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πH^*-Closed Sets in Topological Spaces

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ABSTRACT: The study of generalized closed sets plays a central role in modern topology, particularly in understanding finer variations of separation axioms and closure operators. In this paper, we introduce and investigate the concept of πH^*-closed sets, as a natural extension of H^*-closed, δg-closed, πg-closed, and related structures. Several illustrative examples are presented using discrete, indiscrete, and cofinite topologies to clarify the distinctions and interrelations among these classes of sets. Fundamental properties of πH^*-closed sets are established, showing that while finite unions of πH^*-closed sets preserve the property, their finite intersections may fail to do so. We further define the dual notion of πH^*-open sets and derive equivalent characterizations in terms of h-closure and h-interior operators. The study also introduces the class of πH^*-$T_{1/2}$ spaces, where πH^*-closed sets coincide with h-closed sets, and provides several equivalent formulations. Our results unify and extend earlier investigations on generalized closed sets, offering a comprehensive framework for analyzing closure-based generalizations and separation axioms. This contributes to a deeper understanding of the structural richness of topological spaces and opens new directions for further research in generalized topology.

Keywords: H^*-closed, r^*-closed, h-closed, H^*g-closed, gH^*-closed.

Contents

1. Introduction

The study of generalized open and closed sets in topology has evolved significantly over the decades. In 1937, Stone [11] introduced the concept of regular open sets, marking the beginning of this progression. Levine [14] introduced s-open sets in 1963, followed by Njåstad [18], who defined α-open sets in 1965. In 1968, Zaitsev [25] introduced π-open sets, which are a weaker form of regular open sets, while Velicko [15] proposed δ-open sets, which are stronger than open sets.

In 1970, Levine [13] initiated the study of generalized closed sets (abbreviated as g-closed sets), laying the groundwork for many subsequent generalizations. In 1982, Mashour [2] introduced p-open sets, and in 1983, Abd El-Monsef et al. [10] presented the concept of β-open sets. Further developments include the introduction of δg-closed sets by Dontchev and Ganster in 1996 [7], and πg-closed sets by Dontchev and Noiri in 2000 [8].

In 2006, Park [9] introduced πgp-closed sets, while Aslim et al. [1] defined πgs-closed sets. In 2009, Arockiarani and Janaki [4] proposed $\pi g\alpha$-closed sets, followed by πgsp-closed sets introduced by Sarsak and Rajesh in 2010 [12]. In 2012, Sudha and Sivakamasundari [22] introduced the concept of δg^*-closed sets, and in 2016, Pious and Annalakshmi [23] proposed regular*-open sets.

Recent advancements include the work of Meenakshi et al. [20,21], who introduced η^*-open and J-closed sets in 2019, followed by J^*-closed and J^{**}-closed sets in 2020 [19]. Continuing this line of research,

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in 2024, Neeraj Tomar and M. C. Sharma [17] introduced a new class called πSC^*-closed sets, examining their properties and interrelationships. In the same year, Neeraj Kumar Tomar et al. [16] also defined new separation axioms in topological spaces using H^*-closed and SC^*-closed sets, contributing further to the development of generalized topology.

The paper is structured as follows:

- **Section 2:** Preliminaries and definitions of generalized closed sets, h-closure, H^*-closure, and related operators.
- **Section 3:** Introduction of πH^*-closed sets with illustrative examples and basic properties.
- **Section 4:** Further properties, including unions, intersections, and relationships with other classes of sets.
- **Section 5:** Conclusion summarizing the findings and directions for future research.

2. Preliminaries

Unless mentioned otherwise, the term *space* will always refer to a topological space, and no form of separation axiom is presumed throughout the discussion. Let

$$f : (X, \tau) \to (Y, \sigma) \quad \text{and} \quad g : (Y, \sigma) \to (Z, \rho)$$

be mappings between topological spaces, often written in the simpler form

$$f : X \to Y \quad \text{and} \quad g : Y \to Z.$$

If $A \subseteq X$, the closure and interior of A are represented by $\mathrm{cl}(A)$ and $\mathrm{int}(A)$, respectively. A subset A of a topological space X is said to be δ-open [15] if it can be expressed as a union of regular open [11] subsets of X. The complement of a δ-open set is known as a δ-closed [15] set.

We now define some basic notions that will be used throughout. For a detailed understanding of these concepts, readers are referred to [2,3,5,10,11,14,18,24].

Definition 2.1 *A subset A of a topological space X is said to be*

1. ***regular closed*** *[11] if $A = \mathrm{cl}(\mathrm{int}(A))$.*
2. ***semi closed*** *[14] if $\mathrm{int}(\mathrm{cl}(A)) \subseteq A$.*
3. ***pre-closed*** *[2] if $\mathrm{cl}(\mathrm{int}(A)) \subseteq A$.*
4. *w**-closed** [24] if $\mathrm{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is semi-open in X.*
5. *α**-closed** [18] if $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(A))) \subseteq A$.*
6. *β**-closed** [10] if $\mathrm{int}(\mathrm{cl}(\mathrm{int}(A))) \subseteq A$.*
7. *α^***-set** [5] if $\mathrm{int}(\mathrm{cl}(\mathrm{int}(A))) = \mathrm{int}(A)$.*
8. ***C-set*** *[5] if $A = U \cap V$, where U is open and V is an α^*-set in X.*
9. ***h-closed*** *[3] if s-$\mathrm{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is w-open in X.*
10. ***gh-closed*** *[3] if h-$\mathrm{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is h-open in X.*
11. ***regular-h-open*** *[3] if there exists a regular open set U such that $U \subseteq A \subseteq h$-$\mathrm{cl}(U)$.*
12. ***rgh-closed*** *[3] if h-$\mathrm{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is regularly h-open in X.*
13. ***hCg-closed*** *[3] if h-$\mathrm{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is a C-set in X.*

In a topological space X, the complement of a regular-closed set [11] is known as a regular-open set [11]. Likewise, the complements of semi-closed [14], pre-closed [2], w-closed [24], α-closed [18], β-closed [10], h-closed [3], gh-closed [3], rgh-closed [3], and hCg-closed [3] sets are termed as semi-open [14], pre-open [2], w-open [24], α-open [18], β-open [10], h-open [3], gh-open [3], rgh-open [3], and hCg-open [3] sets, respectively.

The complement of a regular-h-open set is called a regular-h-closed set [3].

For any subset $A \subseteq X$, the *h-closure* of A, denoted by $h\text{-}\operatorname{cl}(A)$, is the intersection of all h-closed sets that contain A. In the same manner, the *semi-closure* of A, written as $s\text{-}\operatorname{cl}(A)$, is the intersection of all semi-closed sets that include A.

Similarly, the *h-interior* of A, expressed as $h\text{-}\operatorname{int}(A)$, refers to the union of all h-open subsets contained in A. Correspondingly, the *semi-interior* of A, represented by $s\text{-}\operatorname{int}(A)$, is the union of all semi-open [14] subsets that lie inside A.

Definition 2.2 *Let X be a topological space and let $A \subseteq X$. The set A is said to be* generalized closed *[13] (abbreviated as g-closed [13]) if for every open set $U \in \tau$ satisfying $A \subseteq U$, it follows that*

$$\operatorname{cl}(A) \subseteq U.$$

The complement of a g-closed [13] set is referred to as a *g-open* [13] set.

The *generalized closure* of a subset A, denoted by $\operatorname{cl}^*(A)$, is defined as the intersection of all g-closed [13] sets of X that contain A.

Similarly, the *generalized interior* of A, denoted by $\operatorname{int}^*(A)$, is the union of all g-open [13] subsets of X that are contained in A.

Definition 2.3 *A subset A of X is said to be* δ-open *[15] if*

$$\delta\text{-}\operatorname{int}(A) = A;$$

equivalently, a δ-open set can be expressed as the union of regular open [11] sets.

Let A be a subset of a topological space X. The *δ-interior* of A, denoted by $\delta\text{-}\operatorname{int}(A)$, is defined as the union of all regular open [11] subsets of X that are contained within A.

The complement of a δ-open set is referred to as a *δ-closed* set [15]. Conversely, a subset $A \subseteq X$ is said to be *δ-closed* [15] if

$$A = \delta\text{-}\operatorname{cl}(A),$$

where $\delta\text{-}\operatorname{cl}(A)$ represents the intersection of all regular closed sets [11] of X that contain A.

Definition 2.4 *Let X be a topological space. A subset $A \subseteq X$ is said to be* regular*-open *[23] (or* r*-open *[23]) if*

$$A = \operatorname{int}(\operatorname{cl}^*(A)).$$

The complement of an r*-open set is called an *r*-closed* set [23].

The *r*-interior* of a subset A of X, denoted by $\text{r}^*\text{-}\operatorname{int}(A)$, is the union of all r*-open [23] sets contained in A.

Similarly, the *r*-closure* of A, denoted by $\text{r}^*\text{-}\operatorname{cl}(A)$, is defined as the intersection of all r*-closed [23] sets of X that contain A.

Definition 2.5 *Let X be a topological space. A subset $A \subseteq X$ is said to be η^*-open [20,21] if it can be expressed as a union of r^*-open [23] sets of X.*

The complement of an η^*-open set is referred to as an η^*-closed [20,21] set. For any subset A of X, the η^*-interior, denoted by $\eta^*\text{-}\operatorname{int}(A)$, is defined as the union of all η^*-open [20,21] sets contained within A.

Similarly, the η^*-closure of A, denoted by $\eta^*\text{-}\operatorname{cl}(A)$, is the intersection of all η^*-closed [20,21] sets of X that include A.

Remark 2.1 *[7] The hierarchy among various generalized open and closed sets in a topological space can be summarized as follows:*

- Every regular open [11] set is necessarily π-open, which in turn implies δ-open [15], η^*-open [20,21], and finally open, α-open [18], s-open [14], and β-open [10] sets.
- Dually, every regular closed [11] set is π-closed, hence δ-closed [15], η^*-closed [20,21], closed, α-closed, s-closed, and β-closed [10].
- Similarly, the chain of inclusions for other open and closed variants can be expressed as:

$$\text{r-open} \Rightarrow \pi\text{-open} \Rightarrow \delta\text{-open} \Rightarrow \eta^*\text{-open} \Rightarrow \text{open} \Rightarrow \alpha\text{-open} \Rightarrow p\text{-open} \Rightarrow \beta\text{-open},$$

$$\text{r-closed} \Rightarrow \pi\text{-closed} \Rightarrow \delta\text{-closed} \Rightarrow \eta^*\text{-closed} \Rightarrow \text{closed} \Rightarrow \alpha\text{-closed} \Rightarrow p\text{-closed} \Rightarrow \beta\text{-closed}.$$

This demonstrates that each successive class of sets generalizes the preceding one, forming a hierarchy of openness and closedness properties.

Remark 2.2 *[7] For any subset $U \subseteq X$ in a topological space, the following inclusion relations among various generalized closures hold:*

1. $$\beta\text{-}\operatorname{cl}(U) \subseteq s\text{-}\operatorname{cl}(U) \subseteq \alpha\text{-}\operatorname{cl}(U) \subseteq \operatorname{cl}(U) \subseteq \eta^*\text{-}\operatorname{cl}(U) \subseteq \delta\text{-}\operatorname{cl}(U) \subseteq \pi\text{-}\operatorname{cl}(U) \subseteq r\text{-}\operatorname{cl}(U).$$
2. $$g\text{-}\operatorname{cl}(U) \subseteq \operatorname{cl}(U) \subseteq \eta^*\text{-}\operatorname{cl}(U) \subseteq \delta\text{-}\operatorname{cl}(U) \subseteq \pi\text{-}\operatorname{cl}(U) \subseteq r\text{-}\operatorname{cl}(U).$$
3. $$\beta\text{-}\operatorname{cl}(U) \subseteq p\text{-}\operatorname{cl}(U) \subseteq \alpha\text{-}\operatorname{cl}(U) \subseteq \operatorname{cl}(U) \subseteq \eta^*\text{-}\operatorname{cl}(U) \subseteq \delta\text{-}\operatorname{cl}(U) \subseteq \pi\text{-}\operatorname{cl}(U) \subseteq r\text{-}\operatorname{cl}(U).$$
4. $$\eta\text{-}\operatorname{cl}(U) \subseteq \alpha\text{-}\operatorname{cl}(U) \subseteq \operatorname{cl}(U) \subseteq \eta^*\text{-}\operatorname{cl}(U) \subseteq \delta\text{-}\operatorname{cl}(U) \subseteq \pi\text{-}\operatorname{cl}(U) \subseteq r\text{-}\operatorname{cl}(U).$$

Remark 2.3 *[7] The following chain of implications holds among various types of closed sets in a topological space:*

$$\pi\text{-closed} \;\Rightarrow\; \text{closed} \;\Rightarrow\; \alpha\text{-closed} \;\Rightarrow\; h\text{-closed} \;\Rightarrow\; H^*\text{-closed} \;\Rightarrow\; gh\text{-closed} \;\Rightarrow\; rgh\text{-closed}.$$

Furthermore, the following relations also hold:

$$H^*\text{-closed} \;\Rightarrow\; gH^*\text{-closed} \;\Rightarrow\; rgH^*\text{-closed}.$$

However, the converses of these implications do not necessarily hold, as can be verified through suitable counterexamples.

Example 2.1 (Counterexample to Remark 2.3)
Consider the topological space $X = \{k, l, m\}$ with topology

$$\tau = \{\emptyset, \{k\}, \{k, l\}, X\}.$$

In this space, the converse implications in Remark 2.3 fail for several classes of closed sets, demonstrating that the chains of implications do not hold in the reverse direction.

Example 2.2 Let $X = \{k, l, m\}$ and define a topology

$$\tau = \{\emptyset, X, \{k\}, \{k, l\}\}.$$

Consider the subset $A = \{l\} \subseteq X$. Then:

- A is H^*-closed, because its closure under H^* coincides with A itself.
- However, A is not closed, since its complement $X \setminus A = \{k, m\} \notin \tau$.
- Thus, the converse "H^*-closed $\Rightarrow$ closed" fails in this topology.

This demonstrates that although the above implications hold in one direction, their converses are not generally true.

Definition 2.6 *Let X be a topological space and let $A \subseteq X$. Then A is said to possess the following properties:*

1. ***J-closed*** *[20,21] if for every η^*-open set $U \subseteq X$ containing A,*
$$\mathrm{cl}(A) \subseteq U.$$
2. ***J^*-closed*** *[19] if for every open set $U \in \tau$ such that $A \subseteq U$,*
$$\eta^*\text{-}\,\mathrm{cl}(A) \subseteq U.$$
3. ***δ-generalized closed*** *(briefly, δg-closed) [7] if*
$$\delta\text{-}\,\mathrm{cl}(A) \subseteq U$$
whenever $A \subseteq U \subseteq X$.
4. ***δg^*-closed*** *[22] if for each g-open set $U \subseteq X$ containing A,*
$$\delta\text{-}\,\mathrm{cl}(A) \subseteq U.$$
5. ***J^{**}-closed*** *[19] if for every η^*-open set $U \subseteq X$ with $A \subseteq U$,*
$$\eta^*\text{-}\,\mathrm{cl}(A) \subseteq U.$$
6. ***πg-closed*** *[8] if for all π-open sets $U \subseteq X$ satisfying $A \subseteq U$,*
$$\mathrm{cl}(A) \subseteq U.$$
7. ***πgp-closed*** *[9] if for every π-open set U containing A,*
$$p\text{-}\,\mathrm{cl}(A) \subseteq U.$$
8. ***πgs-closed*** *[1] if for all π-open sets U with $A \subseteq U$,*
$$s\text{-}\,\mathrm{cl}(A) \subseteq U.$$
9. ***$\pi g\alpha$-closed*** *[4] if for each π-open set $U \subseteq X$,*
$$\alpha\text{-}\,\mathrm{cl}(A) \subseteq U.$$
10. ***πgsp-closed*** *[12] if for every π-open set U with $A \subseteq U$,*
$$sp\text{-}\,\mathrm{cl}(A) \subseteq U.$$
11. ***$\pi g\eta$-closed*** *[6] if*
$$\eta\text{-}\,\mathrm{cl}(A) \subseteq U$$
whenever $A \subseteq U$ and U is π-open in X.

12. ***generalized*** H^****-closed*** *(briefly, gH^*-closed) [16] if*

$$H^*\text{-}\mathrm{cl}(A) \subseteq U$$

whenever $A \subseteq U$ and U is H^*-open in X.

13. H^*g***-closed*** *[16] if*

$$H^*\text{-}\mathrm{cl}(A) \subseteq U$$

whenever $A \subseteq U$ and U is open in X.

14. ***regular-***H^****-open*** *[16] if there exists a regular open set $U \subseteq X$ such that*

$$U \subseteq A \subseteq H^*\text{-}\mathrm{cl}(U).$$

The complement of a J-closed subset (respectively J^*-closed, δg-closed, δg^*-closed, J^{**}-closed, πg-closed, πgp-closed, πgs-closed, $\pi g\alpha$-closed, πgsp-closed, $\pi g\eta$-closed, gH^*-closed, or H^*g-closed) subset of a topological space X is defined as a J-open (respectively J^*-open, δg-open, δg^*-open, J^{**}-open, πg-open, πgp-open, πgs-open, $\pi g\alpha$-open, πgsp-open, $\pi g\eta$-open, gH^*-open, or H^*g-open) set.

In other words, each of these open sets is characterized as the complement of its corresponding closed set within the same topological structure.

Moreover, the complement of a regular-H^*-open set is termed an *r-H^*-closed* set, preserving the duality between openness and closedness under the H^*-operator.

Definition 2.7 *A subset A of a topological space X is said to be* H^*-closed *[16] if*

$$h\text{-}\mathrm{cl}(A) \subseteq U$$

whenever $A \subseteq U$ and U is hCg-open [3] in X.

The complement of an H^*-closed set is called an *H^*-open* set [16].

The *H^*-closure* of a subset A, denoted by

$$H^*\text{-}\mathrm{cl}(A),$$

is defined as the intersection of all H^*-closed sets of X that contain A.

Similarly, the *H^*-interior* of A, written as

$$H^*\text{-}\mathrm{int}(A),$$

is the union of all H^*-open sets contained within A.

The collections of various types of sets in a space X are denoted as follows:

$H^*O(X)$	: the class of all H^*-open sets, [16]
$H^*C(X)$	: the class of all H^*-closed sets, [16]
$RO(X)$	: the class of all regular open sets [11],
$RC(X)$	: the class of all regular closed sets [11],
$SO(X)$	: the class of all semi-open sets [14],
$SC(X)$	: the class of all semi-closed sets [14].

3. πH^*-Closed Sets

In this section, we introduce a new class of generalized closed sets, referred to as πH^*-closed sets. This concept serves as an extension of several existing generalized closed set structures in topology. Furthermore, the relationships and distinctions between πH^*-closed sets and other well-established types of generalized closed sets are carefully examined and discussed.

Definition 3.1 *A subset U of a topological space X is said to be πH^*-closed if*

$$h\text{-}\operatorname{cl}(U) \subseteq M \quad \textit{whenever } U \subseteq M,$$

and M is a π-open subset of X.

In other words, a set is πH^*-closed when its h-closure remains contained within every π-open set that contains it.

Theorem 3.1 *Every H^*-closed set is πH^*-closed, but the converse does not necessarily hold.*

Proof: Let U be an H^*-closed subset of a topological space X, and let M be any π-open set in X such that $U \subseteq M$. Since every π-open set is an open set [26], it follows that M is open in X.

Because U is H^*-closed, we have

$$h\text{-}\operatorname{cl}(U) \subseteq M.$$

Hence, by definition, U is πH^*-closed.

However, the converse need not be true in general, since there exist πH^*-closed sets that are not H^*-closed. □

Theorem 3.2 *Every gH^*-closed set is πH^*-closed, but the converse does not necessarily hold.*

Proof: Let U be a gH^*-closed subset of a topological space X, and let M be any π-open set in X such that $U \subseteq M$. By definition [26], every π-open set is also h-open.

Since U is gH^*-closed, we have

$$h\text{-}\operatorname{cl}(U) \subseteq M.$$

Thus, U satisfies the defining condition of a πH^*-closed set. Hence, every gH^*-closed set is πH^*-closed.

However, the converse need not be true in general, since a πH^*-closed set may fail to be gH^*-closed. □

Theorem 3.3 *Every h-closed set is πH^*-closed, but the converse need not be true.*

Proof: Let U be an h-closed subset of a topological space X, and let M be any π-open set such that $U \subseteq M$. Since U is h-closed, we have

$$h\text{-}\operatorname{cl}(U) = U.$$

Clearly,

$$U = h\text{-}\operatorname{cl}(U) \subseteq M.$$

Thus, U satisfies the defining condition of a πH^*-closed set [26]. Therefore, every h-closed set is πH^*-closed.

However, the converse does not always hold, as there exist πH^*-closed sets that are not h-closed. □

Corollary 3.1 *Every r^*-closed set is πH^*-closed, but the converse does not necessarily hold.*

Proof: It is known that every r^*-closed set is h-closed. Furthermore, we have already established that every h-closed set is πH^*-closed. Hence, every r^*-closed set must be πH^*-closed. However, the converse is not always true. □

Corollary 3.2 *Every regular closed set is πH^*-closed, but the converse does not necessarily hold.*

Proof: Since each regular closed set is h-closed, and every h-closed set is πH^*-closed, it follows directly that every regular closed set is πH^*-closed [26]. Conversely, a πH^*-closed set need not be regular closed. □

Corollary 3.3 *Every π-closed set is πH^*-closed, but the converse does not necessarily hold.*

Proof: Every π-closed set is also h-closed, and since every h-closed set has been shown to be πH^*-closed, the result follows immediately [26]. Nevertheless, the reverse implication does not hold in general. □

Remark 3.1 The converses of Corollaries 3.1,3.2 and 3.3 are not generally valid. Indeed, a set may be πH^*-closed without being r^*-closed, regular closed, or π-closed. The following example illustrates this fact.

Example 3.1 Let $X = \{k, l, m\}$ be a finite topological space with topology

$$\tau = \{\varnothing, \{k\}, \{k, l\}, X\}.$$

We define:

- the family of π-open sets as
$$\pi O(X) = \{\varnothing, \{k\}, \{k, l\}, X\},$$
- the h-closure of a subset $A \subseteq X$ as
$$h\text{-}cl(A) = \bigcap\{U \in \tau : A \subseteq U\}.$$

Now consider the subset $A = \{k, l\} \subseteq X$.

- Since $h\text{-}cl(A) = \{k, l\} \subseteq X$, it follows that A is πH^*-closed.
- However, A is not π-closed, because its complement $\{m\}$ is not π-open.
- Similarly, A is neither regular closed nor r^*-closed, since it does not coincide with the closure of its interior.

Thus, A provides a concrete example of a πH^*-closed set which is neither π-closed, nor regular closed, nor r^*-closed. This verifies that the implications

$$r^*\text{-closed} \Rightarrow h\text{-closed} \Rightarrow \pi H^*\text{-closed}$$

and

$$\pi\text{-closed} \Rightarrow \pi H^*\text{-closed}$$

hold, but none of their converses are true in general.

Theorem 3.4 *Every δ-closed set is πH^*-closed, but the converse does not necessarily hold.*

Proof: Let U be a δ-closed subset of a topological space X, and let M be any π-open set of X such that $U \subseteq M$. By definition [7], we have

$$\delta\text{-}cl(U) = U \subseteq M.$$

Since the h-closure of any set is always contained in its δ-closure, i.e.,

$$h\text{-}cl(U) \subseteq \delta\text{-}cl(U),$$

it follows that

$$h\text{-}cl(U) \subseteq \delta\text{-}cl(U) \subseteq M.$$

Thus U satisfies the defining condition of a πH^*-closed set. Hence every δ-closed set is πH^*-closed. However, the converse is not true in general, as there exist πH^*-closed sets which are not δ-closed. □

Example 3.2 Let $X = \{k, l, m\}$ be a finite topological space with the topology

$$\tau = \{\varnothing, X, \{k\}, \{k, l\}\}.$$

The π-open sets in this topology are:

$$\varnothing,\ \{k\},\ \{k, l\},\ X.$$

The regular open sets are also:

$$\varnothing,\ \{k\},\ \{k, l\},\ X.$$

Thus, every regular open set is π-open.

A set $A \subseteq X$ is δ-closed if $\delta\text{-}cl(A) = A$, where $\delta\text{-}cl(A)$ denotes the intersection of all regular closed sets containing A. The regular closed sets in this topology are:

$$\varnothing,\ \{k, l\},\ X.$$

Hence,

$$\delta\text{-}cl(\{k\}) = \{k, l\}, \qquad \delta\text{-}cl(\{k, l\}) = \{k, l\}, \qquad \delta\text{-}cl(X) = X.$$

Thus, the δ-closed sets are:

$$\varnothing,\ \{k, l\},\ X.$$

A set $U \subseteq X$ is πH^*-closed if

$$h\text{-}cl(U) \subseteq M \quad \text{whenever } U \subseteq M \text{ and } M \text{ is } \pi\text{-open}.$$

Consider $U = \{k\}$. Since $\{k\} \subseteq \{k, l\}$, where $\{k, l\}$ is π-open, and

$$h\text{-}cl(\{k\}) = \{k\} \subseteq \{k, l\},$$

it follows that $\{k\}$ is πH^*-closed.

However, $\{k\}$ is *not* δ-closed, since

$$\delta\text{-}cl(\{k\}) = \{k, l\} \neq \{k\}.$$

Thus, this example shows that a πH^*-closed set need not be δ-closed.

Theorem 3.5 *Every δg^*-closed set is πH^*-closed, but the converse need not hold.*

Proof: Let U be a δg^*-closed subset of a topological space X, and let M be any π-open set such that $U \subseteq M$.

Since every π-open set is also g-open [13], it follows that M is g-open. Given that U is δg^*-closed, we have

$$\delta\text{-}cl(U) \subseteq M.$$

Moreover, by the standard inclusion among closure operators, we obtain

$$h\text{-}cl(U) \subseteq \delta\text{-}cl(U) \subseteq M.$$

Thus, U satisfies the defining condition of a πH^*-closed set. Therefore, every δg^*-closed set is πH^*-closed.

However, the converse does not hold in general, since there exist πH^*-closed sets that are not δg^*-closed. □

Example 3.3 Let $X = \{k, l, m\}$ and define a topology

$$\tau = \{\varnothing, X, \{k\}, \{k, l\}\}.$$

The π-open sets in (X, τ) are

$$\pi O(X) = \{\varnothing, \{k\}, \{k, l\}, X\},$$

since each of these sets is equal to the interior of its closure, and therefore satisfies the definition of π-openness.

The δg^*-closed sets in this space are

$$\varnothing,\ \{k\},\ \{k, l\},\ X,$$

because each of these sets satisfies the property that its δ-closure lies inside every g-open set containing it.

Consider the set $U = \{k\}$. Since U is δg^*-closed, and for any π-open set M containing U (for instance, $M = \{k, l\}$), we have

$$h\text{-}cl(U) \subseteq \delta\text{-}cl(U) = \{k, l\} \subseteq M,$$

it follows that U is πH^*-closed.

Now consider the set $V = \{l\}$. The set V is πH^*-closed because for every π-open set M with $V \subseteq M$ (here $M = X$), we obtain

$$h\text{-}cl(V) = \{l, m\} \subseteq X.$$

However, V is not δg^*-closed, since

$$\delta\text{-}cl(V) = \{l, m\} \neq V.$$

Thus, this example shows that while every δg^*-closed set is πH^*-closed, the converse does not hold in general.

Theorem 3.6 *Every δg-closed set is πH^*-closed, but the converse need not hold.*

Proof: Let U be a δg-closed subset of a topological space X, and let M be any π-open set such that $U \subseteq M$. Since every π-open set is open, it follows that M is an open subset of X. Given that U is δg-closed, we have

$$\delta\text{-}cl(U) \subseteq M.$$

Furthermore, by the fundamental relation between the h-closure and the δ-closure,

$$h\text{-}cl(U) \subseteq \delta\text{-}cl(U) \subseteq M.$$

Thus, U satisfies the defining condition of a πH^*-closed set. Therefore, every δg-closed set is πH^*-closed.

However, the converse does not necessarily hold in general, as there exist πH^*-closed sets that are not δg-closed. □

Example 3.4 Let $X = \{k,\ l,\ m\}$ be a finite topological space with the topology

$$\tau = \{\varnothing,\ \{k\},\ \{k, l\},\ X\}.$$

Step 1: Identify π-open sets.
In this topology, every open set is also π-open. Hence,

$$\pi O(X) = \{\varnothing,\ \{k\},\ \{k, l\},\ X\}.$$

Step 2: Consider the subset $U = \{l\}$.
The δ-closure of U is

$$\delta\text{-}cl(U) = X,$$

since the smallest regular closed set containing l is the whole space X. Thus,

$$\delta\text{-}cl(U) = X \neq U,$$

which shows that U is *not* δg-closed.

Step 3: Verify that U is πH^*-closed.

The π-open supersets of U are $\{k, l\}$ and X. The h-closure of U is

$$h\text{-}cl(U) = \{k, l\}.$$

For each π-open set M with $U \subseteq M$, we have

$$h\text{-}cl(U) = \{k, l\} \subseteq M.$$

Hence, U satisfies the defining condition of a πH^*-closed set.

Therefore, $U = \{l\}$ is πH^*-closed but not δg-closed, demonstrating that the converse of Theorem 3.6 does not hold in general.

Theorem 3.7 *Every α-closed set is πH^*-closed, but the converse does not necessarily hold.*

Proof: Let U be an α-closed subset of a topological space X, and let M be any π-open set in X such that

$$U \subseteq M.$$

Since U is α-closed, we have

$$\alpha\text{-cl}(U) = U \subseteq M.$$

Moreover, the h-closure of any set is always contained in its α-closure, hence

$$h\text{-cl}(U) \subseteq \alpha\text{-cl}(U) = U \subseteq M.$$

Thus h-cl$(U) \subseteq M$ for every π-open set M containing U, which shows that U is πH^*-closed.

Therefore, every α-closed set is πH^*-closed. However, the converse does not hold in general. □

Example 3.5 Consider the finite topological space $X = \{k, l, m\}$ with topology

$$\tau = \{\emptyset, \{k\}, \{k, l\}, X\}.$$

Step 1: Identify π-open sets. Since each open set in this topology is also π-open, we have

$$\pi O(X) = \{\emptyset, \{k\}, \{k, l\}, X\}.$$

Step 2: Choose the subset $U = \{l\}$. Compute its α-closure:

$$\alpha\text{-cl}(U) = \{k, l\}.$$

Since α-cl$(U) \neq U$, the set U is not α-closed.

Step 3: Check πH^*-closedness. Let M be any π-open set containing U. The possible π-open supersets of U are $\{k, l\}$ and X.

The h-closure of U is

$$h\text{-cl}(U) = \{k, l\}.$$

Clearly,

$$h\text{-cl}(U) \subseteq M$$

for each π-open superset M of U. Thus, U satisfies the defining condition of a πH^*-closed set.

Therefore, the subset $U = \{l\}$ of X is πH^*-closed but not α-closed. Hence, the converse of Theorem 3.7 does not hold in general.

Theorem 3.8 *Every πg-closed set is πH^*-closed, but the converse need not hold.*

Proof: Let $U \subseteq X$ be a πg-closed set and let M be any π-open set such that $U \subseteq M$. By the definition [26] of πg-closedness, we have

$$\mathrm{cl}(U) \subseteq M.$$

Since the h-closure of any set is always contained in its ordinary closure, i.e.,

$$h\text{-}\mathrm{cl}(U) \subseteq \mathrm{cl}(U),$$

it follows that

$$h\text{-}\mathrm{cl}(U) \subseteq \mathrm{cl}(U) \subseteq M.$$

Hence U satisfies the defining condition of a πH^*-closed set. Therefore, every πg-closed set is πH^*-closed [26].

However, the converse is not true in general: there exist sets that are πH^*-closed but not πg-closed. □

Example 3.6 Let $X = \{k, l, m\}$ and consider the topology

$$\tau = \{\varnothing, \{k\}, \{k, l\}, X\}.$$

Take the subset $U = \{l\}$.

Check πH^*-closedness: Every π-open superset of U is either $\{k, l\}$ or X. Since

$$h\text{-}\mathrm{cl}(U) = \{k, l\},$$

we have

$$h\text{-}\mathrm{cl}(U) \subseteq M$$

for each π-open [26] set M with $U \subseteq M$. Hence, U is πH^*-closed.

Check πg-closedness: The usual closure of U is

$$\mathrm{cl}(U) = \{k, l\},$$

and clearly

$$\mathrm{cl}(U) \not\subseteq U.$$

Thus, U is not πg-closed.

This example shows that a set may be πH^*-closed without being πg-closed, proving that the converse of Theorem 3.8 does not hold in general.

Lemma 3.1 *Let A be a subset of a topological space X and let $x \in X$. The H^*-closure operator satisfies the following fundamental properties:*

1. *Express the **Pointwise characterization:** of H^*-closure a point belongs to the closure of A precisely when it cannot be separated from A by any H^*-open neighborhood.*
2. ***H^*-closedness:** Identifies H^*-closed sets as the fixed points of the closure operator.*
3. ***Monotonicity:** enlarging a set cannot reduce its H^*-closure.*
4. ***Idempotency:** applying the closure operation twice yields no further enlargement.*
5. ***Closure of the closure:** Ensures that the closure of any set under the H^* operator is itself closed with respect to the same operator.*

Lemma 3.2 *A subset A of a topological space X is gH^*-open in X if and only if every H^*-closed subset F of X contained in A satisfies*

$$F \subseteq H^*\text{-}int(A).$$

Remark 3.2 [26] From the above definitions, theorems, and established results, we observe several important relationships that describe how πH^*-closed sets interact with other classes of generalized closed sets in a topological space (X, τ). These relationships may be summarized as follows:

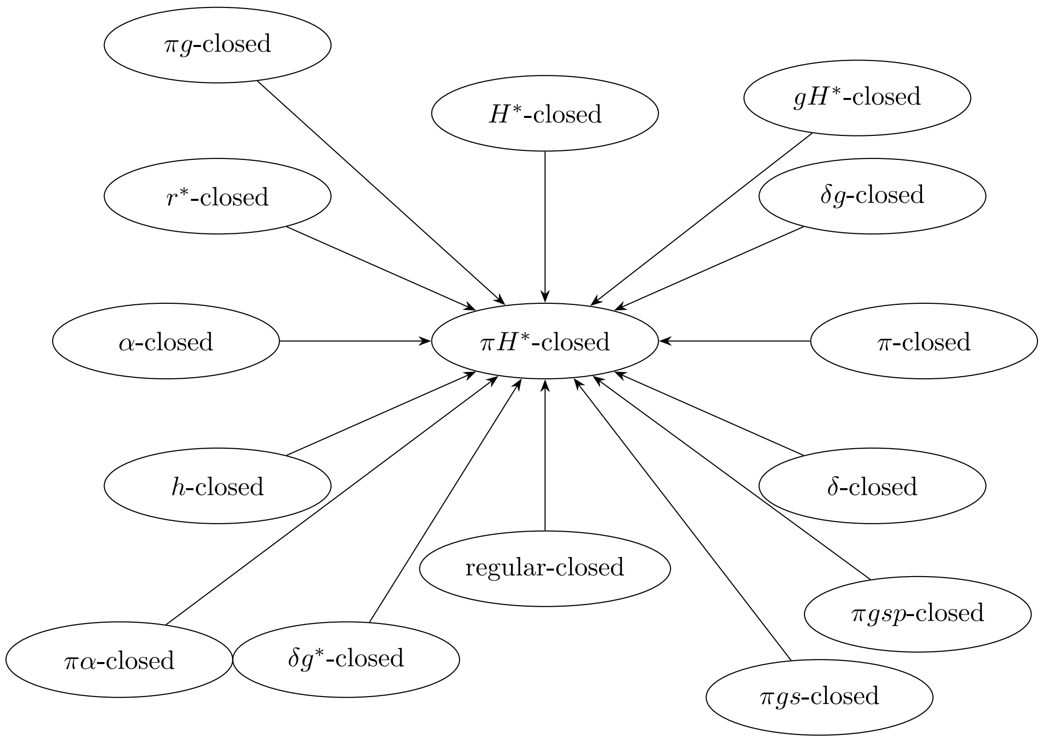

Figure 1: Relationships of various closed sets implying πH^*-closedness.

Example 3.7 Let $X = \{k, l, m\}$ be a topological space with the **discrete topology**.

Note: In a discrete topology, every subset of X is both open and closed.

Hence, all special closed sets, such as regular-closed, δg-closed, H^*-closed, etc., reduce to the same: *every subset of* X.

Therefore, in this case, **all the above types of closed sets trivially imply πH^*-closedness**.

Table 1: Examples of Various Closed Sets and Their πH^*-Closedness in a Discrete Topology

Type of Set	**Example in Discrete Topology**	**Is it πH^*-closed?**
regular closed	$\{k, l\}$	Yes
δg-closed	$\{l, m\}$	Yes
δg^*-closed	$\{k\}$	Yes
H^*-closed	$\{k, m\}$	Yes
h-closed	$\{l\}$	Yes
α-closed	$\{k, l, m\}$	Yes
δ-closed	$\{m\}$	Yes
δH^*-closed	$\emptyset$	Yes
πg-closed	$\{k, l\}$	Yes

Remark 3.3 In discrete topology, everything collapses to the same situation: Every subset of X is πH^*-closed.

Example 3.8 Let $X = \{k, l, m\}$ with the topology $\tau = \{\emptyset, X\}$ (the **trivial topology**).

Table 2: Examples of Various Closed Sets and Their πH^*-Closedness in a Trivial Topology

Type of Set	**Possible Examples**	**Is it πH^*-closed?**
regular closed	$\emptyset, X$	Yes
δg-closed	$\emptyset, X, \{k\}, \{l\}, \{m\}$	Yes
δg^*-closed	$X, \{k\}, \{l\}, \{m\}$	Yes
H^*-closed	$X, \{k\}, \{l\}, \{m\}, \emptyset$	Yes
h-closed	X	Yes
α-closed	$X, \emptyset$	Yes
δ-closed	$\{k\}, \{l\}, \{m\}$	Yes
δH^*-closed	$X, \{k\}, \{l, m\}$	Yes
πg-closed	$X, \{k\}, \{l\}, \{m\}$	Yes

Example 3.9 Let $X = \mathbb{N}$ (the set of natural numbers) with the **cofinite topology**

$$\tau = \{\emptyset\} \cup \{U \subseteq \mathbb{N} \mid \mathbb{N} \setminus U \text{ is finite}\}.$$

Table 3: Examples of Various Closed Sets and Their πH^*-Closedness in a Cofinite Topology

Type of Set	**Typical Examples**	**Is it πH^*-closed?**
regular closed	$\emptyset, \mathbb{N}, \mathbb{N} \setminus \{1, 2\}$	Yes
δg-closed	Finite sets $\{1\}, \{2, 3\}$, plus $\mathbb{N}$	Yes
δg^*-closed	Finite sets and cofinite sets, e.g. $\{5\}, \mathbb{N} \setminus \{1\}$	Yes
H^*-closed	Finite sets, cofinite sets, $\mathbb{N}$	Yes
h-closed	Only $\mathbb{N}$	Yes
α-closed	$\emptyset, \mathbb{N}$	Yes
δ-closed	Finite sets	Yes
δH^*-closed	Finite sets and cofinite sets	Yes
πg-closed	Cofinite sets, e.g. $\mathbb{N} \setminus \{1, 2, 3\}$	Yes

4. Properties of πH^*-Closed Sets

In this section, we obtained properties of πH^*-closed sets.

Theorem 4.1 *[[26]] Let X be a topological space. If a subset A of X is both π-open and πH^*-closed, then A is h-closed.*

Proof: Suppose $A \subseteq X$ is π-open and πH^*-closed. Consider any π-open set M such that $A \subseteq M$. Since A is πH^*-closed, by definition [16] we have

$$h\text{-cl}(A) \subseteq M.$$

Taking $M = A$ (as A is π-open) [26], we get

$$h\text{-cl}(A) \subseteq A.$$

But always $A \subseteq h\text{-cl}(A)$ for any subset A. Hence,

$$A = h\text{-cl}(A).$$

Therefore, A is h-closed. □

Example 4.1 Let $X = \{k, l, m\}$ and define a topology

$$\tau = \{\emptyset, \{k\}, \{k, l\}, X\}.$$

Now we verify the claim step by step:

1. **Open and closed sets:**
   (a) Open sets: $\emptyset, \{k\}, \{k, l\}, X$.
   (b) Closed sets (complements): $X, \{m\}, \{l, m\}, \emptyset$.

2. **Consider the set $A = \{l, m\}$:**
   (a) The closure of A is
   $$\text{cl}(A) = \{l, m\} = A.$$
   Hence, A is h-closed, because $h\text{-cl}(A) = A$.

3. **Check whether A is πH^*-closed:** The collection of π-open sets is
   $$\pi O(X) = \{\{k\}, \{k, l\}, X\}.$$
   Consider the π-open set $M = X$ that contains A. For A to be πH^*-closed, we must have
   $$h\text{-cl}(A) \subseteq M$$
   for every π-open set M containing A.
   Now take another π-open set $M = \{k, l\}$, which contains l but not m. Then
   $$A \nsubseteq M \quad \text{and} \quad h\text{-cl}(A) = A \nsubseteq M.$$
   Hence, the condition fails.
   Therefore, A is h-closed but not πH^*-closed.

Theorem 4.2 *Let X be a topological space. If a subset A of X is regular-open and πH^*-closed, then A is h-closed.*

Proof: Assume that $A \subseteq X$ is both r-open and πH^*-closed.
Since every r-open set is necessarily π-open [26], it follows that A is also a π-open set.
Given that A is πH^*-closed, we have
$$h\text{-cl}(A) \subseteq A.$$
On the other hand, by the definition [16] of the h-closure operator,
$$A \subseteq h\text{-cl}(A).$$
Combining both inclusions yields
$$A = h\text{-cl}(A).$$
Therefore, A is h-closed in X [16]. □

Example 4.2 Let $X = \{k, l, m\}$ and define the topology

$$\tau = \{\emptyset, \{k\}, \{k, l\}, X\}.$$

Consider the subset $A = \{l, m\}$.

1. A is h-closed since
$$h\text{-cl}(A) = A.$$

2. However, A is not πH^*-closed, because for the π-open set $M = \{k, l\}$, we have
$$h\text{-cl}(A) = A \nsubseteq M.$$

Hence, the converse of Theorem 4.2 does not hold.

Theorem 4.3 *Let X be a topological space. If a subset $A \subseteq X$ is both π-open and πH^*-closed, then A is h-closed, and consequently, clopen.*

Proof: Assume that $A \subseteq X$ is π-open and πH^*-closed. Since A is πH^*-closed, by definition we have
$$h\text{-cl}(A) \subseteq A.$$

On the other hand, by the property of the h-closure operator [26],
$$A \subseteq h\text{-cl}(A).$$

Combining these two inclusions gives
$$A = h\text{-cl}(A),$$
which shows that A is h-closed in X.

Moreover, it is well known that every π-open and h-closed set [16,26] is closed. Therefore, A is both open (being π-open) and closed (being h-closed); that is,
$$A \text{ is clopen in } X.$$
□

Example 4.3 Let $X = \{k, l, m\}$ and define the topology
$$\tau = \{\emptyset, \{k\}, \{k, l\}, X\}.$$

Consider the subset $A = \{k, l\}$. Then:

- A is π-open because it belongs to every minimal open family containing its points.
- A is πH^*-closed, since for any π-open set M containing A, we have
$$h\text{-cl}(A) = A \subseteq M.$$
- Hence, A is h-closed, and being both open and closed, A is clopen in X.

Remark 4.1 *Let X be a topological space. It is important to note that the class of πH^*-closed sets is not necessarily closed under finite intersections. That is, even if A and B are both πH^*-closed subsets of X, their intersection $A \cap B$ may fail to be πH^*-closed.*

This observation shows that the family of πH^*-closed sets does not, in general, form a closed system under intersection operations, distinguishing it from the class of ordinary closed sets in standard topological structures.

Theorem 4.4 *Let X be a topological space. Then the finite union of πH^*-closed sets is itself a πH^*-closed set.*

Proof: Let $A_1, A_2, \dots, A_n$ be πH^*-closed subsets of X, and let

$$A = \bigcup_{i=1}^{n} A_i.$$

Consider any π-open set M in X such that $A \subseteq M$. Since $A_i \subseteq A \subseteq M$ and each A_i is πH^*-closed, we have

$$h\text{-cl}(A_i) \subseteq M \qquad \text{for each } i = 1, 2, \dots, n.$$

Using the basic property of the h-closure operator, we obtain

$$h\text{-cl}\left(\bigcup_{i=1}^{n} A_i\right) = \bigcup_{i=1}^{n} h\text{-cl}(A_i) \subseteq M.$$

Hence h-cl$(A) \subseteq M$, which shows that A is πH^*-closed.

Therefore, the finite union of πH^*-closed sets is always πH^*-closed. □

Example 4.4 Let $X = \{k, l, m\}$ and define the topology

$$\tau = \{\emptyset, \{k\}, \{k, l\}, X\}.$$

Consider

$$A_1 = \{k\} \qquad \text{and} \qquad A_2 = \{k, l\}.$$

Both A_1 and A_2 are πH^*-closed in X, since for every π-open set M containing them, we have

$$h\text{-cl}(A_i) \subseteq M.$$

Their union

$$A_1 \cup A_2 = \{k, l\}$$

is also πH^*-closed, confirming Theorem 4.4.

However, the intersection

$$A_1 \cap A_2 = \{k\}$$

may not necessarily be πH^*-closed under all possible topologies on X, illustrating the statement of Remark 4.1.

5. Conclusion

In this work, we have introduced and systematically investigated a new class of closed sets in topology, termed πH^*-closed sets, defined through h-open sets. Our analysis revealed their rich structural properties, strong connections with several established generalized closed sets, and their capacity to unify and extend existing notions in topological theory.

Unlike many previously known classes, πH^*-closed sets strike a balance between generality and tractability: they preserve important properties under finite unions, while their behavior under intersections distinguishes them as a genuinely novel construct.

The dual notion of πH^*-open sets and the introduction of πH^*-$T_{1/2}$ spaces further underscore the depth of this framework, offering new insights into generalized separation axioms and closure operators. These results not only enrich the taxonomy of generalized closed sets but also provide fertile ground for expanding the theory of continuity, compactness, and separation within topological spaces.

6. Applications and Future Directions

The versatility of πH^*-closed sets positions them as a promising tool for both pure and applied research. In particular:

- **Advanced topological frameworks:** Their extension to bitopological spaces, fuzzy topological spaces, and other generalized settings could yield stronger unifying principles.
- **Separation theory:** The study of πH^*-$T_{1/2}$ spaces may lead to refined hierarchies of generalized separation axioms.
- **Mapping theory:** Concepts such as πH^*-closed maps, πH^*-open maps, and πH^*-continuity may serve as robust tools for analyzing topological transformations.
- **Applied disciplines:** The role of generalized closed sets in digital topology, information systems, and theoretical computer science suggests potential practical applications of πH^*-closed sets in modeling discrete structures, approximation theory, and decision-making systems.

In conclusion, πH^*-closed sets open new horizons in the study of generalized closed sets and separation axioms, establishing themselves as a unifying and forward-looking concept in topology. Their study not only strengthens the theoretical foundations of the subject but also paves the way for significant future applications across mathematics and related disciplines.

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