



A Study on Domination in Lower Deg-Centric Graphs

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ABSTRACT: The lower deg-centric graph of a simple connected graph G , denoted by G_{ld} , is a graph constructed from G such that $V(G_{ld}) = V(G)$ and $E(G_{ld}) = \{v_i v_j : d_G(v_i, v_j) < deg_G(v_i)\}$. A dominating set in a graph G with vertex set $V(G)$ is a set S of vertices of G such that every vertex in $V(G) - S$ is adjacent to at least one vertex in S . The domination number of G , denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G . This paper presents the domination properties and domination number of lower deg-centric graphs. In addition, investigate the properties and structural characteristics of this type of graph.

Keywords: Distance, degree, lower deg-centric graph, domination, domination number.

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1. Introduction

For a basic terminology of graph theory, we refer to [1]. For further topics on graph classes, [4]. Throughout this paper, a graph is assumed to be simple, connected, and undirected. The number of edges of a graph G is denoted by $\varepsilon(G)$. Recall that the distance between two distinct vertices v_i and v_j of G , denoted by $d_G(v_i, v_j)$, is the length of the shortest path joining them. The eccentricity of a vertex $v_i \in V(G)$, denoted by $e(v_i)$, is the farthest distance from v_i to some vertex of G . A particular type of newly derived graphs based on the vertex degrees and distances in graphs called *deg-centric graphs* have been introduced in (see [2]) as follows, The *degree centric graph* or *deg-centric graph* of a graph G is the graph G_d with $V(G_d) = V(G)$ and $E(G_d) = \{v_i v_j : d_G(v_i, v_j) \leq deg_G(v_i)\}$ (see [2]). Let G be a graph and G_d be the deg-centric graph of G . Then, the successive iteration *deg-centric graph* of G , denoted by G_{d^k} , is defined as the derived graph obtained by taking the deg-centric graph successively k times, that is, $G_{d^k} = ((G_d)_d \dots)_d$, (k -times). This process is known as *deg-centrication process* (see [2]).

The *exact degree centric graph* or *exact deg-centric graph* of a graph G and denoted by G_{ed} , is the graph with $V(G_{ed}) = V(G)$ and $E(G_{ed}) = \{v_i v_j : d_G(v_i, v_j) = deg_G(v_i)\}$. This graph transformation is called exact deg-centrication (see [3]). The *upper degree centric graph* or *upper deg-centric graph* of a graph G and denoted by G_{ud} , is the graph with $V(G_{ud}) = V(G)$ and $E(G_{ud}) = \{v_i v_j : d_G(v_i, v_j) \geq deg_G(v_i)\}$. This graph transformation is called upper deg-centrication (see [5]). The *coarse degree centric graph* or *coarse deg-centric graph* of a graph G , denoted by G_{cd} , is the graph with $V(G_{cd}) = V(G)$ and $E(G_{cd}) = \{v_i v_j : d_G(v_i, v_j) > deg_G(v_i)\}$. This graph transformation is called coarse deg-centrication (see [7]). The *lower degree centric graph* or *lower deg-centric graph* of a graph G , denoted by G_{ld} , is the graph with $V(G_{ld}) = V(G)$ and $E(G_{ld}) = \{v_i v_j : d_G(v_i, v_j) < deg_G(v_i)\}$ (see [6]).

A dominating set in a graph G with vertex set $V(G)$ is a set S of vertices of G such that every vertex in $V(G) - S$ is adjacent to at least one vertex in S . The domination number of G , denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G . A dominating set of G of cardinality $\gamma(G)$ is called a $\gamma(G)$ -set (see [9]).

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Motivated by the above-mentioned studies, we investigate the domination and some properties of the lower deg-centric graphs. Furthermore, we analyze how the degree-based structural variations influence the domination parameters and overall graph behavior.

Definition 1.1 [6] *The lower deg-centric graph of a simple connected graph G , denoted by G_{ld} , is a graph constructed from G such that $V(G_{ld}) = V(G)$ and $E(G_{ld}) = \{v_i v_j : d_G(v_i, v_j) < \deg_G(v_i)\}$.*

Definition 1.2 [6] *The iterated lower deg-centric graph of a graph G , denoted by G_{ld^k} , is defined as the graph obtained by applying *deg-centrication* successively k -times. That is, $G_{ld^k} = ((G_{ld})_{ld\dots})_{ld}$, (k -times).*

A graph G is D -completable if, after a finite number of iterated lower deg-centrication, the resultant graph is complete. Let $\varphi(G)$ denote the number of iterations required to transform a D -completable graph G to complete. By convention $\varphi(K_n) = 0$, $n \geq 1$ and $\varphi(K_{1,n}) = \infty$, $n \geq 2$.

Note that in the lower deg graph G_{ld} , if a vertex has degree one in G , that vertex should not have any edge contributions in G_{ld} , this vertex would get a loop attached on the first iteration of G_{ld} but in our study we don't want loops in our graph.

Proposition 1.1 [6] *For a connected graph G of order n , the lower deg-centric graph $G_{ld} \cong K_n$ if and only if $\deg_G(v_i) > e_G(v_i)$, for all $v_i \in V(G)$.*

2. Domination of Lower Deg-Centric Graphs

In this section, we will discuss the domination and domination number of the lower deg-centric graphs. Additionally, we examine how these parameters vary with changes in vertex degree distributions and explore their implications for graph optimization and structural analysis.

Proposition 2.1 *For a connected graph G of order n , if $\deg_G(v_i) > e_G(v_i)$, for all $v_i \in V(G)$, $\gamma(G_{ld}) = 1$.*

Proof: Let G be a connected graph of order n . If $\deg_G(v_i) > e_G(v_i)$, then, in view of Proposition 1.1, we have $G_{ld} \cong K_n$. The lower deg-centric graph in this case is a complete graph with n vertices. Therefore, any vertex v_i can serve as a dominating vertex since it is adjacent to all other vertices. Hence, $\gamma(G_{ld}) = 1$. \square

Proposition 2.2 *If G_{ld} is a graph of order $n \geq 3$ which contains a vertex of degree $n-1$, then $\gamma(G_{ld}) = 1$.*

Proof: The result is a direct consequence of the Definition of domination. \square

Proposition 2.3 *For any lower deg-centric graph G_{ld} of order n , $G_{ld} = \overline{K_n}$, then, $\gamma(G_{ld}) = n$.*

Proof: The result is a direct consequence of the Definition of domination. \square

Remark 2.1 Let G_{ld} be a lower deg-centric graph with no isolated vertices. If D is a γ -set of G_{ld} , then $V(G_{ld}) - D$ is also a dominating set.

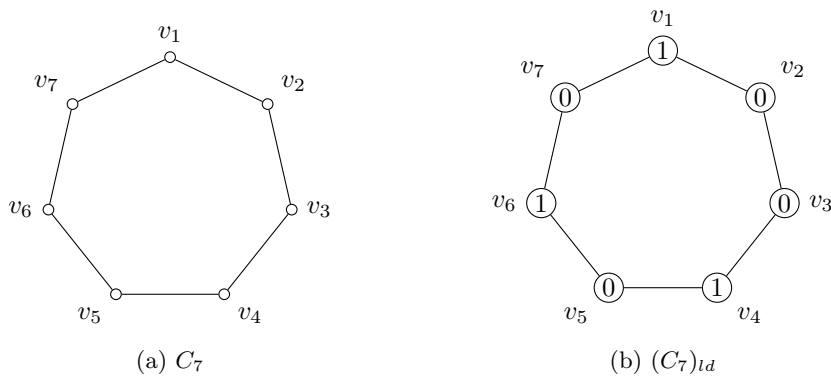
Remark 2.2 If a lower deg-centric graph G_{ld} has no isolated vertices, then $\gamma(G_{ld}) \leq \frac{n}{2}$.

Remark 2.3 For a lower deg-centric graph G_{ld} ,

- (i) $\gamma(G_{ld}) + \gamma(\overline{G_{ld}}) \leq n + 1$.
- (ii) $\gamma(G_{ld})\gamma(\overline{G_{ld}}) \leq n$.

Proposition 2.4 *For $n \geq 3$, $\gamma((K_n)_{ld}) = 1$.*

Proof: For a complete graph K_n , since $\delta(K_n) > e_G(v_i)$, the lower deg-centric graph of K_n of order $n \geq 3$ is always isomorphic to the complete graph K_n . In view of Proposition 2.1, we have $\gamma((K_n)_{ld}) = 1$. \square

Figure 2: The lower deg-centric graph of C_7 with $\gamma((C_7)_{ld}) = 3$.

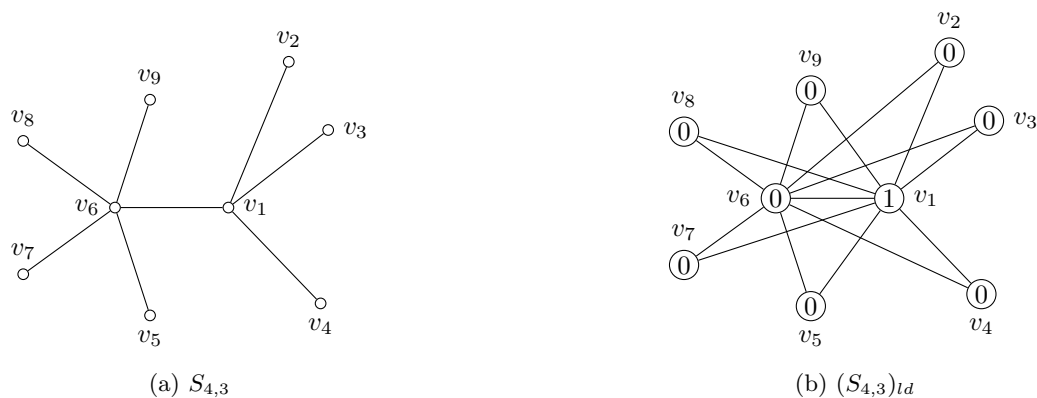
Proof: The lower deg-centric graph of a star graph $K_{1,n}$, where $n \geq 0$, is always isomorphic to the star graph itself. For $n \geq 0$, the central vertex v_0 is included in the minimum dominating set, since it is adjacent to all other vertices. Hence, $\gamma((K_{1,n})_{ld}) = 1$. \square

A non-trivial *bistar graph*, denoted by $S_{a,b}$, is a graph obtained by joining the centers of two non-trivial star graphs $k_{1,a}$, $a \geq 1$ and $k_{1,b}$, $b \geq 1$ with the edge v_0u_0 .

Proposition 3.4 For $a, b \geq 2$, $\gamma((S_{a,b})_{ld}) = 1$.

Proof: Let $S_{a,b}$, where $a, b \geq 1$, be a bistar graph. Let the pendant vertices of $K_{1,a}$ be the set $X = \{v_1, v_2, \dots, v_a\}$ and the pendant vertices of $K_{1,b}$ be the set $Y = \{u_1, u_2, \dots, u_b\}$. Let $W = \{v_0, u_0\}$ denote the two central vertices. By Definition 1.1, both v_0 and u_0 are adjacent to all other $a + b + 1$ vertices. In terms of domination, either of these two central vertices can be included in the minimum dominating set. Hence, $\gamma((S_{a,b})_{ld}) = 1$. \square

An illustration of proposition 3.4 is given in Figure 3.

Figure 3: : The lower deg-centric graph of $S_{4,3}$ with $\gamma((S_{4,3})_{ld}) = 1$.

A *wheel graph*, denoted by $W_{1,n}$ for $n \geq 3$, is obtained by taking a cycle C_n (called the *rim*, with rim vertices) and adding a central vertex v_0 connected to each rim vertex by *spokes*, that is, by the edges v_0v_i , where $1 \leq i \leq n$.

Proposition 3.5 For $n \geq 3$, $\gamma((W_{1,n})_{ld}) = 1$.

Proof: For a wheel graph $W_{1,n}$, where $n \geq 3$, note that $\deg(v_i) > e(v_i)$ for all $v_i \in V(W_{1,n})$. Hence, by Definition 1.1, the lower deg-centric graph $(W_{1,n})_{ld}$ is isomorphic to the complete graph K_{n+1} . In view of Proposition 2.2, we have $\gamma((W_{1,n})_{ld}) = 1$. \square

Proposition 3.6 For a complete bipartite graph $K_{n,m}$, $n, m \geq 3$, $\gamma((K_{n,m})_{ld}) = 1$.

Proof: Let $K_{n,m}$ be a complete bipartite graph with $n, m \geq 3$. By Definition 1.1, the lower deg-centric graph $(K_{n,m})_{ld}$ is complete, which implies that $(K_{n,m})_{ld} \cong K_{n+m}$. In view of Proposition 2.2, we have $\gamma((K_{n,m})_{ld}) = 1$. \square

A *helm graph*, denoted by $H_{1,n}$, $n \geq 3$ is a graph obtained from a wheel graph $W_{1,n}$ by attaching a pendant vertex u_i to the corresponding *rim vertex* v_i .

Proposition 3.7 For $n \geq 3$, $\gamma((H_{1,n})_{ld}) = 1$.

Proof: The helm graph $H_{1,n}$, where $n \geq 3$, is of order $2n + 1$. Let

$$V(H_{1,n}) = \{v_0, v_1, v_2, \dots, v_n, \underbrace{u_1, u_2, \dots, u_n}_{\text{pendant vertices}}\}.$$

In the lower deg-centric graph $(H_{1,n})_{ld}$, each of the vertices v_0 and v_i is incident with $2n$ edges. In terms of domination, either v_0 or any v_i can be chosen as the dominating vertex, since all $2n$ vertices are adjacent to them. Hence, $\gamma((H_{1,n})_{ld}) = 1$. \square

A *closed helm graph* denoted by $CH_{1,n}$, $n \geq 3$ is the graph obtained from a helm graph $H_{1,n}$ by cyclically joining the pendant vertices to form an outer rim.

Proposition 3.8 For $n \geq 3$, $\gamma((CH_{1,n})_{ld}) = 1$.

Proof: Consider a closed helm graph $CH_{1,n}$, where $n \geq 3$. The graph is clearly of order $2n + 1$. Let

$$V(CH_{1,n}) = \{v_0, v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}.$$

For all $CH_{1,n}$ with $n < 6$, we have $\delta(CH_{1,n}) = 3$. For $n = 3, 4, 5$, by Proposition 1.1, the lower deg-centric graph of a closed helm graph $CH_{1,n}$ is complete. Hence, by Proposition 2.1, $\gamma((CH_{1,n})_{ld}) = 1$.

If $n \geq 6$, then $\delta(CH_{1,n}) = 3$ and $\text{diam}(CH_{1,n}) = 4$. In $CH_{1,n}$, the central vertex v_0 satisfies $\deg(v_0) = n$. In the lower deg-centric graph, $\deg(v_0) = 2n$. We can therefore include vertex v_0 in the minimum dominating set, since all other $2n$ vertices are adjacent to v_0 . Hence, $\gamma((CH_{1,n})_{ld}) = 1$. \square

A *djembe graph*, denoted by $D_{1,n}$, is obtained by joining the vertices u_i 's; $1 \leq i \leq n$ of a closed helm graph $CH_{1,n}$ to its central vertex v_0 .

Proposition 3.9 For $n \geq 3$, $\gamma((D_{1,n})_{ld}) = 1$.

Proof: Consider a djembe graph $D_{1,n}$, where $n \geq 3$. Clearly, $D_{1,n}$ is of order $2n + 1$. Let

$$V(D_{1,n}) = \{v_0, v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}.$$

In $D_{1,n}$, the central vertex v_0 has degree $\deg(v_0) = 2n > \varepsilon(v_0) = 1$, and each of the remaining vertices satisfies $\deg(v_i) = \deg(u_i) = 4 > \varepsilon(v_i) = 1$. Consequently, there are $2n$ edges incident on each of the $2n + 1$ vertices in $(D_{1,n})_{ld}$. Hence, $(D_{1,n})_{ld} \cong K_{2n+1}$.

By Proposition 2.1, it follows that $\gamma((D_{1,n})_{ld}) = 1$. \square

A *double wheel* DW_n is obtained by taking two copies of a wheel W_n $n \geq 3$ and *merging* the two central vertices.

Proposition 3.10 For $n \geq 3$, $\gamma((DW_n)_{ld}) = 1$.

Proof: Consider a double wheel graph DW_n , where $n \geq 3$. Clearly, DW_n is of order $2n + 1$. Let

$$V(DW_n) = \{v_0, v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}.$$

In DW_n , the central vertex v_0 has degree $\deg(v_0) = 2n > \varepsilon(v_0) = 1$, and each of the rim vertices satisfies $\deg(v_i) = \deg(u_i) = 3 > \varepsilon(v_i) = 1$. Hence, each of the $2n + 1$ vertices in $(DW_n)_{ld}$ is incident to $2n$ edges. Therefore, $(DW_n)_{ld} \cong K_{2n+1}$.

By Proposition 2.1, it follows that $\gamma((DW_n)_{ld}) = 1$. \square

A *sunlet graph*, denoted by Sl_n , $n \geq 3$, is a graph obtained by attaching a pendant vertex to every vertex of a cycle graph c_n , $n \geq 3$. In other words, a sunlet graph on $2n$ vertices is obtained by taking the corona product $C_n \circ K_1$.

Proposition 3.11 For $n \geq 3$, $\gamma((Sl_n)_{ld}) = \lceil \frac{n}{3} \rceil$

Proof: Consider a sunlet graph Sl_n , where $n \geq 3$. The graph Sl_n is of order $2n$, with

$$V(Sl_n) = \{v_1, v_2, \dots, v_n, \underbrace{u_1, u_2, \dots, u_n}_{\text{pendant vertices}}\}.$$

For Sl_3 , we have $\deg(v_i) = 3 > \varepsilon(v_i) = 2$. According to Definition 1.1, each vertex v_i is adjacent to all other $2n - 1$ vertices in the lower deg-centric graph. Hence, by including any one vertex v_i in the dominating set, we obtain a minimum dominating set. Therefore, $\gamma((Sl_3)_{ld}) = 1$.

For Sl_4 , by Definition 1.1, each vertex v_i is adjacent to $2n - 2$ vertices. In the domination process, adding the vertex v_1 to the minimum dominating set covers all but one vertex, namely u_3 . By including u_3 as well, the set becomes dominating. Hence, $\gamma((Sl_4)_{ld}) = 2$.

Now, consider $n \geq 5$. In Sl_n , $\deg(v_i) = 3$ and $\deg(u_i) = 1$. By Definition 1.1, each v_i is adjacent to seven vertices in $(Sl_n)_{ld}$, while each pendant vertex u_i has no incident edge in $(Sl_n)_{ld}$. Thus, in $(Sl_n)_{ld}$, $\deg(v_i) = 7$ and $\deg(u_i) = 3$.

Let the rim vertices be consecutively labeled as $\{v_1, v_2, v_3, \dots, v_n\}$. Each vertex v_i is adjacent to $v_{i-2}, v_{i-1}, v_{i+1}$, and v_{i+2} in $(Sl_n)_{ld}$. Therefore, the minimum dominating set can be formed by selecting the vertices v_{3i+1} , where $i = 0, 1, 2, \dots, n$. Consequently,

$$\gamma((Sl_n)_{ld}) = \lceil \frac{n}{3} \rceil.$$

\square

An illustration of a proposition 3.11 is given in Figure 4.

A *gear graph*, denoted by G_n , $n \geq 3$, is a graph obtained by inserting an extra vertex between each pair of adjacent vertices on the perimeter of a wheel graph W_n .

Proposition 3.12 For $n \geq 3$, $\gamma((G_n)_{ld}) = 1$.

Proof: Consider a gear graph G_n , where $n \geq 3$. The graph G_n is of order $2n + 1$, with

$$V(G_n) = \{v_0, v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}.$$

Since $\deg_G(v_0) = n > \varepsilon(v_0) = 2$, by Definition 1.1, the vertex v_0 is adjacent to all other vertices in $(G_n)_{ld}$. By adding v_0 to the dominating set, all remaining vertices are dominated. Therefore,

$$\gamma((G_n)_{ld}) = 1.$$

\square

A *web graph*, denoted by $Wb_{1,n}$, $n \geq 3$ is the graph obtained by attaching a pendant edge to each vertex of the outer cycle (or rim) of the closed helm graph $CH_{1,n}$.

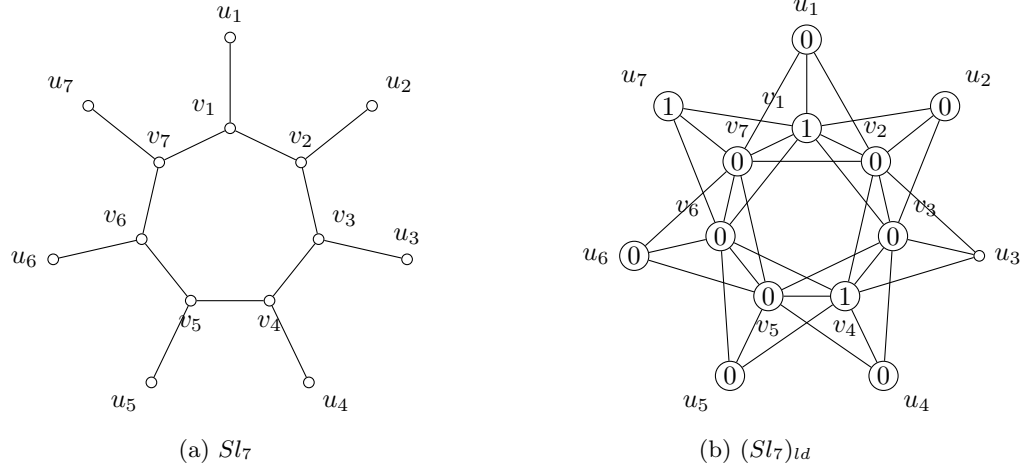


Figure 4: $\gamma(Sl_7)_{ld} = 3$.

Proposition 3.13 For $n \geq 3$, $\gamma((Wb_{1,n})_{ld}) = 1$.

Proof: Consider a web graph $Wb_{1,n}$, where $n \geq 3$. The graph $Wb_{1,n}$ is of order $3n + 1$, with

$$V(Wb_{1,n}) = \{v_0, v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n, \underbrace{w_1, w_2, \dots, w_n}_{\text{pendant vertices}}\}.$$

For $n \geq 3$, since $\deg(v_0) = n \geq \varepsilon(v_0) = 2$ in $Wb_{1,n}$, by Definition 1.1, the vertex v_0 is adjacent to all other vertices in $(Wb_{1,n})_{ld}$. Hence, adding v_0 to the dominating set ensures that all other vertices are dominated. Therefore,

$$\gamma((Wb_{1,n})_{ld}) = 1.$$

□

A *flower graph*, $F_{1,n}$, $n \geq 3$ is a graph obtained from a helm graph $H_{1,n}$, by joining each of its pendant vertices u_i 's to its central vertex v_0 .

Proposition 3.14 For $n \geq 3$, $\gamma((F_{1,n})_{ld}) = 1$.

Proof: Consider a flower graph $F_{1,n}$, where $n \geq 3$. The graph $F_{1,n}$ is of order $2n + 1$, with

$$V(F_{1,n}) = \{v_0, v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}.$$

Since $\delta(F_{1,n}) = 2$ and $\text{diam}(F_{1,n}) = 2$, we observe that $\deg(v_0) = 2n \geq \deg(v_i) = n \geq \varepsilon(v_0) = 2$ in $F_{1,n}$. According to Definition 1.1, each of the vertices v_0 and v_i has $2n$ incident edges in $(F_{1,n})_{ld}$.

In the domination process, adding the vertex v_0 to the dominating set ensures that all other $2n$ vertices are adjacent to v_0 . Therefore,

$$\gamma((F_{1,n})_{ld}) = 1.$$

□

The *sunflower graph*, denoted by $SF_{1,n}$, $n \geq 3$ is obtained from the wheel $W_{1,n}$ by attaching n vertices u_i , $1 \leq i \leq n$ such that each u_i is adjacent to v_i and v_{i+1} and count the suffix is taken modulo n .

Proposition 3.15 For $n \geq 3$, $\gamma((SF_{1,n})_{ld}) = 1$.

Proof: Consider a sunflower graph $SF_{1,n}$, where $n \geq 3$. The graph $SF_{1,n}$ is of order $2n + 1$, with

$$V(SF_{1,n}) = \{v_0, v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}.$$

In $SF_{1,n}$, we have $\deg(v_0) = n \geq \varepsilon(v_0) = 2$ and $\deg(v_i) = n + 1 \geq \varepsilon(v_i) = 2$. By Definition 1.1, each of the vertices v_0 and v_i has $2n$ incident edges in $(SF_{1,n})_{ld}$.

In the domination process, we may include either v_0 or any vertex v_i in the dominating set, since all other $2n$ vertices are adjacent to these vertices. Therefore,

$$\gamma((SF_{1,n})_{ld}) = 1.$$

□

An illustration of a proposition 3.15 is given in Figure 5.

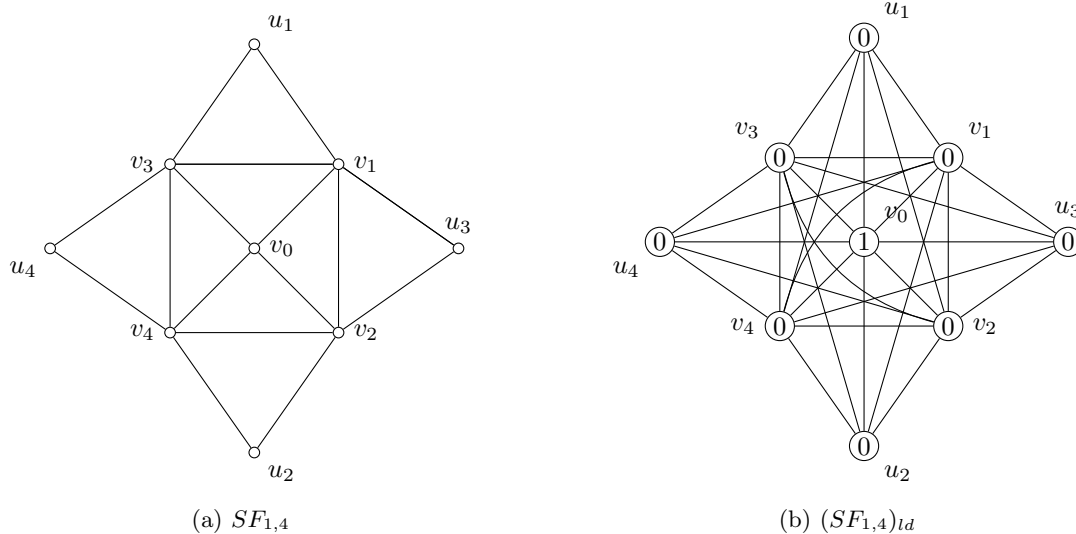


Figure 5: $\gamma((SF_{1,4})_{ld}) = 1$.

A closed sunflower graph $CSF_{1,n}$ is obtained by adding the edge $u_i u_{i+1}$ of the sunflower graph.

Proposition 3.16 For $n \geq 3$, $\gamma((CSF_{1,n})_{ld}) = 1$.

Proof: Consider the closed sunflower graph $CSF_{1,n}$, which is of order $2n + 1$. Let

$$V(CSF_{1,n}) = \{v_0, v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}.$$

For $n \geq 3$, we have $\delta(CSF_{1,n}) \geq \text{diam}(CSF_{1,n})$. By Definition 1.1, the graph $(CSF_{1,n})_{ld}$ is complete. Hence, by Proposition 2.2,

$$\gamma((CSF_{1,n})_{ld}) = 1.$$

□

A blossom graph, denoted by $Bl_{1,n}$, is obtained by making each u_i adjacent to the central vertex of the closed sunflower graph.

Proposition 3.17 For a blossom graph $Bl_{1,n}$, $n \geq 3$, $\gamma((Bl_{1,n})_{ld}) = 1$.

Proof: Consider a blossom graph $Bl_{1,n}$, where $n \geq 3$. The blossom graph is of order $2n + 1$. Let

$$V(Bl_{1,n}) = \{v_0, v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}.$$

By Proposition 1.1, the graph $(Bl_{1,n})_{ld}$ is complete. Hence, by Proposition 2.2,

$$\gamma((Bl_{1,n})_{ld}) = 1.$$

□

Consider a complete graph K_n with the vertex set $V = v_1, v_2, v_3, \dots, v_n$. Let $U = u_1, u_2, u_3, \dots, u_n$ be a copy of $V(G)$ such that u_i corresponds to v_i . The *sun graph*, denoted by S_n , is a graph with vertex set $V \cup U$ and two vertices x and y are adjacent in S_n if $x \sim y$ in K_n and $x = u_i, y \in v_i, v_{i+1}$.

Proposition 3.18 For $n \geq 3$, $\gamma((S_n)_{ld}) = 1$.

Proof: The sun graph S_n , where $n \geq 3$, is of order $2n$. Let

$$V(S_n) = \{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}.$$

Since $\deg_{S_n}(v_i) = n + 1 > e(v_i) = 2$, by Definition 1.1, $(2n - 1)$ edges are incident from each vertex v_i in $(S_n)_{ld}$. Therefore, by selecting any vertex v_i in the dominating set, all other $2n - 1$ vertices become adjacent to v_i . Hence,

$$\gamma((S_n)_{ld}) = 1.$$

□

A *closed sun graph* CS_n is the graph obtained by adding the edges $u_i u_{i+1}$ to the sun graph S_n . In view of Definition 1.1, the lower deg-centric graph of a closed sun graph CS_n , where $n \geq 3$, is complete. This implies that

$$\varepsilon((CS_n)_{ld}) = \varepsilon(K_{2n}).$$

Hence,

$$\gamma((CS_n)_{ld}) = 1.$$

A *friendship graph*, denoted by F_n ($n \geq 1$), is obtained by joining n copies of the complete graph K_3 with a common vertex. In view of Definition 1.1,

$$\gamma((F_n)_{ld}) = 1.$$

A *butterfly graph*, denoted by BF_n ($n \geq 1$), is constructed by joining two copies of the cycle graph C_n with a common vertex v , and is therefore isomorphic to the friendship graph F_n . In view of Definition 1.1,

$$\gamma((BF_n)_{ld}) = 1.$$

An *antiprism graph*, denoted by A_n , $n \geq 3$ is a graph obtained two cycles C_n and C'_n of order n with vertex sets $V = \{v_1, v_2, v_3, \dots, v_n\}$ and $U = \{u_1, u_2, u_3, \dots, u_n\}$ respectively. Join the vertices $u_i v_i$ and $u_i v_{i+1}$ to form the additional edges.

Proposition 3.19 For $n \geq 3$, $\gamma((A_n)_{ld}) = \lceil \frac{n}{6} \rceil$.

Proof: Consider an antiprism graph A_n , where $n \geq 3$. The graph is of order $2n$ with

$$V(A_n) = \{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}.$$

For $3 \leq n \leq 6$, we have $\deg(v_i) = \deg(u_i) = 4 > e(v_i) = e(u_i)$ in A_n . By Definition 1.1, $(A_n)_{ld} \cong K_{2n}$, which implies

$$\gamma((A_n)_{ld}) = 1 = \lceil \frac{n}{6} \rceil.$$

For $n \geq 7$, $\deg(v_i) = \deg(u_i) = 4$ in A_n , and by Definition 1.1, $\deg(v_i) = \deg(u_i) = 12$ in $(A_n)_{ld}$. In the lower deg-centric graph $(A_n)_{ld}$, the rim vertices are labeled consecutively as $\{v_1, v_2, v_3, \dots, v_n\}$. Each vertex v_i is adjacent to six vertices on the rim:

$$v_{i-3}, v_{i-2}, v_{i-1}, v_{i+1}, v_{i+2}, \text{ and } v_{i+3}.$$

To construct a minimum dominating set, we can select the vertex set

$$\{v_{6i+1} \mid i = 0, 1, 2, 3, \dots, n\}.$$

Hence,

$$\gamma((A_n)_{ld}) = \left\lceil \frac{n}{6} \right\rceil.$$

□

4. Conclusion

In this study, the domination properties of lower deg-centric graphs were examined, and the domination numbers for several classes of such graphs were determined. The presented results provide a foundational framework for further exploration in this emerging area of graph theory. Future research may focus on extending the investigation of various graph-theoretical parameters to lower deg-centric graphs of other classes, thereby uncovering new patterns and relationships. Additionally, studies can explore different types of domination and their potential applications within this framework. Beyond domination, examining the structural characteristics of lower deg-centric graphs may yield deeper combinatorial insights and enrich the theoretical understanding of deg-centric structures. Overall, the findings of this work open promising directions for advancing domination theory in the context of deg-centric graphs.

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