Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2026 (44) 4** : 1–16.
ISSN-0037-8712
doi:10.5269/bspm.80081

A Model for Solving Uncertain Multi-Objective Solid Transportation Problem

Chaitra M.*, Vatsala G. A., K. J. Ghanashyam, Prathyusha G.

ABSTRACT: Solid transportation problem (STP) extends the classic transportation model into three dimensions by incorporating sources, destinations, and different modes of transport, known as conveyances. This paper addresses the Uncertain Multi-objective Solid Transportation Problem (UMOSTP), a complex variant where the goal is to simultaneously minimize total transportation cost and time under conditions of uncertainty. Fuzzy cost coefficients are used here to represent the ambiguity introduced by variables such as route availability and fluctuating fuel prices. Using the α-cut method, we first translate these fuzzy costs into precise numerical values. We then use a Modified Vogels Approximation Method (MVAM) to identify a basic feasible initial solution that satisfies all supply, demand, and conveyance constraints while optimising cost and time. There is a comparison with the conventional Vogels Approximation Method (VAM). A numerical example illustrating the efficacy of the suggested model reveals that MVAM offers a more practical and efficient solution for the UMOSTP.

Key Words: VAM, α-cut Method, triangular fuzzy numbers, fuzzy cost coefficients, UMOSTP.

Contents

1. Introduction

Transporting goods from several sources to different locations while meeting supply and demand restrictions is the primary goal of the transportation problem, a particular kind of linear programming problem (LPP). The main objective is to reduce the total cost of transportation while maintaining the capacity constraints of each source and the needs of each destination.

This idea is expanded into a three-dimensional model by the solid transportation problem (STP), which takes conveyance, demand, and supply aspects into account [5,11,13]. Various forms of transportation are considered in this formulation. STP is known as a multi-objective solid transportation problem (MOSTP) when it takes into account several objectives [7,12,18,19].

Transporting goods between sources and destinations while minimizing the overall fuzzy transportation cost is the focus of a fuzzy transportation issue [4]. Identifying one or more fuzzy optimum solutions

* Corresponding author.
2020 *Mathematics Subject Classification:* 90B06, 90C08.
Submitted November 11, 2025. Published February 05, 2026

to such issues involving several objective functions is referred to as a fuzzy multi-objective transportation problem [3].

Transporting heterogeneous goods from warehouses to customer locations via a variety of conveyances while optimizing many competing objectives is the goal of multi-objective fuzzy solid transportation (MOFSTP) [1]. This approach is very useful for enterprises that transport a variety of items at the same time.

The emphasis moves from identifying a single ideal solution to establishing an ideal compromise or effective answer since the goals of such challenges frequently clash.

Finding the best solution to the MOFSTP is the main goal of this work. Both the classic VAM and the MVAM are used to solve and evaluate the problem for comparison.

2. Literature Review

A basic challenge in linear programming (LPP) is the transportation problem, which focuses on distributing goods as efficiently as possible from multiple sources to multiple destinations. The main goal is to satisfy supply and demand constraints while minimising the overall cost of shipping.

This model is advanced by the STP, which adds a third dimension—conveyances, or the various forms of transportation that are available. When an STP involves the optimization of more than one objective, such as both cost and time, it is classified as a MOSTP.

This paper focuses on developing a robust methodology to find such a solution for the FMOSTP. The performance of the established VAM is compared with a MVAM to demonstrate a more effective approach.

The STP represents an advanced form of the traditional transportation model by incorporating three dimensions—sources, destinations, and modes of conveyance—within a single framework. This multidimensional extension enables the simultaneous consideration of multiple routes, commodities, and capacities.

Over time, researchers have developed a variety of strategies to optimize STP performance under different conditions, including deterministic, fuzzy, and uncertain environments.

Shanthi Pandian and Dhanapal [1] introduced the Zero Point Method to obtain optimal solutions for STP instances, interpreting STP as a balanced transportation problem and applying systematic reduction and coverage operations to identify zero-cost allocations that meet all constraints.

Sobana and Anuradha [2] formulated a fuzzy solid transportation problem with interval-valued costs, employing fractional goal programming and converting fuzzy data to crisp equivalents using the α-cut approach, along with heuristic algorithms for solutions.

Rani and Gulati [3]proposed a fully fuzzy multi-objective multi-item STP and used trapezoidal fuzzy numbers combined with fuzzy programming to broaden the applicability of transportation and solid transportation frameworks in multi-item decision-making scenarios.

Dalman and Sivri [5] transformed interval coefficients into deterministic values through weighted-factor techniques and developed fuzzy goal programming and global criteria approaches to identify optimal solutions. Their findings showed effective management of uncertain STPs with fuzzy mathematical programming.

Uddin, Miah, Khan, and AlArjani [7] proposed a goal programming approach tailored to uncertain multi-objective transportation problems by using uncertain normal distributions and fuzzy membership functions to derive compromise solutions at various confidence

2.1. Synopsis of the Review

The publications are grouped based on the type of transportation problem the year they were published and the MCDM method used. The table below gives a summary of all the studies related to STP, FSTP and UMOSTP

Year	Reference	TP	STP	FSTP	MOTP	UMOSTP	Fuzzy Set	Membership Function	Multi-Obj/Dec	B/UB TP	Algorithm/Tech
2010	1	✓	✓							B	Zero Point Method
2018	2		✓	✓			✓	✓		UB	Frac. Goal Prog.
2016	3			✓	✓		✓	✓	✓	B	Fuzzy Programming
2023	4			✓	✓		✓	✓		B	Goal Programming
2017	5		✓		✓	✓	✓	✓	✓	UB	Membership func.
2020	6	✓			✓				✓	B	MVAM
2021	7				✓	✓	✓		✓	UB	Goal Prog.
2016	8	✓								B	VAM
2017	9		✓	✓			✓			UB	Fuzzy Programming
2019	10				✓				✓	B	Budget constraint
2022	11		✓						✓	UB	Int. fuzzy logic
2023	12				✓	✓			✓	B/UB	Rough MOFC TP
2017	13	✓			✓					UB	Profit maximization
2021	14		✓						✓	B	Hierarchical GP
2020	15									B	Waste mgmt TP
2012	16								✓	UB	GP, imprecise env.
2021	17								✓	B	Fuzzy GP
2023	18				✓	✓	✓	✓		UB	Int. fuzzy FCSTP
2013	19								✓	UB	Alt. methods comp.

Table 1: Summary of key literature addressing transportation and solid transportation problems. B=Balanced; UB=Unbalanced; TP=Transportation Problem; etc.

3. Preliminaries and Definitions

3.1. Triangular Fuzzy Number:

A triangular fuzzy number is one of the most commonly used types of fuzzy numbers, represented by three points

$$A = \mu_A(y) = \begin{cases} 0, & y < a_1 \\ \dfrac{y - a_1}{a_2 - a_1}, & a_1 \leq y < a_2 \\ \dfrac{a_3 - y}{a_3 - a_2}, & a_2 \leq y < a_3 \\ 0, & y \geq a_3 \end{cases} \tag{3.1}$$

a_1=The lower limit (the smallest possible value).
a_2: The modal value (the most likely value or the peak of the triangle).
a_3: The upper limit (the largest possible value).

3.2. α-cut Method:

Let A be a fuzzy set and $\alpha \in [0, 1]$ a real number. The α-cut set A_α is a crisp set defined as:

$$A_\alpha = [a_1 + \alpha(a_2 - a_1),\ a_3 - \alpha(a_3 - a_2)]$$

where $a_1 + \alpha(a_2 - a_1)$ is the lower bound and $a_3 - \alpha(a_3 - a_2)$ is the upper bound. This technique is used to convert fuzzy coefficients into crisp values for computation and graphical representation is as shown in Figure1.

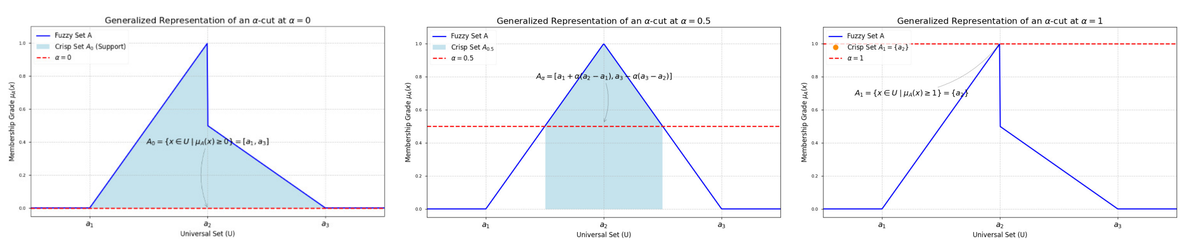

Figure 1: α-cut Method: (a) $\alpha = 0$, (b) $\alpha = 0.5$, (c) $\alpha = 1$

3.3. Vogel's Approximation Method (VAM):

VAM is used to compute the initial basic feasible solution for transportation problems, including STP. The penalty for each row and column is calculated as the difference between the two smallest shipping costs. The variable with the lowest cost in the row or column with the largest penalty receives as much allocation as possible, and the satisfied row or column is crossed out. This iterative process continues, recalculating penalties at each step, until only one row or column is left for allocation.

3.4. Modified Vogel's Approximation Method (MVAM):

MVAM enhances VAM by yielding better, more optimal initial solutions. The process involves:

- **Phase 1: Problem Formulation.** Define the origins (sources), destinations, and conveyances, along with their respective supply, demand, and capacity constraints. Construct the three-dimensional cost matrix c_{ijk}. Ensure the problem is balanced (total supply = total demand = total conveyance); if not, add dummy variables.

- **Phase 2: Cost Matrix Reduction.** To identify the most cost-effective routes, reduce the cost matrix. For each row (origin), find the minimum cost element and subtract this value from all other elements in that row. This creates a new reduced cost matrix where the most favourable routes have a cost of zero.

- **Phase 3: Iterative Allocation.** Systematically assign quantities x_{ijk} to cells until all constraints are met.

  - *Select a Cell:* Choose a cell (i, j, k) for allocation, typically starting with a cell that has a zero cost in the reduced matrix.
  - *Determine Allocation Quantity:* Allocate the maximum possible quantity, which is the minimum of the remaining supply at origin i, demand at destination j, and capacity of conveyance k.

  $$\text{Allocation} = \min(\text{Current Supply}_i, \text{Current Demand}_j, \text{Current Capacity}_k)$$

  - *Update Constraints:* Subtract the allocated amount from the respective supply, demand, and capacity.
  - *Eliminate Satisfied Constraints:* If a row, column, or conveyance's capacity is reduced to zero, remove it from further consideration.
  - *Repeat:* Continue this process until all supplies and demands are fully allocated.

- **Phase 4: Calculation of Total Cost.** Once the allocation is complete, calculate the total transportation cost by summing the products of the allocated quantities and their corresponding costs from the original cost matrix:

$$\text{Total Cost} = \sum_{i,j,k} (x_{ijk} \times c_{ijk})$$

4. Mathematical Model for Multi-Objective Fuzzy Solid Transportation Problem (MOFSTP)

The mathematical model for a MOFSTP is defined as follows:
Objective Function:

$$\min Z_t = \sum_{u=1}^{p} \sum_{v=1}^{q} \sum_{w=1}^{r} a_{uvw} x_{uvw}$$

where $t = 1, 2, \ldots, T$ corresponds to different objective functions (e.g., cost, time).

Subject to the Constraints:

$$\sum_{v=1}^{q}\sum_{w=1}^{r} x_{uvw} = S_u, \quad u = 1, 2, \ldots, p$$
$$\sum_{u=1}^{p}\sum_{w=1}^{r} x_{uvw} = D_v, \quad v = 1, 2, \ldots, q$$
$$\sum_{u=1}^{p}\sum_{v=1}^{q} x_{uvw} = C_w, \quad w = 1, 2, \ldots, r$$
$$x_{uvw} \geq 0 \text{ for all } u, v, w$$

where:

- a_{uvw} : fuzzy or uncertain transportation cost (or time, etc.)
- x_{uvw} : amount shipped from source u to destination v by conveyance w
- S_u : supply at source u
- D_v : demand at destination v
- C_w : capacity of conveyance w
- p, q, r : total number of sources, destinations, and conveyances, respectively

The problem is said to be balanced if

$$\sum_{u=1}^{p} S_u = \sum_{v=1}^{q} D_v = \sum_{w=1}^{r} C_w$$

Note: The coefficients a_{uvw}, S_u, D_v, and C_w may all be fuzzy or uncertain.

The tabular form of the MOFSTP is usually constructed with all these coefficients placed for each source-destination-conveyance cell as shown in table2

	D1			**D2**			**D3**			
	C1	C2	C3	C1	C2	C3	C1	C2	C3	
S1	a_{111}	a_{112}	a_{113}	a_{121}	a_{122}	a_{123}	a_{131}	a_{132}	a_{133}	S1′
S2	a_{211}	a_{212}	a_{213}	a_{221}	a_{222}	a_{223}	a_{231}	a_{232}	a_{233}	S2′
S3	a_{311}	a_{312}	a_{313}	a_{321}	a_{322}	a_{323}	a_{331}	a_{332}	a_{333}	S3′
	C1′	C2′	C3′	C1′	C2′	C3′	C1′	C2′	C3′	
	D1′			D2′			D3′			

Table 2: Symbolic representation of the Solid Transportation Problem

Where, a_{ijk} are all transportation cost values, S1′, S2′, S3′ are all supply values, D1′, D2′, D3′ are all destination values, and C1′, C2′, C3′ are conveyance capacity values.

5. Illustrative Example

Example: Consider a multi-objective fuzzy solid transportation that has two objective functions.

$$
\begin{aligned}
\min Z_1 =& (2,4,6)x_{111} + (3,5,9)x_{112} + (3,7,11)x_{113} + (3,5,7)x_{121} + (7,9,11)x_{122} + (5,9,13)x_{123} \\
&+ (5,7,9)x_{131} + (9,11,17)x_{132} + (7,11,19)x_{133} + (4,6,8)x_{211} + (6,10,16)x_{212} + (4,8,18)x_{213} \\
&+ (3,7,11)x_{221} + (10,14,18)x_{222} + (8,10,20)x_{223} + (2,6,12)x_{231} + (7,11,19)x_{232} + (8,12,20)x_{233} \\
&+ (1,5,11)x_{311} + (3,5,13)x_{312} + (7,19,23)x_{313} + (3,9,15)x_{321} + (7,11,13)x_{322} + (7,15,21)x_{323} \\
&+ (4,10,16)x_{331} + (6,14,22)x_{332} + (10,12,14)x_{333}
\end{aligned}
$$

$$
\begin{aligned}
\min Z_2 =& (2,6,8)x_{111} + (3,7,9)x_{112} + (4,8,12)x_{113} + (4,6,10)x_{121} + (8,10,14)x_{122} + (7,9,13)x_{123} \\
&+ (6,8,14)x_{131} + (8,14,20)x_{132} + (9,11,21)x_{133} + (3,7,11)x_{211} + (6,12,16)x_{212} + (3,5,9)x_{213} \\
&+ (2,6,8)x_{221} + (7,15,19)x_{222} + (6,12,14)x_{223} + (4,6,10)x_{231} + (8,12,16)x_{232} + (9,13,17)x_{233} \\
&+ (3,5,9)x_{311} + (2,8,14)x_{312} + (9,15,19)x_{313} + (4,8,10)x_{321} + (7,9,13)x_{322} + (10,14,16)x_{323} \\
&+ (6,10,16)x_{331} + (8,12,14)x_{332} + (10,14,16)x_{333}
\end{aligned}
$$

Subject to the Constraints:

- Sources: $S_1 = (20,40,60)$, $S_2 = (40,70,90)$, $S_3 = (10,50,70)$
- Destinations: $D_1 = (10,40,80)$, $D_2 = (50,60,70)$, $D_3 = (10,60,70)$
- Conveyances: $C_1 = (10,50,70)$, $C_2 = (30,50,70)$, $C_3 = (20,60,80)$

Now, applying the α-cut method to the above two objective functions, the lower bound and upper bound (LB & UB) of classic cost coefficient values of MOFSTP are given below in Tables (2)-(25), which satisfy the equilibrium condition. Applying the VAM and MVAM to each objective function, we obtain an optimal solution. Below presents the optimal solution values for the respective objective function under each table.

Objective Function Z_1

Lower bound values

Transportation cost values when $\alpha = 0$

	D1			D2			D3			
	C1	C2	C3	C1	C2	C3	C1	C2	C3	Supply
S1	2	3	3	3	7	5	5	9	7	20
S2	4	6	4	3	10	8	2	7	8	40
S3	1	3	7	3	7	7	4	6	10	10
Conveyance	20	30	20	20	30	20	20	30	20	
Demand	10			50			10			

Table 3: Transportation cost values.

Reduced Transportation Table

	D1			D2			D3			
	C1	C2	C3	C1	C2	C3	C1	C2	C3	Supply
S1	0	1	1	1	5	3	3	7	5	20
S2	2	4	2	1	8	6	0	5	6	40
S3	0	2	6	2	6	6	3	5	9	10
Conveyance	20	30	20	20	30	20	20	30	20	
Demand	10			50			10			

Table 4: Reduced transportation cost values.

The optimal solution of the lower bound classic values of Z_1 when $\alpha = 0$ is

- Applying the general procedure of VAM, the optimal solution is 420
- Using the methodology of MVAM, the optimal solution to the given MOFSTP is $x_{121} = 20, x_{212} = 10, x_{222} = 20, x_{223} = 10, x_{333} = 10$ and the total minimum transportation cost is 460

Transportation cost values when $\alpha = 0.5$

	D1			D2			D3			
	C1	C2	C3	C1	C2	C3	C1	C2	C3	Supply
S1	3	4	5	4	8	7	6	10	9	30
S2	5	8	6	5	12	9	4	9	10	55
S3	3	4	13	6	9	11	7	10	11	30
Conveyance	35	40	40	35	40	40	35	40	40	
Demand	25			55			35			

Table 5: Transportation cost values.

Reduced transportation Table is

	D1			D2			D3			
	C1	C2	C3	C1	C2	C3	C1	C2	C3	Supply
S1	0	1	2	1	5	4	3	7	6	30
S2	1	4	2	1	8	5	0	5	6	55
S3	0	1	10	3	6	8	4	7	8	30
Conveyance	35	40	40	35	40	40	35	40	40	
Demand	25			55			35			

Table 6: Reduced transportation cost values.

The optimal solution of the lower bound classic values of Z_1 when α =0.5 is

- Applying the general procedure of VAM, the optimal solution is 860
- Using the methodology of MVAM, the optimal solution to the given MOFSTP is $x_{121} = 30, x_{231} = 5, x_{222} = 15, x_{213} = 25, x_{223} = 10, x_{332} = 30$ and the total minimum transportation cost is 850

Transportation cost values when $\alpha = 1$

	D1			D2			D3			Supply
	C1	C2	C3	C1	C2	C3	C1	C2	C3	
S1	4	5	7	5	9	9	7	11	11	40
S2	6	10	8	7	14	10	6	11	12	70
S3	5	5	19	9	11	15	10	14	12	50
Conveyance	50	50	60	50	50	60	50	50	60	
Demand	40			60			60			

Table 7: Transportation cost values.

Reduced transportation Table is

	D1			D2			D3			Supply
	C1	C2	C3	C1	C2	C3	C1	C2	C3	
S1	0	1	3	1	5	5	3	7	7	40
S2	0	4	2	1	8	4	0	5	6	70
S3	0	0	14	4	6	10	5	9	7	50
Conveyance	50	50	60	50	50	60	50	50	60	
Demand	40			60			60			

Table 8: Reduced transportation cost values.

The optimal solution of the lower bound classic values of Z1 when α =1 is

- Applying the general procedure of VAM, the optimal solution is 1490
- Using the methodology of MVAM, the optimal solution to the given MOFSTP is $x_{121} = 40, x_{231} = 10, x_{213} = 40, x_{223} = 20, x_{332} = 50$ and the total minimum transportation cost is 1470

Upper Bound Values
Transportation cost values when α =0

	D1			D2			D3			Supply
	C1	C2	C3	C1	C2	C3	C1	C2	C3	
S1	6	9	11	7	11	13	9	17	19	60
S2	8	16	18	11	18	20	12	19	20	90
S3	11	13	23	15	13	21	16	22	14	70
Conveyance	70	70	80	70	70	80	70	70	80	
Demand	80			70			70			

Table 9: Transportation cost values.

Reduced transportation table is

	D1			D2			D3			
	C1	C2	C3	C1	C2	C3	C1	C2	C3	Supply
S1	0	3	5	1	5	7	3	11	13	60
S2	0	8	10	3	10	12	4	11	12	90
S3	0	2	12	4	2	10	5	11	3	70
Conveyance	70	70	80	70	70	80	70	70	80	
Demand	80			70			70			

Table 10: Reduced transportation cost values.

The optimal solution of the upper bound classic values of Z1 when α =0 is

- Applying the general procedure of VAM, the optimal solution is 2880
- Using the methodology of MVAM, the optimal solution to the given MOFSTP is $x_{112} = 60, x_{221} = 70, x_{212} = 10, x_{213} = 10, x_{333} = 70$ and the total minimum transportation cost is 2750

Transportation cost values when α = 0.5

	D1			D2			D3			
	C1	C2	C3	C1	C2	C3	C1	C2	C3	Supply
S1	5	7	9	6	10	11	8	14	15	50
S2	7	13	13	9	16	15	9	15	16	80
S3	8	9	21	12	12	18	13	18	13	60
Conveyance	60	60	70	60	60	70	60	60	70	
Demand	60			65			65			

Table 11: Transportation cost values.

Reduced transportation Table is

	D1			D2			D3			
	C1	C2	C3	C1	C2	C3	C1	C2	C3	Supply
S1	0	2	4	1	5	6	3	9	10	50
S2	0	6	6	2	9	8	2	8	9	80
S3	0	1	13	4	4	10	5	10	5	60
Conveyance	60	60	70	60	60	70	60	60	70	
Demand	60			65			65			

Table 12: Reduced Transportation cost values.

The optimal solution of the upper bound classic values of Z1 when α =0.5 is

- Applying the general procedure of VAM, the optimal solution is 2215
- Using the methodology of MVAM, the optimal solution to the given MOFSTP is $x_{121} = 50, x_{212} = 60, x_{222} = 5, x_{232} = 5, x_{213} = 50, x_{333} = 60$ and the total minimum transportation cost is 1925

Transportation cost values when α = 1

	D1			D2			D3			Supply
	C1	C2	C3	C1	C2	C3	C1	C2	C3	
S1	4	5	7	5	9	9	7	11	11	40
S2	6	10	8	7	14	10	6	11	12	70
S3	5	5	19	9	11	15	10	14	12	50
Conveyance	50	50	60	50	50	60	50	50	60	
Demand	40			60			60			

Table 13: Transportation cost values.

Reduced transportation Table is

	D1			D2			D3			Supply
	C1	C2	C3	C1	C2	C3	C1	C2	C3	
S1	0	1	3	1	5	5	3	7	7	40
S2	0	4	2	1	8	4	0	5	6	70
S3	0	0	14	4	6	10	5	9	7	50
Conveyance	50	50	60	50	50	60	50	50	60	
Demand	40			60			60			

Table 14: Reduced Transportation cost values.

The optimal solution of the upper bound classic values of Z1 when α =1 is

- Applying the general procedure of VAM, the optimal solution is 1490
- Using the methodology of MVAM, the optimal solution to the given MOFSTP is $x_{121} = 40, x_{231} = 10, x_{213} = 40, x_{223} = 20, x_{332} = 50$ and the total minimum transportation cost is 1470

Objective function Z_2
Lower Bound Values
Transportation cost values when α =0

	D1			D2			D3			Supply
	C1	C2	C3	C1	C2	C3	C1	C2	C3	
S1	2	3	4	4	8	7	6	8	9	20
S2	3	6	3	2	7	6	4	8	9	40
S3	3	2	9	4	7	10	6	8	10	10
Conveyance	20	30	20	20	30	20	20	30	20	
Demand	10			50			10			

Table 15: Transportation cost values.

Reduced transportation Table is

	D1			D2			D3			Supply
	C1	C2	C3	C1	C2	C3	C1	C2	C3	
S1	0	1	2	2	6	5	4	6	7	20
S2	1	4	1	0	5	4	2	6	7	40
S3	1	0	7	2	5	8	4	6	8	10
Conveyance	20	30	20	20	30	20	20	30	20	
Demand	10			50			10			

Table 16: Reduced Transportation cost values.

The optimal solution of the lower bound classic values of Z2 when α =0 is

- Applying the general procedure of VAM, the optimal solution is 370
- Using the methodology of MVAM, the optimal solution to the given MOFSTP is $x_{121} = 20, x_{212} = 10, x_{222} = 20, x_{223} = 10, x_{333} = 10$ and the total minimum transportation cost is 400.

Transportation cost values when α = 0.5

	D1			D2			D3			Supply
	C1	C2	C3	C1	C2	C3	C1	C2	C3	
S1	4	5	6	5	9	8	7	11	10	30
S2	5	9	4	4	11	9	5	10	11	55
S3	4	5	12	6	8	12	8	10	12	30
Conveyance	35	40	40	35	40	40	35	40	40	
Demand	25			55			35			

Table 17: Transportation cost values.

Reduced transportation Table is

	D1			D2			D3			Supply
	C1	C2	C3	C1	C2	C3	C1	C2	C3	
S1	0	1	2	1	5	4	3	7	6	30
S2	1	5	0	0	7	5	1	6	7	55
S3	0	1	8	2	4	8	4	6	8	30
Conveyance	35	40	40	35	40	40	35	40	40	
Demand	25			55			35			

Table 18: Reduced Transportation cost values.

The optimal solution of the lower bound classic values of Z2 when α =0.5 is

- Applying the general procedure of VAM, the optimal solution is 860
- Using the methodology of MVAM, the optimal solution to the given MOFSTP is $x_{121} = 30, x_{231} = 5, x_{222} = 15, x_{212} = 25, x_{232} = 10, x_{333} = 30$ and the total minimum transportation cost is 895

Transportation cost values when $\alpha = 1$

	D1			D2			D3			Supply
	C1	C2	C3	C1	C2	C3	C1	C2	C3	
S1	6	7	8	6	10	9	8	14	11	40
S2	7	12	5	6	15	12	6	12	13	70
S3	5	8	15	8	9	14	10	12	14	50
Conveyance	50	50	60	50	50	60	50	50	60	
Demand	40			60			60			

Table 19: Transportation cost values.

Reduced transportation Table is

	D1			D2			D3			Supply
	C1	C2	C3	C1	C2	C3	C1	C2	C3	
S1	0	1	2	0	4	3	2	8	5	40
S2	2	7	0	1	10	7	1	7	8	70
S3	0	3	10	3	4	9	5	7	9	50
Conveyance	50	50	60	50	50	60	50	50	60	
Demand	40			60			60			

Table 20: Reduced Transportation cost values.

The optimal solution of the lower bound classic values of Z2 when α =1 is

- Applying the general procedure of VAM, the optimal solution is 1580
- Using the methodology of MVAM, the optimal solution to the given MOFSTP is $x_{121} = 40, x_{231} = 10, x_{213} = 40, x_{223} = 20, x_{332} = 50$ and the total minimum transportation cost is 1510

Upper Bound Values

Transportation cost values when α =0

	D1			D2			D3			Supply
	C1	C2	C3	C1	C2	C3	C1	C2	C3	
S1	8	9	12	10	14	13	14	20	21	60
S2	11	16	9	8	19	14	10	16	17	90
S3	9	14	19	10	13	16	16	14	16	70
Conveyance	70	70	80	70	70	80	70	70	80	
Demand	80			70			70			

Table 21: Transportation cost values.

Reduced transportation Table is

	D1			D2			D3			Supply
	C1	C2	C3	C1	C2	C3	C1	C2	C3	
S1	0	1	4	2	6	5	6	12	13	60
S2	3	8	1	0	11	6	2	8	9	90
S3	0	5	10	1	4	7	7	5	7	70
Conveyance	70	70	80	70	70	80	70	70	80	
Demand	80			70			70			

Table 22: Reduced Transportation cost values.

The optimal solution of the upper bound classic values of Z2 when α =0 is

- Applying the general procedure of VAM, the optimal solution is 3030
- Using the methodology of MVAM, the optimal solution to the given MOFSTP is $x_{111} = 60, x_{231} = 10, x_{222} = 70, x_{213} = 10, x_{333} = 70$ and the total minimum transportation cost is 2950

Transportation cost values when α = 0.5

	D1			D2			D3			Supply
	C1	C2	C3	C1	C2	C3	C1	C2	C3	
S1	7	8	10	8	12	11	11	17	16	50
S2	9	14	7	7	17	13	8	14	15	80
S3	7	11	17	9	11	15	13	13	15	60
Conveyance	60	60	70	60	60	70	60	60	70	
Demand	60			65			65			

Table 23: Transportation cost values.

Reduced transportation Table is

	D1			D2			D3			Supply
	C1	C2	C3	C1	C2	C3	C1	C2	C3	
S1	0	1	3	1	5	4	4	10	9	50
S2	2	7	0	0	10	6	1	7	8	80
S3	0	4	10	2	4	8	6	6	8	60
Conveyance	60	60	70	60	60	70	60	60	70	
Demand	60			65			65			

Table 24: Reduced Transportation cost values.

The optimal solution of the upper bound classic values of Z2 when α =0.5 is

- Applying the general procedure of VAM, the optimal solution is 2195
- Using the methodology of MVAM, the optimal solution to the given MOFSTP is $x_{121} = 50, x_{221} = 10, x_{213} = 60, x_{223} = 5, x_{233} = 5, x_{332} = 60$ and the total minimum transportation cost is 2065

Transportation cost values when $\alpha = 1$

	D1			D2			D3			Supply
	C1	C2	C3	C1	C2	C3	C1	C2	C3	
S1	6	7	8	6	10	9	8	14	11	40
S2	7	12	5	6	15	12	6	12	13	70
S3	5	8	15	8	9	14	10	12	14	50
Conveyance	50	50	60	50	50	60	50	50	60	
Demand	40			60			60			

Table 25: Transportation cost values.

Reduced transportation Table is

	D1			D2			D3			Supply
	C1	C2	C3	C1	C2	C3	C1	C2	C3	
S1	0	1	2	0	4	3	2	8	5	40
S2	2	7	0	1	10	7	1	7	8	70
S3	0	3	10	3	4	9	5	7	9	50
Conveyance	50	50	60	50	50	60	50	50	60	
Demand	40			60			60			

Table 26: Reduced Transportation cost values.

The optimal solution of the upper bound classic values of Z2 when α =1 is

- Applying the general procedure of VAM, the optimal solution is 1580
- Using the methodology of MVAM, the optimal solution to the given MOFSTP is $x_{121} = 40, x_{231} = 10, x_{213} = 40, x_{223} = 20, x_{332} = 50$ and the total minimum transportation cost is 1510

6. Illustrations of Results

The Multi-Objective Fuzzy Solid Transportation gives the feasible and compromise solutions for both the lower and upper bound cost coefficient values of multi-objective function.

Illustrative Example	Cost Values	VAM			MVAM		
		$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$
1	Lower Bound of Z1	420	860	1490	460	850	1470
	Lower Bound of Z2	370	860	1580	400	895	1510
	Upper Bound of Z1	2880	2215	1490	2750	1925	1470
	Upper Bound of Z2	3030	2195	1580	2950	2065	1510

Table 27: Illustrations of results.

Visualization of Results

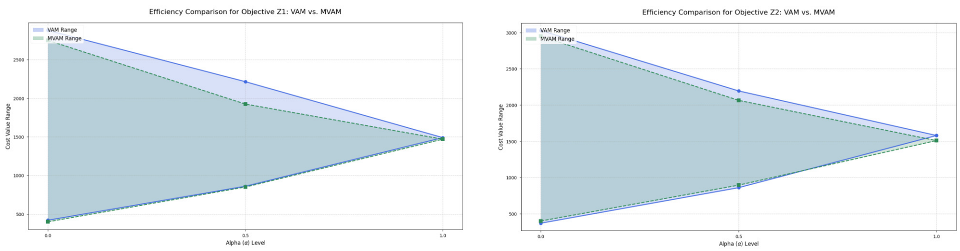

Figure 2: Comparision Results between VAM and MVAM

7. Conclusion

The obtained results clearly demonstrate that the MOFSTP provides both feasible and compromise solutions corresponding to the lower and upper bound cost coefficients of the multi-objective functions. A comparative analysis between the conventional VAM and MVAM reveals that the MVAM consistently yields improved (lower) transportation costs across all $\alpha-$ levels ($\alpha = 0$, 0.5 and 1). This indicates that the MVAM produces solutions that are not only closer to the optimal but also more efficient in addressing the uncertainty and imprecision inherent in fuzzy transportation parameters. Therefore the MVAM can be considered a more reliable and efficient heuristic for obtaining superior initial feasible solutions in MOFSTP.

Acknowledgments

The authors would like to thank the referees for their valuable comments and suggestions, which helped me to improve the quality and clarity of this paper. Also I would like to thank my guide and Friends who helped to prepare the paper

References

1. P. Shanthi Pandian, A. Dhanapal, *A new approach for solving solid transportation problems,* Journal of Applied Mathematical Sciences, **4** (2010) no. 72, 3603–3610.

2. V. E. Sobana, D. Anuradha, *Solution of solid transportation problems in fuzzy approach,* International Journal of Pure and Applied Mathematics, **119** No. 9 (2018) 313–321.

3. D. Rani, T. R. Gulati, *Uncertain multi-objective multi-product solid transportation problems,* Journal of Indian Academy of Sciences, **41** No. 5 (2016) 531–539, DOI: 10.1007/s12046-016-0491-x.

4. P. G., G. A. Vatsala, K. N. Uday Kumar, *Fuzzy solid transportation problem via goal programming with linear and non-linear membership functions.*

5. H. Dalman, M. Sivri, *Multi-Objective solid transportation problem in uncertain environment,* Iran J Sci Technol Trans Sci, DOI: 10.1007/s40995-017-0254-5.

6. J. Pratihar, R. K. S. A., A. Edalatpanah, and A. Dey, *Modified Vogel's approximation method for transportation problem under uncertain environment,* Complex & Intelligent Systems, https://doi.org/10.1007/s40747-020-00153-4.

7. M. S. Uddin, M. Miah, M. A. Khan, A. AlArjani, *Goal Programming tactic for uncertain multi-objective transportation problem using fuzzy linear membership functions,* Alexandria Engineering Journal, **60** (2021) 2525–2533.

8. M. Wali Ullah, M. Alhaz Uddin, R. Kawser, *A modified Vogel's approximation method for obtaining a good primal solution of transportation problems,* Annals of Pure and Applied Mathematics, **11** No. 1 (2016) 63–71.

9. S. A. Jalil, S. Sadia, S. Javaid, Q. M. Ali, *A solution approach for solving fully fuzzy multi-objective solid transportation problems,* International Journal on Agriculture Statistics Sci., **13** No. 1 (2017) 75–84.

10. P. Kundu, S. Kar, M. Maiti, *Multi-Objective solid transportation problems with budget constraint in uncertain environment,* International Journal of Systems Science, ISSN: 0020-7721 (Print), 1464-5319 (Online).

11. M. K. Sharma, Kamini, N. Dhiman, *A new Goal Programming approach for multi-objective solid transportation problems with interval-valued Intuitionistic fuzzy logic,* Journal of International Academy of Physical Sciences, **23** No. 3 (2019) 251–268.

12. Shivani, D. Rani, A. Ebrahimnejad, *An approach to solve an unbalanced fully rough multi-objective fixed-charge transportation problem,* Computational and Applied Mathematics, **41** (2022) 129, https://doi.org/10.1007/s40314-022-01830-5.

13. O. Niluminda, U. Ekanayake, *The novel efficient method to solve balanced and unbalanced profit maximization in transportation problems,* American Journal of Mathematical and Computer Modeling, **8** No. 2 (2023) 17–24, doi: 10.11648/j.ajmcm.20230802.11.

14. K. B. Jagtap, S. V. Kawale, *Optimizing transportation problem with multiple objectives by Hierarchical order Goal Programming model,* Global Journal of Pure and Applied Mathematics, **13** No. 9 (2017) 5333–5339.

15. Y. Y. Tadesse, *Mathematical Modeling on unbalanced transportation problem for solid waste management for the reduction of cost: Case of Jimma town, South west Ethiopia,* International Journal of Scientific and Academic Research (IJSAR), **1** Issue 10 (2021).

16. R. Luo, S. Ji, T. Ji, *An effective chaos-driven differential evolution for multi-objective unbalanced transportation problem considering fuel consumption,* Applied Soft Computing Journal (2020).

17. M. Ahmadpour, S. Chitgar, *Using Goal Programming for transformation planning decision problem in Imprecise Environment,* Australian Journal of Basic and Applied Sciences, **6** No. 2 (2012) 57–65.

18. S. Ghosh, S. K. Roy, A. Ebrahimnejad, J. L. Verdegay, *Multi-Objective fully Intuitionistic fuzzy fixed-charge solid transportation problem,* Complex & Intelligent Systems, **7** (2021) 1009–1023, https://doi.org/10.1007/s40747-020-00251-3.

19. M. H. Abdelati, A. M. Abd-El-Tawwab, E. E. M. Ellimony, M. Rabie, *Solving a multi-objective solid transportation problem: a comparative study of alternative methods for decision making,* Journal of Engineering and Applied Science, **70** (2023) 82, https://doi.org/10.1186/s44147-023-00247-z.

Chaitra M.,
Department of Mathematics, BGS College of Engineering and Technology
Research Scholar, Visvesvaraya Technological University, Belagavi
India.
E-mail address: chaitramg16@gmail.com

and

Vatsala G A.,
Associate Professor,
Department of Mathematics, Dayananda Sagar Academy of Technology and Management
Visvesvaraya Technological University, Belagavi
India.
E-mail address: dr.vatsala.ga@gmail.com

and

K. J. Ghanashyam.,
Assistant Professor,
Department of Data Analytics and Mathematical Sciences,
Jain (Deemed-To-Be) University, Bangalore
India.
E-mail address: ghanashyamkj@gmail.com

and

Prathyusha G.,
Assistant Professor,
Department of Mathematics, BVRIT Hyderabad College of Engineering for Women , Hyderabad
India.
E-mail address: prathyusha665g@gmail.com