Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2025 (43) 3** : 1–7.
ISSN-0037-8712
doi:10.5269/bspm.80106

A Novel Methodology for Finding the Shortest Path in Z-Graph by Applying Modifiers

P. Vijayalakshmi*, S. Devibala, M. A. Gopalan

ABSTRACT: In this paper, we transform the Z-shortest path problem (ZSPP) into a fuzzy shortest path problem (FSPP) by applying the modifiers technique. Then the fuzzy shortest path problem can be solved using suitable existing methods.

Key Words: Z-number, ranking function, triangular Z-number, fuzzy shortest path problem (FSPP), z-shortest path problem (ZSPP), modifiers.

Contents

1. Introduction

Numerous application problems, modelled as mathematical programming problems often involve uncertainty. Bellman and Zadeh [1] introduced the concept of decision making in fuzzy environments. FSPP was first introduced by Dubois and Prade [2] and Okada and Gen [3] proposed an algorithm based on Dijkstra's Algorithm for finding the shortest path. Various papers have been published on shortest path problem in fuzzy environment using Dijkstra's Algorithm.

A graph with Z - weighted edges is known as Z-graph. Very few authors have researched the problem of finding the shortest path in a Z- graph using various algorithms.

Z-number and their modifiers have been explored by Shahila Bhanu M and Velammal G [7] and Rani P and Velammal G [8]. This paper specifically focuses on applying modifiers to the first component in Z-valuation. Now, we convert the ZSPP to FSPP using the modifier technique, and FSPP can be addressed using suitable existing methods.

2. Preliminaries

Definition 2.1: Shortest Path in Graph Let $\mathrm{G} = (\mathrm{V}, \mathrm{E})$ be a weighted graph, directed or undirected. Weight of path $\mathrm{P} =< \mathrm{v}_0, \mathrm{v}_1, \mathrm{v}_2, \cdots, \mathrm{v}_\mathrm{k} >$ is $\mathrm{w}(\mathrm{P}) = \sum \mathrm{w}(\mathrm{v}_{\mathrm{i}-1}, \mathrm{v}_\mathrm{i})$.

Shortest path $\delta(\mathrm{u}, \mathrm{v})$ from u to v has weight

$$\delta(u,v) = \begin{cases} \min\{w(P) : P \text{ is path from } u \text{ to } v\} \text{ if path exists} \\ \infty \quad \text{Otherwise} \end{cases}$$

Definition 2.2: Formal definition of Z-number A Z-number is an ordered pair of fuzzy numbers (A, B) where A is a fuzzy set defined on the real line and B is a fuzzy number whose support is contained in $[0, 1]$.

* Corresponding author.
2010 *Mathematics Subject Classification:* 03E72, 52B05, 05C72.
Submitted November 12, 2025. Published December 20, 2025

Definition 2.3: Z-weight of an edge A Z-graph is a graph with Z-number weighted edges. The Z-weight of an edge is the Z-distance between the corresponding two nodes.

Definition 2.4: Z-length of the path Consider a path P from a vertex u to a vertex v , consists of edges $e_1, e_2 \dots e_n$. Suppose the weights of these edges are the Z-numbers $(A_1, B_1), (A_2, B_2) \dots (A_n, B_n)$. The Z-length of the path P is denoted by ZL (P)

$$ZL(P) = (A_1, B_1)(+, \min)(A_2, B_2) \dots (+, \min)(A_n, B_n)$$

The shortest path problem in Z- graphs is the problem of finding a path between two specified nodes such that the Z-length of the path is minimum.

Definition 2.5: Operation on fuzzy numbers Let $A_1 = (a, b, c), A_2 = (d, e, f)$ be two triangular fuzzy numbers. Then $A_1 + A_2 = (a + d, b + e, c + f)$.

Definition 2.6: Binary operation on Z-numbers [9] Let (A_1, B_1) and (A_2, B_2) be two Z -numbers. Then

$$(A_1, B_1)(+, \min)(A_2, B_2) = (A_1 + A_2, \min(B_1, B_2))$$

Definition 2.7: Linguistic Hedge and Modifier [7] Linguistic hedges are special linguistic terms by which other linguistic terms are modified. Linguistic terms such as very, more or less, fairly, or extremely are examples of linguistic hedges. A linguistic hedge, H, may be applied to a fuzzy set by using a unary operation, h defined on the unit interval $[0, 1]$.

A unary operation, which is an increasing bijection, representing a linguistic hedge are called modifier.

3. Main Result

3.1. Converting ZSPP to FSPP using Modifiers

Consider the statement "It is almost sure that the journey will take about 3 to 4 hours". This can be expressed in the form of Z -valuation as X isz (A, B), where X denotes the variate "journey timing," A denotes the fuzzy set "about 3 to 4 hours" and B denotes the fuzzy set "almost sure".

Now, if we want to rewrite the statement as "It is very sure that journey will take A' ", we want to find A' such that X isz (A', B') where B' is the fuzzy set representing "very sure." This involves applying modifiers to the first component in Z-valuation. M. Shahila Bhanu has studied this problem, indicating that X isz (A, B) is considered equivalent to the statement X isz (A', B'). So, we can replace the Z -valuation (A, B) by (A', B').

Consider a path P , consisting of the edges $e_1, e_2, \dots e_n$, with edge Z -weights $(A_1, B_1), (A_2, B_2) \dots (A_n, B_n)$. The Z-length of the path P is

$$ZL(P) = (A_1, B_1)(+, \min)(A_2, B_2) \dots (+, \min)(A_n, B_n)$$

Here, we apply modifier technique by converting all the second component equal to B.
(i.e.) we find $A_1', A_2' \dots A_n'$ so that

$$(A_1, B_1) \sim (A_1', B), (A_2, B_2) \sim (A_2', B) \dots (A_n, B_n) \sim (A_n', B)$$

Then, $ZL(P) = (A_1', B)(+, \min)(A_2', B) \dots (+, \min)(A_n', B)$

$$= (A_1' + A_2' + \dots + A_n', B)$$

Hence, it is enough to concentrate only on the first component of the edge weight.
Then by the modifier technique, the ZSPP is transformed to FSPP.

3.2. Modified Dijkstra's Algorithm for FSPP

We establish a suitable ranking function, denoted by R for comparing two fuzzy numbers. The source vertex is represented by 's.' P is used to store all currently unvisited vertices in the graph G.
Algorithm: Inputs: V = $\{1, 2, 3 \ldots \mathrm{n}\}$// the set of vertices in the given connected graph G
s // the source vertex
$d(i,j)$ for $i,j = 1,2,\ldots n$// $d(i,j)$ = fuzzy length of arc (i,j)

and $d(i,j) \geq 0, i,j \in V$

Initialisation: d[s] = $(0,0,0)$
for every vertex i in $V, i \neq s$
{
d[i] = (∞,∞,∞)
Predecessor[i] = φ
}
d min = (∞,∞,∞)
Main part:

- P = V$\backslash\{$s$\}$// unvisited vertex set
- N = s
- While P $\neq \varnothing$
  {
- for every vertex i in P
  {
- if R(d[i]) > R(d[N] + d(i, j))
- {
- predecessor[i] = N,
- $d[i] = d[N] + d(i,j)$
- }
- if R(d[i]) < R(d min), then alt = i, dmin = d[i]
  }
- N = alt
- Delete N from P
- d min = (∞,∞,∞)
  }

Output: d [i] // length of FSP from s to i
3.3. Example Consider a graph G in figure -1 with Z -number weighted edges.
We need to find the shortest path from A to E. The Z-weights of the edges are represented as follows.
w (AB) - (around 160, very likely)
w(AC) – (around 320 , very likely)
w (BC) - (around 150, likely)
w (BD) - (around 290, very likely)
w (BE) - (around 390, likely)
w (CD) - (around 220, likely)

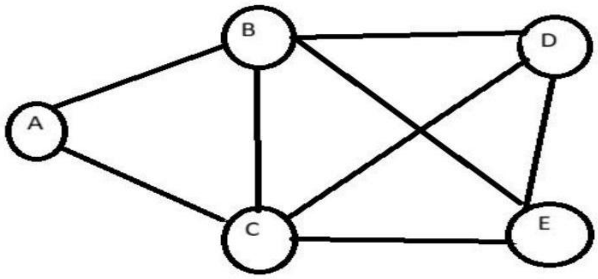

Figure - 1

w (CE) - (around 200, very likely)
w (DE) - (around 320, very likely)

For mathematical representation, we consider "likely" as approximately 80% (i.e.) (. 75, .8, .85) and "very likely" as 90% (i.e.) (.85, .9, .95). Therefore, the Z- weight of the edges is provided in the table -1 .

Edges	Z-weight
AB	((155, 160, 165), (.85, .9, .95))
AC	((312, 320, 328), (.85, .9, .95))
BC	((142, 150, 158), (.75, .8, .85))
BD	((282, 290, 300), (.85, .9, .95))
BE	((382, 390, 398), (.75, .8, .85))
CD	((212, 220, 228), (.75, .8, .85))
CE	((190, 200, 210), (.85, .9, .95))
DE	((312, 320, 328), (.85, .9, .95))

Table - 1

Here, except for the edges BC, BE and CD , all others have the second component as "very likely." Our goal is to make the second component of all the edges equal to "very likely." To achieve this, we use the modifiers technique to ensure uniformity in the second component. According to this technique, the first component of the corresponding weight of the edges is slightly modified. Hence, we obtain a ZSPP where all parameters have "very likely" as their reliability component.

Since all the second components are equal, we have
min(the second component of all the edges) = (.85, .9, .95)

After applying the technique of modifiers to ensure uniformity in the second component, we obtain the following modified table.

Edges	Z-weight
AB	((155, 160, 165), (.85, .9, .95))
AC	((312, 320, 328), (.85, .9, .95))
BC	((142, 151, 160), (.85, .9, .95))
BD	((282, 290, 300), (.85, .9, .95))
BE	((382, 391, 400), (.85, .9, .95))
CD	((212, 221, 230), (.85, .9, .95))
CE	((190, 200, 210), (.85, .9, .95))
DE	((312, 321, 330), (.85, .9, .95))

Table - 2 In the modified "Z- weight table," the first component of the edges BC, BE and CD altered and the second components of all the vertices are made equal.

So, we concentrate only on the first component for finding SPP. Hence, the ZSPP is converted to FSPP.

We consider the ranking function as $\mathrm{R}((\mathrm{x},\mathrm{y},\mathrm{z})) = \frac{x+y+z}{3}$

Let $\mathrm{V} = \{\mathrm{A},\mathrm{B},\mathrm{C},\mathrm{D},\mathrm{E}\}$

Take $s = A$

Take $\mathrm{d(A)} = (0,0,0)$ and $\mathrm{dmin} = (\infty,\infty,\infty)$

$\mathrm{N} = \mathrm{A}$

Then $\mathrm{P} = \{\mathrm{B},\mathrm{C},\mathrm{D},\mathrm{E}\}$

Now $\mathrm{d(A)} + \mathrm{d(A,B)} = (0,0,0) + (155,160,165)$

$$= (155,160,165)$$

Here, $\mathrm{R(d(A)+d(A,B))} < \mathrm{R(d(B))}$

Then, Predecessor $[\mathrm{B}] = \mathrm{A}, \mathrm{d(B)} = (155,160,165)$

Now, $\mathrm{d(A)} + \mathrm{d(A,C)} = (0,0,0) + (312,320,328)$

$$= (312,320,328)$$

Here, $\mathrm{R(d(A)+d(A,C))} < \mathrm{R(d(C))}$

$\therefore$ Predecessor $[\mathrm{C}] = \mathrm{A}, \mathrm{d(C)} = (312,320,328)$

By ranking method, $\mathrm{R(d(B))} < \mathrm{R(d(C))}$

So, alt $= B$ and $\mathbf{d}(B) = (155,160,165)$ and B is removed from P

Now, $\mathrm{N} = \mathrm{B}$ and $\mathrm{P} = \{\mathrm{C},\mathrm{D},\mathrm{E}\}$

$\mathrm{d(B)} + \mathrm{d(B,C)} = (155,160,165) + (142,151,160)$

$$= (297,311,325)$$

Here, $\mathrm{R(d(B)+d(B,C))} < \mathrm{R(d(C))}$

Then, Predecessor $[\mathrm{C}] = \mathrm{B}, \mathrm{d(C)} = (297,311,325)$

Now, $\mathrm{d(B)} + \mathrm{d(B,D)} = (155,160,165) + (282,290,300)$

$$= (437,450,465)$$

$\mathrm{R(d(B)+d(B,D))} < \mathrm{R(d(D))}$

Then, Predecessor[D] = B, d (D) = (437,450, 465)

$\mathrm{d(B)} + \mathrm{d(B,E)} = (155,160,165) + ((382,391,400)$

$$= (537,551,565)$$

$\mathrm{R(d(B)+d(B,E))} < \mathrm{R(d(E))}$

$\therefore$ Predecessor $[\mathrm{E}] = \mathrm{B}$ and $\mathrm{d(E)} = (537,551,565)$

By ranking method, $\mathrm{R(d(C))} < \mathrm{R(d(D))}, \mathrm{R(d(C))} < \mathrm{R(d(E))}$ and $\mathrm{R(d(D))} < \mathrm{R(d(E))}$

So, alt $= \mathbf{C}$ and $\mathbf{d}(\mathbf{C}) = (297,311,325)$ and $\mathbf{C}$ is removed

Hence, $\mathrm{N} = \mathrm{C}$, and $\mathrm{P} = \{\mathrm{D},\mathrm{E}\}$

$\mathrm{d(C)} + \mathrm{d(C,D)} = (297,311,325) + (212,221,230),$

$$= (509,542,555)$$

$\mathrm{R(d(C)+d(C,D))}$ not $< \mathrm{R(d(D))}$

$\mathrm{d(C)} + \mathrm{d(C,E)} = (297,311,325) + (190,200,210),$

$$= (487,511,535)$$

$\mathrm{R(d(C)+d(C,E))} < \mathrm{R(d(E))}$

Predecessor $[\mathrm{E}] = \mathrm{C}$ and $\mathrm{d(E)} = (487,511,535)$

So, alt $= \mathbf{E}$ and $\mathbf{d}(\mathbf{E}) = (487,511,525)$ and $\mathbf{E}$ is removed

Hence, N = E, and P = {D}
d(E) + d(E.D) = $(487, 511, 525) + (312, 321, 330)$

$$= (799, 832, 855)$$

R(d(E) + d(E, D)) not $<$ R(d(D))
Output: The shortest path in a graph with fuzzy weighted edge is $\mathbf{A} \rightarrow \mathbf{B} \rightarrow \mathbf{C} \rightarrow \mathbf{E}$, and its length is $(487, 511, 525)$) Hence, the length of shortest Z - path is $((487, 511, 525), (.85, .9, .95))$. So, it is "sure" that the shortest Z-path length is "around 511".

$$d[A] = (0, 0, 0)$$

- Predecessor$[B] = A$ and d[B] $= (155, 160, 165)$
- SP from source to B $= \{B\}U\{AB\} = \{AB\}$
- Predecessor $[C] = B$ and $d[C] = (297, 311, 325)$
- SP from source to $C = \{AB\}U\{BC\} = \{AB, BC\}$
- Predecessor $[D] = B$ and $d[D] = (437, 450, 465)$
- SP from source to $D = \{AB\}U\{BD\} = \{AB, BD\}$
- Predecessor $[E] = C$ and d[E] $= (487, 511, 525)$
- SP from source to $E = \{AB\}U\{BC\}U\{CE\} = \{AB.BC, CE\}$
- Hence the shortest path from source " A " to destination " E " is $\mathbf{A} \rightarrow \mathbf{B} \rightarrow \mathbf{C} \rightarrow \mathbf{E}$.

4. Conclusion

An innovative approach for solving ZSPP to FSPP by using modifier has been provided in this paper. We have demonstrated this approach by a numerical example.

References

1. Belman R E, Zadeh L A, "Decision making in a fuzzy environment", Management Science, 1970, Vol. 17, N0. 4. https://doi.org/10.1287/mnsc.17.4.B141
2. Dubois D and Prade H, Fuzzy Sets and Systems, Theory and Applications, Academic Press, New York, NY, USA, 1980. DOI:10.2307/2273604
3. Okada S and Gen M, Fuzzy shortest path problem, Computers, and Industrial Engineering 27 (1994) 465-468.
4. Zadeh L A, A note on Z-numbers, Information science, 181, 29232932,2011. https://doi.org/10.1016/j.ins.2011.02.022
5. Siddhartha Sankar Biswas, Z-Dijkstra's Algorithm to solve SPP in Z-Graph, Oriental Journal of Computer Science and Technology, Vol 10, no 1, 180-186.:http://dx.doi.org/10.13005/ojcst/10.01.24
6. Parameswari K, Lexicographic Order Based Ranking for Z-numbers, Advances In Mathematics: Scientific Journal 9 (2020), no 5, 3075-3083. 10.37418/amsj.9.5.67
7. Shahila Bhanu M and Velammal G, "Applying Hedges to the first component of Z-numbers," AIJRSTEM, 2005, issue10, vol 10(3), 200-203.
8. Rani P, Velammal G, "Applying modifier to the first component of Z-numbers in the fuzzy if- then rules", AIJRSTEM, 15(1), pp 165 168.
9. Stephen S, Novel Binary Operations on Z-numbers and their application in Fuzzy Critical Path Method, Advances in Mathematics: Scientific Journal 9 (2020), no 5, 3111-3120. DOI:10.37418/amsj.9.5.70
10. Okada S and Soper T, A shortest path problem on a network with fuzzy arc lengths, Fuzzy Sets and Systems,109(1),2000, pp,129-140. https://doi.org/10.1016/S0165-0114(98)00054-2

11. Okada S, Fuzzy shortest path problems incorporating interactivity among paths, Fuzzy Sets and Systems, Vol. 142(3)2004, PP 335-357.
DOI:10.1016/S0165-0114(03)00225-2

12. Kung J Y, Chuang T N and Lin C T, Decision making on network problem With fuzzy arc lengths, IMACS Multi Conference on Computational Engineering in Systems Applications (CESA) (2006), 578-580.
DOI:10.1109/CESA.2006.4281719

13. Zadeh L A, A Fuzzy-Set-Theoretic Interpretation of Linguistic Hedges, Journal Of Cybernetics Volume 2, 1972- Issue 3.
https://doi.org/10.1080/01969727208542910

P. Vijayalakshmi (Corresponding author),
Part-time Research Scholar, School of Mathematics,
Madurai Kamaraj University, Madurai-625021, Tamil Nadu, India.
E-mail address: shreevijiharish@gmail.com

and

S. Devibala,
Associate Professor, Department of Mathematics,
Sri Meenakshi Government Arts College for Women(A),
Affiliated to Madurai Kamaraj University, Madurai-625002, Tamil Nadu, India.
E-mail address: devibalasubburaman@gmail.com

and

M. A. Gopalan,
Professor, Department of Mathematics,
Shrimati Indira Gandhi College,
Affiliated to Bharathidasan University, Tiruchirappalli-620002, Tamil Nadu, India.
E-mail address: mayilgopalan@gmail.com