



An Efficient Hybrid Equation (EHFM) for Face Recognition Technique: Comparison with Statistical Recognition

Rusul Mohsen Washeel*, Zahraa Fadhil Abd Alhussain and Asmhan Flied Hassan

ABSTRACT: In this research we propose a novel equation (technique) for face recognition. Face recognition is very important task in digital image processing (image similarity), which have many applications in security, biometrics, social media, and entertainment. However, that task is still challenging and limited by several factors. The new technique, we call it, a hybrid equation (EHFM), which has been tested versus the famous measure (SSIM). Face recognition via (EHFM) is depend on a test (reference) image and images of database. The new propose mathematical equation (EHFM) is performed by MATLAB R2020a using the very famous database AT&T images, we chose eight subjects and ten views (poses) per subject with various facial expressions. The essential goal of this our new idea is to introduce more accuracy equation (technique) good for face recognition technique that can be perform in real-time environment. Performance of new our proposed (EHFM) equation for face recognition, which is proved experimentally, also it is evidenced to outperform the (SSIM) statistical-based and famous face recognition.

Keywords: Hybrid Equation (EHFM), Statistical Similarity (SSIM), Features Similarity Index Matrix (FSIM), Entropic-equation (EE), Histogramic-equation (HE), face recognition technique.

Contents

| | |
|--|-----------|
| 1 Introduction | 1 |
| 2 Theoretical Background | 2 |
| 2.1 Image Similarity | 2 |
| 2.2 Image Similarity technologies | 2 |
| 2.2.1 Techniques based on the statistical approach | 2 |
| 2.2.2 Techniques based on the Information-Theoretic approach | 3 |
| 3 The Hybrid Equation (EHFM) for Face Recognition | 4 |
| 3.1 The Proposed Hybrid Equation (EHFM) | 5 |
| 3.2 Face Recognition Based on the Equation (EHFM) | 5 |
| 3.3 Recognition Confidence for Faces | 6 |
| 3.4 High-Performance of (EHFM) in Face Recognition | 6 |
| 3.4.1 Experimental Results and Performance | 6 |
| 4 Conclusion | 11 |

1. Introduction

Face recognition technique is growing rapidly in the pattern recognition and digital image processing. Applications via face recognition techniques are manifold and various, consisting of the access-control, the security and the video surveillance, the credit-card user identification, the forensic, the entertainment, and the video indexing. These techniques or applications have made it a quite popular article domain in latest three decades; the reader can view [9,12] for the survey. Recently, various techniques about face recognition are created and developed, the sources [6,7] overview the several techniques of face recognition. Also, many beneficial databases are created with effectively uses for evaluation of performance to many approaches [3]. Distinct matching algorithm to face recognition system is created or designed depend on the image-similarity measurements such as the standard (SSIM) [10]. And there are distinct measures of similarity introduced and used to different purposes, survey [5,2,1].

* Corresponding author.

2020 *Mathematics Subject Classification*: 46E22, 46K15.

Submitted November 26, 2025. Published March 13, 2026

This work introduces a new mathematical equation used for face recognition which based on three measures (laws), and the comparison is made with the statistical-(SSIM). Our new face recognition method, which we named as a hybrid mathematical equation (EHFM), which is meet the requirement to face recognition accuracy.

Automatic recognition designs to recognize the identity of any given face image with comparison among images in the memory. Several face recognition designs, the memory of a face-recognizer is exemplified by a set of training images, and saved as a face-image data-base. In our work, the set of training images are represented by the very famous face image database AT&T. Therefore, the task of the face-recognizer is to find the more similar feature-vector among all the images in the training-set to the feature-vector for a given test face image.

The reference [10] has been shown that the standard (statistical) SSIM accords a good scale of similarity under noise-free conditions, while it falls when noise start to increase. In addition, it may afford a non-negligible similarity between any two different images. This is attributed to the fact that the statistical-SSIM is entirely based on the statistical features of any two images, and may contain hidden correlations.

The above reason motivates us to introduce a new mathematical equation, which is more accurate in face recognition. Simulation results viewed that our new designed equation (EHFM) achieves higher performance than the statistical-(SSIM).

The rest of our research is arranged as follows: Sec. two introduces the foundations of image similarity measures and face recognition. Sec. three describes the Forming of novel mathematical equation (EHFM) for face recognition (measure of image similarity). Sec. four shows experimental (efficient and accuracy) results, discussion, and concluding remarks.

2. Theoretical Background

2.1. Image Similarity

The notion of similarity is outlined because it reveals the distance between completely different information structures. The similarity is the quantity that reflects the strength of correlative properties between the 2 pieces of the information, the un similarity measures, and the distinction between the 2 things of data [12,6]. The range of similarity should fall inside the interval between zero and one.

2.2. Image Similarity technologies

Many matching algorithms for biometric authentication systems and object are designed supporting the measure of image similarity. Where it can be divided via the following sides.

2.2.1. Techniques based on the statistical approach. There are many similarity measures which depend on statistical concepts, but the most famous (standard) one is the measure (SSIM), the following is clarification of it.

1. The Structural Similarity Index (SSIM) measure

The structural or statistical similarity measure (SSIM) gaps the visual of quality via comparing the three main statistical measures of the reference and noisy images: mean (corresponds to luminance), variance (corresponds to contrast), cross-correlation (corresponds to structure). The purpose of utilizing a description of the fulfilling invariance under any small changes in image [10]. This concepts applied in the spatial domain as in (SSIM) [4], in the multi-domain representations as in (MSSIM) [?] and (CW-SSIM) [11]. The standard SSIM was designed as the formula below.

$$I(F_1, F_2) = \frac{(2\mu_{F_1}\mu_{F_2} + C_1)(2\sigma_{F_1F_2} + C_2)}{(\mu_{F_1}^2 + \mu_{F_2}^2 + C_1)(\sigma_{F_1}^2 + \sigma_{F_2}^2 + C_2)} \quad (2.1)$$

Where $I(F_1, F_2)$ is the SSIM-equation between two-images (variables) F_1 (reference or test image) and F_2 (corrupted image), $\mu_{F_1}, \mu_{F_2}, \sigma_{F_1}^2$, and $\sigma_{F_2}^2$ are the statistical concepts; the means and variances of any pixel (intensity) values of any two images F_1, F_2 , and $\sigma_{F_1F_2}$ refers to the statistical covariance of the two images F_1 and F_2 , the distinct constants C_1 and C_2 are specified

by $C_1 = (W_1L)^2$ and $C_2 = (W_2L)^2$, with W_1 and W_2 are very small constants, with $L = 255$ (represents the max- value of pixel).

(SSIM) has been used for evaluating image similarity results in a rapidly increasing number of exciting applications.

2. Features Similarity Index Matrix (FSIM)

The famous method (Feature-Similarity-Index) maps the features, and it measures of the similarity between any two images. To view the method (FSIM) we must to view two criteria obviously. They are: the Phase-Congruency (P_C) and the Gradient-Magnitude (M_G). The (P_C): A new way for determination the features of image is phase-Congruency. One of the more necessary characteristics of (P_C) that it is never changing to light alteration. Aside from, it is capable to determinate features which are more interesting and necessary. The Gradient-magnitude (M_G): The calculation of image-gradient. Let F be an image, and M_x, M_y represents its vertical and horizontal gradients, resp., then gradient magnitude of the image F is defined as [8] (FSIM).

$$(\sqrt{M_x + M_y})$$

In our research we will calculate the amount of similarity between any two images to determine the quality. If there are two images F_1 and F_2 with their phase-congruency, symbolize by P_{C1} and P_{C2} , resp., the Phase-Congruency (P_C) maps taken from the images F_1, F_2 and Magnitude-Gradient (M_G) maps M_1 and M_2 taken from any two images likewise. (FSIM) is computed depend on P_{C1}, P_{C2}, M_1 and M_2 . At first, we must compute the amount of similarity to any two images as below;

$$S_P = \frac{2P_{C1}P_{C2} + R_1}{P_{C1}^2 + P_{C2}^2 + R_1} \quad (2.2)$$

where R_1 represent any positive constant utilize to increase the stability of S_P . Partially R_1 can be computed depend on P_C . The Eq. (2.2) above characterizes the measurement to specify the similarity of any two positive-real-numbers, and the range of it is within limits 0 to 1. And similarly, we can compute amount of similarity of M_1 and M_2 as below:

$$S_M = \frac{2M_1M_2 + R_2}{M_1^2 + M_2^2 + R_2} \quad (2.3)$$

where R_2 is any positive constant which bases on the dynamic range of gradient-magnitude values. In this method, both R_1, R_2 are constant, thus the (FSIM) measure can be conveniently used.

Now S_P, S_M are combined with other to compute the similarity SI of the two images F_1 and F_2 . The similarity SI defined as;

$$SI = [S_P]^\alpha \cdot [S_M]^\beta \quad (2.4)$$

Where the two parameters α, β are used to adjust the relative-importance to P_C and M . In the research, we take $\alpha = \beta = 1$, via Eqs. (2.2) and (2.3) it is clear that the method (FSIM) is normalized (i.e. between 0 to 1).

2.2.2. Techniques based on the Information-Theoretic approach. In fact, utilizing of information - theoretic concepts in field of image (similarity) processing is very possible when we suppose that the image is 2D random variable.

1. Entropic-Equation (EE)

The basic field of interest in this equation is the Shannon-entropy theory (i.e. entropy, joint entropy). The foundation for entropy is in the information-theory. The most significant uses of the entropy in the application is the Shannon entropy, and the mathematical form of it:

Entropy \mathbb{E} of a discrete random-variable x with the values in $\{x_1, x_2, \dots, x_n\}$ is;

$$\mathbb{E}(F_1) = - \sum_{i=1}^n p_i \log p_i \quad (2.5)$$

where $p_i = p_r [F_1 = x_i]$.

We can extend the entropy from a single random-variable to a marriage of random variables. The entropy of a pair of random-variables is said to be joint-entropy and symbolized by $\mathbb{E}(x, y)$ in this our research, with p_{ij} (joint-distribution) as fellows.

$$\mathbb{E}(F_1, F_2) = - \sum_{i=1}^n \sum_{j=1}^m p_{ij} \log p_{ij} \quad (2.6)$$

Now entropic-equation (EE) [1]:

$$EE = \frac{(\mathbb{E}_{F_1} \cdot \mathbb{E}_{F_2} + a) \cdot (2 \cdot \mathbb{E}_{F_1 F_2} + a)}{(\mathbb{E}_{F_1}^2 + \mathbb{E}_{F_2}^2 + a) \cdot (\mathbb{E}_{F_1 F_2}^2 + a)} \quad (2.7)$$

Where \mathbb{E}_{F_1} is the entropy to the image F_1 which calculate via using reference image-histogram $H(F_1)$, \mathbb{E}_{F_2} is the entropy to the image F_2 which calculate via using histogram of distorted image $H(F_2)$ and the symbol $\mathbb{E}_{F_1 F_2}$ is the joint-entropy of a pair of images F_1 and F_2 which estimate via using reference-image and distorted version of it, image-joint-histogram $H(F_1, F_2)$, and a is a very small constant equal to 0.05.

2. Histogram Equation (HE)

The histogram of any digital image is a function characterizing the distribution of color image intensity-level. For any intensity -level, the corresponding histogram value equal to the no. of the pixels take the same intensity value.

The joint histogram defines as the joint-probability of any two random-variables (we mean images in this case;). In the two-dimensional-joint-histogram, the value of pixel to any two images are represented the coordinate-axis. If we take any two images F_1, F_2 and their values intensity i and j , then the joint hist. $H_{ij}(F_1, F_2)$ represents the pixel probability to have the value i on image F_1 and j on image F_2 . Thus, all entry in joint hist. is the number of times of intensity value i in one-image corresponds to value j of intensity in the second-image. When the joint-hist. is normalized, it turn up to a measure of the joint-probability-distribution-function (PDF) of intensities values in the two images [1].

The image histogram takes the following mathematical formula:

$$H(F_1) = H_i(F_1), i \in (1, 2, \dots, N) \quad (2.8)$$

The most important in the applications is joint-histogram, whose formula is $H_{ij}(F_1, F_2)$. Of $i = j$ axis then; $H_{ij}(F_1, F_1) = H_{ji}(F_1, F_1)$. Thus, normalized joint-histogram is defined for two images F_1 and F_2 with size $M \times N$ as: $H(F_1, F_2) = [H_{ij}]$

Where:

$$H_{ij} = \frac{\text{Number of joint events } (X = i, Y = j)}{M \times N} \quad (2.9)$$

Now, The Histogram Equation (HE) [1]:

$$HE(F_1, F_2) = \frac{(H_{F_1} \cdot H_{F_2}) \cdot (2 \cdot H_{F_1 F_2})}{(H_{F_1}^2 + H_{F_2}^2) \cdot (H_{F_1 F_2}^2)} \quad (2.10)$$

Where H_{F_1} is the hist. of first (reference) image, H_{F_2} is the hist. of second (noisy version) image and $H_{F_1 F_2}$ is a joint-hist. of the F_1 and F_2 .

3. The Hybrid Equation (EHFM) for Face Recognition

This section is proposed a modern Face Recognition technique depend on the combination of three equations (statistic equation and two information theoretic equations) for measure face image similarity (face recognition), the basic ranges of interest are three mathematically-structures. The first is the information theoretic equation (entropic equation) (EE) measure [1]. The second is (histogram-equation) (HE) [1] depend on image-histogram and joint-histogram. The third is the well-known similarity measure (FSIM) based on statistical measures [8].

3.1. The Proposed Hybrid Equation (EHFM)

First, we must propose an error-estimate between reference F_1 and F_2 noisy copy of it. The error estimation is depended on the above two information-theoretic equations and the above statistical measure, as follows;

$$\mathbf{E}(F_1, F_2) = \frac{(3 \cdot EE(F_1, F_2) \cdot HE(F_1, F_2) \cdot FSIM(F_1, F_2))}{(EE(F_1, F_2))^3 + (HE(F_1, F_2))^3 + (FSIM(F_1, F_2))^3 + C} \quad (3.1)$$

Where $C = 0.001$ is constant that is very small used for balance the equation, to avoid dividing by 0 and adding it to raise equation stability-ratio. The symbol (EE) corresponds to the Entropic Equation as in Eq. (2.7) in section one, and the symbol (HE) corresponds to the (Histogram Equation) as in Eq. (2.10) in section one. And (FSIM) corresponds to the feature Similarity Index Matrix as in Eq. (2.4).

Ordinarily, F_1 and F_2 are a reference and a distorted copy of it, resp.

Actually, what distinguishes new proposed equation (EHFM), worth while and unique is utilizing of all its terms simultaneously when we implementing it to detect similarities of face images or distinguish face images.

Clearly

$$0 \leq \mathcal{E}(F_1, F_2) \leq 1. \quad (3.2)$$

Depending on the equation of (EHFM) error estimate, the similarity-measure (EHFM) can be proposed as:

$$EHFM = E(F_1, F_2) = 1 - \mathcal{E}(F_1, F_2) \quad (3.3)$$

Where:

$$0 \leq E(F_1, F_2) \leq 1 \quad (3.4)$$

Notice that, our algorithm consists of all the above equations and can run all these methods to find the similarities between face images with very high accuracy, reliable in security and many purposes, this is one of the distinctive properties of (EHFM measure).

3.2. Face Recognition Based on the Equation (EHFM)

The equation (EHFM) utilizes for face recognition technique between a reference image F_1 and many face images $\{F_{2pt}\}$ for a no. Q of persons, and each person (whose no. in the data-base is p s. t. $p \leq Q$) has T (poses) different images (any pose is indexed via t); so that a total of $N = Q \cdot T$ different images in the dataset. One of poses is chosen with a number $t = x$ which indicates to the person with no. p to do as a reference face image. Then two examinations are performed: the first exam keeps the pose x^{th} in the face image dataset then starts the process of new method of recognition. This is an easy mission, but helps us in examining and comparing the performances of our proposed equation. The second examination removes the x^{th} pose and then starts the process of new method of recognition, which is considered as a hard mission since the different poses of faces may be treated as different images. In this section we characterize a face recognition algorithm utilizing the equation (EHFM) error estimate as mentions in Eq. (3.1) above.

Between a reference image F_1 and a person p in the face dataset AT&T, and comparison via the equation (EHFM) as the following:

$$E(F_1, p) = \max_t \{E(F_1, F_{2pt})\} \quad (3.5)$$

The value of $E(F_1, p)$ refers to the quantity or value of confidence that we can put in the recognition process. Surely, for the ideal case $E(F_1, p) = 1$, that occurs only if the reference pose exists in the face dataset being used. The face recognition depends on the mathematical formula below.

$$p_i = \arg \left[\max_p \{E(F_1, p)\} \right] \quad (3.6)$$

Where the function $\arg[\cdot]$ represents the ordinal no. of any person in the face database whose certain pose gives maximal value of similarity with the reference face image F_1 .

Then similar process to find the best match via statistical-(SSIM) based. And written as mathematical formula

$$I(F_1, p) = \max_t I(F_1, F_{2pt}) \quad (3.7)$$

The above mathematical formula with the face recognition decision, as the formula:

$$p_j = \arg \left[\max_p \{I(F_1, p)\} \right] \quad (3.8)$$

3.3. Recognition Confidence for Faces

In plurality of cases: (EHFM), and (SSIM) agree such that:

$$p_i = p_j,$$

And so on, the confidence in face recognition varies a large amount. Via “confidence in face recognition” we denote over there is a “very good” distance among the peaks as in Eq. (3.6) and Eq. (3.8) the following-in-altitude peaks who maybe hesitate the resolution when the two peaks (using the same method) are near in magnitude.

Present, confidence in recognizing the trial face image F_1 utilizing a measure μ . First, in the measure we find the second peak (2nd -max) $\{\mu(F_1, p) | p = 1, \dots, Q\}$. Subsequently, we find the differ between the max. and the second-max. for each method, which we named thither the Max-difference, and referred via $\partial(\mu, F_1)$, which may be $\partial(E, F_1)$ or $\partial(I, F_1)$. And the Max-Max-difference of our new method is the resulting amount for this.

The mathematical definition of the confidence in recognizing the reference face image F_1 utilize the amount of similarity μ is:

$$\partial(E, F_1) = \max_p \{E(F_1, p)\} - \max_{2p} \{E(F_1, p)\} \quad (3.9)$$

Whereas $\max_{2p} \{E(F_1, p)\}$ represents the 2nd-max of the similarities-curve between the reference face image F_1 and other faces of persons in the AT&T dataset (notice exhausted that all face poses for any person in $\mu(F_1, p)$ as the Eq. (3.5)).

We will notice in the following sec. that for all the cases through testing, the new proposed equation (EHFM) outperforms the statistical similarity equation (SSIM) as we always have:

$$\partial(E, F_1) > \partial(I, F_1) \forall F_1 \quad (3.10)$$

Therefore, we expect as successful role for the equation (EHFM) in face image recognition wherever the decision gives is very clear in a most cases. This difference can run to (0.5) in cases, wherever it represents half the amount of the total measure (1). Surely, we can notice and take the best results and most clear when the reference face image is included in the AT&T dataset, though this case is not realistic.

3.4. High-Performance of (EHFM) in Face Recognition

This section views the efficient results and discussion of our new proposed measure (EHFM), and before that we view the database, ways and cases which we followed.

3.4.1. Experimental Results and Performance.

1. The database

The dataset where we used a dataset for gray face images known as the AT&T dataset that was used in designing many face recognition methods. This dataset was divided into two sets, for disconnect testing-purposes and for training-purposes over the training period, we utilized in this our work (80) face image, which consists of eight person images, and ten poses via different facial expressions to each person. We will display a sample of this principle. The reader can see Figure 1 in below. And in this our work, we will fit the size of face images (i.e. dimensions) to be as (92×92) pixels.

2. Testing

We utilized MATLAB-R2020a [4] for training and to test the new proposed mathematical equation for face recognition which presented in this research.

Performance of the proposed mathematical equation (EHFM) is tested and then compared with the famous statistical (SSIM) . The efficient (algorithm) equation (EHFM) is tested in two different cases, and the results viewed its accuracy and effectiveness of the results in facial recognition.

(a) Case 1- Test Image Included in the Dataset

The experiment appeared excellent results and a high estimation for statistical (SSIM) and our proposed equation (EHFM). In this case, where the test face image F_1 (reference-face image) belongs to the dataset utilized for training, though this case is considered ideal and less realistic than the second case.

(b) Case 2- Test Image Excluded from the Database

In this case, the test image (reference face image) is removed from the database utilized in training. As though, the recognition process remains very good and our proposed equation (EHFM) is superior in comparison with (SSIM) for recognizing the face image in despite of the difference in poses and expressive poses. We utilized the distance between the second and first maximum peaks of similarity curve for each of the face recognition systems mentioned in our work. To compare the performance, the more distance, the more confident closely the decision have to recognize the person.

(c) Discussion

Figures 1 views sample of two person poses from the AT and T dataset, utilized for testing. The reference face pose, considered as test face image, is indicated. Figure 2 views results of image similarity test between the reference face image and other faces of people in the dataset, where, for each person, the maximum amount of similarity among all of his or her poses is considered. Notice that, for almost all cases, the proposed equation (EHFM) obtains and keeps greater difference between the test face image and other images(persons), also the reader can see the Table 1 which describes the numerical comparison of a test image which is included the Reference F_1 in the AT&T dataset. The Figure 3 is more realistic as the test excludes the face image on test from the dataset, also the reader can see the Table 2 which describe the numerical comparison. In this case the maximum amount of similarity is less than 1 for both method of recognition, but our proposed equation (EHFM) gives much better results. And in Figure 4 The comparison between our new equation (EHFM), and the famous (SSIM), of test image which excluded the Reference (F_1) in the AT&T database. And clearly that (EHFM) $[E(F_1, p)]$ achieves more confidence than statistical-SSIM $[I(F_1, p)]$ as $\partial(E, F_1) > \partial(I, F_1)$, also the reader can see the Table 3 which describes the numerical results for the comparison.

The figures below show the sample of main results of the face recognition and high performance of our proposed equation (EHFM).

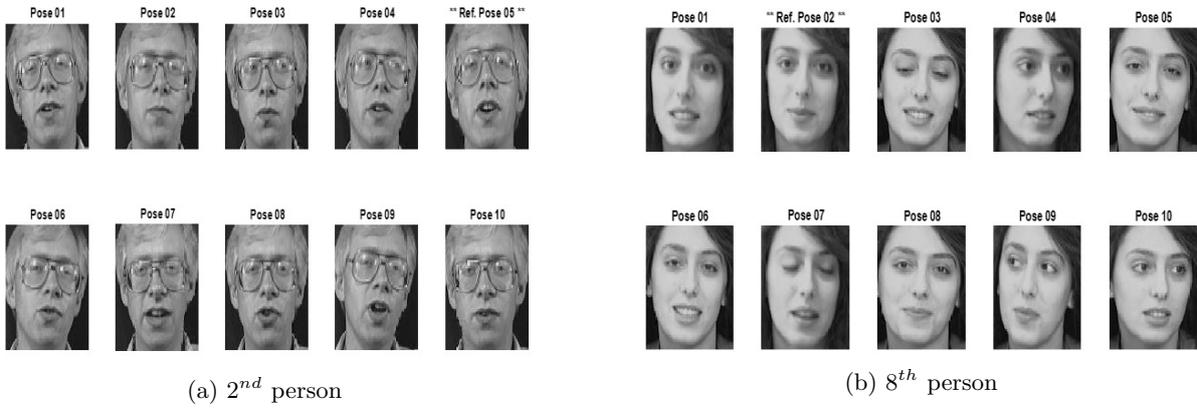


Figure 1: The ten poses in the AT&T database, the reference F_1 is indicated (the fifth pose).

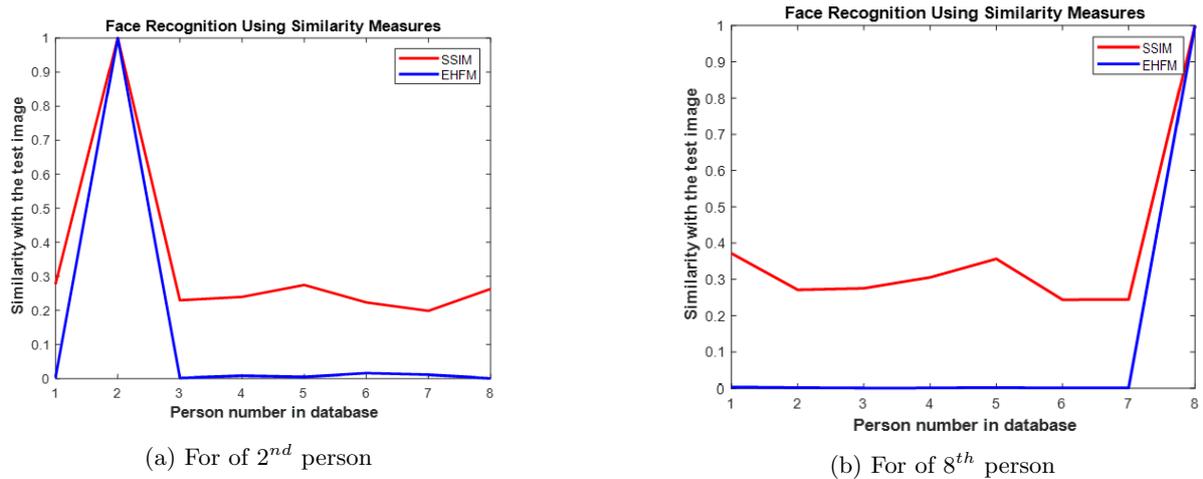
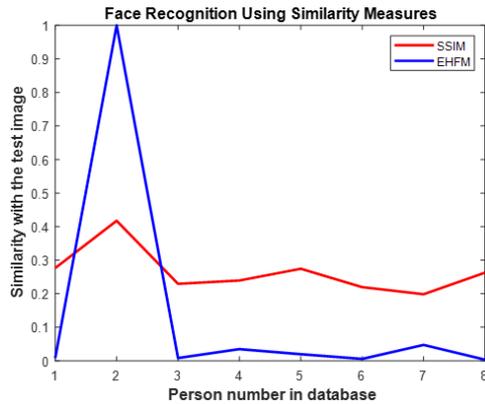


Figure 2: The comparison between (EHFM) our proposed equation, and the well-known (SSIM). Test image, where included the Reference F_1 (the fifth pose) in AT&T database.

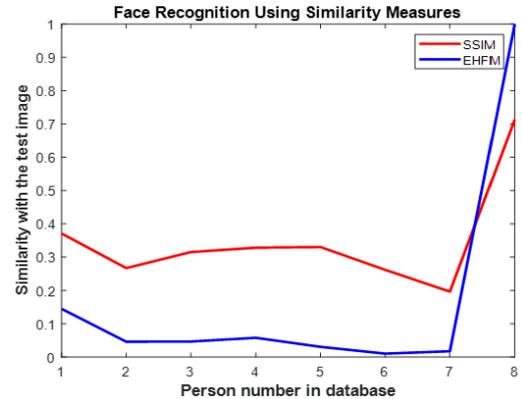
Table 1: Numerical results of the comparison between the (EHFM) our proposed measure, and the statistical (SSIM) of a test image which is included the Reference F_1 in the AT&T dataset.

| Ref. image | Measure | | |
|--------------------------------|------------|--------|--------|
| | Person No. | | |
| 2^{nd} person as in Figure 1 | 1 | 0.2764 | 0.0020 |
| | 2 | 1.0000 | 1.0000 |
| | 3 | 0.2295 | 0.0019 |
| | 4 | 0.2394 | 0.0081 |
| | 5 | 0.2745 | 0.0046 |
| | 6 | 0.2231 | 0.0155 |
| | 7 | 0.1985 | 0.0110 |
| | 8 | 0.2628 | 0.0008 |
| 8^{th} person as in Figure 1 | 1 | 0.3715 | 0.0040 |
| | 2 | 0.2711 | 0.0022 |
| | 3 | 0.2752 | 0.0011 |
| | 4 | 0.3055 | 0.0008 |
| | 5 | 0.3566 | 0.0016 |
| | 6 | 0.2439 | 0.0005 |
| | 7 | 0.2446 | 0.0007 |
| | 8 | 1.0000 | 1.0000 |

*Through the illustrations and table of the numerical results of the identification values, it becomes clear that the proposed equation (EHFM) results are superior to other technique (SSIM) in terms of accuracy in this case (the reference image within the database).



(a) For of 2^{nd} person



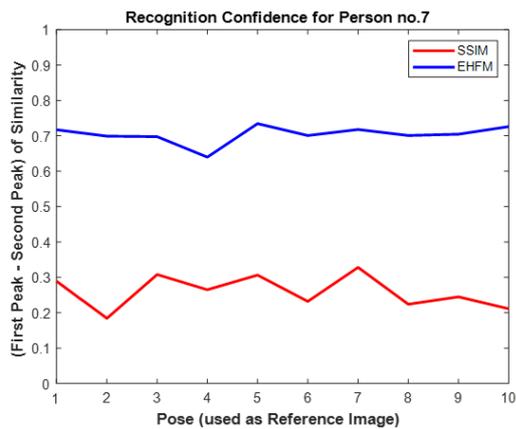
(b) For of 8^{th} person

Figure 3: The comparison between (EHFM) our proposed equation, and the well-known (SSIM). Test image of, where excluded F_1 the Reference (the fifth pose) in AT&T database.

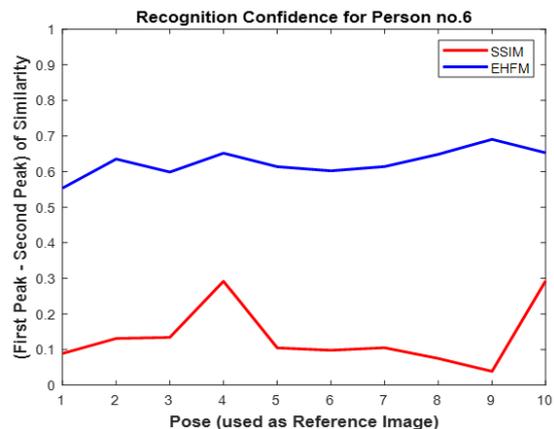
Table 2: Numerical results of the comparison between the proposed mathematical equation (EHFM), and the famous statistical (SSIM) of a test image which is excluded the Reference F_1 in the AT&T dataset.

| Ref. image | Measure | | |
|--------------------------------|------------|--------|--------|
| | Person No. | | |
| 2^{nd} person as in Figure 1 | 1 | 0.2764 | 0.0084 |
| | 2 | 0.4175 | 1.0000 |
| | 3 | 0.2295 | 0.0081 |
| | 4 | 0.2394 | 0.0348 |
| | 5 | 0.2745 | 0.0196 |
| | 6 | 0.2196 | 0.0056 |
| | 7 | 0.1985 | 0.0474 |
| | 8 | 0.2628 | 0.0033 |
| 8^{th} person as in Figure 1 | 1 | 0.3715 | 0.1449 |
| | 2 | 0.2711 | 0.0460 |
| | 3 | 0.2752 | 0.0466 |
| | 4 | 0.3055 | 0.0578 |
| | 5 | 0.3566 | 0.0307 |
| | 6 | 0.2439 | 0.0103 |
| | 7 | 0.2446 | 0.0174 |
| | 8 | 0.7131 | 1.0000 |

*Through the illustrations and table of the numerical results of the identification values, it becomes clear that the proposed equation (EHFM) results are superior to other technique (SSIM) in terms of accuracy in this case (the reference image within the database).



(a) For of 7^{th} person



(b) For of 6^{th} person

Figure 4: The comparison between (EHFM) our proposed equation, and the well-known (SSIM). Test image of is excluded the Reference (F_1) in AT&T database. It is clear that EHFM $[E(F_1, p)]$ achieves more confidence than statistical-SSIM $[I(F_1, p)]$ as $\partial(E, F_1) > \partial(I, F_1)$.

Table 3: Numerical results of the comparison between the proposed mathematical equation (EHFM), and the famous statistical- (SSIM), of a test image which is excluded the Reference F_1 in the AT&T dataset. It is clear that EHFM [$E(F_1, p)$] achieves more confidence than the statistical-SSIM [$I(F_1, p)$] as $\partial(E, F_1) > \partial(I, F_1)$.

| Ref. image | Measure | | SSIM | EHFM |
|------------------------|------------|--|--------|--------|
| | Person No. | | | |
| 6 th Person | 1 | | 1.0000 | 0.1036 |
| | 2 | | 0.9221 | 0.2022 |
| | 3 | | 0.1087 | 0.0390 |
| | 4 | | 0.1263 | 0.0841 |
| | 5 | | 0.1960 | 0.0843 |
| | 6 | | 0.1414 | 1.0000 |
| | 7 | | 0.0686 | 0.1811 |
| | 8 | | 0.1054 | 0.0390 |
| 7 th Person | 1 | | 0.0296 | 0.2250 |
| | 2 | | 1.0000 | 1.0000 |
| | 3 | | 0.0026 | 0.0182 |
| | 4 | | 0.0445 | 0.3021 |
| | 5 | | 0.0327 | 0.0719 |
| | 6 | | 0.2528 | 0.5683 |
| | 7 | | 0.0264 | 0.6791 |
| | 8 | | 0.0061 | 0.0415 |

*Through the illustrations and tables of the numerical results of the identification values, it is clear from the preference of the proposed technology results over other techniques in terms of accuracy by relying on the observation that indicates the greater the distance between the second and first maximum peaks, the greater the amount of confidence in the recognize.

4. Conclusion

This research presents a novel mathematical equation for facial recognition (named EHFM) that utilizes statistic and information – theoretic concepts in image similarity. The equation (EHFM) was simulated numerically via MATLAB R2020a utilizing 80 face images of AT&T database; eight subjects (persons) with each having ten different poses with many different facial expressions. Results viewed that the novel mathematical equation (EHFM) outperforms the well-known (standard measure) (SSIM) in face-recognition technique in two cases (included and excluded the test image). The basic performance of (EHFM) that was considered (and utilized for comparison) is the distance between the maximum amount of similarity (found by each certain method) and the second maximum amount of similarity. As no more complications are used in the proposed equation, it is evident that this equation is well-suited for real-time hardware, low-cost, or software implementation. As a Future Work we can; designing another similarity equation via the combination with different similarity measures: we can design and study a combination of two or more information theoretic similarity with statistical measures used in the different methods, also we can recognition under rotation: rotation can affect image recognition. An attempt to mix the above measure with another measures (which are resistive against rotation and Gaussian noise) would be worthwhile.

References

1. A.M.R. Abdulhassan and A.F. Hassan. Efficient information-theoretic-statistical (itsm) equation for face recognition technique: Comparison with statistical technique and information-theoretic technique. *IOP Conference Series: Materials Science and Engineering*, 928(4):042036, 2020.
2. A. Alhussain, Z. Fadhill, and A.F. Hassan. A binary relation fuzzy soft matrix-theoretic approach to image quality measurement: Comparison with statistical similarity metrics. *Mathematical Modelling of Engineering Problems*, 10(3):799–804, 2023.
3. K. Almazrouei and T. Rabie. Survey of face recognition using eigenfaces. In *2023 5th International Conference on Robotics and Computer Vision (ICRCV)*, pages 169–174, Nanjing, China, September 2023. IEEE.

4. R.C. Gonzalez, R.E. Woods, and S.L. Eddins. *Digital Image Processing Using MATLAB*. Pearson Education India, 2004.
5. A.F. Hassan, D. Cai-lin, and Z.M. Hussain. An information-theoretic image quality measure: Comparison with statistical similarity. *Journal of Computer Science*, 10:2269–2283, 2014.
6. A.K. Jain, A. Ross, and S. Prabhakar. An introduction to biometric recognition. *IEEE Transactions on Circuits and Systems for Video Technology*, 14(1):4–20, 2004.
7. T. Kanade. *Computer Recognition of Human Faces*, volume 47. Birkhäuser, Basel, Germany, 1977.
8. R. Kumar and V. Moyal. Visual image quality assessment technique using fsm. *International Journal of Computer Applications Technology and Research*, 2(3):250–254, 2013.
9. R.F. Paguirigan, M.B. Camero, M.A. Equias, M. Abisado, and G.A. Sampedro. Machine learning approaches to facial recognition: A survey. In *2023 International Conference on Electronics, Information, and Communication (ICEIC)*, pages 1–6, Singapore, 2023. IEEE.
10. Z. Wang, A.C. Bovik, H.R. Sheikh, and E.P. Simoncelli. Image quality assessment: From error visibility to structural similarity. *IEEE Transactions on Image Processing*, 13(4):600–612, 2004.
11. Z. Wang and E.P. Simoncelli. Translation insensitive image similarity in complex wavelet domain. In *Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, volume 2, pages ii–573, Philadelphia, PA, USA, 2005. IEEE.
12. W. Zhao, R. Chellappa, P.J. Phillips, and A. Rosenfeld. Face recognition: A literature survey. *ACM Computing Surveys (CSUR)*, 35(4):399–458, 2003.

Rusul Mohsen Washeel,
Department of Mathematics,
Faculty of Basic Education, University of Kufa,
Iraq.
E-mail address: rusulm.alziady@uokufa.edu.iq

and

Zahraa Fadhil Abd Alhussain,
Department of Mathematics,
Faculty of Education for Women, University of Kufa,
Iraq.
E-mail address: zahraaf.aldabbagh@uokufa.edu.iq

and

Asmhan Flieh Hassan,
Department of Mathematics,
Faculty of Education for Women, University of Kufa,
Iraq.
E-mail address: asmhanf.alzuhairy@uokufa.edu.iq