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Multi-Criteria Decision-Making for Urban Green Space Planning Using Intuitionistic Fuzzy AHP and Aggregation Operators

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ABSTRACT: Urban green spaces are essential components of smart cities, contributing to environmental sustainability, public health, and social well-being. Selecting optimal locations for green space development requires balancing multiple, often conflicting criteria such as environmental impact, accessibility, cost, and maintenance requirements. To address the inherent uncertainty and subjectivity in such multi-criteria decision-making (MCDM) problems, this study proposes a novel Intuitionistic Fuzzy Analytic Hierarchy Process (AHP) integrated with advanced aggregation operators, namely Frank aggregation operators, Power Geometric operators, and the Ordered Weighted Averaging (OWA) operator for robust criteria weighting and evaluation. The proposed framework combines Intuitionistic Fuzzy AHP to effectively capture experts' hesitancy and partial membership, while the aggregation operators enhance the expressiveness and flexibility of criteria synthesis under fuzziness. Subsequently, the Fuzzy Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is applied to rank potential urban green space locations based on their relative closeness to the ideal solution. The Python-based experimental tool is utilized to implement and evaluate the proposed approach, ensuring reproducibility and computational efficiency. A case study conducted in a smart city context demonstrates superior performance and stability compared to conventional techniques.

Keywords: MCDM, OWA, IF-AHP, TOPSIS, UGS.

Contents

1. Introduction

Urban green spaces (UGSs) are critically important in an increasingly urbanized world, in terms of their contribution to ecosystem services and the quality of life of urban residents [1]. It is essential to choose sites for UGSs with care and mindful design because their values are complex from ecological, socioeconomic, and psychological perspectives [2]. UGSs provide members of the public with sites to relax and socialize but growing concerns surrounding the number of available UGSs, particularly in urbanized regions, has emerged. UGSs have been increasingly acknowledged for their ability to enhance life expectancy and improve outcomes related to public health [3]. In addition, UGSs can be examined for developing understanding of urban lifestyles and trends [4]. Urban areas are unique, being composed of a built environment that integrates natural sites within the surrounding area, in which urban forests

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are the most diverse form of UGS, a collection of "networks or systems of all woodlands, groups of trees, and individual trees in an urban and peri-urban area" [5].

Increased population growth, rapid urbanisation, and unsustainable development are placing tremendous pressure on the world's ecosystems, which threatens planetary systems [6]. Population increases result in increased demands on natural resources; for example, over 13 million hectares of forests are lost each year as a result of land use changes, such as agriculture and urbanisation [7]. The ecological transformations are linked urbanisation, deforestation, and climate change exemplify the Anthropocene as a period of mass ecological alteration that includes biodiversity loss, counting insects, and an increasing threat of a sixth mass extinction [8]. Insects are disappearing from many levels of our ecosystems with many converging drivers at play—to name a few, habitat loss and destruction, pollution, pesticides, invasive species, artificial night illumination, climate change, and overexploitation all cause stress to insect populations [9]. The decline of our insects poses a very real risk to basic ecosystem functions and services, such as pollination, decomposition, and biological control of pest species critical components of ecosystem function to achieve balance, support human wellbeing including global food security, not to mention the maintenance of healthy ecosystems through well-considered planning and forward-thinking, is of primary importance at prospective sustainable socio-economic development and a basic provision for global health[10].

Effective urban green space planning relies on sophisticated geospatial technologies, such as remote sensing and Geographic Information Systems (GIS) [11]. When combined with a structured decision-making approach, like the Analytic Hierarchy Process (AHP), these geospatial technologies can allow urban planners to rank many criteria and sub-criteria in order to evaluate potential sites for urban green space [12]. Using the AHP, criteria can be systematically assigned weights based on their importance, and can help urban planners consider all pertinent factors at the same time [13]. Urban planners are required to include various decision-making factors into one decision making process, such as distance to existing residential areas, accessibility, land use, soil characteristics, and topographically challenged areas. Through the application of GIS and the AHP, urban planners can develop evidence-based and context-sensitive strategies for improving urban green spaces that suit the specific needs and limitations of global cities, like Gondar, that are becoming more urbanized [14]. Afterward this method of multi-criteria decision-making is deduced in the Cartesian framework of the complex intuitionistic fuzzy set, with expected aggregation operators. This novel approach is later followed by applying the developed multi-criteria decision-making process to strategically assess an urban green space environment [15].

This research and study address the need for a systematic data-driven approach using Geographic Information System (GIS), analytic hierarchy process (AHP), and fuzzy multi-criteria decision making to find and prioritize possible UGS sites in fast urbanizing cities. Rapid urbanization, population rise, and unsustainable land use is increasing stress on ecosystems, and access to important urban green spaces (UGS). Urban green spaces provide ecological balance, public health, and quality of life. UGS planning is still complex and not fully optimized.

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1.1. Objectives of the Paper

To optimize urban green space allocation in smart cities by evaluating multiple criteria such as environmental impact, accessibility, cost, and maintenance requirements. The proposed approach combines Fuzzy AHP to handle subjective uncertainties and weight the criteria, and Fuzzy TOPSIS to rank potential locations based on their closeness to the ideal solution.

1.2. Contribution of the Paper

1. Constructed a structurally sound decision-making framework using Intuitionistic Fuzzy Analytic Hierarchy Process (IF-AHP) to represent uncertainty, hesitancy, and subjectivity in expert judg-

ments.

2. Incorporated Frank aggregation, Power Geometric, and Ordered Weighted Averaging (OWA) operators to aggregate multi-expert evaluations while producing a more reliable decision model.

3. Evaluated nine different urban green space options across 21 criteria that focused on environmental, economic, accessibility, and performance factors relevant to smart city planning.

4. Used a clearly outlined intuitionistic fuzzy linguistic scale to represent different degrees of satisfaction, dissatisfaction, and hesitancy, as provided by experts.

5. Utilized Fuzzy TOPSIS to provide a ranking of alternatives based on their closeness to ideal solutions, fostering transparency and actionable decision-making support.

6. Completed an IF-AHP and aggregation operations using a Python-based decision support tool that was efficient and reproducible.

7. Provided a scalable and flexible approach for urban planners and policymakers to prioritize and make investments in green infrastructure within a smart city context.

2. Literature Review

In 2025, Cui et al.[16] investigated a decision-making framework based on the Analytical Hierarchy Process (AHP) combined with an intuitionistic fuzzy set (IFS) for a multi-criteria decision-making (MCDM) algorithm. Their proposed model organizes aspects and uncertainties of urban land or urban-area development and development planning in general, considering several criteria which include infrastructure, socio-economic, economic, institutional, and environmental criteria. Their model is based on the use of Intuitionistic Fuzzy Sets (IFS). An intuitionistic fuzzy set is a sophisticated, advanced model of ordinary fuzzy sets that fits within an interval by considering not only the degree of membership (DoM), but also the degree of non-membership (DoNM). To assist in decision-making, Analytic Hierarchy Process (AHP) and Sugeno-Weber-based t-norm (the TNM) and t-conorm (the TCNM) aggregation operators are used to develop the Intuitionistic Fuzzy Sugeno-Weber Weighted Averaging (IFSWWA) operators.

In 2024, Anjum et al.[17] explored the complexities of urban planning and addressed major issues by carefully weighing four options for smart city technology, community-based development, green infrastructure investment, and transit-oriented development. Unlike traditional evaluations, their study applies the Intuitionistic Fuzzy AHP and Aggregation Operators, thus identifying and navigating the uncertainty present in decision making. This methodological approach improves the accuracy of assessment by providing detailed information about the advantages and disadvantages of each option. Their study offers useful insights for urban policymakers and planners using carefully weighted criteria and employing a methodical ranking procedure.

In 2025, Romel et al.[18] developed a multi-criteria decision-making (MCDM) framework based on the intuitionistic fuzzy analytic hierarchy process (IF-AHP) to evaluate land-based hydrogen transportation alternatives across Canada. Seven factors, their subsequent thirty-three subfactors, and three alternatives to hydrogen transportation were identified through a literature review. Pairwise comparison was aggregated among factors, subfactors, and alternatives from three decision makers using an intuitionistic fuzzy weighted average, and priority weights were computed using entropy-based weight. Their results show that safety and economic efficiency emerged as the most influential factors in the evaluation of hydrogen transportation alternatives, followed by environmental impact, security, and social impact and public health in ascending order.

In 2025, Yazdani et al.[19] proposed a comprehensive framework for group decision analysis employing Analytic Hierarchy Process (AHP) and Combined Compromise Solution to address the optimal site selection problem for waste disposal facilities. Innovatively integrating fuzzy methodology, the authors optimize decision-makers' preference inputs. Through their proposed method, decision-makers' weights and criteria weights are calculated, while fuzzy CoCoSo is utilized to determine the final collective decision ranking. This study advances urban waste management strategies, offering a systematic approach that accounts for the diverse perspectives of stakeholders and the complex dynamics inherent in waste management decision-making.

In 2025, Kraiem et al.[20] introduced a decision-making framework to optimize energy efficiency in smart manufacturing environments, integrating Intuitionistic Fuzzy Sets (IFS) with Multi-Criteria Decision-Making (MCDM) techniques. The approach addresses key challenges, including reducing carbon footprints, managing operating costs, and adhering to stringent environmental standards. Eight essential criteria are identified, such as the use of renewable energy, the efficiency of production, and the health and safety of workers, to evaluate energy performance. Using the entropy method for criterion weighting and the CRADIS technique for alternative ranking, the study prioritizes a range of energy-efficient solutions. Their empirical results, validated through sensitivity analysis, show that alternative 5 delivers the most significant improvement in energy efficiency.

In 2021, Hanine et al.[21] investigated and evaluated the most influential dimensions and criteria for smart city development (SCD) in the Moroccan context. To reach this goal, this study proposed a new integrated Multi-Criteria Decision-Making (MCDM) model based on Intuitionistic Fuzzy Analytical Hierarchy Process (IF-AHP) and Intuitionistic Fuzzy Decision-Making Trial and Evaluation Laboratory (IF-DEMATEL). The use of intuitionistic fuzzy set theory helps in dealing with the linguistic imprecision and the ambiguity of experts' judgment. Results reveal that 'Smart Living and Governance' and 'Smart Economy' are major dimensions impacting the SCD in the Moroccan context. The proposed model focuses on enhancing the understanding of different dimensions/criteria and situations in smart cities compared to traditional cities and elevates their decision-making capability.

In 2024, Bačić et al.[22] developed a valuation model for urban green infrastructure in land management, focusing on urban cities. The fuzzy AHP method was used to calculate the weighting coefficients for a suitable set of criteria, and the TOPSIS method was used to select the priority city districts for implementing green infrastructure. The research results are relevant to decision makers, who can utilize them to prioritize areas for the development and implementation of green infrastructure. The green infrastructure index calculated in this study can be compared with other spatial and land data for effective spatial planning.

In 2024, Tran et al.[23] presented an effective MCDM model that integrates Fuzzy-AHP-TOPSIS to evaluate and choose the best robot. The Fuzzy-AHP is utilized to establish a set of weights for the evaluation criteria. Subsequently, the proposed technique analyzes, prioritizes, and chooses the best robot option from the ranking list for the factory. The experimental results demonstrate that by employing the integrated fuzzy analytical hierarchy process, taking into account parameter weights and expert judgment, the robots are identified in order of best to worst alternatives to factories.

In 2025, Mahajan et al.[24] introduced an innovative multi-criteria group decision-making (MCGDM) framework tailored to evaluate and enhance women empowerment policies. Acknowledging the interconnectedness of gender equality, organizational effectiveness, and economic growth as pivotal goals, the framework employs spherical fuzzy sets (SFS) to accommodate both qualitative and quantitative information, addressing the uncertainties characteristic of real-world decision-making scenarios. The framework's evaluation process incorporated assessments by five experts who analyzed eight women empowerment policies across six critical factors: Social and Cultural Factors, Educational Factors, Economic Factors, Health and Well-being, Political and Legal Factors, and Technological Access.

In 2025, Nomura et al.[25] proposed a fuzzy inference system (FIS)-based framework composed of seven interconnected modules designed to assess diverse criteria, including flood vulnerability, water quality, habitat connectivity, vegetation condition, and social vulnerability. The model was applied in the urban watersheds of São José dos Campos, Brazil, a municipality recognized for its smart city initiatives and urban environmental complexity. Through the integration of multi-criteria spatial data, the framework effectively prioritized urban areas, highlighting critical zones for extreme event mitigation, water quality preservation, habitat conservation, and recreational space provision. The system's modular structure, use of triangular membership functions, and incorporation of the gamma operator allow for adaptable prioritization across different planning horizons.

2.1. Problem Statement

Urban green space planning is a complex, multifaceted issue, which can support smart city objectives with regards to sustainability, public health, and livability. Previous studies have utilized fuzzy and intuitionistic fuzzy models in combination with multi-criteria decision making methods (MCDM) for urban development, energy efficiency, waste management, urban service infrastructure planning, and cycling infrastructure planning. However, no studies have applied the Intuitionistic Fuzzy AHP in combination with advanced aggregation operators to address uncertainty and hesitance improvements in expert decisions and equally, only a few other studies have specifically applied the Fuzzy TOPSIS MCDM method, which benchmarks alternatives against the ideal solution when ranking methods exist for other MCDM methods. This study aims to exploit the positives from these studies, with the first aim of the study being to outline an integrated MCDM framework using Intuitionistic Fuzzy AHP with the Frank, Power Geometric, and OWA operators and then applying the Fuzzy TOPSIS ranking method for the final ranking of the green space sites. In so doing, it is our intention to provide a more valid, interpretable, and context specific decision-making approach to optimise how urban green spaces are optimally allocated in smart cities.

3. Preliminaries

Definition 3.1: Fuzzy Set [26]
Assume Y be a nonempty set. A fuzzy set B from Y is defined as

$$B = \{(y, \nu_B(y)) : y \in Y\} \tag{3.1}$$

where $\nu_B(y) : Y \to [0,1]$ is the membership function of the fuzzy set B.

A fuzzy set is a collection of items with graded membership, often known as degrees of membership.

Definition 3.2: Fuzzy Number [27]
A fuzzy number B on the real line $\mathbb{R}$ must satisfy the following requirements:

1. $\nu_B(y_0)$ is piecewise continuous.
2. There exists at least one $y_0 \in \mathbb{R}$ with $\nu_B(y_0) = 1$.
3. B must be normal and convex.

Definition 3.3: Intuitionistic Fuzzy Set [28]
Assume Y is a nonempty set. An intuitionistic fuzzy set B in Y is an object having the form:

$$B = \{(y, \nu_B(y), \mu_B(y)) : y \in Y\} \tag{3.2}$$

where the functions $\nu_B(y), \mu_B(y) : Y \to [0,1]$ define respectively the degree of membership and degree of non-membership of the element $y \in Y$ to the set B, which is a subset of Y, and for every element $y \in Y$,

$$0 \le \nu_B(y) + \mu_B(y) \le 1.$$

Furthermore, we obtain:

$$\phi_B(y) = 1 - \nu_B(y) - \mu_B(y) \tag{3.3}$$

which is called the *intuitionistic fuzzy set index* or *hesitation margin* of y in B. $\phi_B(y)$ is the degree of indeterminacy of $y \in Y$ in the IFS B and $\phi_B(y) \in [0,1]$, i.e., $\phi_B(y) : Y \to [0,1]$ and $0 \le \phi_B(y) \le 1$ for all $y \in Y$. $\phi_B(y)$ indicates uncertainty about whether y belongs to the IFS B.
Example: Let B be an intuitionistic fuzzy set with $\nu_B(y) = 0.7$ and $\mu_B(y) = 0.1$, then

$$\phi_B(y) = 1 - (0.7 + 0.1) = 0.2.$$

It can be interpreted as: the degree that the object y belongs to IFS B is 0.7, the degree that the object y does not belong to IFS B is 0.1, and the degree of hesitancy is 0.2.

Definition 3.4: Intuitionistic Fuzzy Number (IFN)[29]

Intuitionistic fuzzy numbers (IFN) extend fuzzy numbers by including both membership and non-membership degrees. Each element has a membership function that indicates how much it belongs to the set, as well as a non-membership function that indicates how little it belongs. The total of these two numbers is less than or equal to one, providing for an extra level of uncertainty.

IFNs are useful in cases involving ambiguity or inadequate knowledge, such as medical diagnosis and risk assessments, since they provide a more comprehensive framework for reasoning under uncertainty.

An intuitionistic fuzzy set I on the real line is considered an intuitionistic fuzzy number if it meets the following three conditions:

1. It is *normal*, if at least one point y exists with $\nu_1(y) = 1$ and $\mu_1(y) = 0$.
2. It is *convex*.
3. The support, i.e., $\{y \in Y : \nu_1(y) > 0, \mu_1(y) < 1\}$, is both closed and bounded.

Definition 3.5: Analytic Hierarchy Process (AHP)[30]
Saaty invented the Analytic Hierarchy Process (AHP), which is one of the best solutions for the multi-criteria decision-making problem. The AHP assists in breaking down difficult problems into a multi-level hierarchical structure, sub-criteria, and alternatives. The use of hierarchical order helps to simplify the problem into a condition that is more clearly comprehended. Weights are calculated for each attribute and verified for consistency. Based on the weights of each attribute, the decision maker can make a reasonable decision.

Definition 3.6: Fuzzy Analytic Hierarchy Process (FAHP)[31]
The Fuzzy Analytical Hierarchy Process (FAHP) is the result of combining the Analytic Hierarchy Process with fuzzy set theory. It was proposed by Laarhoven and Pedrycz in 1983. In the FAHP method, fuzzy comparison ratios can better handle ambiguity than AHP values.

4. Intuitionistic Fuzzy Set

This part provides basic IFS definitions to help understand the implementation of IF-AHP, which is explained in the following section. The definitions in this section are primarily derived from Atanassov (1999).

The IFS B can be defined as:

$$B = \{(y, \nu_y, \mu_y) \mid y \in Y\} \tag{4.1}$$

The membership (ν_y) and non-membership (μ_y) functions for an element y are defined as follows:

$$\nu_y : Y \to [0,1] \quad \text{and} \quad \mu_y : Y \to [0,1] \tag{4.2}$$

where Y is the probable range of values for a variable y, and B is an IFS constructed over Y using membership and non-membership functions in $[0,1]$.

For any value of y in an IFS B, the statement:

$$0 \leq (\nu_y + \mu_y) \leq 1$$

is true. However, if the expression is reduced to $(\nu_y + \mu_y) = 1$, the IFS becomes a normal fuzzy set.

For IFS, the degree of non-determinacy ϕ_y (or non-specificity) of the element y in IFS B is defined as follows:

$$\phi_y = 1 - \nu_y - \mu_y; \quad \phi_y : Y \to [0,1] \tag{4.3}$$

Therefore, for ordinary fuzzy sets, the degree of non-determinacy $\phi_y = 0$.

5. Intuitionistic Fuzzy Analytic Hierarchy Process (IF-AHP)

The Intuitionistic Fuzzy Analytic Hierarchy Process (IF-AHP) integrates intuitionistic fuzzy set theory with the traditional Analytic Hierarchy Process to handle uncertainty and hesitancy in expert judgments. The steps involved in the IF-AHP methodology are as follows:

Step 1: Define the Problem Hierarchy

Divide the decision-making problem into three stages:

1. The **goal** (e.g., ideal green space selection),
2. The **criteria** (21 in total across environmental, economic, accessibility, and technical aspects),
3. The **alternatives** (nine potential locations).

Step 2: Perform Pairwise Comparisons Using IFVs

Experts analyze criteria and alternatives by converting linguistic concepts into *Intuitionistic Fuzzy Values (IFVs)*, which capture the degree of membership, non-membership, and hesitation (indeterminacy).

Step 3: Check Consistency

Ensure consistency in expert evaluations by applying relevant consistency indices or by re-evaluating pairwise comparisons to maintain logical coherence.

Step 4: Calculate Priority Weights

Determine the priority weights (ψ) of each criterion and sub-criterion using the IF-AHP formulas such as the *intuitionistic fuzzy geometric mean method*.

Step 5: Aggregate Weights

Combine weights across hierarchical levels through *intuitionistic fuzzy arithmetic aggregation*, ensuring the inclusion of both membership and non-membership information from multiple experts.

Step 6: Form the Final Decision Matrix

Integrate the aggregated weights with expert ratings to construct the *intuitionistic fuzzy weighted matrix*, representing the comprehensive evaluation structure of the alternatives.

Step 7: Rank Alternatives

Apply the **Fuzzy Technique for Order Preference by Similarity to Ideal Solution (Fuzzy TOPSIS)** to rank alternatives based on their relative closeness to the ideal and anti-ideal solutions. The highest-ranked option corresponds to the most suitable location for urban green space development.

6. Intuitionistic Fuzzy Analytic Hierarchy Process (IF-AHP) Methodological Steps

Step 1: Constructing the Hierarchical Structure

The decision-making hierarchy is organized into three levels:

- **Level 1:** Defines the goal of selecting the best green space alternative.

- **Level 2:** Includes 21 criteria ($C_1, C_2, \ldots, C_{21}$) categorized under environmental, economic, accessibility, and technical domains.
- **Level 3:** Includes nine potential alternative locations (A_1–A_9) for evaluation.

Step 2: Construct the Intuitionistic Fuzzy Pairwise Comparison Matrix

Pairwise comparisons between criteria are made using linguistic terms that are translated into intuitionistic fuzzy numbers (IFNs). Each entry b_{ij} in the comparison matrix is represented as:

$$b_{ij} = (\nu_{ij}, \mu_{ij}) \tag{6.1}$$

with hesitancy defined as:

$$\phi_{ij} = 1 - \nu_{ij} - \mu_{ij} \tag{6.2}$$

Reciprocals of comparisons follow the relation:

$$b_{ji} = (\nu_{ji}, \mu_{ji}) = (\mu_{ij}, \nu_{ij}) \tag{6.3}$$

Step 3: Compute the Geometric Mean for Each Criterion

For each criterion C_i, the intuitionistic fuzzy geometric mean over all pairwise comparisons is computed as:

$$GM_i = \left(\prod_{j=1}^{m} \nu_{ij}\right)^{1/m}, \quad \left(\prod_{j=1}^{m} \mu_{ij}\right)^{1/m} \tag{6.4}$$

Step 4: Normalize the Fuzzy Weights

The normalized intuitionistic fuzzy weight x_i for each criterion is calculated using:

$$\nu_i = \frac{GM_{\nu_i}}{\sum_{l=1}^{m} GM_{\nu_l}}, \quad \mu_i = \frac{GM_{\mu_i}}{\sum_{l=1}^{m} GM_{\mu_l}} \tag{6.5}$$

and the hesitancy:

$$\phi_i = 1 - \nu_i - \mu_i \tag{6.6}$$

Step 5: Consistency Verification

While standard AHP uses a consistency ratio, consistency checking in IF-AHP can be performed through expert re-validation or the development of updated consistency indices as necessary.

Application in Urban Green Space Planning

In this study, three experts (D_1, D_2, and D_3) compared the 21 criteria pairwise. The verbal assessments were matched to IFNs using a preset scale. The intuitionistic fuzzy weights for each criterion were calculated and utilized to generate the weighted decision matrix. These fuzzy weights captured the experts' uncertainty and partial agreement, allowing for more rigorous ranking of characteristics such as environmental effect, accessibility, and cost for selecting the best urban green space locations.

Step 6: Aggregation of Individual Decision Matrices

Aggregation operators are used to combine judgments from multiple decision makers (D_1, D_2, D_3). Using advanced aggregation methods, the intuitionistic fuzzy decision matrices of each expert are merged into a single consensus matrix.

Frank Aggregation Operator

Let $q(\xi_i) = (q(\xi_i), \overline{q}(\xi_i))$ $(i = 1, 2, \ldots, m)$ be a set of IFRSs in Y. An IFRFWA operator of dimension m is defined as:

$$IFRFWA(q(\xi_1), q(\xi_2), \ldots, q(\xi_m)) = \psi_{i-1}^{m} \psi_i q(\xi_i), \quad \psi_{i-1}^{m} \psi_i \overline{q}(\xi_i) \tag{6.7}$$

where the weight vector of ξ_i $(i = 1, 2, \ldots, m)$ is $\psi = (\psi_1, \psi_2, \ldots, \psi_m)^U$, with $\sum_{i=1}^{m} \psi_i = 1$ and $\psi_i \in [0, 1]$.

Using induction on m, the IFRFWA operator can be expressed as:

$$IFRFWA(q(\xi_1), q(\xi_2), \ldots, q(\xi_m)) = \left\{ 1 - \log_i \left(1 + \prod_{i=1}^{m} (\eta^{1\delta - 1})^{\psi_i} \right), \log_i \left(1 + \prod_{i=1}^{m} (\eta^{t} - 1)^{\psi_i} \right) \right\} \tag{6.8}$$

Power Average and Geometric Operators

Yager (2001) presented a nonlinear weighted average aggregation operator known as the Power Average (PA) operator, expressed as:

$$PA(b_1, b_2, \ldots, b_m) = \frac{\sum_{i=1}^{m} (1 + U(b_i)) b_i}{\sum_{j=1}^{m} (1 + U(b_j))} \tag{6.9}$$

where

$$U(b_i) = \sum_{j=1}^{m} Sup(b_i, b_j) \tag{6.10}$$

and $Sup(b_i, b_j)$ is the support for b_i from b_j, satisfying:

- $Sup(b_i, b_j) \in [0, 1]$
- $Sup(b_i, b_j) = Sup(b_j, b_i)$
- $Sup(b_i, b_j) \geq Sup(b_t, b_u)$ if $e(b_i, b_j) \leq e(b_t, b_u)$

where $e(b_i, b_j)$ is a distance measure.

Based on the geometric mean (Buckley, 1985) and the PA operator, Xu and Yager (2010) proposed the Power Geometric (PG) operator:

$$PG(b_1, b_2, \ldots, b_m) = \prod_{i=1}^{m} b_i^{\sum_{i=1}^{m} (b_i + U(b_i))} \tag{6.11}$$

Ordered Weighted Averaging (OWA) Operator

Let R be the set of real numbers and X be a weight vector such that

$$X = (\psi_1, \psi_2, \ldots, \psi_m),$$

where $\psi_l \in [0, 1]$ and $\sum_{l=1}^{m} \psi_l = 1$.

An Ordered Weighted Averaging Operator (OWAO) of dimension m, as defined by Yager, is:

$$\text{OWAO}_X(b_1, b_2, \ldots, b_m) = \sum_{l=1}^{m} \psi_l b_{(s(l))} \tag{6.12}$$

where $s : \{1, 2, \ldots, m\} \to \{1, 2, \ldots, m\}$ is a permutation such that $b_{(s(l))} \geq b_{(s(l+1))}$ for all $l = 1, 2, \ldots, m-1$.

These aggregation steps ensure a balanced integration of expert judgments, accounting for individual hesitancy and uncertainty.

Step 7: Final Intuitionistic Fuzzy Weighted Decision Matrix

The aggregated intuitionistic fuzzy decision matrix is multiplied with the IF-AHP derived weights to obtain the final IF-weighted decision matrix. Let $X = (x_1, x_2, \ldots, x_m)$ be the vector of criterion weights:

$$\overline{\nu}_{ij}^{final} = \overline{\nu}_{ij} \cdot x_j, \quad \overline{\mu}_{ij}^{final} = \overline{\mu}_{ij} \cdot x_j \tag{6.13}$$

The hesitation degree is calculated as:

$$\overline{\phi}_{ij}^{final} = 1 - \overline{\nu}_{ij}^{final} - \overline{\mu}_{ij}^{final} \tag{6.14}$$

This matrix reflects the weighted performance of each alternative across all criteria, integrating both expert input and criterion importance under uncertainty.

Step 8: Application of Fuzzy TOPSIS

The final IF-weighted decision matrix is evaluated using Fuzzy TOPSIS to rank alternatives.

Ideal Solutions:

$$\begin{cases} \nu_j^+ = \max_i(\overline{\nu}_{ij}^{final}), & \mu_j^+ = \min_i(\overline{\mu}_{ij}^{final}) \\ \nu_j^- = \min_i(\overline{\nu}_{ij}^{final}), & \mu_j^- = \max_i(\overline{\mu}_{ij}^{final}) \end{cases} \tag{6.15}$$

Separation Measures:

$$\begin{cases} SM_i^+ = \sqrt{\sum_j [(\overline{\nu}_{ij}^{final} - \nu_j^+)^2 + (\overline{\mu}_{ij}^{final} - \mu_j^+)^2]} \\ SM_i^- = \sqrt{\sum_j [(\overline{\nu}_{ij}^{final} - \nu_j^-)^2 + (\overline{\mu}_{ij}^{final} - \mu_j^-)^2]} \end{cases} \tag{6.16}$$

Closeness Coefficient:

$$CC_i = \frac{SM_i^-}{SM_i^+ + SM_i^-} \tag{6.17}$$

Alternatives are then ranked in descending order of CC_i, identifying the most suitable location for urban green space development.

7. Illustrative Example

Consider the evaluation of nine alternatives (A1 to A9) over 21 criteria (C1 to C21) using the Intuitionistic Fuzzy AHP method. This method associates each alternative with intuitionistic fuzzy sets that include both membership (positive preference) and non-membership (negative preference) values. This dual representation enables the effective modeling of uncertainty and hesitation in expert judgments, resulting in more reliable and informed decision-making in complicated settings.

Table 1: Number of Alternatives

Alternatives	
A1	Bowen Hills Green Zone
A2	Springfield Central Open Space
A3	Coorparoo Nature Reserve Extension
A4	Redbank Plains Creek Corridor
A5	Southport Riverfront Parklands
A6	Caboolture Eco-Recreation Site
A7	Upper Mount Gravatt Community Park
A8	Chermside Urban Oasis
A9	Logan Industrial Greening Project

Table 2: Number of Criteria

Criterion ID	**Code**	**Criterion Description**
C1	ENV-1	PM2.5 Air Quality ($\mu g/m^3$)
C2	ENV-2	Urban Heat Island Effect (°C above baseline)
C3	ENV-3	Biodiversity Score (0–1)
C4	ENV-4	Noise Levels (dB)
C5	ENV-5	Flood Hazard (probability, 0–1)
C6	ENV-6	Slope (%)
C7	ENV-7	Distance to Protected Area (km)
C8	SOC-1	Land Use Compatibility (score 0–1)
C9	SOC-2	Distance to Residential Areas (km)
C10	SOC-3	Accessibility (Public Transport, minutes)
C11	SOC-4	SEIFA Index (higher = less disadvantaged)
C12	SOC-5	Recreational Potential (0–1)
C13	SOC-6	Distance to Water Body (km)
C14	ECO-1	Land Cost ($/m^2)
C15	ECO-2	Development Cost Estimate ($k)
C16	ECO-3	Maintenance Cost Estimate ($k/year)
C17	ECO-4	Property Value Uplift Potential (0–1)
C18	UPT-1	Infrastructure Availability (0–1)
C19	UPT-2	Minimum Required Area Met? (Yes=1, No=0)
C20	UPT-3	Parcel Status (Vacant=1, Underused=0.5, Used=0)
C21	UPT-4	Zoning Compliance (Compliant=1, Not=0)

In this study, twenty-one criteria are used to evaluate the alternatives, with assessments provided by three decision-makers (D_1, D_2, and D_3). To effectively capture the inherent uncertainty and subjectivity in their judgments, a seven-level linguistic scale is adopted, defined as:

$$S = \{\text{Extreme Low (EL)}, \text{Very Low (VL)}, \text{Low (L)}, \text{Medium (M)}, \\ \text{High (H)}, \text{Very High (VH)}, \text{Extreme High (EH)}\}$$

Each linguistic term corresponds to an Intuitionistic Fuzzy Set (IFS) over the closed interval [0,1], incorporating values for membership (μ), non-membership (ν), and hesitation (π) as detailed in Table 3.

These IFS-based linguistic evaluations allow for a richer representation of expert opinions by accounting not only for the degree of support or opposition but also for indecisiveness or hesitancy. This enables a more robust and realistic multi-criteria decision-making process in the context of urban green space planning.

Table 3: Linguistic Variables

Abbreviation	**Linguistic Term**	**IFS Values** (μ, ν, π)
EL	Extreme Low	(0.10, 0.85, 0.05)
VL	Very Low	(0.20, 0.75, 0.05)
L	Low	(0.35, 0.60, 0.05)
M	Medium	(0.50, 0.45, 0.05)
H	High	(0.65, 0.30, 0.05)
VH	Very High	(0.80, 0.15, 0.05)
EH	Extreme High	(0.90, 0.05, 0.05)

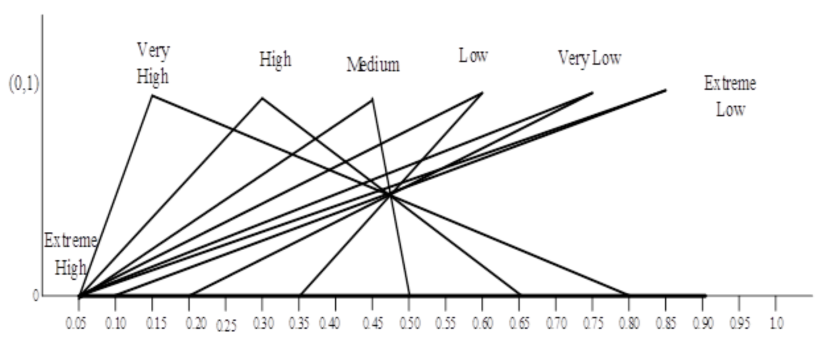

Figure 1: Satisfaction and dissatisfaction degree for linguistic values

Each decision matrix (D_1, D_2, D_3) independently evaluates nine alternatives against twenty-one criteria using predefined linguistic terms. These linguistic terms are then converted into Intuitionistic Fuzzy Numbers (IFNs) to form the initial decision matrices for further aggregation and analysis.

Table 4: Performance Rating of Alternatives and Weighted Decision Matrix D_1 for 21 Criteria

Decision Maker	D_1								
Alternatives	A1	A2	A3	A4	A5	A6	A7	A8	A9
C1	M	L	M	H	VH	L	VL	M	H
C2	H	M	H	VH	H	L	L	H	VH
C3	VH	H	VH	H	VH	M	M	VH	EH
C4	M	L	VH	M	M	VL	L	H	VH
C5	L	VL	M	VH	L	L	L	M	H
C6	VL	L	VH	H	VL	L	VL	VH	VH
C7	H	VH	H	VH	H	M	L	H	EH
C8	VH	M	H	H	VH	L	M	VH	VH
C9	M	VL	M	VH	M	VL	VL	M	H
C10	VL	EL	L	M	VL	EL	EL	L	VH
C11	EL	VL	VL	L	EL	VL	VL	VL	M
C12	M	H	M	H	VH	L	M	M	H
C13	H	M	H	VH	H	M	L	H	VH
C14	M	VL	VH	M	M	VL	VL	VH	H
C15	L	L	M	H	L	VL	VL	M	VH
C16	VL	L	VL	VH	VL	VL	VL	VL	VH
C17	H	M	H	H	H	L	L	H	VH
C18	VH	H	VH	VH	VH	M	M	VH	EH
C19	M	VL	M	H	M	VL	VL	M	H
C20	VL	VL	L	M	VL	VL	VL	L	VH
C21	L	L	M	H	L	L	VL	M	H

Table 5: Performance Rating of Alternatives and Weighted Decision Matrix D_2 for 21 Criteria

Decision Maker	D_2								
Alternatives	A1	A2	A3	A4	A5	A6	A7	A8	A9
C1	VH	L	VH	VH	H	L	VL	VH	VH
C2	M	VL	H	H	VH	L	L	H	H
C3	H	M	VH	VH	H	M	L	VH	EH
C4	VH	L	VH	VH	M	VL	L	VH	VH
C5	L	VL	M	H	L	L	VL	M	H
C6	L	L	H	VH	L	VL	VL	VH	VH
C7	VH	M	VH	H	VH	M	L	H	EH
C8	H	VH	H	VH	H	L	M	VH	VH
C9	M	VL	VH	VH	M	VL	VL	VH	H
C10	VL	EL	L	M	VL	EL	EL	L	VH
C11	L	VL	VL	L	EL	VL	VL	VL	M
C12	M	M	VH	VH	VH	L	M	VH	H
C13	H	VL	H	H	H	M	VL	H	VH
C14	VH	L	VH	VH	M	VL	VL	VH	H
C15	M	VL	M	VH	L	VL	VL	M	VH
C16	L	VL	VL	VH	VL	VL	VL	VL	VH
C17	VH	M	H	H	VH	L	L	H	VH
C18	H	H	VH	VH	H	M	M	VH	EH
C19	M	VL	M	VH	M	VL	VL	VH	H
C20	VL	VL	L	M	VL	VL	VL	L	VH
C21	L	L	VH	H	L	L	VL	VH	VH

Table 6: Performance Rating of Alternatives and Weighted Decision Matrix D_3 for 21 Criteria

Decision Maker	D_3								
Alternatives	A1	A2	A3	A4	A5	A6	A7	A8	A9
C1	M	VL	H	VH	H	VL	VL	VH	H
C2	VH	L	VH	H	VH	L	L	H	VH
C3	VH	VH	VH	EH	H	M	L	VH	EH
C4	H	L	H	VH	M	VL	L	VH	VH
C5	VH	L	VH	H	L	L	VL	M	H
C6	M	VL	VH	VH	VL	VL	VL	VH	VH
C7	H	VH	H	EH	VH	M	L	H	EH
C8	VH	M	VH	VH	H	L	M	VH	VH
C9	M	VL	VH	H	M	VL	VL	VH	H
C10	L	EL	L	M	VL	EL	EL	L	VH
C11	VL	VL	VL	L	EL	VL	VL	VL	M
C12	VH	VH	VH	VH	VH	L	M	VH	H
C13	H	M	VH	EH	H	M	VL	H	VH
C14	VH	VL	VH	H	M	VL	VL	VH	H
C15	M	L	M	VH	L	VL	VL	M	VH
C16	L	VL	VL	VH	VL	VL	VL	VL	VH
C17	H	M	VH	VH	VH	L	L	H	VH
C18	VH	H	EH	EH	H	M	M	VH	EH
C19	M	VL	VH	H	M	VL	VL	VH	H
C20	L	VL	L	VH	VL	VL	VL	L	VH
C21	M	L	VH	VH	L	L	VL	VH	VH

The performance ratings from each decision matrix are transformed into weighted decision matrices by applying the **Intuitionistic Fuzzy Analytic Hierarchy Process (IF-AHP)**, which calculates criterion weights based on expert-driven pairwise comparisons under uncertainty.

Table 7: Fuzzy Weight Matrix from Operators

Criterion	**Fuzzy Weight** [μ, ν, π]
C1	[0.56, 0.39, 0.06]
C2	[0.59, 0.35, 0.05]
C3	[0.71, 0.24, 0.05]
C4	[0.57, 0.38, 0.05]
C5	[0.47, 0.47, 0.06]
C6	[0.50, 0.44, 0.05]
C7	[0.68, 0.27, 0.06]
C8	[0.67, 0.28, 0.05]
C9	[0.49, 0.45, 0.05]
C10	[0.31, 0.64, 0.05]
C11	[0.24, 0.71, 0.05]
C12	[0.64, 0.30, 0.06]
C13	[0.60, 0.34, 0.05]
C14	[0.53, 0.41, 0.05]
C15	[0.45, 0.50, 0.05]
C16	[0.35, 0.60, 0.05]
C17	[0.62, 0.33, 0.05]
C18	[0.72, 0.22, 0.05]
C19	[0.48, 0.47, 0.05]
C20	[0.35, 0.59, 0.05]
C21	[0.51, 0.44, 0.06]

Expert judgments from D_1, D_2, and D_3 are then aggregated into a unified decision matrix using the **Frank**, **Power Geometric Mean**, and **OWA** operators, ensuring balanced integration while effectively capturing uncertainty and hesitancy.

Table 8: Frank Aggregation Operator

Criteria	A1	A2	A3	A4	A5	A6	A7	A8	A9
C1	[0.6,0.35,0.05]	[0.3,0.65,0.05]	[0.65,0.3,0.05]	[0.75,0.2,0.05]	[0.7,0.25,0.05]	[0.3,0.65,0.05]	[0.2,0.75,0.05]	[0.7,0.25,0.05]	[0.7,0.25,0.05]
C2	[0.65,0.3,0.05]	[0.35,0.6,0.05]	[0.7,0.25,0.05]	[0.7,0.25,0.05]	[0.75,0.2,0.05]	[0.35,0.6,0.05]	[0.35,0.6,0.05]	[0.65,0.3,0.05]	[0.75,0.2,0.05]
C3	[0.75,0.2,0.05]	[0.65,0.3,0.05]	[0.8,0.15,0.05]	[0.783,0.167,0.05]	[0.7,0.25,0.05]	[0.5,0.45,0.05]	[0.4,0.55,0.05]	[0.8,0.15,0.05]	[0.9,0.05,0.05]
C4	[0.65,0.3,0.05]	[0.35,0.6,0.05]	[0.75,0.2,0.05]	[0.7,0.25,0.05]	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.35,0.6,0.05]	[0.75,0.2,0.05]	[0.8,0.15,0.05]
C5	[0.5,0.45,0.05]	[0.25,0.7,0.05]	[0.6,0.35,0.05]	[0.7,0.25,0.05]	[0.35,0.6,0.05]	[0.35,0.6,0.05]	[0.25,0.7,0.05]	[0.5,0.45,0.05]	[0.65,0.3,0.05]
C6	[0.35,0.6,0.05]	[0.3,0.65,0.05]	[0.75,0.2,0.05]	[0.75,0.2,0.05]	[0.25,0.7,0.05]	[0.25,0.7,0.05]	[0.2,0.75,0.05]	[0.8,0.15,0.05]	[0.8,0.15,0.05]
C7	[0.7,0.25,0.05]	[0.7,0.25,0.05]	[0.7,0.25,0.05]	[0.783,0.167,0.05]	[0.75,0.2,0.05]	[0.5,0.45,0.05]	[0.35,0.6,0.05]	[0.65,0.3,0.05]	[0.9,0.05,0.05]
C8	[0.75,0.2,0.05]	[0.6,0.35,0.05]	[0.7,0.25,0.05]	[0.75,0.2,0.05]	[0.7,0.25,0.05]	[0.35,0.6,0.05]	[0.5,0.45,0.05]	[0.8,0.15,0.05]	[0.8,0.15,0.05]
C9	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.7,0.25,0.05]	[0.75,0.2,0.05]	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.7,0.25,0.05]	[0.65,0.3,0.05]
C10	[0.25,0.7,0.05]	[0.1,0.85,0.05]	[0.35,0.6,0.05]	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.1,0.85,0.05]	[0.1,0.85,0.05]	[0.35,0.6,0.05]	[0.8,0.15,0.05]
C11	[0.22,0.73,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.35,0.6,0.05]	[0.1,0.85,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.5,0.45,0.05]
C12	[0.6,0.35,0.05]	[0.65,0.3,0.05]	[0.7,0.25,0.05]	[0.75,0.2,0.05]	[0.8,0.15,0.05]	[0.35,0.6,0.05]	[0.5,0.45,0.05]	[0.7,0.25,0.05]	[0.65,0.3,0.05]
C13	[0.65,0.3,0.05]	[0.4,0.55,0.05]	[0.7,0.25,0.05]	[0.783,0.167,0.05]	[0.65,0.3,0.05]	[0.5,0.45,0.05]	[0.25,0.7,0.05]	[0.65,0.3,0.05]	[0.8,0.15,0.05]
C14	[0.7,0.25,0.05]	[0.25,0.7,0.05]	[0.8,0.15,0.05]	[0.65,0.3,0.05]	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.8,0.15,0.05]	[0.65,0.3,0.05]
C15	[0.45,0.5,0.05]	[0.3,0.65,0.05]	[0.5,0.45,0.05]	[0.75,0.2,0.05]	[0.35,0.6,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.5,0.45,0.05]	[0.8,0.15,0.05]
C16	[0.3,0.65,0.05]	[0.25,0.7,0.05]	[0.2,0.75,0.05]	[0.8,0.15,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.8,0.15,0.05]
C17	[0.7,0.25,0.05]	[0.5,0.45,0.05]	[0.7,0.25,0.05]	[0.7,0.25,0.05]	[0.75,0.2,0.05]	[0.35,0.6,0.05]	[0.35,0.6,0.05]	[0.65,0.3,0.05]	[0.8,0.15,0.05]
C18	[0.75,0.2,0.05]	[0.65,0.3,0.05]	[0.833,0.117,0.05]	[0.833,0.117,0.05]	[0.7,0.25,0.05]	[0.5,0.45,0.05]	[0.5,0.45,0.05]	[0.8,0.15,0.05]	[0.9,0.05,0.05]
C19	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.6,0.35,0.05]	[0.7,0.25,0.05]	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.7,0.25,0.05]	[0.65,0.3,0.05]
C20	[0.25,0.7,0.05]	[0.2,0.75,0.05]	[0.35,0.6,0.05]	[0.6,0.35,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.35,0.6,0.05]	[0.8,0.15,0.05]
C21	[0.4,0.55,0.05]	[0.35,0.6,0.05]	[0.7,0.25,0.05]	[0.7,0.25,0.05]	[0.35,0.6,0.05]	[0.35,0.6,0.05]	[0.2,0.75,0.05]	[0.7,0.25,0.05]	[0.75,0.2,0.05]

Table 9: Power Geometric Aggregation

Criteria	A1	A2	A3	A4	A5	A6	A7	A8	A9
C1	[0.58,0.31,0.1]	[0.29,0.65,0.06]	[0.64,0.27,0.09]	[0.75,0.19,0.06]	[0.7,0.24,0.07]	[0.29,0.65,0.06]	[0.2,0.75,0.05]	[0.68,0.22,0.1]	[0.7,0.24,0.07]
C2	[0.64,0.27,0.09]	[0.33,0.59,0.09]	[0.7,0.24,0.07]	[0.7,0.24,0.07]	[0.75,0.19,0.06]	[0.35,0.6,0.05]	[0.35,0.6,0.05]	[0.65,0.3,0.05]	[0.75,0.19,0.06]
C3	[0.75,0.19,0.06]	[0.64,0.27,0.09]	[0.8,0.15,0.05]	[0.78,0.13,0.09]	[0.7,0.24,0.07]	[0.5,0.45,0.05]	[0.39,0.55,0.06]	[0.8,0.15,0.05]	[0.9,0.05,0.05]
C4	[0.64,0.27,0.09]	[0.35,0.6,0.05]	[0.75,0.19,0.06]	[0.68,0.22,0.1]	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.35,0.6,0.05]	[0.75,0.19,0.06]	[0.8,0.15,0.05]
C5	[0.46,0.38,0.16]	[0.24,0.7,0.06]	[0.58,0.31,0.1]	[0.7,0.24,0.07]	[0.35,0.6,0.05]	[0.35,0.6,0.05]	[0.24,0.7,0.06]	[0.5,0.45,0.05]	[0.65,0.3,0.05]
C6	[0.33,0.59,0.09]	[0.29,0.65,0.06]	[0.75,0.19,0.06]	[0.75,0.19,0.06]	[0.24,0.7,0.06]	[0.24,0.7,0.06]	[0.2,0.75,0.05]	[0.8,0.15,0.05]	[0.8,0.15,0.05]
C7	[0.7,0.24,0.07]	[0.68,0.22,0.1]	[0.7,0.24,0.07]	[0.78,0.13,0.09]	[0.75,0.19,0.06]	[0.5,0.45,0.05]	[0.35,0.6,0.05]	[0.65,0.3,0.05]	[0.9,0.05,0.05]
C8	[0.75,0.19,0.06]	[0.58,0.31,0.1]	[0.7,0.24,0.07]	[0.75,0.19,0.06]	[0.7,0.24,0.07]	[0.35,0.6,0.05]	[0.5,0.45,0.05]	[0.8,0.15,0.05]	[0.8,0.15,0.05]
C9	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.68,0.22,0.1]	[0.75,0.19,0.06]	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.68,0.22,0.1]	[0.65,0.3,0.05]
C10	[0.24,0.7,0.06]	[0.1,0.85,0.05]	[0.35,0.6,0.05]	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.1,0.85,0.05]	[0.1,0.85,0.05]	[0.35,0.6,0.05]	[0.8,0.15,0.05]
C11	[0.19,0.73,0.08]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.35,0.6,0.05]	[0.1,0.85,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.5,0.45,0.05]
C12	[0.58,0.31,0.1]	[0.64,0.27,0.09]	[0.68,0.22,0.1]	[0.75,0.19,0.06]	[0.8,0.15,0.05]	[0.35,0.6,0.05]	[0.5,0.45,0.05]	[0.68,0.22,0.1]	[0.65,0.3,0.05]
C13	[0.65,0.3,0.05]	[0.37,0.53,0.1]	[0.7,0.24,0.07]	[0.78,0.13,0.09]	[0.65,0.3,0.05]	[0.5,0.45,0.05]	[0.24,0.7,0.06]	[0.65,0.3,0.05]	[0.8,0.15,0.05]
C14	[0.68,0.22,0.1]	[0.24,0.7,0.06]	[0.8,0.15,0.05]	[0.64,0.27,0.09]	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.8,0.15,0.05]	[0.65,0.3,0.05]
C15	[0.44,0.5,0.06]	[0.29,0.65,0.06]	[0.5,0.45,0.05]	[0.75,0.19,0.06]	[0.35,0.6,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.5,0.45,0.05]	[0.8,0.15,0.05]
C16	[0.29,0.65,0.06]	[0.24,0.7,0.06]	[0.2,0.75,0.05]	[0.8,0.15,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.8,0.15,0.05]
C17	[0.7,0.24,0.07]	[0.5,0.45,0.05]	[0.7,0.24,0.07]	[0.7,0.24,0.07]	[0.75,0.19,0.06]	[0.35,0.6,0.05]	[0.35,0.6,0.05]	[0.65,0.3,0.05]	[0.8,0.15,0.05]
C18	[0.75,0.19,0.06]	[0.65,0.3,0.05]	[0.83,0.1,0.06]	[0.83,0.1,0.06]	[0.7,0.24,0.07]	[0.5,0.45,0.05]	[0.5,0.45,0.05]	[0.8,0.15,0.05]	[0.9,0.05,0.05]
C19	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.58,0.31,0.1]	[0.7,0.24,0.07]	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.68,0.22,0.1]	[0.65,0.3,0.05]
C20	[0.24,0.7,0.06]	[0.2,0.75,0.05]	[0.35,0.6,0.05]	[0.58,0.31,0.1]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.35,0.6,0.05]	[0.8,0.15,0.05]
C21	[0.39,0.55,0.06]	[0.35,0.6,0.05]	[0.68,0.22,0.1]	[0.7,0.24,0.07]	[0.35,0.6,0.05]	[0.35,0.6,0.05]	[0.2,0.75,0.05]	[0.68,0.22,0.1]	[0.75,0.19,0.06]

Table 10: OWA Aggregation

Criteria	A1	A2	A3	A4	A5	A6	A7	A8	A9
C1	[0.68,0.27,0.05]	[0.34,0.62,0.05]	[0.73,0.22,0.05]	[0.78,0.16,0.05]	[0.74,0.21,0.05]	[0.34,0.62,0.05]	[0.2,0.75,0.05]	[0.77,0.18,0.05]	[0.74,0.21,0.05]
C2	[0.73,0.22,0.05]	[0.42,0.52,0.05]	[0.74,0.21,0.05]	[0.74,0.21,0.05]	[0.78,0.16,0.05]	[0.35,0.6,0.05]	[0.35,0.6,0.05]	[0.65,0.3,0.05]	[0.78,0.16,0.05]
C3	[0.78,0.16,0.05]	[0.73,0.22,0.05]	[0.8,0.15,0.05]	[0.84,0.1,0.05]	[0.74,0.21,0.05]	[0.5,0.45,0.05]	[0.44,0.51,0.05]	[0.8,0.15,0.05]	[0.9,0.05,0.05]
C4	[0.73,0.22,0.05]	[0.35,0.6,0.05]	[0.78,0.16,0.05]	[0.77,0.18,0.05]	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.35,0.6,0.05]	[0.78,0.16,0.05]	[0.8,0.15,0.05]
C5	[0.62,0.33,0.05]	[0.29,0.66,0.05]	[0.68,0.27,0.05]	[0.74,0.21,0.05]	[0.35,0.6,0.05]	[0.35,0.6,0.05]	[0.29,0.66,0.05]	[0.5,0.45,0.05]	[0.65,0.3,0.05]
C6	[0.42,0.52,0.05]	[0.34,0.62,0.05]	[0.78,0.16,0.05]	[0.78,0.16,0.05]	[0.29,0.66,0.05]	[0.29,0.66,0.05]	[0.2,0.75,0.05]	[0.8,0.15,0.05]	[0.8,0.15,0.05]
C7	[0.74,0.21,0.05]	[0.77,0.18,0.05]	[0.74,0.21,0.05]	[0.84,0.1,0.05]	[0.78,0.16,0.05]	[0.5,0.45,0.05]	[0.35,0.6,0.05]	[0.65,0.3,0.05]	[0.9,0.05,0.05]
C8	[0.78,0.16,0.05]	[0.68,0.27,0.05]	[0.74,0.21,0.05]	[0.78,0.16,0.05]	[0.74,0.21,0.05]	[0.35,0.6,0.05]	[0.5,0.45,0.05]	[0.8,0.15,0.05]	[0.8,0.15,0.05]
C9	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.77,0.18,0.05]	[0.78,0.16,0.05]	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.77,0.18,0.05]	[0.65,0.3,0.05]
C10	[0.29,0.66,0.05]	[0.1,0.85,0.05]	[0.35,0.6,0.05]	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.1,0.85,0.05]	[0.1,0.85,0.05]	[0.35,0.6,0.05]	[0.8,0.15,0.05]
C11	[0.28,0.67,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.35,0.6,0.05]	[0.1,0.85,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.5,0.45,0.05]
C12	[0.68,0.27,0.05]	[0.73,0.22,0.05]	[0.77,0.18,0.05]	[0.78,0.16,0.05]	[0.8,0.15,0.05]	[0.35,0.6,0.05]	[0.5,0.45,0.05]	[0.77,0.18,0.05]	[0.65,0.3,0.05]
C13	[0.65,0.3,0.05]	[0.47,0.48,0.05]	[0.74,0.21,0.05]	[0.84,0.1,0.05]	[0.65,0.3,0.05]	[0.5,0.45,0.05]	[0.29,0.66,0.05]	[0.65,0.3,0.05]	[0.8,0.15,0.05]
C14	[0.77,0.18,0.05]	[0.29,0.66,0.05]	[0.8,0.15,0.05]	[0.73,0.22,0.05]	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.8,0.15,0.05]	[0.65,0.3,0.05]
C15	[0.48,0.46,0.05]	[0.34,0.62,0.05]	[0.5,0.45,0.05]	[0.78,0.16,0.05]	[0.35,0.6,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.5,0.45,0.05]	[0.8,0.15,0.05]
C16	[0.34,0.62,0.05]	[0.29,0.66,0.05]	[0.2,0.75,0.05]	[0.8,0.15,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.8,0.15,0.05]
C17	[0.74,0.21,0.05]	[0.5,0.45,0.05]	[0.74,0.21,0.05]	[0.74,0.21,0.05]	[0.78,0.16,0.05]	[0.35,0.6,0.05]	[0.35,0.6,0.05]	[0.65,0.3,0.05]	[0.8,0.15,0.05]
C18	[0.78,0.16,0.05]	[0.65,0.3,0.05]	[0.86,0.09,0.05]	[0.86,0.09,0.05]	[0.74,0.21,0.05]	[0.5,0.45,0.05]	[0.5,0.45,0.05]	[0.8,0.15,0.05]	[0.9,0.05,0.05]
C19	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.68,0.27,0.05]	[0.74,0.21,0.05]	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.77,0.18,0.05]	[0.65,0.3,0.05]
C20	[0.29,0.66,0.05]	[0.2,0.75,0.05]	[0.35,0.6,0.05]	[0.68,0.27,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.35,0.6,0.05]	[0.8,0.15,0.05]
C21	[0.44,0.51,0.05]	[0.35,0.6,0.05]	[0.77,0.18,0.05]	[0.74,0.21,0.05]	[0.35,0.6,0.05]	[0.35,0.6,0.05]	[0.2,0.75,0.05]	[0.77,0.18,0.05]	[0.78,0.16,0.05]

Table 11: Final Aggregation Matrix of 3 Operators

Criteria	A1	A2	A3	A4	A5	A6	A7	A8	A9
C1	[0.62,0.31,0.07]	[0.31,0.64,0.05]	[0.67,0.27,0.06]	[0.76,0.18,0.05]	[0.71,0.23,0.06]	[0.31,0.64,0.05]	[0.2,0.75,0.05]	[0.72,0.22,0.07]	[0.71,0.23,0.06]
C2	[0.67,0.27,0.06]	[0.37,0.57,0.06]	[0.71,0.23,0.06]	[0.71,0.23,0.06]	[0.76,0.18,0.05]	[0.35,0.6,0.05]	[0.35,0.6,0.05]	[0.65,0.3,0.05]	[0.76,0.18,0.05]
C3	[0.76,0.18,0.05]	[0.67,0.27,0.06]	[0.8,0.15,0.05]	[0.8,0.13,0.06]	[0.71,0.23,0.06]	[0.5,0.45,0.05]	[0.41,0.54,0.05]	[0.8,0.15,0.05]	[0.9,0.05,0.05]
C4	[0.67,0.27,0.06]	[0.35,0.6,0.05]	[0.76,0.18,0.05]	[0.72,0.22,0.07]	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.35,0.6,0.05]	[0.76,0.18,0.05]	[0.8,0.15,0.05]
C5	[0.53,0.39,0.09]	[0.26,0.69,0.05]	[0.62,0.31,0.07]	[0.71,0.23,0.06]	[0.35,0.6,0.05]	[0.35,0.6,0.05]	[0.26,0.69,0.05]	[0.5,0.45,0.05]	[0.65,0.3,0.05]
C6	[0.37,0.57,0.06]	[0.31,0.64,0.05]	[0.76,0.18,0.05]	[0.76,0.18,0.05]	[0.26,0.69,0.05]	[0.26,0.69,0.05]	[0.2,0.75,0.05]	[0.8,0.15,0.05]	[0.8,0.15,0.05]
C7	[0.71,0.23,0.06]	[0.72,0.22,0.07]	[0.71,0.23,0.06]	[0.8,0.13,0.06]	[0.76,0.18,0.05]	[0.5,0.45,0.05]	[0.35,0.6,0.05]	[0.65,0.3,0.05]	[0.9,0.05,0.05]
C8	[0.76,0.18,0.05]	[0.62,0.31,0.07]	[0.71,0.23,0.06]	[0.76,0.18,0.05]	[0.71,0.23,0.06]	[0.35,0.6,0.05]	[0.5,0.45,0.05]	[0.8,0.15,0.05]	[0.8,0.15,0.05]
C9	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.72,0.22,0.07]	[0.76,0.18,0.05]	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.72,0.22,0.07]	[0.65,0.3,0.05]
C10	[0.26,0.69,0.05]	[0.1,0.85,0.05]	[0.35,0.6,0.05]	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.1,0.85,0.05]	[0.1,0.85,0.05]	[0.35,0.6,0.05]	[0.8,0.15,0.05]
C11	[0.23,0.71,0.06]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.35,0.6,0.05]	[0.1,0.85,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.5,0.45,0.05]
C12	[0.62,0.31,0.07]	[0.67,0.27,0.06]	[0.72,0.22,0.07]	[0.76,0.18,0.05]	[0.8,0.15,0.05]	[0.35,0.6,0.05]	[0.5,0.45,0.05]	[0.72,0.22,0.07]	[0.65,0.3,0.05]
C13	[0.65,0.3,0.05]	[0.41,0.52,0.07]	[0.71,0.23,0.06]	[0.8,0.13,0.06]	[0.65,0.3,0.05]	[0.5,0.45,0.05]	[0.26,0.69,0.05]	[0.65,0.3,0.05]	[0.8,0.15,0.05]
C14	[0.72,0.22,0.07]	[0.26,0.69,0.05]	[0.8,0.15,0.05]	[0.67,0.27,0.06]	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.8,0.15,0.05]	[0.65,0.3,0.05]
C15	[0.46,0.49,0.05]	[0.31,0.64,0.05]	[0.5,0.45,0.05]	[0.76,0.18,0.05]	[0.35,0.6,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.5,0.45,0.05]	[0.8,0.15,0.05]
C16	[0.31,0.64,0.05]	[0.26,0.69,0.05]	[0.2,0.75,0.05]	[0.8,0.15,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.8,0.15,0.05]
C17	[0.71,0.23,0.06]	[0.5,0.45,0.05]	[0.71,0.23,0.06]	[0.71,0.23,0.06]	[0.76,0.18,0.05]	[0.35,0.6,0.05]	[0.35,0.6,0.05]	[0.65,0.3,0.05]	[0.8,0.15,0.05]
C18	[0.76,0.18,0.05]	[0.65,0.3,0.05]	[0.84,0.1,0.05]	[0.84,0.1,0.05]	[0.71,0.23,0.06]	[0.5,0.45,0.05]	[0.5,0.45,0.05]	[0.8,0.15,0.05]	[0.9,0.05,0.05]
C19	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.62,0.31,0.07]	[0.71,0.23,0.06]	[0.5,0.45,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.72,0.22,0.07]	[0.65,0.3,0.05]
C20	[0.26,0.69,0.05]	[0.2,0.75,0.05]	[0.35,0.6,0.05]	[0.62,0.31,0.07]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.2,0.75,0.05]	[0.35,0.6,0.05]	[0.8,0.15,0.05]
C21	[0.41,0.54,0.05]	[0.35,0.6,0.05]	[0.72,0.22,0.07]	[0.71,0.23,0.06]	[0.35,0.6,0.05]	[0.35,0.6,0.05]	[0.2,0.75,0.05]	[0.72,0.22,0.07]	[0.76,0.18,0.05]

The final intuitionistic fuzzy weighted decision matrix combines aggregated expert evaluations, reflecting each alternative's overall performance across all criteria. It accounts for membership, non-membership, and hesitation values from all decision-makers.

Table 12: Continued

Table 12: Final Aggregation Matrix of 3 Operators for the Criteria

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
C1	[1, 1, 1]	[1.72, 1.92, 2.11]	[2.26, 2.51, 2.76]	[2.64, 2.94, 3.23]	[2.39, 2.65, 2.92]	[3.53, 3.92, 4.31]	[3.03, 3.36, 3.70]	[6.37, 7.08, 7.78]	[5.59, 6.21, 6.83]	[6.75, 7.50, 8.26]	[7.99, 8.87, 9.76]
C2	[0.47, 0.52, 0.57]	[1, 1, 1]	[1.43, 1.59, 1.75]	[1.61, 1.79, 1.97]	[2.02, 2.24, 2.47]	[2.60, 2.89, 3.18]	[3.18, 3.53, 3.88]	[3.29, 3.66, 4.02]	[6.72, 7.46, 8.21]	[7.34, 8.15, 8.97]	[5.04, 5.60, 6.16]
C3	[0.36, 0.40, 0.44]	[0.57, 0.63, 0.69]	[1, 1, 1]	[1.61, 1.78, 1.96]	[2.16, 2.39, 2.63]	[1.92, 2.13, 2.35]	[3.49, 3.87, 4.26]	[3.38, 3.76, 4.13]	[3.41, 3.78, 4.16]	[5.42, 6.02, 6.62]	[5.22, 5.79, 6.37]
C4	[0.31, 0.34, 0.37]	[0.50, 0.56, 0.61]	[0.50, 0.56, 0.62]	[1, 1, 1]	[2.09, 2.33, 2.56]	[2.99, 3.32, 3.65]	[2.98, 3.32, 3.65]	[2.77, 3.07, 3.38]	[3.43, 3.82, 4.20]	[5.14, 5.71, 6.28]	[7.13, 7.93, 8.72]
C5	[0.34, 0.38, 0.41]	[0.40, 0.45, 0.49]	[0.38, 0.42, 0.46]	[0.39, 0.43, 0.47]	[1, 1, 1]	[1.22, 1.35, 1.49]	[2.00, 2.22, 2.44]	[2.73, 3.03, 3.33]	[3.32, 3.69, 4.06]	[4.12, 4.58, 5.03]	[4.51, 5.02, 5.52]
C6	[0.23, 0.26, 0.28]	[0.31, 0.35, 0.38]	[0.42, 0.47, 0.52]	[0.27, 0.30, 0.33]	[0.67, 0.74, 0.81]	[1, 1, 1]	[1.61, 1.79, 1.97]	[2.75, 3.06, 3.37]	[2.62, 2.91, 3.20]	[3.52, 3.91, 4.31]	[2.10, 2.33, 2.56]
C7	[0.27, 0.30, 0.33]	[0.25, 0.28, 0.31]	[0.23, 0.26, 0.28]	[0.27, 0.30, 0.33]	[0.41, 0.45, 0.50]	[0.50, 0.56, 0.62]	[1, 1, 1]	[1.95, 2.17, 2.39]	[2.28, 2.53, 2.79]	[2.08, 2.31, 2.54]	[2.05, 2.28, 2.50]
C8	[0.13, 0.14, 0.16]	[0.25, 0.27, 0.30]	[0.24, 0.27, 0.29]	[0.29, 0.33, 0.36]	[0.30, 0.33, 0.36]	[0.29, 0.33, 0.36]	[0.41, 0.46, 0.51]	[1, 1, 1]	[1.97, 2.19, 2.41]	[1.97, 2.19, 2.40]	[2.33, 2.59, 2.85]
C9	[0.14, 0.16, 0.18]	[0.12, 0.13, 0.15]	[0.24, 0.26, 0.29]	[0.24, 0.26, 0.29]	[0.24, 0.27, 0.30]	[0.31, 0.34, 0.38]	[0.36, 0.39, 0.43]	[0.41, 0.46, 0.50]	[1, 1, 1]	[1.55, 1.73, 1.90]	[2.12, 2.36, 2.60]
C10	[0.12, 0.13, 0.15]	[0.11, 0.12, 0.13]	[0.15, 0.17, 0.18]	[0.16, 0.18, 0.19]	[0.20, 0.22, 0.24]	[0.23, 0.26, 0.28]	[0.39, 0.43, 0.48]	[0.41, 0.46, 0.50]	[0.52, 0.58, 0.64]	[1, 1, 1]	[1.09, 1.21, 1.33]
C11	[0.10, 0.11, 0.12]	[0.16, 0.18, 0.20]	[0.16, 0.17, 0.19]	[0.11, 0.13, 0.14]	[0.18, 0.20, 0.22]	[0.39, 0.43, 0.47]	[0.40, 0.44, 0.48]	[0.35, 0.39, 0.42]	[0.38, 0.42, 0.47]	[0.74, 0.83, 0.91]	[1, 1, 1]
C12	[0.07, 0.07, 0.08]	[0.09, 0.10, 0.11]	[0.13, 0.14, 0.16]	[0.13, 0.15, 0.16]	[0.21, 0.23, 0.25]	[0.16, 0.18, 0.20]	[0.37, 0.41, 0.45]	[0.44, 0.49, 0.54]	[0.46, 0.52, 0.57]	[0.60, 0.66, 0.73]	[0.37, 0.41, 0.45]
C13	[0.07, 0.08, 0.09]	[0.10, 0.11, 0.12]	[0.07, 0.08, 0.09]	[0.07, 0.08, 0.09]	[0.15, 0.16, 0.18]	[0.20, 0.22, 0.24]	[0.19, 0.21, 0.23]	[0.22, 0.24, 0.26]	[0.22, 0.25, 0.27]	[0.45, 0.50, 0.55]	[0.56, 0.62, 0.68]
C14	[0.05, 0.05, 0.06]	[0.07, 0.08, 0.09]	[0.07, 0.08, 0.09]	[0.08, 0.09, 0.10]	[0.11, 0.12, 0.14]	[0.10, 0.11, 0.13]	[0.21, 0.24, 0.26]	[0.14, 0.15, 0.17]	[0.28, 0.32, 0.35]	[0.18, 0.20, 0.22]	[0.21, 0.24, 0.26]
C15	[0.04, 0.04, 0.05]	[0.04, 0.04, 0.04]	[0.04, 0.04, 0.05]	[0.06, 0.07, 0.08]	[0.09, 0.10, 0.11]	[0.09, 0.10, 0.11]	[0.11, 0.13, 0.14]	[0.12, 0.14, 0.15]	[0.25, 0.27, 0.30]	[0.19, 0.21, 0.23]	[0.20, 0.22, 0.25]
C16	[0.03, 0.03, 0.03]	[0.03, 0.03, 0.04]	[0.03, 0.04, 0.04]	[0.05, 0.05, 0.06]	[0.07, 0.07, 0.08]	[0.07, 0.07, 0.08]	[0.11, 0.13, 0.14]	[0.12, 0.13, 0.14]	[0.21, 0.23, 0.26]	[0.20, 0.22, 0.25]	[0.17, 0.19, 0.21]
C17	[0.03, 0.03, 0.03]	[0.02, 0.02, 0.03]	[0.02, 0.03, 0.03]	[0.04, 0.05, 0.05]	[0.05, 0.05, 0.06]	[0.06, 0.07, 0.08]	[0.08, 0.09, 0.10]	[0.07, 0.08, 0.09]	[0.10, 0.11, 0.13]	[0.19, 0.21, 0.23]	[0.12, 0.13, 0.14]
C18	[0.03, 0.03, 0.03]	[0.03, 0.03, 0.04]	[0.04, 0.04, 0.05]	[0.03, 0.03, 0.03]	[0.04, 0.05, 0.05]	[0.07, 0.08, 0.08]	[0.07, 0.08, 0.09]	[0.08, 0.09, 0.10]	[0.09, 0.10, 0.10]	[0.11, 0.12, 0.13]	[0.17, 0.18, 0.20]
C19	[0.02, 0.02, 0.02]	[0.02, 0.02, 0.02]	[0.03, 0.03, 0.03]	[0.02, 0.02, 0.02]	[0.03, 0.03, 0.04]	[0.05, 0.05, 0.06]	[0.05, 0.05, 0.06]	[0.05, 0.06, 0.06]	[0.09, 0.10, 0.11]	[0.06, 0.07, 0.07]	[0.09, 0.10, 0.11]
C20	[0.01, 0.02, 0.02]	[0.02, 0.02, 0.02]	[0.01, 0.01, 0.02]	[0.03, 0.03, 0.03]	[0.02, 0.02, 0.02]	[0.02, 0.03, 0.03]	[0.03, 0.03, 0.04]	[0.04, 0.05, 0.05]	[0.05, 0.06, 0.06]	[0.05, 0.06, 0.06]	[0.08, 0.09, 0.10]
C21	[0.01, 0.01, 0.01]	[0.01, 0.01, 0.01]	[0.01, 0.01, 0.01]	[0.02, 0.02, 0.02]	[0.01, 0.01, 0.02]	[0.02, 0.02, 0.02]	[0.03, 0.03, 0.04]	[0.04, 0.04, 0.04]	[0.03, 0.03, 0.04]	[0.03, 0.03, 0.04]	[0.05, 0.06, 0.06]

Criteria	C12	C13	C14	C15	C16	C17	C18	C19	C20	C21
C1	[12.42,13.8,15.18]	[11.33,12.59,13.84]	[16.76,18.62,20.49]	[20.57,22.86,25.15]	[30.44,33.83,37.21]	[32.32,35.91,39.5]	[32.19,35.76,39.34]	[42.11,46.79,51.47]	[58.88,65.42,71.96]	[113.88,126.53,139.18]
C2	[8.97,9.97,10.97]	[8.35,9.28,10.2]	[11.46,12.73,14.01]	[22.1,24.56,27.02]	[27.15,30.17,33.19]	[36.26,40.29,44.32]	[26.89,29.87,32.86]	[45.71,50.79,55.87]	[51.17,56.86,62.54]	[74.75,83.05,91.36]
C3	[6.31,7.01,7.71]	[11.61,12.9,14.19]	[11.06,12.29,13.52]	[20.17,22.41,24.65]	[24.95,27.72,30.49]	[34.37,38.19,42.01]	[21.6,24.0,26.39]	[29.67,32.97,36.27]	[64.62,71.8,78.98]	[80.1,89.0,97.9]
C4	[6.09,6.76,7.44]	[11.24,12.49,13.74]	[9.61,10.68,11.75]	[12.81,14.23,15.65]	[17.3,19.22,21.15]	[19.18,21.31,23.44]	[30.6,34.01,37.41]	[42.05,46.72,51.4]	[31.02,34.47,37.91]	[53.85,59.83,65.81]
C5	[3.95,4.38,4.82]	[5.58,6.2,6.82]	[7.27,8.08,8.88]	[8.95,9.94,10.94]	[12.31,13.68,15.04]	[16.37,18.19,20.01]	[18.45,20.5,22.55]	[28.07,31.19,34.31]	[42.49,47.21,51.93]	[64.15,71.28,78.41]
C6	[4.97,5.52,6.07]	[4.15,4.61,5.08]	[7.9,8.78,9.66]	[8.91,9.9,10.89]	[12.29,13.65,15.02]	[12.73,14.15,15.56]	[11.72,13.03,14.33]	[17.77,19.74,21.71]	[33.89,37.65,41.42]	[41.5,46.11,50.72]
C7	[2.21,2.46,2.71]	[4.21,4.68,5.15]	[3.8,4.22,4.65]	[7.06,7.85,8.63]	[7.08,7.86,8.65]	[10.04,11.15,12.27]	[11.28,12.53,13.78]	[17.19,19.1,21.01]	[27.73,30.81,33.89]	[26.64,29.6,32.56]
C8	[1.84,2.04,2.24]	[3.75,4.16,4.58]	[5.9,6.55,7.21]	[6.58,7.31,8.04]	[7.02,7.8,8.58]	[11.63,12.92,14.21]	[10.12,11.25,12.37]	[16.13,17.92,19.71]	[18.82,20.91,23.0]	[22.12,24.58,27.04]
C9	[1.74,1.94,2.13]	[3.64,4.04,4.45]	[2.84,3.16,3.47]	[3.31,3.67,4.04]	[3.88,4.31,4.74]	[7.89,8.77,9.65]	[9.44,10.48,11.53]	[8.76,9.74,10.71]	[16.28,18.09,19.9]	[27.62,30.69,33.75]
C10	[1.35,1.51,1.66]	[1.8,2.0,2.2]	[4.57,5.08,5.59]	[4.33,4.81,5.29]	[4.03,4.48,4.93]	[4.37,4.85,5.34]	[7.56,8.4,9.24]	[13.32,14.8,16.28]	[16.06,17.84,19.63]	[25.78,28.64,31.51]
C11	[2.19,2.44,2.68]	[1.46,1.62,1.78]	[3.81,4.24,4.66]	[4.03,4.48,4.93]	[4.8,5.34,5.87]	[6.91,7.68,8.45]	[4.88,5.42,5.96]	[8.93,9.93,10.92]	[9.7,10.78,11.85]	[16.32,18.13,19.95]
C12	[1,1,1]	[1.25,1.39,1.53]	[1.86,2.06,2.27]	[1.89,2.1,2.31]	[3.27,3.63,4.0]	[4.82,5.35,5.89]	[4.75,5.28,5.8]	[8.68,9.65,10.61]	[8.65,9.61,10.58]	[15.39,17.1,18.81]
C13	[0.65,0.72,0.79]	[1,1,1]	[2.08,2.32,2.55]	[2.63,2.92,3.21]	[3.4,3.78,4.16]	[4.52,5.02,5.52]	[3.69,4.1,4.51]	[6.26,6.96,7.65]	[6.08,6.75,7.43]	[9.27,10.3,11.33]
C14	[0.44,0.48,0.53]	[0.39,0.43,0.48]	[1,1,1]	[1.61,1.79,1.97]	[2.34,2.59,2.85]	[2.84,3.16,3.47]	[3.72,4.13,4.55]	[5.47,6.07,6.68]	[4.74,5.27,5.79]	[5.75,6.39,7.03]
C15	[0.43,0.48,0.52]	[0.31,0.34,0.38]	[0.5,0.56,0.61]	[1,1,1]	[1.78,1.98,2.18]	[3.06,3.4,3.74]	[1.74,1.93,2.12]	[2.28,2.54,2.79]	[3.25,3.61,3.97]	[6.48,7.2,7.92]
C16	[0.25,0.28,0.3]	[0.24,0.26,0.29]	[0.35,0.39,0.42]	[0.45,0.5,0.55]	[1,1,1]	[1.39,1.54,1.7]	[2.73,3.03,3.33]	[1.88,2.09,2.3]	[4.4,4.89,5.38]	[3.81,4.23,4.65]
C17	[0.17,0.19,0.21]	[0.18,0.2,0.22]	[0.29,0.32,0.35]	[0.26,0.29,0.32]	[0.58,0.65,0.71]	[1,1,1]	[1.7,1.89,2.08]	[2.6,2.89,3.18]	[2.95,3.27,3.6]	[2.79,3.1,3.41]
C18	[0.17,0.19,0.21]	[0.22,0.24,0.27]	[0.22,0.24,0.27]	[0.47,0.52,0.57]	[0.3,0.33,0.36]	[0.48,0.53,0.58]	[1,1,1]	[1.82,2.03,2.23]	[3.07,3.41,3.75]	[2.57,2.86,3.15]
C19	[0.09,0.1,0.11]	[0.13,0.14,0.16]	[0.15,0.16,0.18]	[0.35,0.39,0.43]	[0.43,0.48,0.53]	[0.31,0.35,0.38]	[0.44,0.49,0.54]	[1,1,1]	[2.19,2.44,2.68]	[3.55,3.94,4.34]
C20	[0.09,0.1,0.11]	[0.13,0.15,0.16]	[0.17,0.19,0.21]	[0.25,0.28,0.3]	[0.18,0.2,0.22]	[0.27,0.31,0.34]	[0.26,0.29,0.32]	[0.37,0.41,0.45]	[1,1,1]	[1.98,2.2,2.42]
C21	[0.05,0.06,0.06]	[0.09,0.1,0.11]	[0.14,0.16,0.17]	[0.13,0.14,0.15]	[0.21,0.24,0.26]	[0.29,0.32,0.35]	[0.31,0.35,0.38]	[0.23,0.25,0.28]	[0.41,0.46,0.5]	[1,1,1]

Fuzzy TOPSIS is applied to the final intuitionistic fuzzy weighted matrix to determine the rankings. The Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) are identified, and the closeness coefficients are computed to evaluate each alternative's relative performance under the defined criteria.

Table 13: Fuzzy Weight Matrix from Operators

Criteria	**Fuzzy Weight [L, M, U]**
C1	[0.1566, 0.1896, 0.2296]
C2	[0.1321, 0.1599, 0.1935]
C3	[0.1253, 0.1517, 0.1836]
C4	[0.0868, 0.1050, 0.1271]
C5	[0.0715, 0.0865, 0.1047]
C6	[0.0534, 0.0647, 0.0783]
C7	[0.0450, 0.0545, 0.0660]
C8	[0.0309, 0.0374, 0.0453]
C9	[0.0266, 0.0322, 0.0390]
C10	[0.0237, 0.0287, 0.0348]
C11	[0.0178, 0.0216, 0.0261]
C12	[0.0147, 0.0178, 0.0216]
C13	[0.0107, 0.0129, 0.0156]
C14	[0.0086, 0.0104, 0.0126]
C15	[0.0060, 0.0073, 0.0089]
C16	[0.0051, 0.0061, 0.0074]
C17	[0.0034, 0.0042, 0.0050]
C18	[0.0029, 0.0035, 0.0042]
C19	[0.0021, 0.0026, 0.0031]
C20	[0.0017, 0.0020, 0.0024]
C21	[0.0011, 0.0013, 0.0016]

Cosine similarity was applied to determine the relative importance of each criterion by measuring how closely its evaluation pattern aligned with the ideal preference vector. A higher cosine value indicates stronger alignment with expert judgments, thereby assigning greater weight to the criterion. This approach ensures an objective, consistent, and interpretable prioritization framework within the multi-criteria decision-making process.

Table 14: Criteria Weight (Cosine Similarity)

Criteria	Weight
C1	0.75909186
C2	0.74028846
C3	0.66985528
C4	0.75185610
C5	0.78395266
C6	0.77448539
C7	0.68911382
C8	0.69510688
C9	0.77664497
C10	0.75563334
C11	0.72918471
C12	0.71290973
C13	0.73390024
C14	0.76516118
C15	0.78009475
C16	0.77009390
C17	0.72589605
C18	0.65744094
C19	0.77986455
C20	0.77199509
C21	0.77412182

Using the computed closeness coefficients from the Fuzzy TOPSIS method, all nine alternatives are ranked in descending order, indicating their relative suitability for urban green space development under the evaluated criteria.

Table 15: Relative Closeness Rank

Alternative	Relative Closeness	Rank
A1	0.5099	6
A2	0.8114	2
A3	0.2598	8
A4	0.8138	1
A5	0.2024	9
A6	0.6984	5
A7	0.3267	7
A8	0.7612	4
A9	0.7976	3

8. Conclusion

This study developed a comprehensive multi-criteria decision-making framework for optimizing urban green space planning through the Intuitionistic Fuzzy Analytic Hierarchy Process (IF-AHP), integrated with advanced aggregation operators—Frank, Power Geometric, and Ordered Weighted Averaging (OWA)—to effectively address uncertainty and hesitation in expert assessments. The proposed Intuitionistic Fuzzy Analytic Hierarchy Process framework enables a robust prioritization of nine alternative locations across twenty-one diverse criteria categorized under environmental, economic, accessibility, and technical aspects.

The incorporation of Intuitionistic Fuzzy Sets (IFS) provides a more realistic representation of expert input by allowing for ambiguity, while the aggregation operators ensure balanced integration of multiple decision-maker perspectives. Subsequently, the Fuzzy Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) was employed to rank the alternatives based on their relative closeness to the ideal solution. The outcomes were further validated through sensitivity analysis.

The effectiveness of the proposed approach in urban planning decisions was demonstrated by its ability to integrate subjective judgments, handle complex criteria, and maintain computational efficiency through Python-based implementation. This framework can be extended to future studies incorporating real-time GIS data, dynamic stakeholder input, and additional sustainability dimensions such as biodiversity and climate resilience. Overall, the flexibility of the proposed model offers significant potential to support policymakers, urban planners, and environmental managers in promoting sustainable and smart city development that prioritizes ecological balance and public well-being.

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