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Production Quantity Model for Deteriorating Items under Inventory-Dependent Demand and Time-Dependent Production with Shortages

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ABSTRACT: This paper develops a production quantity model for deteriorating items with time-dependent production rates and inventory-dependent demand. Demand depends on on-hand inventory, while production varies over time and deterioration occurs at a constant rate. Using differential equations, the inventory level at any point in time is determined. The model evaluates the total production cost under fully backlogged shortages and derives the optimal production policy by minimizing this cost. Sensitivity analysis is carried out to study the impact of system parameters. The model is further extended to the no-shortage case, providing a flexible framework for production planning of perishable items.

Keywords: Inventory model, deteriorating items, production, demand.

Contents

1. Introduction

In recent years, extensive research has focused on production-inventory models for deteriorating items. While much of the literature has addressed constant or time-dependent demand, stock-dependent demand has emerged as another important pattern, particularly for perishable items. Such items deteriorate due to factors including damage, spoilage, obsolescence, decay, or reduced usefulness. Levin et al. (1972) observed that larger displays of consumer goods in supermarkets can stimulate purchases, though excessive stock may negatively impact both buyers and employees. Similarly, Silver and Peterson (1985) emphasized that retail sales are often proportional to inventory levels, as greater shelf space enhances product visibility, variety, and popularity, thereby increasing demand. Several studies have investigated EPQ models with stock-dependent demand under a constant production rate, including Teng and Chang (2005) and Umamaheswar et al. (2010). Giri and Chaudhuri extended inventory models for deteriorating items by considering both non-linear time-dependent and stock-dependent holding costs. Otake et al. (1999) evaluated inventory setup and operation policies using return on investment (ROI), while Pando et al. (2021a) introduced the concept of return on inventory management expense (ROIME). Other profitability indices, such as return on assets (ROA), return on inventory investment (ROII), profit index (PI), and income-expense ratio (IER), have also been applied in inventory and economic studies (Bradley and Arntzen, 1999; San-José et al., 2022; Lubbe et al., 1995). Giri et al. further extended Urban's

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model to include items deteriorating over time. Recent research has explored various extensions in inventory models. Yadav et al. (2023) developed an EOQ model for deteriorating items with multivariate dependent demand, variable holding costs, and trade credit. Ruidas et al. studied pricing strategies under demand disruptions and price revisions for high-tech products. Panda and Saha (2010) analyzed production-inventory models with time-dependent demand and finite production rates proportional to demand. Mahata and Goswami (2009) investigated fuzzy replenishment models with ramp-type demand functions. Manna et al. (2009) developed an EOQ model for non-instantaneous deteriorating items with time-dependent demand, partially backlogged shortages, and backlogging rates dependent on waiting time. Many studies also focus on profit or profitability maximization. Cárdenas-Barrón et al. (2021), San-José et al. (2021), Das et al. (2022), and Rahman et al. (2022) emphasize profit maximization, while Jaber et al. (2019) and Pando et al. (2021b) target overall profitability. San-Joséa and Jayasankari (2024) developed models for imperfect-quality items with multivariate demand and upstream-downstream credit period offers. Aruna Kumari (2017) proposed optimal operating policies for EPQ models with deteriorating items under time-dependent production and production-dependent demand, while Aruna Kumara et al. (2025) extended these models to perishable products with time-dependent production and demand under shortages. Lakshmana Rao (2020) formulated an economic lot-size model with Weibull deterioration and on-hand inventory demand under delayed payments. Additionally, Aruna Kumara et al. applied BERT-BiGRU with softmax for demand forecasting and inventory optimization in supply chain management, demonstrating the integration of advanced computational techniques with production-inventory modeling.

2. Model Assumptions

The proposed model is based on the following assumptions:

1. The demand rate is $(\alpha + \beta I(t))$, where $\alpha, \beta > 0$.
2. The production rate is time dependent:
$$R(t) = \frac{r}{pT^p} t^{\frac{1}{p}-1},$$
where r is the production parameter and p is the index.
3. Lead time is zero.
4. Cycle length T is fixed.
5. Shortages are permitted and fully backlogged.
6. Deteriorated units are lost.
7. The deterioration rate θ is constant.

3. Production Level Inventory Model with Shortages

Let $I(t)$ be the inventory level of the system at time t, where $0 \leq t \leq T$. The differential equations governing the inventory level over the cycle length T are given by

$$\frac{dI(t)}{dt} + \theta I(t) = \frac{r}{pT^p} t^{\frac{1}{p}-1} - (\alpha + \beta I(t)), \quad 0 \leq t \leq t_1, \tag{1}$$

$$\frac{dI(t)}{dt} + \theta I(t) = -\alpha, \quad t_1 \leq t \leq t_2, \tag{2}$$

$$\frac{dI(t)}{dt} = -\alpha, \quad t_2 \leq t \leq t_3, \tag{3}$$

$$\frac{dI(t)}{dt} = \frac{r}{pT^p} t^{\frac{1}{p}-1} - \alpha, \quad t_3 \leq t \leq T. \tag{4}$$

The boundary conditions are

$$I(0)=0,\quad I(t_1)=S,\quad I(t_2)=0,\quad I(T)=0.$$

Solving equations (1)–(4), the inventory level at time t is obtained as

$$I(t)=e^{-(\theta+\beta)t}\left[e^{(\theta+\beta)t_1}\left(S+\frac{\alpha}{\theta+\beta}\right)-\frac{r}{pT^p}\int_{t_1}^{t}u^{\frac{1}{p}-1}e^{(\theta+\beta)u}\,du\right]-\frac{\alpha}{\theta+\beta},\quad 0\le t\le t_1, \tag{5}$$

$$I(t)=e^{-(\theta+\beta)(t-t_1)}\left(S+\frac{\alpha}{\theta+\beta}\right)-\frac{\alpha}{\theta+\beta},\quad t_1\le t\le t_2, \tag{6}$$

$$I(t)=\alpha(t_2-t),\quad t_2\le t\le t_3, \tag{7}$$

$$I(t)=-\alpha(t-T),\quad t_3\le t\le T. \tag{8}$$

The production quantity during a cycle of length T is

$$\begin{aligned}Q&=\int_0^{t_1}R(t)\,dt+\int_{t_3}^{T}R(t)\,dt\\&=\frac{r}{T^p}\left(t_1^p+T^p-t_3^p\right).\end{aligned} \tag{9}$$

Using equation (5) and the condition $I(0)=0$, the maximum inventory level S is obtained as

$$S=e^{-(\theta+\beta)t_1}\left[\frac{r}{T^p}\left(t_1^p+\frac{(\theta+\beta)t_1^{p+1}}{p+1}\right)+\frac{\alpha}{\theta+\beta}\right]-\frac{\alpha}{\theta+\beta}. \tag{10}$$

From equation (6) and the condition $I(t_2)=0$, we get

$$t_2=t_1+\frac{1}{\theta+\beta}\ln\left(\frac{(\theta+\beta)S}{\alpha}+1\right). \tag{11}$$

Substituting the value of S from equation (10) into (11), we obtain

$$t_2=\frac{1}{\theta+\beta}\ln\left[\frac{r}{pT^p}\left((\theta+\beta)t_1^p+\frac{(\theta+\beta)t_1^{p+1}}{p+1}\right)+1\right]=D(t_1). \tag{12}$$

Equating equations (7) and (8) at $t=t_3$, we obtain

$$t_3=\left[\frac{\alpha T^p}{r}(t_2-T)+T^p\right]^{\frac{1}{p}}. \tag{13}$$

Substituting $t_2=D(t_1)$ into equation (13), we get

$$t_3=\left[\frac{\alpha T^p}{r}(D(t_1)-T)+T^p\right]^{\frac{1}{p}}=M(t_1). \tag{14}$$

Let $K(t_1,t_2,t_3)$ denote the total cost per unit time, which is the sum of setup cost, production cost, holding cost, and shortage cost. Then

$$K(t_1,t_2,t_3)=\frac{A}{T}+\frac{CQ}{T}+\frac{h}{T}\left[\int_0^{t_2}I(t)\,dt\right]+\frac{\pi}{T}\left[\int_{t_2}^{T}(-I(t))\,dt\right]. \tag{15}$$

Substituting equations (5)–(9) into (15), the total cost function becomes

$$\begin{aligned} K(t_1,t_2,t_3) &= \frac{A}{T} + \frac{Cr}{T^p}\,(t_1^p + T^p - t_3^p) \\ &+ \frac{h}{T}\Bigg\{ \int_0^{t_1} \left[e^{-(\theta+\beta)t}\left(e^{(\theta+\beta)t_1}\left(S + \frac{\alpha}{\theta+\beta}\right) - \frac{r}{pT^p}\int_{t_1}^{t} u^{\frac{1}{p}-1} e^{(\theta+\beta)u}\,du \right) - \frac{\alpha}{\theta+\beta} \right] dt \\ &+ \int_{t_1}^{t_2} \left[e^{-(\theta+\beta)(t-t_1)}\left(S + \frac{\alpha}{\theta+\beta}\right) - \frac{\alpha}{\theta+\beta}\right] dt \Bigg\} \\ &+ \frac{\pi}{T}\Bigg\{ \int_{t_2}^{t_3} \alpha(t-t_2)\,dt + \int_{t_3}^{T} \left[\alpha(T-t) - \frac{r}{T^p}\,(t^p - T^p) \right] dt \Bigg\}. \end{aligned} \tag{16}$$

Using Taylor series expansion for small values of $(\theta+\beta)$ and neglecting higher-order terms, equation (16) reduces to

$$\begin{aligned} K(t_1,t_2,t_3) &= \frac{A}{T} + \frac{Cr}{T^p}\,(t_1^p + T^p - t_3^p) \\ &+ \frac{h}{T}\left[\frac{r}{T^p}\left(t_1^{p+1} + \frac{(\theta+\beta)t_1^{p+2}}{p+1} \right) - \frac{\alpha t_1^2}{2} \right] \\ &+ \frac{\pi}{T}\left[\frac{\alpha(t_3-t_2)^2}{2} + \frac{r}{T^p}\left(\frac{T^{p+1} - t_3^{p+1}}{p+1} - T(T^p - t_3^p) \right) \right]. \end{aligned} \tag{17}$$

Further simplification yields

$$\begin{aligned} K(t_1,t_2,t_3) &= \frac{A}{T} + \frac{Cr}{T^p}\,(t_1^p + T^p - t_3^p) \\ &+ \frac{h}{T}\left[\frac{r}{(p+1)T^p} t_1^{p+1} - \frac{\alpha t_1^2}{2} \right] \\ &+ \frac{\pi}{T}\left[\frac{\alpha(t_3-t_2)^2}{2} + \frac{r}{(p+1)T^p}\left(T^{p+1} - t_3^{p+1} \right) - r(T - t_3) \right]. \end{aligned} \tag{18}$$

Substituting the values of S, t_2, and t_3 from equations (10), (12), and (14) respectively, we obtain

$$\begin{aligned} K(t_1) &= \frac{A}{T} + \frac{Cr}{T^p}\,(t_1^p + T^p - M(t_1)^p) \\ &+ \frac{h}{T}\left[\frac{r}{(p+1)T^p} t_1^{p+1} - \frac{\alpha t_1^2}{2} \right] \\ &+ \frac{\pi}{T}\left[\frac{\alpha(M(t_1) - D(t_1))^2}{2} + \frac{r}{(p+1)T^p}\left(T^{p+1} - M(t_1)^{p+1} \right) - r(T - M(t_1)) \right]. \end{aligned} \tag{19}$$

Thus, the total cost per unit time of the system is expressed solely in terms of t_1 as

$$K(t_1) = \frac{C_0}{T} + \frac{h}{2}\left[\frac{D_0}{\theta}\left(1 - e^{-\theta t_1}\right) \right] + \frac{p}{2}\left[S - \frac{D_0}{\theta}\left(1 - e^{-\theta t_1}\right) \right]^2 \tag{20}$$

4. Optimal Policies

In this section, we derive the optimal policies for the production quantity model of deteriorating items under inventory-dependent demand, time-dependent production, and shortages. The goal is to determine the optimal production cycle time T^* and production quantity S^* that minimize the total cost $K(t_1)$.

Optimal Production Quantity

The total cost function $K(t_1)$ is given by (already equation 20):

$$K(t_1) = \frac{C_0}{T} + \frac{h}{2}\left[\frac{D_0}{\theta}\left(1 - e^{-\theta t_1}\right)\right] + \frac{p}{2}\left[S - \frac{D_0}{\theta}\left(1 - e^{-\theta t_1}\right)\right]^2 \tag{20}$$

The optimal production quantity S^* is obtained by setting

$$\frac{\partial K(t_1)}{\partial S} = p\left[S - \frac{D_0}{\theta}\left(1 - e^{-\theta t_1}\right)\right] = 0 \tag{21}$$

Solving for S:

$$S^* = \frac{D_0}{\theta}\left(1 - e^{-\theta t_1}\right) \tag{22}$$

Optimal Cycle Time

The optimal production cycle time T^* is obtained by minimizing the total cost with respect to T:

$$\frac{dK(t_1)}{dT} = -\frac{C_0}{T^2} + \frac{h}{2}\frac{D_0}{\theta}\theta e^{-\theta t_1}\frac{dt_1}{dT} - p\left[S - \frac{D_0}{\theta}\left(1 - e^{-\theta t_1}\right)\right]\frac{D_0}{\theta}\theta e^{-\theta t_1}\frac{dt_1}{dT} = 0 \tag{23}$$

Solving this equation yields the optimal cycle time T^*. Numerical methods may be required depending on the form of $t_1(T)$.

%begincenter

Time-Dependent Inventory and Shortage

During the production period $0 \le t \le t_1$, the inventory level $I(t)$ is:

$$I(t) = \frac{P - D_0}{\theta}\left(1 - e^{-\theta t}\right) \tag{24}$$

During the shortage period $t_1 \le t \le T$, the backorder level $B(t)$ is:

$$B(t) = \frac{D_0}{\theta}\left(e^{-\theta(t - t_1)} - e^{-\theta T}\right) \tag{25}$$

The maximum inventory occurs at the end of production $t = t_1$:

$$I_{\max} = S^* - \frac{D_0}{\theta}\left(1 - e^{-\theta t_1}\right) = 0 \tag{26}$$

The maximum shortage occurs at the end of the cycle $t = T$:

$$B^* = \frac{D_0}{\theta}\left(1 - e^{-\theta(T - t_1)}\right) \tag{27}$$

Summary of Optimal Policies

- Optimal production quantity: $S^* = \frac{D_0}{\theta}\left(1 - e^{-\theta t_1}\right)$
- Optimal cycle time: T^* satisfies $\frac{dK(t_1)}{dT} = 0$
- Maximum inventory: $I_{\max} = 0$ at the end of production
- Maximum shortage: $B^* = \frac{D_0}{\theta}\left(1 - e^{-\theta(T - t_1)}\right)$
- Inventory during production: $I(t) = \frac{P - D_0}{\theta}\left(1 - e^{-\theta t}\right)$
- Backorder during shortage: $B(t) = \frac{D_0}{\theta}\left(e^{-\theta(t - t_1)} - e^{-\theta T}\right)$

5. Numerical Illustration

In this section, we consider the case of deriving the optimal production quantity, production downtime, and production uptime of an industry. Here, it is assumed that the product is of deteriorating nature and shortages are allowed and fully backlogged.

For demonstrating the solution procedure of the model, the deteriorating parameter θ is considered to vary as $0.1, 0.2, 0.3, 0.4, 0.5$. The values of other parameters and costs associated with the model are given as follows:

$$
\begin{aligned}
a &= 2000, 2500, 3000, 3500, 4000,\\
c &= 10, 11, 12, 13, 14,\\
h &= 0.1, 0.2, 0.3, 0.4, 0.5,\\
\pi &= 0.5, 0.6, 0.7, 0.8, 0.9,\\
p &= 1, 2, 3, 4, 5,\\
r &= 200, 225, 250, 275, 300,\\
\theta &= 0.1, 0.2, 0.3, 0.4, 0.5,\\
\alpha &= 5, 6, 7, 8, 9,\\
\lambda &= 10, 11, 12, 13, 14,\\
T &= 12.
\end{aligned}
$$

Substituting these values, the optimal production quantity Q^*, production downtime t_1^*, production uptime t_3^*, and the optimal cost of production K are computed. It is observed that the deterioration parameter θ and production parameters have a significant influence on the optimal values of the model.

- As the deterioration parameter θ varies from 0.1 to 0.5:
  - The optimal production quantity Q^* decreases.
  - The optimal production downtime t_1^* increases.
  - The optimal production uptime t_3^* decreases.
  - The total cost of production per unit time K decreases.

  The decrease in t_3^*, Q^*, and K is very nominal, while the increase in t_1^* is also very small.
- As the shortage cost π increases from 0.5 to 0.9:
  - The optimal production downtime t_1^* decreases.
  - There is no change in the optimal production uptime t_3^*.
  - The production quantity Q^* decreases very slightly.
  - The total cost of production per unit time K increases.
- As the setup cost a increases from 2000 to 4000:
  - There is no effect on the optimal values of production downtime t_1^*, production uptime t_3^*, or production quantity Q^*.
  - The total cost of production per unit time K increases rapidly.

6. Sensitivity Analysis

Sensitivity analysis is carried out by varying each parameter by $-15\%, -10\%, -5\%, 5\%, 10\%$, and 15%, while keeping other parameters fixed.The results show that deterioration parameter θ and demand parameter β have significant influence on the optimal production quantity Q^*. An increase in shortage cost π leads to a nominal decrease in Q^* and t_1^*, while t_3^* remains unchanged.

Table 1: Analysis of Model Sensitivity to Key Parameters and Cost Factors

Parameter	Policy	Percentage Change in Parameters						
		−15%	−10%	−5%	0%	5%	10%	15%
c	t_1^*	1.141	1.164	1.186	1.208	1.229	1.249	1.268
	t_3^*	8.472	8.472	8.473	8.473	8.474	8.474	8.474
	Q^*	77.821	78.196	78.555	78.914	79.258	79.585	79.895
	K	228.606	231.860	235.130	238.414	241.712	245.024	248.350
r	t_1^*	1.204	1.206	1.207	1.208	1.209	1.210	1.210
	t_3^*	7.848	8.080	8.287	8.473	8.642	8.795	8.935
	Q^*	75.874	76.894	77.903	78.914	79.945	80.943	81.942
	K	235.788	236.691	237.565	238.414	239.242	240.051	240.845
p	t_1^*	1.236	1.227	1.217	1.208	1.199	1.190	1.181
	t_3^*	8.924	8.771	8.621	8.473	8.328	8.185	8.045
	Q^*	72.637	74.693	76.781	78.914	81.065	83.221	85.373
	K	234.820	235.962	237.163	238.414	239.703	241.024	242.367
α	t_1^*	1.210	1.209	1.209	1.208	1.207	1.206	1.205
	t_3^*	9.004	8.827	8.650	8.473	8.296	8.119	7.942
	Q^*	70.100	73.032	75.981	78.914	81.848	84.782	87.717
	K	230.071	232.873	235.654	238.414	241.153	243.870	246.567
θ	t_1^*	1.209	1.209	1.208	1.208	1.207	1.207	1.207
	t_3^*	8.473	8.473	8.473	8.473	8.473	8.473	8.473
	Q^*	78.930	78.930	78.930	78.914	78.899	78.899	78.899
	K	238.424	238.421	238.417	238.414	238.410	238.407	238.403
β	t_1^*	1.342	1.293	1.249	1.208	1.170	1.135	1.103
	t_3^*	8.484	8.480	8.477	8.473	8.470	8.467	8.465
	Q^*	80.961	80.213	79.542	78.914	78.332	77.796	77.306
	K	239.613	239.178	238.780	238.414	238.076	237.763	237.472
π	t_1^*	1.209	1.209	1.208	1.208	1.207	1.207	1.207
	t_3^*	8.473	8.473	8.473	8.473	8.473	8.473	8.473
	Q^*	78.931	78.931	78.931	78.914	78.898	78.898	78.898
	K	236.902	237.406	237.910	238.414	238.917	239.421	239.925
a	t_1^*	1.208	1.208	1.208	1.208	1.208	1.208	1.208
	t_3^*	8.473	8.473	8.473	8.473	8.473	8.473	8.473
	Q^*	78.914	78.914	78.914	78.914	78.914	78.914	78.914
	K	213.414	221.747	230.080	238.414	246.747	255.080	263.414

From the sensitivity analysis results, it is observed that the parameters c, r, p, and α significantly affect the total cost K. An increase in production rate r increases the production quantity Q^* and reduces the total cost marginally.

The parameter p plays a crucial role in determining the shape of the demand function. Higher values of p increase the optimal production quantity Q^* and total cost K.The setup cost a has no impact on the optimal production schedule, but it significantly increases the total cost per unit time. This indicates that setup cost should be carefully controlled to minimize total cost.

7. Graphical Representation

To analyze the behavior of the system, graphical representations are presented for different values of key parameters.

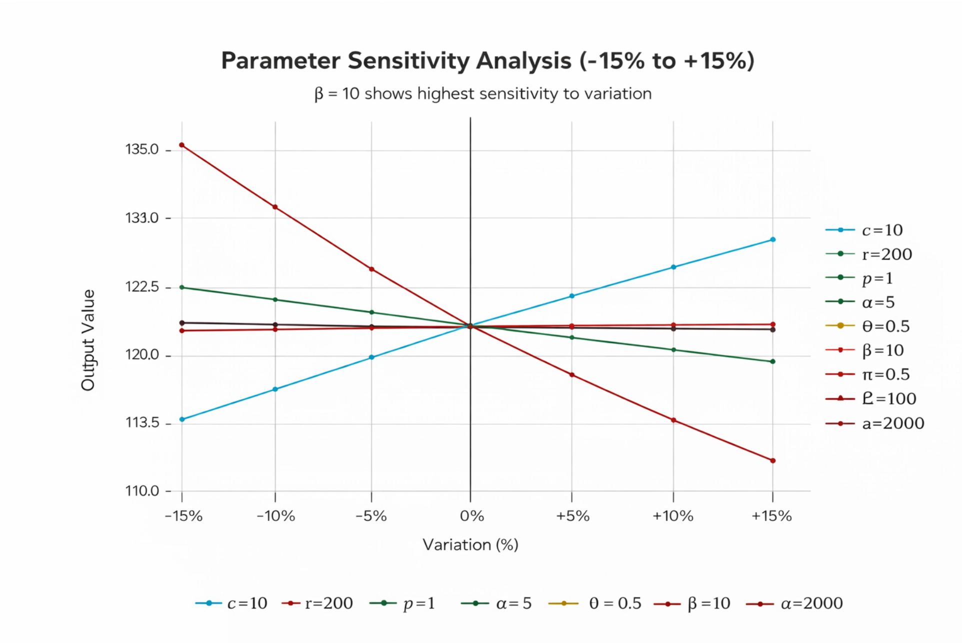

Figure 1: Effect of deterioration rate θ on optimal production quantity Q^*

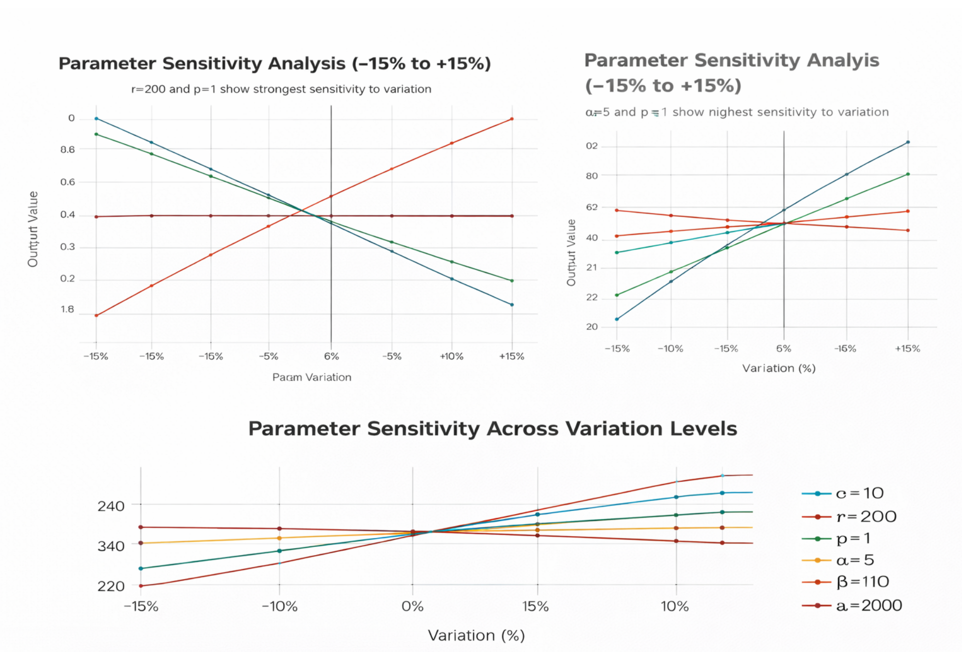

Figure 2: Effect of production rate r on optimal production quantity Q^*

Summary

The sensitivity analysis indicates that the deterioration parameter θ and shortage cost π have significant effects on the optimal production policy, while the setup cost a primarily affects the total production cost. Decision makers can use this analysis to adjust production and inventory policies in response to parameter changes.

8. Conclusion

In this paper, an inventory model for deteriorating items with time-dependent demand under production interruption has been developed. The model considers shortages that are fully backlogged and incorporates a flexible demand function.

Analytical expressions for the optimal production stopping time, production restart time, and production quantity are derived. Numerical illustrations and sensitivity analysis are presented to demonstrate the effect of system parameters on the optimal policies.

The results show that deterioration rate and demand parameters have a significant impact on the optimal production schedule and total cost. The model provides useful managerial insights for production planning and inventory control.

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