly effective for modelling intricate network structures. Concepts such as fuzzy paths, walks, and circuits are utilized in tackling diverse problems, including optimization tasks like the traveling salesman problem, database structuring, and resource allocation in communication systems. Such applications highlight the adaptability of fuzzy networks and encourage the creation of advanced developments along with the exploration of new theoretical foundations.

The study of domination in the networks was introduced by Ore [4] and Berge [3]. Houcine Boumediene Merouane, Mustapha chellali [5] introduced the concepts of secure domination set and 2-dominating set. A.Somasundaram and S.Somasundaram [8] introduced the concept of domination in fuzzy graphs and obtain several bounds for the domination number. Motivated by the notion of dominating number and their applicability [7], we focused on introducing secure fuzzy information number and secure fuzzy information number in the fuzzy networks.

The content of this paper is organized as follows. Section 2 presents the preliminary concepts required for this study. Section 3 introduces and analyses the properties of the Fuzzy Information Number (FIN) and the Secure Fuzzy Information Number (SFIN). In Section 4, a fuzzy graph network is constructed based on the social media content shared by four doctors, focusing on their similarity in discussions related to cardiac problems. This section also demonstrates the proposed concept and highlights the significance of the Secure Fuzzy Information Number. Finally, Section 5 provides the conclusions and outlines potential directions for future work.

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2. Preliminaries

Definition 2.1 ([1]) Let N be the collection of nodes. A fuzzy network is a pair of operators $G = (N, L, \tau, \rho)$ where τ is a node membership value of N and ρ is a symmetric fuzzy connection on τ which is the link membership. i.e. $\tau : N \longrightarrow [0, 1]$ and $\rho : N \times N \longrightarrow [0, 1]$ such that $\rho(u, v) \leq \tau(u) \wedge \tau(v)$ for all u, v in N.

Definition 2.2 ([7]) Let $G = (N, L, \tau, \rho)$ be a fuzzy network. The degree of a vertex u is $d_N(u) = \sum_{u \neq v} \rho(u, v)$.

Definition 2.3 ([1]) A fuzzy network G is complete if $\rho(u, v) = \tau(u) \wedge \tau(v)$ for all $u, v \in N$.

Definition 2.4 ([8]) A fuzzy network G is strong if $\rho(u, v) = \tau(u) \wedge \tau(v)$ for all $u, v \in L$.

Definition 2.5 ([13]) The complement of a fuzzy network $G = (N, L, \tau, \rho)$ is the fuzzy network $\overline{G} = (\overline{N}, \overline{L}, \overline{\tau}, \overline{\rho})$ where $\overline{\tau} = \tau$ and $\overline{\rho(u, v)} = \tau(u) \wedge \tau(v) - \rho(u, v)$ for all $u, v \in L$.

Definition 2.6 ([6]) The fuzzy network $G = (N, L, \tau, \rho)$ is the constant fuzzy network if $\tau(u) = k$, k is a constant, for all $u \in N$.

Definition 2.7 ([9]) The fuzzy network $G = (N, L, \tau, \rho)$ is the regular if $d_N(u) = k$,k is a constant, for all $u \in N$.

Definition 2.8 ([10]) A fuzzy network $G = (N, L, \tau, \rho)$ is said to be strongly regular if it satisfies the following axioms
i) G is a k-regular fuzzy network
ii) Sum of membership values of the nodes common to the adjacent nodes λ is same for all adjacent pair of nodes,
iii) Sum of membership values of the nodes common to the non-adjacent nodes γ is same for all non-adjacent pair of nodes.

Definition 2.9 ([11]) The fuzzy network $G = (N, L, \tau, \rho)$ called a bipartite fuzzy Network if there exists a partition of the nodes set into two disjoint non-empty subsets, $N = N_1 \cup N_2$, $N_1 \cap N_2 = \emptyset$ such that $\rho(u, v) = 0$ whenever $u, v \in N_1$ or $u, v \in N_2$.
That is, there are no fuzzy links (of positive membership) among nodes within the same partition.

Definition 2.10 ([12]) A fuzzy Network $G = (N, L, \tau, \rho)$ is called a wheel fuzzy network(W_4) A fuzzy wheel graph W_4 consists of: One central node n_i: This node is connected to all other nodes (N_p, N_q, N_r) in the network. Three peripheral nodes (N_p, N_q, N_r): These nodes are connected to the central node and to each other, forming a cycle.

Definition 2.11 ([14]) Let $G = (N, L, \tau, \rho)$ be a fuzzy Network, where: N is the set of Nodes, L is the set of Links, $\rho(u, v) \leq \tau(u) \wedge \tau(v)$ for all u, v in N.

The fuzzy incidence matrix M_F is a $N \times L$ matrix defined as: $M_F = [m_{ij}]$ where $\rho(n_i, n_j) = \sigma(n_i) \wedge \sigma(n_j)$,if node n_i is incident to link l_j,0,otherwise.

3. Main results

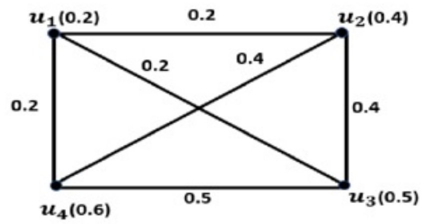
In this section, we have introduced Fuzzy Information Number (FIN) and Secure Fuzzy Information Number (SFIN) based on information available in the incident matrix and also we studied some their properties.

Definition 3.1 *Let $G = (N, L, \tau, \rho)$ be a fuzzy network with nodes $N = \{n_1, n_2, \ldots . n_n\}$ and links $L = \{l_1, l_2, \ldots l_n\}$ then the total information in the network is denoted by $IN(G)$ and it is defined as the sum of all the edge membership across the network and the reduced information in the network is denoted by $RIN(G)$ and it is defined as sum of difference between the nodes with its incidence links. Then the*

Fuzzy Information Number is denoted by $FIN(G)$ *and it is defined by*

$$FIN(G) = \frac{RIN(G)}{IN(G)}$$

The fuzzy incidence matrix(IN) of the below network is given by



$$IN=\left(\begin{array}{c|cccccc} n_1(0.2) & 0.2 & - & - & 0.2 & 0.2 & - \\ n_2(0.4) & 0.2 & 0.4 & - & - & - & 0.4 \\ n_3(0.5) & - & 0.4 & 0.5 & - & 0.2 & - \\ n_4(0.6) & - & - & 0.5 & 0.2 & - & 0.4 \end{array}\right).$$

Total information in the Network is IN(G)=3.8

$$RIN=\left(\begin{array}{c|cccccc} 0.2 & 0. & - & - & 0. & 0. & - \\ 0.4 & 0.2 & 0. & - & - & - & 0. \\ 0.5 & - & 0.1 & 0.0 & - & 0.3 & - \\ 0.6 & - & - & 0.1 & 0.4 & - & 0.2 \end{array}\right)$$

Figure 1: Fuzzy Information Number of a Fuzzy Network

Reduced information in the Network RIN(G)=1.3

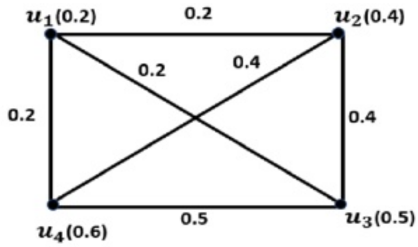
$$FIN(G) = \frac{1.3}{3.8} = 0.342$$

Definition 3.2 *Let* $G = (N, L, \tau, \rho)$ *be a fuzzy network with nodes* $N = \{n_1, n_2, \ldots..n_n\}$ *and links* $L = \{l_1, l_2, \ldots l_n\}$ *then the Secure information in the network is denoted by* $SIN(G)$ *and is defined as the sum of all the edge membership value in the rank reduced incident matrix. The reduced information in the rank reduced incident matrix of the network is denoted by* $SRIN(G)$ *and is defined as sum of difference between the nodes with its incidence links. Then the Secure Fuzzy Information Number is denoted by* $SFIN(G)$ *and it is defined by*

$$SFIN(G) = \frac{SRIN(G)}{SIN(G)}$$

The secure fuzzy incidence matrix(SIN) of the below network is given by

Here rank of the matrix is 4 and the total information in the Network is SIN(G)=2.6



$$SIN=\left(\begin{array}{l|llll} n_1(0.2) & 0.2 & - & - & 0.2 \\ n_2(0.4) & 0.2 & 0.4 & - & - \\ n_3(0.5) & - & 0.4 & 0.5 & - \\ n_4(0.6) & - & -. & 0.5 & 0.2 \end{array}\right)$$

$$SRIN=\left(\begin{array}{l|llll} 0.2 & 0 & - & - & 0 \\ 0.4 & 0.2 & 0. & - & - \\ 0.5 & - & 0.1 & 0 & - \\ 0.6 & - & - & 0.1 & 0.4 \end{array}\right)$$

Figure 2: Secure Fuzzy Information Number of a Fuzzy Network

Reduced information in the Network SRIN(G)=0.8

$$SFIN(G)=\frac{0.8}{2.6}=0.307$$

Definition 3.3 *If $G=(N,L,\tau,\rho)$ be a fuzzy network and $SFIN(G)\geq FIN(G)$ then the network is known as α- Secure fuzzy networks.*

Definition 3.4 *If $G=(N,L,\tau,\rho)$ be a fuzzy network and $SFIN(G)<FIN(G)$ then the network is known as β- Secure fuzzy networks.*

Theorem 3.1 *If $G=(N,L)$ be a constant fuzzy network and strong fuzzy network then $FIN(G)=SFIN(G)=0$.*

Proof:

Let$G=(N,L)$ be a constant fuzzy network with node $N=\{n_1,n_2,\ldots..n_n\}$ and link $L=\{l_1,l_2,\ldots l_n\}$ then by definition $n_i=$ constant for all $n_i\in N$ and since G is strong, we have $\mu(n_i,n_j)=\sigma(n_i)\wedge\sigma(n_j)$ for all$(n_i,n_j)\in L$

Then the total information IN(G)remains same in across the network and reduced information number RIN(G) is zero. Hence FIN(G) = 0

In secure information network then value of all the links l_i in the rank reduced incident matrix are same and the reduced information in the rank reduced incident matrix SRIN(G) is zero hence SFIN(G) = 0

Hence FING = SFING = 0

Theorem 3.2 *If $G=(N,L)$ be a constant fuzzy network and $\mu(n_i,n_j)=constant$ then $FIN(G)=SFIN(G)$.*

Proof:

Let$G=(N,L)$ be a constant fuzzy network with node $N=\{n_1,n_2,\ldots..n_n\}$ and link $L=\{l_1,l_2,\ldots l_n\}$ then by definition $n_i=$ constant for all $n_i\in N$ and since $\mu(n_i,n_j)=constant$ for all$(n_i,n_j)\in L$ then the total information IN(G) remains constant across the network and reduced information number RIN(G) is difference between node and its links.

Hence $FIN(G)=\frac{IN(G)}{RIN(G)}=k$, k is a fuzzy number.

Also in secure information network. The values of all the links l_i in the rank reduced incident matrix are

constant and the reduced information in the rank reduced incident matrix SFIN(G) is difference between nodes and its links in a row reduced matrix $SFIN(G) = \frac{SIN(G)}{SRIN(G)} = k$, k is a fuzzy number.
Hence FING = SFING.

Theorem 3.3 *If $G = (N, L)$ be a strongly regular fuzzy network then fuzzy information number is less than or equal to secure fuzzy information number.*
Proof:
Let$G = (N, L)$ be a strongly regular fuzzy network with node $N = \{n_1, n_2, \ldots .n_n\}$ and link $L = \{l_1, l_2, \ldots l_n\}$ then by definition $d_N(u) = k$,k is a constant. Since sum of membership values of the nodes common to the adjacent and non-adjacent nodes are same then the total information IN(G) remains across the network and reduced information number RIN(G) is difference between node and its links takes a FIN(G) takes a fuzzy number, let it be p.
The values of all the links l_i in the rank reduced incident matrix and the reduced information in the rank reduced incident matrix SFIN(G) is difference between nodes and its links in a row reduced matrix $SFIN(G) = \frac{SIN(G)}{SRIN(G)} = q$, q is a fuzzy number and since G is k regular and preserves adjacency and non-adjacency values then $p \leq q$.

Theorem 3.4 *If G(N,L) be a bipartite fuzzy network then fuzzy information number is greater than or equal to secure fuzzy information number.*
Proof:
Let G(N,L) be a bipartite fuzzy network with node $N = \{n_1, n_2, ...n_n\}$ and link $L = \{l_1, l_2, \ldots l_n\}$ then by definition nodes can divided into two disjoint sets let us take (N_1, N_2) where $N_1, N_2 \in N$ and every link connects a node N_1 to a node N_2 and no links exists inside N_1 or inside N_2 then total information IN(G) remains constant across the network and reduced information number RIN(G) is deference between node and its links.

Hence FIN(G) = IN(G)/RIN(G) = r, r is a fuzzy number. Also in secure information network, the values of all the links l_i in the rank reduced incident matrix are constant and the reduced information in the rank reduced incident matrix SFIN(G) is difference between nodes and its links in a row reduced matrix $SFIN(G) = \frac{SIN(G)}{SRIN(G)} = s$, s is a fuzzy number. Hence $r \geq s$.

Theorem 3.5 *If G(N,L) be a Four-cycle with diagonal fuzzy network then fuzzy information number is greater than or equal to secure fuzzy information number.*
Proof:
Let G(N,L) be a Four-cycle with diagonal fuzzy network with node $N = \{n_1, n_2, \ldots .n_n\}$ and link $L = \{l_1, l_2, \ldots l_{n+1}\}$ by the definition of cycle of four nodes—that is, $L = \{(n_1, n_2), (n_2, n_3), (n_3, n_4), (n_4, 1_1)\}$ —with the addition of one diagonal link connecting two opposite nodes (for instance, (n_1, n_3)).Each nodes n_i and each link $l_{ij} \in L$. the total information IN(G) remains constant across the network and reduced information number RIN(G) is deference between node and its links.

Hence FIN(G) = IN(G)/RIN(G) = u, u is a fuzzy network also in secure information network. The values of all the links l_i in the rank reduced incident matrix are constant and the reduced information in the rank reduced incident matrix SFIN(G) is difference between nodes and its links in a row reduced matrix $SFIN(G) = \frac{SIN(G)}{SRIN(G)} = v$, v is a fuzzy number. Hence $u \geq v$.

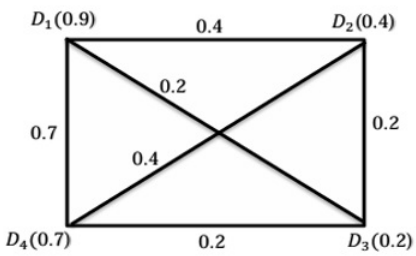
Theorem 3.6 *If G(N,L) be a wheel fuzzy network with four vertices then the fuzzy information Number [FIN(G)] is always Less than or equal to Secure fuzzy information Number [SFIN(G)] .*
Proof:
Let G(N,L) be a wheel fuzzy network with node $N = \{n_1, n_2, \ldots .n_n\}$ and link $L = \{l_1, l_2, \ldots l_{n+1}\}$ by the definition of the structure of G corresponds to a wheel graph W_4— formed by connecting a single central nodes n_0 to all nodes of a four-cycle $C_4 = n_1, n_2, n_3, n_4$— and Formally, $L = (n_1, n_2), (n_2, n_3), (n_3, n_4), (n_4, n_1) \cup (n_0, n_1), (n_0, n_2), (n_0, n_3), (n_0, n_4)$. $L = \{((n_1, n_2), (n_2, n_3), (n_3, n_4)) \cup ((n_0, n_1), (n_0, n_2), (n_0, n_3), (n_0, n_4))\}$

Each nodes $n_i \in N$ and each links $l_{ij} \in L$ then total information IN(G) remains constant across the network and reduced information number RIN(G) is deference between node and its links.

Hence FIN(G) = IN(G)/RIN(G) = u, u is a fuzzy number and also in secure information network the values of all the links l_i in the rank reduced incident matrix are constant and the reduced information in the rank reduced incident matrix SFIN(G) is difference between nodes and its links in a row reduced matrix $SFIN(G) = \frac{SIN(G)}{SRIN(G)} = v$, v is a fuzzy number Hence $u \leq v$.

4. Application of secure fuzzy information networks in the social media platforms.

Social media platforms such as YouTube, Instagram, WhatsApp, Blogs, and Telegram generate and circulate vast volumes of information at a rapid pace. However, much of this information is redundant, unauthenticated, or unreliable, leading to inefficiencies in communication networks. To address this issue, we propose a Secure Fuzzy Information number (SFIN)that effectively eliminates redundant information and ensures that only authenticated and verified data are transmitted across the network. Let us consider 10 parameters that make huge impact of cardiac problems which are Atherosclerosis,Coronary Thrombosis, Hypertension, Diabetes Mellitus,Smoking, Obesity and Sedentary Lifestyle,Stress and Emotional Strain,Family History and Genetic Predisposition, Unhealthy Diet, and Alcohol and Substance Abuse. Suppose we consider discussion of four medical experts(Doctors) on a social media platform focused on problems related to cardiac disease. Each doctor is considered as a node with corresponding membership values discussed among 10 parameters. If common points are discussed between them, the connections are represented as edges with associated edge membership values. The fuzzy incidence matrix(IN) of the network is given by



$$IN = \begin{matrix} D_1(0.9) \\ D_2(0.4) \\ D_3(0.2) \\ D_4(0.7) \end{matrix} \begin{pmatrix} 0.4 & - & - & 0.7 & 0.2 & - \\ 0.4 & 0.2 & - & - & - & 0.4 \\ - & 0.2 & 0.2 & - & 0.2 & - \\ - & - & 0.2 & 0.7 & 0.4 & - \end{pmatrix}$$

Total information in the Network is IN(G)=4.2

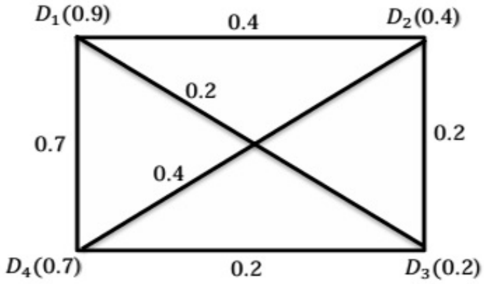
$$RIN = \begin{pmatrix} 0.5 & - & - & 0.2 & 0.7 & - \\ 0.0 & 0.2 & - & - & - & 0.0 \\ - & 0.0 & 0.0 & - & 0.0 & - \\ - & - & 0.5 & 0.2 & 0.3 & - \end{pmatrix}$$

Figure 3: Fuzzy Information Number in the Social Media platforms

Reduced information in the Network RIN(G)=2.6

$$FIN(G) = \frac{2.6}{4.2} = 0.619$$

The secure fuzzy incidence matrix(SIN) of the network is given by
Here rank of the matrix is 4 and the total information in the Network is SIN(G)=3.0



$$\text{SIN} = \begin{matrix} D_1(0.9) \\ D_2(0.4) \\ D_3(0.2) \\ D_4(0.7) \end{matrix} \begin{pmatrix} 0.4 & - & - & 0.7 \\ 0.4 & 0.2 & - & - \\ - & 0.2 & 0.2 & - \\ - & - & 0.2 & 0.7 \end{pmatrix}$$

$$\text{SRIN} = \begin{pmatrix} 0.5 & - & - & 0.2 \\ 0.0 & 0.2 & - & - \\ - & 0.0 & 0.0 & - \\ - & - & 0.5 & 0.2 \end{pmatrix}$$

Figure 4: Secure Fuzzy Information Number in the Social Media platforms

Reduced information in the Network SRIN(G)=1.6

$$SFIN(G) = \frac{1.6}{3.0} = 0.533$$

Metric	Meaning(value)	Results
FIN(G)	Fuzzy Information Number	0.619
SFIN(G)	Secure Fuzzy Information Number	0.533
Redundancy Reduction (%)	$1 - \frac{SFIN(G)}{FIN(G)}$=1 − $\frac{0.533}{0.619}$=1-0.861=0.138	13.8%
Rank(I(G))	Independent information sources	4
SFIN(G) < FIN(G)	Beta Secure Number	Beta Secure Fuzzy Network

Table 1: Result Interpretation of the given fuzzy network

NOTE: If SFIN(G) > FIN(G) then Redundancy Reduction is defined by (%) =$1 - \frac{FIN(G)}{SFIN(G)}$.

5. Conclusion

The fuzzy information number is introduced in this paper, and some of its properties are investigated. For several classes of fuzzy networks, the fuzzy information number (FIN) and secure fuzzy information number (SFIN) have been determined. The proposed Secure Fuzzy Information Network (SFIN) effectively identifies authentic and unique discussion points among doctors on social media platforms, while systematically filtering out approximately 13.8% of redundant or unreliable contributions. By filtering unreliable or redundant content, the network significantly enhances the credibility and precision of shared medical information, thereby promoting a trustworthy and efficient knowledge-sharing environment among healthcare practitioners on social media platforms.

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