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Revan Indices for the F-Sum of Path

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ABSTRACT: Let P_n and P_m be the paths of order n and m respectively. The S, Q, R, and T Sum of these paths forms a graph $P_n +_F P_m$ where F is one of the labels S, Q, R, or T. The sum of minimum and maximum degrees among the vertices of the graph $P_n +_F P_m$ reduced by the degree of the vertex of that graph gives the Revan degree of that vertex. Using this Revan degree, we computed the four Revan topological indices in this paper.

Keywords: Path, F-Sum, Revan Indices.

Contents

1. Introduction

Mathematical chemistry has a topic called chemical graph theory that applies graph theory to chemical objects and phenomena. The physical and biological aspects of chemical substances are modeled using discrete mathematics. A chemical graph is a labelled graph that depicts a chemical compound's structural formula according to chemical graph theory. Chemical bonds and the atoms of the molecule are represented by the edges and vertices of the graph, respectively. Chemical graph theory is applicable to several branches of chemistry, such as computational chemistry, theoretical chemistry, mathematical chemistry, computational biology, and organic chemistry.

[1]

Topological indices also called molecular descriptors are mathematical formulas that can be used to model any graph of a chemical structure. The physicochemical characteristics of a molecule are studied by using topological indices to assess mathematical values. Topological indices are real numbers associated with graphs, and these graphs must be structurally invariant. They are applied in the modeling of molecule's chemical, medicinal, and other properties. On molecular structures, topological indices offer quantitative data. They play a key role in building (QSAR) and (QSPR) modules that correlated structural features with activities and properties. Compounds' physio-chemical properties can be predicted using these correlations.

[2,3]

Degree-based, distance-based, and counting-related indices are among the topological indices used in graph theory. Among other physical and chemical properties, topological indices can be used to assess a chemical compound's melting point, entropy, boiling point, energy generation, and enthalpy of vaporization.

[4]

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A series of topological indices called Revan indices is essential for chemical characterization. They are based on the Revan vertex degree and are used in QSPR analysis. Revan indices are correlated with the physical and chemical properties of various structures. They are employed in both the research of graph decompositions and the analysis of COVID-19 medicines. V.R. Kulli defined the Revan indexes in 2017. They are inspired by Zagreb indices' definitions and varied implementations.

2. Preliminaries

Definition 2.1 *A vertex in H Revan vertex degree is characterized as $r_H(v) = \Delta(H) + \delta(H) - d_H(v)$, where vertices of H have degree that range from $\delta(H)$ to $\Delta(H)$, respectively. The Revan indices of graph H, defined as,*

$$R_1(H) = \sum_{uv \in E(H)} r_H(u) + r_H(v),$$

$$R_2(H) = \sum_{uv \in E(H)} r_H(u) r_H(v),$$

$$R_3(H) = \sum_{uv \in E(H)} |r_H(u) - r_H(v)|,$$

$$R_{01}(H) = \sum_{u \in E(H)} r_H(u)^2.$$

[5]

Definition 2.2 *For a graph H that is connected,*

a) The graph S(H) is produced by appending an additional vertex to every edge of H. An equivalent path having length two is substituted for each edge of H.

b) After adding a new node to each line of H, the graph Q(H) is created by joining these pairs of new vertex on neighboring lines of H with edges.

c) Each edge of H must first have a new vertex added to it, and each new vertex must then be joined to the last vertices of the associated edge in order to produce R(H)

d) The definition of adjacency in T (H) is the incidence or adjacency of the matching elements of H.

e) The graphs S(H) and T(H) are referred to as H subdivision and total graphs, respectively.

We illustrate the above definitions in Fig.1.

[6]

Definition 2.3 *Let the operator F be chooses from the collection S, Q, R, T. A graph with a set of vertices $V(H_1 +_F H_2) = (V(H_1) \cup E(H_1)) \times V(H_2)$ if $[u_1 = v_1 \in V(H_1)$ and $u_2, v_2 \in E(H_2)]$ or $[u_2 = v_2$ and $u_1, v_1 \in E(F(H_1))]$ and two vertices that are near if and only if or is known as the F-sum.*

[7]

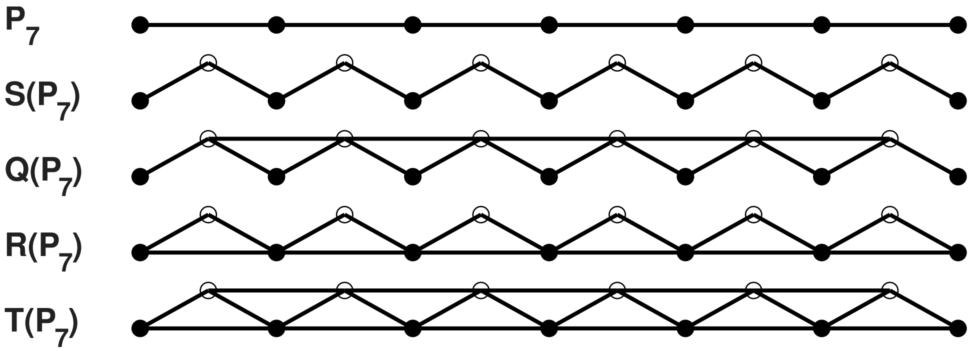

Figure 1: graph H and $S(H)$, $Q(H)$, $R(H)$ and $T(H)$

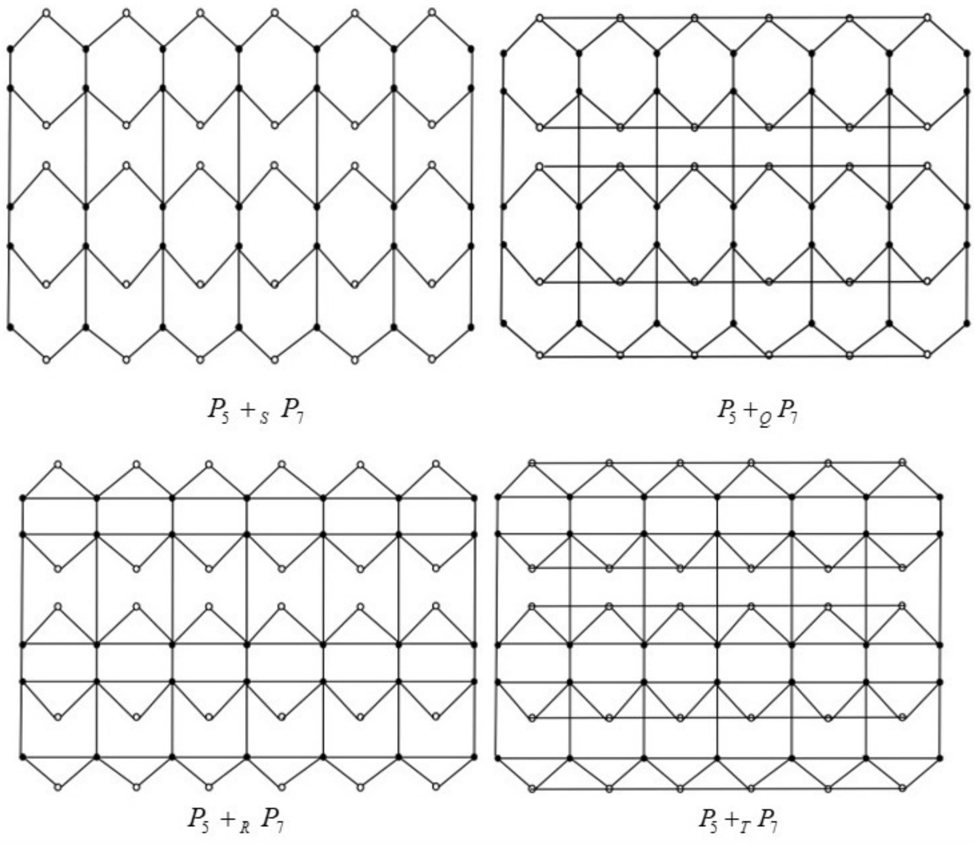

Figure 2: Graph $P_5 +_F P_7$

3. Revan Topological Indices of F-Sum of Paths of Graph

The Revan Topological Indices of Sum of Paths of Graph will be determined in this section:

Theorem 3.1 *Consider H as the graph of $H_1 +_s H_2$ where H_1 and H_2 are the paths of order n and m respectively and $+_s$ is the $S(H)$ sum. Then*

1. $R_1(H) = 16mn + 2n - 6m - 8$
2. $R_2(H) = 20mn + 16n + 2m - 22$

3. $R_3(H) = 4mn - 6m - 2n + 4$

4. $R_01(H) = 20mn - 6m + 10n + 8$

Proof: Let H be the graph of S-Sum of P_n and P_m i.e., $p = P_m +_S P_n$. The total no. of vertices of H is $m(2n-1)$ and edges are $3mn - 2m - n$, respectively.

Table 1: Partition of edges.

$\lvert E\rvert(d_u, d_v)$	$\lvert E\rvert_{(2,2)}$	$\lvert E\rvert_{(2,3)}$	$\lvert E\rvert_{(2,4)}$	$\lvert E\rvert_{(3,3)}$	$\lvert E\rvert_{(3,4)}$	$\lvert E\rvert_{(4,4)}$
The No. of edges	4	$4n+2m-8$	$2(m-3)$	$2(n-2)$	$(2m-4)(n-2)$	$(m-3)(n-2)$

Table 2: Number of vertices of degree $u \in V(H)$.

d_u	2	3	4
The No. of vertices	$nm-m+4$	$2m+2n-8$	$nm-2n-2m+4$

Thus, we have six types of Revan edges based on the degree of the end Revan vertices of each edge as follows, we have $\Delta(H) + \delta(H) = 6$.

Table 3: Partition of Revan edges.

$RE[d_u, d_v]$	$RE_{(4,4)}$	$RE_{(4,3)}$	$RE_{(3,3)}$	$RE_{(3,2)}$	$RE_{(4,2)}$	$RE_{(2,2)}$
The No. of edges	4	$4n+2m-8$	$2(m-3)$	$(2n-4)$	$2(m-2)(n-2)$	$(m-3)(n-2)$

Table 4: Number of vertices of Revan degree $r_H(d_u)$.

$r_H(d_u)$	4	3	2
The No. of vertices of degree	$nm-m+4$	$2m+2n-8$	$nm-2n-2m+4$

1. To compute $R_1(H)$, we can understand that.

$$
\begin{aligned}
R_1(H) = &\sum_{uv\in E(H)} r_H(u) + r_H(v) \\
&= \sum_{RE_{44}} r_H(u) + r_H(v) + \sum_{RE_{43}} r_H(u) + r_H(v) + \sum_{RE_{42}} r_H(u) + r_H(v) + \sum_{RE_{33}} r_H(u) + r_H(v) \\
&+ \sum_{RE_{32}} r_(u) + r_H(v) + \sum_{RE_{22}} r_H(u) + r_H(v) \\
&= 4(4+4) + (4n+2m-8)(4+3) + 2(m-2)(n-2)(4+2) + 2(m-3)(3+3) + 2(3+2)(n-2) \\
&+ (n-2)(m-3)(2+2) \\
&= 32 + 28n + 14m - 56 + 12(nm - 2n - 2m + 4) + 12m - 36 + 10n - 20 + 4(nm - 3n - 2m + 6) \\
&= 32 + 28n + 14m - 56 + 12nm - 24n - 24m + 48 + 12m - 36 + 10n - 20 + 4nm - 12n - 8m + 24 \\
R_1(H) = &\ 16mn + 2n - 6m - 8
\end{aligned}
$$

2. To compute $R_2(H)$, we observe that.

$$
\begin{aligned}
R_2(H) &= \sum_{uv\in E(H)} r_H(u)r_H(v)\\
&= \sum_{RE_{44}} r_H(u)r_H(v) + \sum_{RE_{43}} r_H(u)r_H(v) + \sum_{RE_{42}} r_H(u)r_H(v) + \sum_{RE_{33}} r_H(u)r_H(v)\\
&+ \sum_{RE_{32}} r_H(u)r_H(v) + \sum_{RE_{22}} r_H(u)r_H(v)\\
&= 4(4\times 4) + (4n+2m-8)(4\times 3) + 2(m-2)(n-2)(4\times 2) + 2(m-3)(3\times 3) + 2(n-2)(3\times 2)\\
&+ (n-2)(m-3)(2\times 2)\\
&= 64 + 12(4n+2m-8) + 16(m-2)(n-2) + 18(m-3) + 12(n-2) + 4(mn-3n-2m+6)\\
&= 64 + +488 + 24m - 96 + 16(nm-2n-2m+4) + 18m - 54 + 12n - 24 + 4nm - 12n - 8m + 24\\
R2(H) &= 20mn + 16n + 2m - 22
\end{aligned}
$$

3. To compute $R_3(H)$, we can notice that.

$$
\begin{aligned}
R_3(H) &= \sum_{uv\in E(H)} |r_G(H) - r_H(v)|\\
&= \sum_{RE_{44}} |r_H(u)-r_H(v)| + \sum_{RE_{43}} |r_H(u)-r_H(v)| + \sum_{RE_{42}} |r_H(u)-r_H(v)| + \sum_{RE_{33}} |r_H(u)-r_H(v)|\\
&+ \sum_{RE_{32}} |r_H(u)-r_H(v)| + \sum_{RE_{22}} |r_H(u)-r_H(v)|\\
&= 4(|4-4|) + (4n+2m-8)(|4-3|) + 2(m-2)(n-2)(|4-2|) + 2(m-3)(|3-3|)\\
&+ 2(n-2)(|3-2|) + (n-2)(m-3)(|2-2|)\\
&= 0 + 4n + 2m - 8 + 2(m-2)(n-2)2 + 0 + 2(n-2)(1) + 0\\
&= 4n + 2m - 8 + 4(nm-2n-2m+4) + 2n - 4\\
R_3(H) &= 4nm - 6m - 2n + 4
\end{aligned}
$$

4. To compute $R_{01}(H)$, we can examine that.

$$
\begin{aligned}
R_{01}(H) &= \sum_{u\in V(H)} r_H(H)^2\\
&= \sum_{RE_{44}} r_H(u)^2 + \sum_{RE_{43}} r_H(u)^2 + \sum_{RE_{42}} r_H(u)^2\\
&= [m(m-1)+4][4^2] + [2(m-2)+2(n-2)][3^2] + [(m-2)(n-2)][2^2])\\
&= 16(mn-m+4) + 9(2m+2n-8) + 4(mn-2m-2n+4)\\
R_{01}(H) &= 20mn - 6m + 10n + 8
\end{aligned}
$$

□

Theorem 3.2 *Let H be the graph of $H_1 +_Q H_2$ where H_1 and H_2 are the paths of order n and m respectively and $+Q$ is the $Q_(H)$ sum.Then*

5. $R_1(H) = 12mn + 24m + 6$
6. $R_2(H) = 12mn + 40m + 4n + 22$
7. $R_3(H) = 4m + 4n + 2$
8. $R_{01}(H) = 8mn + 26m + 10n + 8$

Proof: Let H be the graph of Q-Sum of P_n and P_m i.e., $P_n +_Q P_m$. The total no. of vertices of H is $m(2n-1)$ and edges are $4mn - n - 4m$, respectively. We have four distinct types of edges that depend on the degree of each edge's end vertices, as follows,

Table 5: Partition of edges.

$E[d_u, d_v]$	$E_{(2,3)}$	$E_{(3,3)}$	$E_{(3,4)}$	$E_{(4,4)}$
No. of edges	8	$4m-6$	$2(2m+2n-3)$	$3nm-5m-5n+4$

Table 6: Number of vertices of degree $u \in V(H)$.

$d(v)$	2	3	4
Number of vertices	4	$4m+2n-8$	$2nm-2n-5m+4$

Thus, Revan edges are classified into six varieties depending on the degree of the end. The vertices of every edge are as follows, we have $\Delta(H) + \delta(H) = 6$.

Table 7: Partition of Revan edges.

$RE(d_u, d_v)$	$RE_(4,3)$	$RE_(3,3)$	$E_(3,2)$	$E_(2,2)$
Number of edges	8	$4m-6$	$2(2m+2n-3)$	$3nm-5m-5n+4$

Table 8: Number of vertices of Revan degree $r_H(d_u)$.

$r_H(d_u)$	4	3	2
Number of vertices of degree	4	$4m+2n-8$	$2nm-2n-5m+4$

1. To compute $R_1(H)$, we have.

$$\begin{aligned}
R_1(H) &= \sum_{uv \in E(H)} r_H(u) + r_H(v) \\
&= \sum_{RE_{43}} r_H(u) + r_H(v) + \sum_{RE_{33}} r_H(u) + r_H(v) + \sum_{RE_{32}} r_H(u) + r_H(v) + \sum_{RE_{22}} r_H(u) + r_H(v) \\
&= 8(4+3) + (4m-6)(3+3) + (4m+4n-6)(3+2) + (3nm-5m-5n+4)(2+2) \\
&= 56 + 6(4m-6) + 5(4m+4n-6) + 4(3nm-5m-5n+4) \\
&= 56 + 24m - 36 + 20m + 20n - 30 + 12nm - 20m - 20n + 16 \\
R_1(H) &= 12nm + 24m + 6
\end{aligned}$$

2. To compute $R_2(H)$, we can see that.

$$\begin{aligned}
R_2(H) &= \sum_{uv \in E(H)} r_H(u) r_H(v) \\
&= \sum_{RE_{43}} r_H(u) r_H(v) + \sum_{RE_{33}} r_H(u) r_H(v) + \sum_{RE_{32}} r_H(u) r_H(v) + \sum_{RE_{22}} r_H(u) r_H(v) \\
&= 8(4 \times 3) + (4m-6)(3 \times 3) + (4m+4n-6)(3 \times 2) + (3nm-5m-5n+4)(2 \times 2) \\
&= 96 + 9(4m-6) + 6(4m+4n-6) + 4(3nm-5m-5n+4) \\
&= 96 + 36m - 54 + 24m + 24n - 36 + 12nm - 20m - 20n + 16 \\
R_2(H) &= 12nm + 40m + 4n + 22
\end{aligned}$$

3. To compute $R_3(H)$, we observe that.

$$\begin{aligned}
R_3(H) &= \sum_{uv\in E(H)} |r_H(u) - r_H(v)| \\
&= \sum_{RE_{43}} |r_H(u) - r_H(v)| + \sum_{RE_{33}} |r_H(u) - r_H(v)| + \sum_{RE_{32}} |r_H(u) - r_H(v)| + \sum_{RE_{22}} |r_H(u) - r_H(v)| \\
&= 8(|4-3|) + (m-6)(|3-3|) + (4m+4n-6)(|3-2|) + (3nm-5m-5n+4)(|2-2|) \\
&= 8 + 4m + 4n - 6 \\
R_3(H) &= 4n + 4m + 2
\end{aligned}$$

4. To compute $R_{01}(H)$, we can see that.

$$\begin{aligned}
R_{01}(H) &= \sum_{uv\in E(H)} r_H(u)^2 \\
&= [4(4)^2 + (4m+2n-8)(3)^2 + (2nm-2n-5n+4)(2)^2 \\
&= 64 + 9(4m+2n-8) + 4(2nm-2n-5m+4) \\
&= 64 + 36m + 18n - 72 + 8nm - 8n - 10m + 16 \\
R_{01}(H) &= 8nm + 26m + 10n + 8
\end{aligned}$$

□

Theorem 3.3 *Let H be the graph of $H_1 +_R H_2$ where H_1 and H_2 are the paths of order n and m respectively and $+_R$ is the $R(H)$ sum. Then*

$$\begin{aligned}
&9. R_1(H) = 24nm + 6m - 4n - 26 \\
&10. R_2(H) = 32mn + 40m + 22n - 54 \\
&11. R_3(H) = 8mn - 6m - 4n + 2 \\
&12. R_{01}(H) = 40mn - 12m + 10n + 16
\end{aligned}$$

Proof: Let H be the graph of R-Sum of P_n and P_m i.e., $P_n +_R P_m$. The complete number of vertices of H is $m(2n-1)$ and edges are $4mn - 3m - n$, respectively.

Table 9: Partition of edges.

$E(d(u), d(v))$	$E_{(3,2)}$	$E_{(3,4)}$	$E_{(5,3)}$	$E_{(4,2)}$	$E_{(4,4)}$	$E_{(5,2)}$	$E_{(6,5)}$	$E_{(6,2)}$	$E_{(6,6)}$	$E_{(5,5)}$	$E_(6,4)$
Number of edges	4	4	4	$2m-4$	$2(m-3)$	$4n-8$	$2m-6$	$2nm-4n-4m+8$	$2nm-5m-5n+12$	$2n-6$	$2m-4$

Table 10: Number of degree vertices $u \in V(H)$.

$d(u)$	2	3	4	5	6
Number of degree vertices	$nm-m$	4	$2(m-2)$	$2(n-2)$	$nm-2n-2m+4$

Thus, according to the degree of each edge's end Revan vertices, we have six different kinds of Revan edges, we have $\Delta(H) + \delta(H) = 8$.

Table 11: Partition of Revan edges.

$RE[d_(u), d_(v)]$	$RE_{(5,6)}$	$RE_{(4,5)}$	$RE_{(3,5)}$	$RE_{(4,6)}$	$RE_{(4,4)}$	$RE_{(3,6)}$	$E_{(2,3)}$	$E_{(2,6)}$	$E_{(2,2)}$	$E_{(3,3)}$	$E_{(2,4)}$
Number of edges	4	4	4	$2m-4$	$2(m-3)$	$4m-8$	$2m-6$	$2nm-4n-4m+8$	$2nm-5m-5n+12$	$2n-6$	$2m-4$

Table 12: Number of vertices of Revan degree $r_H(d_u)$.

$r_H(d_u)$	6	5	4	3	2
Number of vertices of degree	$nm-m$	4	$2m-4$	$2n-4$	$nm-2n-2m+4$

1. To compute $R_1(H)$, we notice that.

$$
\begin{aligned}
R_1(H) = & \sum_{uv\in RE(H)} r_H(u)+r_H(v) \\
& = \sum_{RE_{56}} r_H(u)+r_H(v) + \sum_{RE_{45}} r_H(u)+r_H(v) + \sum_{RE_{46}} r_H(u)+r_H(v) + \sum_{RE_{44}} r_H(u)+r_H(v) \\
& + \sum_{RE_{36}} r_H(u)+r_H(v) + \sum_{RE_{23}} r_H(u)+r_H(v) + \sum_{RE_{26}} r_H(u)+r_H(v) + \sum_{RE_{22}} r_H(u)+r_H(v) \\
& + \sum_{RE_{33}} r_H(u)+r_H(v) + \sum_{RE_{24}} r_H(u)+r_H(v) + \sum_{RE_{35}} r_H(u)+r_H(v) \\
& = 4(5+6)+4(4+5)+(2m-4)(4+6)+(2m-6)(4+4)+(4m-8)(3+6)+(2m-6)(2+3) \\
& + (2mn-4n-4m+8)(2+6)+(2mn-5n-5m+12)(2+2)+(2n-6)(3+3) \\
& + (2m-4)(2+4)+(3+5)4 \\
& = 44+36+20m-40+16m-48+36n-72+10m-30+16mn-32n-32m+64 \\
& + 8mn-20n-32m+64+8mn-20n-20m+48+12n-36+12m-24+32 \\
R_1(H) = & \ 24mn+6m-4n-26
\end{aligned}
$$

2. To compute $R_2(H)$, we recognize that.

$$
\begin{aligned}
R_2(H) = & \sum_{uv\in RE(H)} r_H(u)r_H(v) \\
& = \sum_{RE_{56}} r_H(u)r_H(v) + \sum_{RE_{45}} r_H(u)r_H(v) + \sum_{RE_{46}} r_H(u)r_H(v) + \sum_{RE_{44}} r_H(u)r_H(v) + \sum_{RE_{36}} r_H(u)r_H(v) \\
& + \sum_{RE_{23}} r_H(u)r_H(v) + \sum_{RE_{26}} r_H(u)r_H(v) + \sum_{RE_{22}} r_H(u)r_H(v) + \sum_{RE_{33}} r_H(u)r_H(v) + \sum_{RE_{24}} r_H(u)r_H(v) \\
& + \sum_{RE_{35}} r_H(u)r_H(v) \\
& = 4(5\times 6)+4(4\times 5)+(2m-4)(4\times 6)+(2m-6)(4\times 4)+(4m-8)(3\times 6) \\
& + (2m-6)(2\times 3)+(2mn-4n-4m+8)(2\times 6)+(2mn-5n-5m+12)(2\times 2) \\
& + (2n-6)(3\times 3)+(2m-4)(2\times 4)+(3\times 5)4 \\
& = 120+80+48m-96+32m-96+72n-144+12m-36+24mn-48m-48n \\
& + 96+8mn-20n-20m+48+18n-54+16m-32+60 \\
R_2(H) = & \ 32mn+40m+22n-54
\end{aligned}
$$

3. To compute $R_3(H)$, we recognize that.

$$
\begin{aligned}
R_3(H) &= \sum_{uv \in RE(H)} |r_H(u) - r_H(v)| \\
&= \sum_{RE_{56}} |r_H(u) - r_H(v)| + \sum_{RE_{45}} |r_H(u) - r_H(v)| + \sum_{RE_{46}} |r_H(u) - r_H(v)| + \sum_{RE_{44}} |r_H(u) - r_H(v)| \\
&+ \sum_{RE_{36}} |r_H(u) - r_H(v)| + \sum_{RE_{23}} |r_H(u) - r_H(v)| + \sum_{RE_{26}} |r_H(u) - r_H(v)| + \sum_{RE_{22}} |r_H(u) - r_H(v)| \\
&+ \sum_{RE_{33}} |r_H(u) - r_H(v)| + \sum_{RE_{24}} |r_H(u) - r_H(v)| + \sum_{RE_{35}} |r_H(u) - r_H(v)| \\
&= 4(5-6) + 4(4-5) + (2m-4)(4-6) + (2m-6)(4-4) + (4m-8)(3-6) + (2m-6)(2-3) + \\
&(2mn - 4n - 4m + 8)(2-6) + (2mn - 5n - 5m + 12)(2-2) + (2n-6)(3-3) + (2m-4)(2-4) + \\
&(3-5)4 \\
&= 4 + 4 + 4m - 8 + 12n - 24 + 2m - 6 + 8nm - 16n - 16m + 32 + 4m - 8 + 8 \\
R_3(H) &= 8mn - 6m - 4n + 2
\end{aligned}
$$

4. To compute $R_{01}(H)$, we recognize that.

$$
\begin{aligned}
R_{01}(H) &= \sum_{u \in V(H)} r_H(u)^2 \\
&= (nm - m)6^2 + (2m-4)4^2 + 4(5^2) + (2n-4)3^2 + (nm - 2n - 2m + 4)2^2 \\
&= 36nm - 36m + 32m - 64 + 100 + 18n - 36 + 4nm - 8n - 8m + 16 \\
R_{01}(H) &= 40mn - 12m + 10n + 16
\end{aligned}
$$

□

Theorem 3.4 *Let H be the graph of $H_1 +_T H_2$ where H_1 and H_2 are the paths of order n and m respectively and $+_T$ is the $T(H)$ sum. Then*

$$
\begin{aligned}
1.R_1(H) &= 38nm - 20m + 4n - 16 \\
2.R_2(H) &= 73nn - m + 31n - 42 \\
3.R_3(H) &= 4mn + 2m - 2n + 6 \\
4.R_{01}(H) &= 34mn + 29m + 14n + 16
\end{aligned}
$$

Proof: Let H be the graph of T-Sum of P_n and P_m i.e., $P_n +_T P_m$. The complete number of vertices of H is $m(2n-1)$ and edges are $5mn - 5m - n$, respectively. Depending on the degree of each edge's end vertices, we have four different kinds of edges,

Table 13: Partition of edges.

$E[d_u, d_v]$	$E_{(3,3)}$	$E_{(3,4)}$	$E_{(3,5)}$	$E_{(3,6)}$	$E_{(4,4)}$	$E_{(4,5)}$	$E_{(4,6)}$	$E_{(5,5)}$	$E_{(5,8)}$	$E_{(6,6)}$
Number of edges	4	$4m$	8	$2(m-2)$	$mn - 2m - -6$	$4n - 12$	$2mn - 4n - 4m + 8$	$2(n-3)$	$2(n-2)$	$2nm - 5n - 5m + 12$

Table 14: Number of degree vertices $u \in V(H)$.

$d(u)$	3	4	5	6
Number of degree vertices	$2m+4$	$nm - m - 4$	$2(n-2)$	$(m-2)(n-2)$

Thus, we have six types of Revan edges based on the degree of the end Revan vertices of each edge as follows, we have $\Delta(H) + \delta(H) = 9$.

Table 15: Partition of Revan edges.

$RE[d_u, d_v]$	$RE_{(6,6)}$	$RE_{(6,5)}$	$RE_{(6,4)}$	$RE_{(6,3)}$	$RE_{(5,5)}$	$RE_{(5,4)}$	$RE_{(5,3)}$	$RE_{(4,4)}$	$RE_{(4,3)}$	$RE_{(3,3)}$
Number of edges	4	$4m$	8	$2(m-2)$	$mn-2m-6$	$4n-12$	$2nm-4n-4m+8$	$2(n-3)$	$2(n-2)$	$2nm-5n-5m+12$

Table 16: Number of vertices of Revan degree $r_H(d_u)$.

$r_H(d_u)$	6	5	4	3
Number of vertices of degree	$2m+4$	$nm-m-4$	$2(m-2)$	$(m-2)(n-2)$

5. To compute $R_1(H)$, we recognize that.

$$\begin{aligned}
R_1(H) &= \sum_{uv\in E(H)} r_H(u)+r_H(v)\\
&= \sum_{RE_{66}} r_H(u)+r_H(v) + \sum_{RE_{65}} r_H(u)+r_H(v) + \sum_{RE_{64}} r_H(u)+r_H(v) + \sum_{RE_{63}} r_H(u)+r_H(v)\\
&+ \sum_{RE_{55}} r_H(u)+r_H(v) + \sum_{RE_{54}} r_H(u)+r_H(v) + \sum_{RE_{53}} r_H(u)+r_H(v) + \sum_{RE_{44}} r_H(u)+r_H(v)\\
&+ \sum_{RE_{43}} r_H(u)+r_H(v) + \sum_{RE_{33}} r_H(u)+r_H(v)\\
&= 4(6+6)+4m(6+5)+8(6+4)+2(m-2)(6+3)+mn-2m-6(5+5)+(4n-12)(5+4)\\
&+ 2nm-4n-4m+8(5+3)+2(n-3)(4+4)+2(n-2)(4+3)+2nm-5n-5m+12(3+3)\\
&= 48+44m+80+18m-36+10mn-20m-60+36n-108+16mn-32n-32m+64\\
&+ 16n-48+14n-28+12mn-30n-30m+72\\
R_1(H) &= 38mn-20m+4n-16
\end{aligned}$$

6. To compute $R_2(H)$, we recognize that.

$$\begin{aligned}
R_2(H) &= \sum_{uv\in E(H)} r_H(u)r_H(v)\\
&= \sum_{RE_{66}} r_H(u)r_H(v) + \sum_{RE_{65}} r_H(u)r_H(v) + \sum_{RE_{64}} r_H(u)r_H(v) + \sum_{RE_{63}} r_H(u)r_H(v) + \sum_{RE_{55}} r_H(u)r_H(v)\\
&+ \sum_{RE_{54}} r_H(u)r_H(v) + \sum_{RE_{53}} r_H(u)r_H(v) + \sum_{RE_{44}} r_H(u)r_H(v) + \sum_{RE_{43}} r_H(u)r_H(v) + \sum_{RE_{33}} r_H(u)r_H(v)\\
&= 4(6\times 6)+4m(6\times 5)+8(6\times 4)+2(m-2)(6\times 3)+mn-2m-6(5\times 5)+(4n-12)(5\times 4)\\
&+ 2nm-4n-4m+8(5\times 3)+2(n-3)(4\times 4)+2(n-2)(4\times 3)+2nm-5n-5m+12(3\times 3)\\
&= 144+120m+192+36m-72+25mn-50m-150+80n-240+30mn-60n-60m\\
&+ 120+32n-96+24n-48+18mn-45+108\\
R_2(H) &= 73mn-m+31n-42
\end{aligned}$$

7. To compute $R_3(H)$, we recognize that.

$$\begin{aligned}
R_3(H) &= \sum_{uv\in E(H)} |r_H(u) - r_H(v)| \\
&= \sum_{RE_{66}} |r_H(u) - r_H(v)| + \sum_{RE_{65}} |r_H(u) - r_H(v)| + \sum_{RE_{64}} |r_H(u) - r_H(v)| + \sum_{RE_{63}} |r_H(u) - r_H(v)| \\
&+ \sum_{RE_{55}} |r_H(u) - r_H(v)| + \sum_{RE_{54}} |r_H(u) - r_H(v)| + \sum_{RE_{53}} |r_H(u) - r_H(v)| + \sum_{RE_{44}} |r_H(u) - r_H(v)| \\
&+ \sum_{RE_{43}} |r_H(u) - r_H(v)| + \sum_{RE_{33}} |r_H(u) - r_H(v)| \\
&= 4|6-6| + 4m|6-5| + 8|6-4| + 2(m-2)|6-3| + mn - 2m - 6|5-5| + (4n-12)|5-4| \\
&+ 12nm - 4n - 4m + 8|5-3| + 2(n-3)|4-4| + 2(n-2)|4-3| + 2nm - 5n - 5m + 12|3-3| \\
&= 4m + 16 + 6(m-2) + 4n - 12 + (2mn - 4n - 4m + 8)2 + 2(n-2) \\
R_3(H) &= 4mn + 2m - 2n + 6
\end{aligned}$$

8. To compute $R_{01}(H)$, we comprehend that.

$$\begin{aligned}
R_{01}(H) &= \sum_{u\in V_(H)} r_H(u)^2 \\
&= (2m+4)6^2 + (nm - -m4)5^2 + 2(n-2)4^2 + (m-2)(n-2)3^2 \\
&= 36(2m+4)25(nm - m - 4) + 32(n-2) + 9(mn - 2n - 2m + 4) \\
&= 72m + 144 + 25nm - 25m - 100 + 32 - 64 + 9mn - 18n - 18m + 36 \\
R_{01}(H) &= 34mn + 29m + 14n + 16
\end{aligned}$$

□

4. Conclusion

Quantitative structures and property (QSPR) and quantitative structure along with activity (QSAR) are analyzed using topological indices. The biological activity and characteristics of a chemical molecule are correlated with molecular structural descriptors using these statistical techniques. The first, second, third, and 01 Revan indices have been computed in this study as part of the examination of the Revan topological indices of the S, Q, R, and T Sums of the Paths.

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