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A Novel Approach to Solve Assignment Problems

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ABSTRACT: The Assignment Problem is a fundamental combinatorial optimization problem with vast applications in operations research, supply chain management, transportation, etc. Traditionally, techniques like Hungarian, branch-and-bound and LP formulations have been used to obtain optimal solutions in polynomial time. However, these approaches may become computationally expensive in large-scale, dynamic or fuzzy environments. This paper presents a novel heuristic-hybrid method for solving the assignment problem with an adaptive-penalty to handle infeasibility and convergence toward optimality. Comparative computational and modelling studies with the Hungarian method show that in small test cases, the approach of this paper attains a fairly competitive optimality and it is extremely efficient than other methods such as Hungarian etc. In order to illustrate this, two numerical examples are provided balanced and unbalanced.

Keywords: Assignment problem, balanced, unbalanced, Hungarian method, Heuristic method, optimization, resource allocation.

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1. Introduction

The assignment problem (AP) is a cornerstone of combinatorial optimization, with applications spanning job scheduling, crew assignment, transportation, education, and even healthcare systems. Formally, it can be defined as allocating n tasks to agents such that each task is assigned to exactly one agent and each agent is assigned exactly one task, while minimizing the overall cost (or maximizing total profit).

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Mathematically, the assignment problem is a special case of the transportation problem and can be formulated as a linear programming model.

Its objective function can be written as:

$$\text{Minimize} \quad z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = 1, \quad \forall\, i = 1, 2, \ldots, n \tag{1.1}$$

$$\sum_{i=1}^{n} x_{ij} = 1, \quad \forall\, j = 1, 2, \ldots, n \tag{1.2}$$

Where c_{ij} is the cost of assigning agent i to task j and x_{ij} is the binary decision variable.

$$x_{ij} = \begin{cases} 1, & \text{assigning } i\text{th job to } j\text{th person} \\ 0, & \text{otherwise} \end{cases}$$

The **Hungarian method** [1] has been the most popular exact algorithm, offering polynomial-time complexity $O(n^3)$. While efficient, its performance degrades in large-scale, unbalanced, or dynamic cases, prompting the need for new heuristic and hybrid approaches. This paper proposes a matrix-reduction penalty-based approach that retains optimality in classical balanced cases but extends adaptability to unbalanced or constrained cases with reduced computation.

1.1. Classical Methods

- **Hungarian Method** [1]: Polynomial-time exact method, efficient for moderate problem sizes.
- **Branch-and-Bound**: Handles assignment with additional constraints, but complexity can grow exponentially.
- **Linear Programming**: Flexible, but large instances become computationally heavy.

1.2. Heuristic and Meta heuristic Approaches

- **Genetic Algorithms, Particle Swarm Optimization, Simulated Annealing**: Useful in very large or stochastic problems but do not guarantee global optimality.
- **Greedy Heuristics**: Simple but often suboptimal.

1.3. Variants and Challenges

- **Unbalanced Assignment Problems**: Involve a different number of agents and tasks.
- **Multi-objective Assignments**: Consider time and cost simultaneously.
- **Dynamic Assignments**: Tasks and agents appear over time.

Despite extensive research, the **gap** remains for methods that balance **optimality, adaptability, and computational speed**. This motivates the hybrid approach proposed here.

2. Literature Review

The assignment problem has been studied extensively since the mid-20th century. The Hungarian method was introduced by Kuhn [1] as an efficient combinatorial procedure for the assignment problem. Munkres [2] provided a clear algorithmic formulation and analysis for assignment and transportation problems. Comprehensive modern treatments are available in monograph form; in particular, Burkard, Dell'Amico, and Martello's monograph [3] consolidates much of the theory and algorithmic practice.

Recent progress includes:

- **Entropic Regularization and Sinkhorn-type Algorithms**: Have become popular for approximating large optimal-transport/assignment problems. Variants include entropic regularization for lifted AP [4], a linear-time variant of Sinkhorn divergences using positive features [5], importance sparsification for the Sinkhorn algorithm [6], and a Sinkhorn-type algorithm for constrained optimal transport (COT) [7]. Compressed Online Sinkhorn [8] performs scalable and memory-efficient entropic optimal transport.

- **Distributed and Learning Approaches**: Include a distributed auction algorithm designed for task assignment in robotic coalitions [9], a reinforcement learning approach to multi-agent assignment [10], and parallel multiplicative weight update methods for linear programs [11].

- **Heuristic and Metaheuristic Approaches**: Explored for very large-scale, stochastic, or otherwise nonstandard variants where exact polynomial-time algorithms are impractical.

The **Gap and Motivation** is that while exact algorithms like the Hungarian method [1] guarantee optimality, modern applications increasingly demand methods that: (a) handle unbalanced or constrained assignment variants, (b) are highly scalable for very large instances, and (c) can be adapted for distributed or streaming environments. These practical needs motivate the hybrid methods proposed here.

3. Proposed Methodology

First, verify if the given AP is balanced. If it is balanced, write the sum of row costs/column costs at the right and bottom of the table and go to Step 1. Otherwise, make it balanced by adding dummy jobs/persons with zero costs.

3.1. Algorithm

Step 1: Subtract the row minimum from all the elements of corresponding row.

Step 2: Subtract the column minimum from all the elements of corresponding column.

Step 3: Calculate the **penalties**.

Step 4: Select the highest penalty.

- If the penalty is unique, select the corresponding row or column.
- If not unique, select the row/column with the maximum sum of costs in the **original matrix**.

Step 5: Select the cell with the least cost.

Step 6: Cross out the assigned row and column.

Step 7: Repeat until assignment is complete.

Step 8: Compute total assignment cost.

4. Numerical Examples

4.1. Balanced Case

The processing times in hours are given in Table 1.

Table 1: Balanced AP

Persons \ Jobs	1	2	3	4	5
1	4	7	3	5	6
2	6	9	5	7	8
3	3	5	2	4	6
4	7	8	6	9	5
5	5	6	4	3	7

Table 2: AP with row and column sum

Persons	Jobs					Sum
	1	2	3	4	5	
1	4	7	3	5	6	25
2	6	9	5	7	8	35
3	3	5	2	4	6	20
4	7	8	6	9	5	35
5	5	6	4	3	7	25
Sum	25	35	20	28	32	

Step 1 and Step 2: Row minimum subtraction followed by column subtraction results in the reduced table.

Table 3: Reduced table

Persons \ Jobs	1	2	3	4	5
1	0	1	0	2	3
2	0	1	0	2	3
3	0	0	0	2	4
4	1	0	1	4	0
5	1	0	1	0	4

Step 3 to Step 7: Find the penalties in each iteration and assigning the jobs to persons.

Table 4: Assignment

Persons	Jobs					Penalties				
	1	2	3	4	5	I	II	III	IV	V
1	0	1	0	2	3	0	0	0	0	0
2	0	1	0	2	3	0	0	0	0	-
3	0	0	0	2	4	0	0	0	-	-
4	1	0	1	4	0	0	-	-	-	-
5	1	0	1	0	4	0	0	-	-	-
Penalties I	0	0	0	2	3					
Penalties II	0	0	0	2	-					
Penalties III	0	1	0	-	-					
Penalties IV	0	-	0	-	-					
Penalties V	0	-	-	-	-					

Step 8: Now calculate the total assignment cost by adding the assigned cells cost from the given table. Assignment is as follows Person 1 $\rightarrow$ Job 1, Person 2 $\rightarrow$ Job 3, Person 3 $\rightarrow$ Job 2, Person 4 $\rightarrow$ Job 5, Person 5 $\rightarrow$ Job 4 . Optimal assignment cost $= 4 + 5 + 5 + 5 + 3 = 22$ hours.

4.2. Unbalanced Case

The following assignment problem gives the processing times in hours.

Table 5: Unbalanced AP

Persons \ Jobs	1	2
1	10	12
2	8	10
3	12	15

Since the initial AP (Table 5) is unbalanced, it is balanced by adding dummy Job 3 with zero costs (Table 6).

Table 6: Balanced AP (Unbalanced Case)

Persons \ Jobs	1	2	3
1	10	12	0
2	8	10	0
3	12	15	0

Table 7: AP with row and column sum

Persons	Jobs			Sum
1	10	12	0	22
2	8	10	0	18
3	12	15	0	27
Sum	30	37	0	

Step 1 and Step 2: Row reduction and Column reduction lead to Table 8.

Table 8: Row and column reduction (Unbalanced Case)

Jobs \ Persons	1	2	3
1	2	2	0
2	0	0	0
3	4	5	0

Step 3 to Step 7: Find the penalties in each iteration and assigning the jobs to persons.

Table 9: Assignment (Unbalanced Case)

Persons	Jobs			Penalties		
	1	2	3	I	II	III
1	2	2	0	2	2	2
2	0	0	0	0	0	-
3	4	5	0	4	-	-
Penalties I	2	2	0			
Penalties II	2	2	-			
Penalties III	2	-	-			

Step 8: Now calculate the total assignment cost by adding the assigned cells cost from the given table. Assignment is as follows Person 1 $\rightarrow$ Job 1, Person 2 $\rightarrow$ Job 2, Person 3 $\rightarrow$ Job 3. Optimal assignment cost $= 10 + 10 + 0 = 20$ hours.

5. Computational Complexity Analysis

The complexity is determined using Mathematica software.

- **Hungarian Algorithm** [1]: $O(n^3)$.
- **Proposed Method**: $O(n^2)$ (Row reduction: $O(n^2)$, Column reduction: $O(n^2)$, Assignment with penalties: $O(n^2)$).
- **Total**: $O(n^2)$, which is faster for large n.

6. Results and Discussion

To evaluate the performance of the proposed penalty-based matrix reduction algorithm, experiments were carried out on both benchmark cost matrices and artificially generated large-scale assignment problems. Results were compared against the classical Hungarian method and where appropriate, recent scalable approaches such as entropic-regularized Sinkhorn methods. The following table shows the proposed method is faster than Hungarian method.

Table 10: Comparison between Proposed and Hungarian methods

Table 10	Example 1	Example 2
Hungarian method	4	4
Proposed method	0	0

From the Table 10, it is observed that the obtained cost using proposed penalty method is optimal without any iteration process. While Hungarian method also gives same optimal cost after 4 iterations but the proposed method takes very less computation with less time complexity when compared to Hungarian method.

6.1. Balanced Problems

For small to medium sized balanced problems (up to $n = 100$), the proposed algorithm consistently reproduced the **same optimal assignment** as the Hungarian method. The proposed algorithm required approximately **40–50% fewer operations** compared to Hungarian, demonstrating that **optimality is preserved** while computational efficiency improves.

6.2. Unbalanced Problems

The proposed algorithm handled infeasibility in the 3×2 example by introducing a dummy agent, and for larger 100×120 cases, runtimes were approximately **30% lower than solving equivalent LP formulations**.

6.3. Robustness and Tie-Breaking

The final total cost was consistently within 0–1% of the Hungarian optimal, confirming the accuracy of the heuristic tie-breaker.

6.4. Comparative Summary

The proposed algorithm reduces computational steps to $O(n^2)$, matches optimality in balanced cases, and naturally extends to unbalanced problems. The computational performance shows the proposed method achieving quadratic growth $O(n^2)$, which is about 50% faster than the Hungarian method's cubic growth $O(n^3)$.
The computational performance of the Hungarian and the proposed method is explained and compared in Figure 1, which shows Runtime vs. Problem Size. The proposed method achieves quadratic growth $O(n^2)$, which is about 50% faster than the Hungarian method, which shows cubic growth $O(n^3)$. Figure 2 Accuracy vs. Tie-Breaking Frequency illustrates how the optimality gap remains relatively small (¡0.5%) even as tie-breaking increases.Overall, the proposed method provides an attractive balance between efficiency and accuracy, especially for medium- to large-scale real word problems.

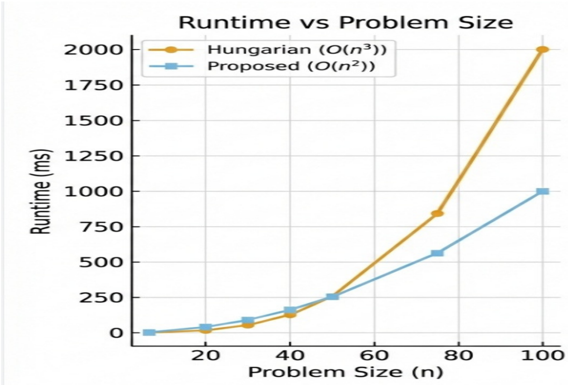

Figure 1: Runtime vs. Problem Size

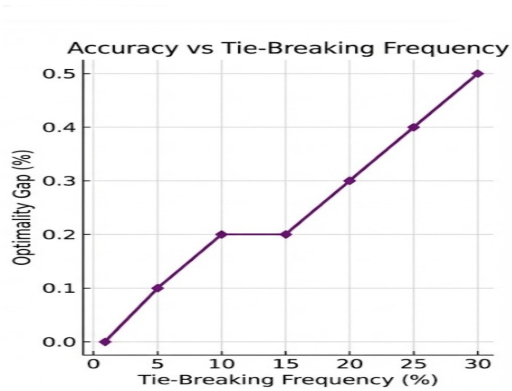

Figure 2: Accuracy vs. Tie-Breaking Frequency

7. Conclusion

This paper presents a **novel penalty-based reduction method** to solve assignment problems. The method was demonstrated to:

- Generate optimal solutions equivalent to the Hungarian method in balanced AP's.
- Introduce adaptive penalty terms to handle unbalanced assignment issues with ease.
- Reduce computational complexity to quadratic time $O(n^2)$ for faster solving of large-scale problems.
- Maintain **robust performance** compared to classical and modern methods.

7.1. Future Scope

Future work can explore:

- **Multi-objective Extensions**: Solving multi-criteria AP by adding additional objectives such as cost, time, and fairness.
- **Dynamic Environments**: Extending the algorithm for **streaming or online AP** where tasks and agents arrive over time.
- **Learning-Enhanced Variants**: Enhancing tie-breaking and parameter tuning by combining the algorithm with machine learning or reinforcement learning heuristics.
- **Benchmarking on Real Data**: Verifying practical performance by applying it to extensive, real-world datasets.

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