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Generalized Integral Transforms of Incomplete R-Functions

Priti Purohit and Ravi Gupta

ABSTRACT: Advanced integral transformations utilizing the incomplete R-functions, a category about generalized special functions, are examined in this study. We thoroughly prove image formulae of the incomplete R-functions using two new integral transforms: the general integral transform and the generalized Bessel-Maitland integral transform. A few existing integral transformations for the range of special functions are assessed using these relations. Our results lay the groundwork for more research into incomplete R-functions and their usefulness in intricate numerical and analytical frameworks.

Keywords: Mellin-Barnes type integral, Rathie's R-function, incomplete R-functions, integral transforms.

Contents

1. Introduction

Initially we consider the additional popular incomplete gamma functions $\gamma(\tau, x)$ and $\Gamma(\tau, x)$, that are given in the following manner:

$$\gamma(\tau, x) = \int_0^x t^{\tau-1} e^{-t} dt \ \ (\Re(\tau) > 0, x \geq 0), \tag{1.1}$$

and

$$\Gamma(\tau, x) = \int_x^\infty t^{\tau-1} e^{-t} dt \ \ (x \geq 0; \Re(\tau) > 0 \text{ when } x = 0). \tag{1.2}$$

On setting $x = 0$ into relation (1.2), it simplifies to the widely acknowledged gamma function. Furthermore, definitions (1.1) and (1.2) naturally lead to the afterwards decomposition formula:

$$\gamma(\tau, x) + \Gamma(\tau, x) = \Gamma(\tau), \ \ (\Re(\tau) > 0). \tag{1.3}$$

Numerous researchers are currently working on a range of impacts for incomplete, such as applications of incomplete I and $\bar{I}$-functions in analysis of the family of fractional integral and differential equations [4,5,7], and fractional calculus images that include incomplete and other extended functions [8,20,21]. The writers of [12,6] recently looked at applications of the incomplete I-function on the impact for Lambert's law, while the authors of [13,14,24] investigated incomplete Yang Y-function class extension through its incomplete functions.

In accordance with recent work of Gupta and Purohit [10], a family of incomplete R-functions that correlate to Rathie's R-function concerning the incomplete gamma functions, they're described as follows:

$$\begin{aligned} {}_{\varepsilon,\,\delta}^{\alpha,\,\beta}\gamma_{p,\,q}^{m,\,n}(z) &= {}_{\varepsilon,\,\delta}^{\alpha,\,\beta}\gamma_{p,\,q}^{m,\,n}\left[z \,\middle|\, \begin{matrix} (\mathfrak{u}_1, \mathfrak{U}_1, x), (\mathfrak{u}_2, \mathfrak{U}_2), \cdots, (\mathfrak{u}_p, \mathfrak{U}_p) \\ (\mathfrak{v}_1, \mathfrak{V}_1), \cdots, (\mathfrak{v}_q, \mathfrak{V}_q) \end{matrix} \right] \\ &= \frac{1}{2\pi i} \int_{\mathcal{L}} \frac{\mathsf{g}(s, x)}{(\alpha + \beta s)^{\varepsilon + \delta s}} \, z^{-s} \, ds, \end{aligned} \tag{1.4}$$

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wherein

$$\mathsf{g}(s,x) = \frac{\gamma(1-\mathfrak{u}_1-\mathfrak{U}_1 s, x) \prod_{j=1}^{m} \Gamma(\mathfrak{v}_j+\mathfrak{V}_j s) \prod_{j=2}^{n} \Gamma(1-\mathfrak{u}_j-\mathfrak{U}_j s)}{\prod_{j=m+1}^{q} \Gamma(1-\mathfrak{v}_j-\mathfrak{V}_j s) \prod_{j=n+1}^{p} \Gamma(\mathfrak{u}_j+\mathfrak{U}_j s)}, \tag{1.5}$$

and

$$\begin{aligned} {}^{\alpha,\,\beta}_{\varepsilon,\,\delta}\Gamma^{m,\,n}_{p,\,q}(z) &= {}^{\alpha,\,\beta}_{\varepsilon,\,\delta}\Gamma^{m,\,n}_{p,\,q}\left[z \;\middle|\; \begin{matrix} (\mathfrak{u}_1,\mathfrak{U}_1,x),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p) \\ (\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q) \end{matrix} \right] \\ &= \frac{1}{2\pi i}\int_{\mathcal{L}} \frac{\mathsf{G}(s,x)}{(\alpha+\beta s)^{\varepsilon+\delta s}}\, z^{-s}\, ds, \end{aligned} \tag{1.6}$$

wherever

$$\mathsf{G}(s,x) = \frac{\Gamma(1-\mathfrak{u}_1-\mathfrak{U}_1 s, x) \prod_{j=1}^{m} \Gamma(\mathfrak{v}_j+\mathfrak{V}_j s) \prod_{j=2}^{n} \Gamma(1-\mathfrak{u}_j-\mathfrak{U}_j s)}{\prod_{j=m+1}^{q} \Gamma(1-\mathfrak{v}_j-\mathfrak{V}_j s) \prod_{j=n+1}^{p} \Gamma(\mathfrak{u}_j+\mathfrak{U}_j s)}. \tag{1.7}$$

The integrers p, q, m and n are such that $0 < m < q, 0 < n < p$, and z cannot be equal to zero. Additionally, the numbers $\mathfrak{U}_j (j = 1, \cdots, p)$ and $\mathfrak{V}_j (j = 1, \cdots, q)$ are positive, whereas the numbers $\mathfrak{u}_j (j = 1, \cdots, p)$ and $\mathfrak{v}_j (j = 1, \cdots, q)$ are complex, see [15,16,17]. In this situation, the variables $\alpha, \beta, \varepsilon$, and δ are chosen to ensure that the right-side integral of (1.8) exists. With the same presumptions and requirements as the R-function in [26], the incomplete R-functions exist for every $x \geq 0$.

For details on the fundamental characteristics of incomplete R-functions, such as formulas for derivative, decomposition, reduction, integral representations, and fractional calculus, one can refer to [10]. [22] also discusses applications to model the effects of environmental contamination on species.

The following decomposition formula is easily obtained from the explanations for lower and upper incomplete R-functions:

$${}^{\alpha,\,\beta}_{\varepsilon,\,\delta}\gamma^{m,\,n}_{p,\,q}(z) + {}^{\alpha,\,\beta}_{\varepsilon,\,\delta}\Gamma^{m,\,n}_{p,\,q}(z) = {}^{\alpha,\,\beta}_{\varepsilon,\,\delta}R^{m,\,n}_{p,\,q}(z).$$

The widely known R-function, which is the include that shows up on the right, was first created by Rathie [26] (see also, [25]) to look into more problems pertaining to evaluating statistical hypotheses. The following is the manner in which it is defined:

$$\begin{aligned} {}^{\alpha,\,\beta}_{\varepsilon,\,\delta}R^{m,\,n}_{p,\,q}(z) &= {}^{\alpha,\,\beta}_{\varepsilon,\,\delta}R^{m,\,n}_{p,\,q}\left[z \;\middle|\; \begin{matrix} (\mathfrak{u}_1,\mathfrak{U}_1),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p) \\ (\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q) \end{matrix} \right] \\ &= \frac{1}{2\pi i}\int_{\mathcal{L}} \frac{\mathsf{g}(s)}{(\alpha+\beta s)^{\varepsilon+\delta s}}\, z^{-s}\, ds, \end{aligned} \tag{1.8}$$

wherein

$$\mathsf{g}(s) = \frac{\prod_{j=1}^{m} \Gamma(\mathfrak{v}_j+\mathfrak{V}_j s) \prod_{j=1}^{n} \Gamma(1-\mathfrak{u}_j-\mathfrak{U}_j s)}{\prod_{j=m+1}^{q} \Gamma(1-\mathfrak{v}_j-\mathfrak{V}_j s) \prod_{j=n+1}^{p} \Gamma(\mathfrak{u}_j+\mathfrak{U}_j s)}. \tag{1.9}$$

To study current work on integral formulations of the R-function, one may specifically utilize [9]. Notably, for $\alpha = 1$ and $\beta = 0$ (or $\varepsilon = 0$ and $\delta = 0$) (see, also [28]), the incomplete R-functions give rise for a set of H-functions that are incomplete, which are characterized in the ordered manner as follows:

$$\begin{aligned} {}^{1,\,0}_{\varepsilon,\,\delta}\gamma^{m,\,n}_{p,\,q}(z) = {}^{\alpha,\,\beta}_{0,\,0}\gamma^{m,\,n}_{p,\,q}(z) &= \gamma^{m,\,n}_{p,\,q}\left[z \;\middle|\; \begin{matrix} (\mathfrak{u}_1,\mathfrak{U}_1,x),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p) \\ (\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q) \end{matrix} \right] \\ &= \frac{1}{2\pi i}\int_{\mathcal{L}} \mathsf{g}(s,x) z^{-s}\, ds, \end{aligned} \tag{1.10}$$

and

$$
{}^{1,\,0}_{\varepsilon,\,\delta}\Gamma^{m,\,n}_{p,\,q}(z) = {}^{\alpha,\,\beta}_{0,\,0}\Gamma^{m,\,n}_{p,\,q}(z) = \Gamma^{m,\,n}_{p,\,q}\left[z \,\middle|\, \begin{matrix} (\mathfrak{u}_1,\mathfrak{U}_1,x),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p) \\ (\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q) \end{matrix} \right]
$$

$$
= \frac{1}{2\pi i}\int_{\mathcal{L}} \mathsf{G}(s,x)\, z^{-s}\, ds, \tag{1.11}
$$

where (1.5) defines $\mathsf{g}(s,x)$ and (1.7) defines $\mathsf{G}(s,x)$. Moreover, these functions are closely connected to Fox's H-function.

Integral transformations are effective tools for simplifying difficult mathematical problems, making it possible to analyze and solve equations in a variety of fields. They promote improvements in research and engineering by bridging the gaps between time, space, and frequency, see [18,27]. In order to comprehend the present investigation, we now define the basic concepts and information on generalized integral transforms.

Definition 1.1 ([11]) For $z \geq 0$, $\phi(\wp) \neq 0$, and $\sigma(\wp)$ positive real functions, let $f(z)$ be an intgrable function. The following formula therefore yields the generalized integral transform of $f(z)$:

$$
\mathcal{T}\{f(z),\wp\} = \phi(\wp)\int_0^\infty e^{-\sigma(\wp)z} f(z)dz. \tag{1.12}
$$

When $f(z) = z^\rho$, in particular, we obtain

$$
\mathcal{T}\{z^\rho,\wp\} = \frac{\phi(\wp)\Gamma(\rho+1)}{\{\sigma(\wp)\}^{\rho+1}}, \quad \rho > 0. \tag{1.13}
$$

For other modern applications of the aforementioned general integral transform, see [1]. Table 1 illustrates the connections between the basic integral transform and general integral transforms currently used in the Laplace transform class; see [11] and references thierin.

Definition 1.2 ([3]) Suppose $\tau,\mu,\nu,\eta,\wp \in \mathbb{C}$, such that

$$
\Re(\tau) \geq 0, \Re(\mu) \geq 0, \Re(\eta) \geq 0, \Re(\nu) \geq -1, \Re(\wp) \geq 0
$$

and $\kappa \in (0,1) \cup \mathbb{N}$, subsequently the generalized Bessel-Maitland integral transform of $f(z)$ is given by:

$$
{}_{\tau}\mathcal{H}^{\mu,\eta}_{\nu,\kappa}\{f(z);\wp\} = \mathcal{H}\{f(z);\wp\} = \int_0^\infty (\wp z)^\tau \mathcal{J}^{\mu,\eta}_{\nu,\kappa}(\wp z) f(z)dz, \tag{1.14}
$$

where the generalized Bessel-Maitland function $\mathcal{J}^{\mu,\eta}_{\nu,\kappa}(z)$, is outlined as under:

$$
\mathcal{J}^{\mu,\eta}_{\nu,\kappa}(z) = \sum_{n=0}^\infty \frac{(\eta)_{\kappa n}(-z)^n}{n!\Gamma(\mu n+\nu+1)}.
$$

If we set $f(z) = z^\rho$, $\Re(\rho) > -1$, then the generalized Bessel-Maitland integral transform of power function is given by (see, [2,3]):

$$
{}_{\tau}\mathcal{H}^{\mu,\eta}_{\nu,\kappa}\{z^\rho;\wp\} = \frac{\Gamma(\tau+\rho+1)\Gamma(\eta-\kappa(\tau+\rho+1))}{\Gamma(\eta)\Gamma(1+\nu-\mu(\tau+\rho+1))}\frac{1}{\wp^{\rho+1}}. \tag{1.15}
$$

Moreover, the connection that follows between the Laplace transform and the Generalized Bessel-Maitland transform (see, [2,3]):

$$
{}_{0}\mathcal{H}^{\kappa,1}_{0,\kappa}\{f(z);\wp\} = \mathcal{L}\{f(z);\wp\}. \tag{1.16}
$$

There is the subsequent connection exists between the Hankel transform and the Generalized Bessel-Maitland transform (see, [2,3]):

$$
{}_{\frac{\nu}{2}-\frac{1}{4}}\mathcal{H}^{1,\eta}_{\nu,0}\{f(z);\wp^2\} = \frac{1}{\wp\sqrt{2}}\mathcal{H}_\nu\{f\left(\frac{z^2}{4}\right);\wp\}. \tag{1.17}
$$

Using two novel integral transforms, the general integral transform (1.12) and the generalized Bessel-Maitland integral transform (1.14), we rigorously verify image formulae of the incomplete R-functions. Several novel and little-known integral transformation formulas involving incomplete H-functions, Rathie's R-function, and several other special functions are evaluated using these key findings. Our findings provide a foundation for further investigation into incomplete R-functions and their use in complex analytical and numerical contexts.

2. Main Results

The general integral transform and the generalized Bessel-Maitland integral transform of the incomplete R-functions are two such instances of potentially significant integral transforms that we establish in this section. The following assumptions are used about the characters Ω, ω, and Δ within this section:

$$\Omega = \sum_{j=1}^{m} \mathfrak{V}_j - \sum_{j=m+1}^{q} \mathfrak{V}_j + \sum_{j=1}^{n} \mathfrak{U}_j - \sum_{j=n+1}^{p} \mathfrak{U}_j,$$

$$\omega = \sum_{j=1}^{p} \mathfrak{v}_j - \sum_{j=1}^{q} \mathfrak{u}_j + \frac{p-q}{2}, \quad \Delta = \sum_{j=1}^{p} \mathfrak{V}_j - \sum_{j=1}^{q} \mathfrak{U}_j$$

Theorem 2.1 *If* $\Delta > 0,\ \rho > 0,\ |arg(z)| < \Omega\pi/2,$

$$-\rho \begin{array}{c} min \\ 1 \leq j \leq m \end{array} \Re(\frac{\mathfrak{v}_j}{\mathfrak{V}_j}) < \Re(\lambda)$$

and $\Re(\wp) > 0$, *thereafter the general integral transform for the lower incomplete R-function is as follows:*

$$\mathcal{T}\{z^{\lambda-1}\ {}^{\alpha,\,\beta}_{\varepsilon,\,\delta}\gamma^{m,\,n}_{p,\,q}\left[cz^{\rho}\ \middle|\ \begin{array}{c} (\mathfrak{u}_1, \mathfrak{U}_1, x), (\mathfrak{u}_2, \mathfrak{U}_2), \cdots, (\mathfrak{u}_p, \mathfrak{U}_p) \\ (\mathfrak{v}_1, \mathfrak{V}_1), \cdots, (\mathfrak{v}_q, \mathfrak{V}_q) \end{array}\right]; \wp\}$$
$$= \phi(\wp)\{\sigma(\wp)\}^{-\lambda}\ {}^{\alpha,\,\beta}_{\varepsilon,\,\delta}\gamma^{m,\,n+1}_{p+1,\,q}\left[c\{\sigma(\wp)\}^{-\rho}\ \middle|\ \begin{array}{c} (\mathfrak{u}_1, \mathfrak{U}_1, x), (1-\lambda, \rho), (\mathfrak{u}_2, \mathfrak{U}_2), \cdots, (\mathfrak{u}_p, \mathfrak{U}_p) \\ (\mathfrak{v}_1, \mathfrak{V}_1), \cdots, (\mathfrak{v}_q, \mathfrak{V}_q) \end{array}\right]. \tag{2.1}$$

Proof: Using (1.4), the left hand side (say L_1), can be written as:

$$L_1 = \mathcal{T}\left[\frac{z^{\lambda-1}}{2\pi i}\int_{\mathcal{L}} \frac{\mathsf{g}(s,x)}{(\alpha+\beta s)^{\varepsilon+\delta s}}\,(cz^{\rho})^{-s}\,ds\right] \tag{2.2}$$

$$= \frac{1}{2\pi i}\int_{\mathcal{L}} \frac{\mathsf{g}(s,x)\ c^{-s}}{(\alpha+\beta s)^{\varepsilon+\delta s}}\,\mathcal{T}\{z^{\lambda-\rho s-1}\}\,ds \tag{2.3}$$

Applying the result (1.13), the euqation above now yields

$$L_1 = \frac{1}{2\pi i}\int_{\mathcal{L}} \frac{\mathsf{g}(s,x)\ c^{-s}}{(\alpha+\beta s)^{\varepsilon+\delta s}}\,\frac{\phi(\wp)\Gamma(\lambda-\rho s)}{\{\sigma(\wp)\}^{\lambda-\rho s}}\,ds \tag{2.4}$$

$$= \frac{\phi(\wp)\{\sigma(\wp)\}^{-\lambda}}{2\pi i}\int_{\mathcal{L}} \frac{\mathsf{g}(s,x)\ c^{-s}\{\sigma(\wp)\}^{\rho s}}{(\alpha+\beta s)^{\varepsilon+\delta s}}\,\Gamma(\lambda-\rho s)\,ds, \tag{2.5}$$

which upon using the definition (1.4), we arrive at the desired result (2.1). □

Theorem 2.2 *If* $\Delta > 0,\ \rho > 0,\ |arg(z)| < \Omega\pi/2,$

$$-\rho \begin{array}{c} min \\ 1 \leq j \leq m \end{array} \Re(\frac{\mathfrak{v}_j}{\mathfrak{V}_j}) < \Re(\lambda)$$

and $\Re(\wp) > 0$, *then the general integral transform for the upper incomplete R-function is as follows:*

$$\begin{aligned}&\mathcal{T}\{z^{\lambda-1}\, {}^{\alpha,\,\beta}_{\varepsilon,\,\delta}\Gamma^{m,\,n}_{p,\,q}\left[cz^{\rho}\,\middle|\,\begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1,x),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right];\wp\}\\&\quad=\phi(\wp)\{\sigma(\wp)\}^{-\lambda}\,{}^{\alpha,\,\beta}_{\varepsilon,\,\delta}\Gamma^{m,\,n+1}_{p+1,\,q}\left[c\{\sigma(\wp)\}^{-\rho}\,\middle|\,\begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1,x),(1-\lambda,\rho),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right].\end{aligned}\tag{2.6}$$

Details of the proof of the above thm have been omitted from this section for reasons of simplicity.

The family of incomplete H-functions arises from the incomplete R-functions and can be defined by [28], where $\alpha = 1$ and $\beta = 0$ (or $\varepsilon = 0$ and $\delta = 0$). In light of the above, one can readily derive the integral transforms of the upper incomplete H-function as follows:

Corollary 2.1 *The following is the general integral transform for the upper incomplete H-function under the conditions of Theorem 2.2:*

$$\begin{aligned}&\mathcal{T}\{z^{\lambda-1}\,\Gamma^{m,\,n}_{p,\,q}\left[cz^{\rho}\,\middle|\,\begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1,x),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right];\wp\}\\&=\phi(\wp)\{\sigma(\wp)\}^{-\lambda}\\&\times\;\Gamma^{m,\,n+1}_{p+1,\,q}\left[c\{\sigma(\wp)\}^{-\rho}\,\middle|\,\begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1,x),(1-\lambda,\rho),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right].\end{aligned}\tag{2.7}$$

Further, the Theorem 2.2 now reduce to the following integral transformation for R-functions if we set $x = 0$:

Corollary 2.2 *The following is the general integral transform for the Rathie's R-function under the conditions of Theorem 2.2:*

$$\begin{aligned}&\mathcal{T}\{z^{\lambda-1}\,{}^{\alpha,\,\beta}_{\varepsilon,\,\delta}R^{m,\,n}_{p,\,q}\left[cz^{\rho}\,\middle|\,\begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right];\wp\}\\&=\phi(\wp)\{\sigma(\wp)\}^{-\lambda}\\&\times\;{}^{\alpha,\,\beta}_{\varepsilon,\,\delta}R^{m,\,n+1}_{p+1,\,q}\left[c\{\sigma(\wp)\}^{-\rho}\,\middle|\,\begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1),(1-\lambda,\rho),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right].\end{aligned}\tag{2.8}$$

Additionally, if $x = 0$ is set in Corollary 2.1, else $\alpha = 1$ and $\beta = 0$ (or $\varepsilon = 0$ and $\delta = 0$) in in Corollary 2.2, we derive the following novel finding regarding Fox's H-function:

Corollary 2.3 *The following is the general integral transform for the Fox's H-function under the conditions of Theorem 2.2:*

$$\begin{aligned}&\mathcal{T}\{z^{\lambda-1}\,H^{m,\,n}_{p,\,q}\left[cz^{\rho}\,\middle|\,\begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right];\wp\}\\&=\phi(\wp)\{\sigma(\wp)\}^{-\lambda}\\&\times\;H^{m,\,n+1}_{p+1,\,q}\left[c\{\sigma(\wp)\}^{-\rho}\,\middle|\,\begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1),(1-\lambda,\rho),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right].\end{aligned}\tag{2.9}$$

Theorem 2.3 *If* $\Delta > 0$, $\rho > 0$, $|arg(z)| < \Omega\pi/2$,

$$-\rho\;\underset{1\leq j\leq m}{min}\;\Re\left(\frac{\mathfrak{v}_j}{\mathfrak{V}_j}\right) < \Re(\lambda)$$

and $\Re(\wp) > 0$, *then the generalized Bessel-Maitland integral transform for the lower incomplete R-function is as follows:*

$$
\begin{aligned}
&{}_{\tau}\mathcal{H}^{\mu,\eta}_{\nu,\kappa}\{z^{\lambda-1}\;{}^{\alpha,\,\beta}_{\varepsilon,\,\delta}\gamma^{m,\,n}_{p,\,q}\left[cz^{\rho}\;\middle|\;\begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1,x),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right];\wp\}\\
&=\frac{\wp^{-\lambda}}{\Gamma(\eta)}\;{}^{\alpha,\,\beta}_{\varepsilon,\,\delta}\gamma^{m+1,\,n+1}_{p+1,\,q+2}\left[c\wp^{-\rho}\;\middle|\right.\\
&\left.\begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1,x),(1-\tau-\lambda,\rho),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\eta-\kappa(\tau+\lambda),\kappa\rho),(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q),(1+\nu-\mu(\tau+\lambda),\mu\rho)\end{matrix}\right].
\end{aligned}
\tag{2.10}
$$

Proof: Using (1.4), the left hand side (say L_2), can be written as:

$$
\begin{aligned}
L_2 &= {}_{\tau}\mathcal{H}^{\mu,\eta}_{\nu,\kappa}\left[\frac{z^{\lambda-1}}{2\pi i}\int_{\mathcal{L}}\frac{\mathsf{g}(s,x)}{(\alpha+\beta s)^{\varepsilon+\delta s}}\,(cz^{\rho})^{-s}\,ds\right] &(2.11)\\
&=\frac{1}{2\pi i}\int_{\mathcal{L}}\frac{\mathsf{g}(s,x)\;c^{-s}}{(\alpha+\beta s)^{\varepsilon+\delta s}}\;{}_{\tau}\mathcal{H}^{\mu,\eta}_{\nu,\kappa}\{z^{\lambda-\rho s-1}\}\,ds. &(2.12)
\end{aligned}
$$

Using the image formula, which is the generalized Bessel-Maitland integral transform of the power function, namely

$$
{}_{\tau}\mathcal{H}^{\mu,\eta}_{\nu,\kappa}\{z^{\rho};\wp\}=\frac{\Gamma(\tau+\rho+1)\Gamma(\eta-\kappa(\tau+\rho+1))}{\Gamma(\eta)\Gamma(1+\nu-\mu(\tau+\rho+1))}\frac{1}{\wp^{\rho+1}},
$$

we now yields

$$
\begin{aligned}
L_2 &= \frac{1}{2\pi i}\int_{\mathcal{L}}\frac{\mathsf{g}(s,x)\;c^{-s}}{(\alpha+\beta s)^{\varepsilon+\delta s}}\,\frac{\Gamma(\tau+\lambda-\rho s)\Gamma(\eta-\kappa(\tau+\lambda-\rho s))}{\Gamma(\eta)\Gamma(1+\nu-\mu(\tau+\lambda-\rho s))}\,\frac{1}{\wp^{\lambda-\rho s}}\,ds &(2.13)\\
&=\frac{1}{2\pi i}\int_{\mathcal{L}}\frac{\mathsf{g}(s,x)\;c^{-s}}{(\alpha+\beta s)^{\varepsilon+\delta s}}\,\frac{\Gamma(\tau+\lambda-\rho s)\Gamma(\eta-\kappa(\tau+\lambda)+\kappa\rho s)}{\Gamma(\eta)\Gamma(1+\nu-\mu(\tau+\lambda)+\mu\rho s)}\,\frac{1}{\wp^{\lambda-\rho s}}\,ds. &(2.14)
\end{aligned}
$$

which upon using the definition (1.4), we arrive at the desired result (2.10). □

Theorem 2.4 *If* $\Delta > 0$, $\rho > 0$, $|arg(z)| < \Omega\pi/2$,

$$
-\rho\;\begin{matrix}min\\1\leq j\leq m\end{matrix}\;\Re\Big(\frac{\mathfrak{v}_j}{\mathfrak{V}_j}\Big)<\Re(\lambda)
$$

and $\Re(\wp) > 0$, *then the generalized Bessel-Maitland integral transform for the upper incomplete R-function is as follows:*

$$
\begin{aligned}
&{}_{\tau}\mathcal{H}^{\mu,\eta}_{\nu,\kappa}\{z^{\lambda-1}\;{}^{\alpha,\,\beta}_{\varepsilon,\,\delta}\Gamma^{m,\,n}_{p,\,q}\left[cz^{\rho}\;\middle|\;\begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1,x),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right];\wp\}\\
&=\frac{\wp^{-\lambda}}{\Gamma(\eta)}\;{}^{\alpha,\,\beta}_{\varepsilon,\,\delta}\Gamma^{m+1,\,n+1}_{p+1,\,q+2}\left[c\wp^{-\rho}\;\middle|\right.\\
&\left.\begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1,x),(1-\tau-\lambda,\rho),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\eta-\kappa(\tau+\lambda),\kappa\rho),(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q),(1+\nu-\mu(\tau+\lambda),\mu\rho)\end{matrix}\right].
\end{aligned}
\tag{2.15}
$$

Specializing the parameters to the values indicated yields the following results, which arise as immediate consequences of Theorem 2.4:

Corollary 2.4 *The following is the generalized Bessel-Maitland integral transform for the upper incomplete H-function under the conditions of Theorem 2.4:*

$$
\begin{aligned}
&{}_{\tau}\mathcal{H}^{\mu,\eta}_{\nu,\kappa}\{z^{\lambda-1}\ \Gamma^{m,\,n}_{p,\,q}\left[cz^{\rho}\ \middle|\ \begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1,x),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right];\wp\}\\
&=\frac{\wp^{-\lambda}}{\Gamma(\eta)}\ \Gamma^{m+1,\,n+1}_{p+1,\,q+2}\left[c\wp^{-\rho}\ \middle|\ \begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1,x),(1-\tau-\lambda,\rho),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\eta-\kappa(\tau+\lambda),\kappa\rho),(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q),(1+\nu-\mu(\tau+\lambda),\mu\rho)\end{matrix}\right].
\end{aligned}
\tag{2.16}
$$

Corollary 2.5 *The following is the generalized Bessel-Maitland integral transform for the Rathie's R-function under the conditions of Theorem 2.4:*

$$
\begin{aligned}
&{}_{\tau}\mathcal{H}^{\mu,\eta}_{\nu,\kappa}\{z^{\lambda-1}\ {}^{\alpha,\,\beta}_{\varepsilon,\,\delta}R^{m,\,n}_{p,\,q}\left[cz^{\rho}\ \middle|\ \begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right];\wp\}\\
&=\frac{\wp^{-\lambda}}{\Gamma(\eta)}\ {}^{\alpha,\,\beta}_{\varepsilon,\,\delta}R^{m+1,\,n+1}_{p+1,\,q+2}\left[c\wp^{-\rho}\ \middle|\ \begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1),(1-\tau-\lambda,\rho),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\eta-\kappa(\tau+\lambda),\kappa\rho),(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q),(1+\nu-\mu(\tau+\lambda),\mu\rho)\end{matrix}\right].
\end{aligned}
\tag{2.17}
$$

Corollary 2.6 *The following is the generalized Bessel-Maitland integral transform for the Fox's H-function under the conditions of Theorem 2.4:*

$$
\begin{aligned}
&{}_{\tau}\mathcal{H}^{\mu,\eta}_{\nu,\kappa}\{z^{\lambda-1}\ H^{m,\,n}_{p,\,q}\left[cz^{\rho}\ \middle|\ \begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right];\wp\}\\
&=\frac{\wp^{-\lambda}}{\Gamma(\eta)}\ H^{m+1,\,n+1}_{p+1,\,q+2}\left[c\wp^{-\rho}\ \middle|\ \begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1),(1-\tau-\lambda,\rho),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\eta-\kappa(\tau+\lambda),\kappa\rho),(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q),(1+\nu-\mu(\tau+\lambda),\mu\rho)\end{matrix}\right].
\end{aligned}
\tag{2.18}
$$

By considering Theorems 2.1 and 2.3, similar findings using the lower incomplete H-function may also be reached.

The work of [19,23] might be consulted for further information about integral transforms of incomplete H and I-functions.

3. Special Cases & Concluding Remarks

The following section addresses some of the consequences of the main findings presented in the preceding section. The widely recognized integral transforms available in the literature can be obtained by specializing the suitable functions $\phi(\wp)$ and $\sigma(\wp)$ in the general integral transform, see Table 1. For example, if we assume $\phi(\wp)=1$ and $\sigma(\wp)=\wp$ in Theorems 2.1 and 2.2, the Laplace transform of the family of incomplete R-functions yields the following results (see, [10]):

Corollary 3.1 *The following are the general integral transform for the family of incomplete R-functions under the conditions of Theorems 2.1 and 2.2:*

$$
\begin{aligned}
&\mathcal{L}\{z^{\lambda-1}\ {}^{\alpha,\,\beta}_{\varepsilon,\,\delta}\gamma^{m,\,n}_{p,\,q}\left[cz^{\rho}\ \middle|\ \begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1,x),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right];\wp\}\\
&=\wp^{-\lambda}\ {}^{\alpha,\,\beta}_{\varepsilon,\,\delta}\gamma^{m,\,n+1}_{p+1,\,q}\left[c\wp^{-\rho}\ \middle|\ \begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1,x),(1-\lambda,\rho),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right],
\end{aligned}
\tag{3.1}
$$

and

$$\mathcal{L}\{z^{\lambda-1}\,{}^{\alpha,\,\beta}_{\varepsilon,\,\delta}\Gamma^{m,\,n}_{p,\,q}\left[cz^{\rho}\,\middle|\,\begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1,x),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right];\wp\}$$

$$=\wp^{-\lambda}\,{}^{\alpha,\,\beta}_{\varepsilon,\,\delta}\Gamma^{m,\,n+1}_{p+1,\,q}\left[c\wp^{-\rho}\,\middle|\,\begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1,x),(1-\lambda,\rho),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right],\tag{3.2}$$

provided the incomplete R-functions involved in the statement converge completely.

However, the relationship (1.16) with Theorems 2.3 and 2.4 can also be used to get the aforementioned result.

Again, if we assume $\phi(\wp)=1/\wp$ and $\sigma(\wp)=1/\wp$ in Theorems 2.1 and 2.2, the Sumudu transform of the family of incomplete R-functions yields the following results:

Corollary 3.2 *The following are the general integral transform for the family of incomplete R-functions under the conditions of Theorems 2.1 and 2.2:*

$$\mathcal{S}\{z^{\lambda-1}\,{}^{\alpha,\,\beta}_{\varepsilon,\,\delta}\gamma^{m,\,n}_{p,\,q}\left[cz^{\rho}\,\middle|\,\begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1,x),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right];\wp\}$$

$$=\wp^{\lambda-1}\,{}^{\alpha,\,\beta}_{\varepsilon,\,\delta}\gamma^{m,\,n+1}_{p+1,\,q}\left[c\wp^{\rho}\,\middle|\,\begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1,x),(1-\lambda,\rho),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right],\tag{3.3}$$

and

$$\mathcal{S}\{z^{\lambda-1}\,{}^{\alpha,\,\beta}_{\varepsilon,\,\delta}\Gamma^{m,\,n}_{p,\,q}\left[cz^{\rho}\,\middle|\,\begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1,x),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right];\wp\}$$

$$=\wp^{\lambda-1}\,{}^{\alpha,\,\beta}_{\varepsilon,\,\delta}\Gamma^{m,\,n+1}_{p+1,\,q}\left[c\wp^{\rho}\,\middle|\,\begin{matrix}(\mathfrak{u}_1,\mathfrak{U}_1,x),(1-\lambda,\rho),(\mathfrak{u}_2,\mathfrak{U}_2),\cdots,(\mathfrak{u}_p,\mathfrak{U}_p)\\(\mathfrak{v}_1,\mathfrak{V}_1),\cdots,(\mathfrak{v}_q,\mathfrak{V}_q)\end{matrix}\right],\tag{3.4}$$

provided the incomplete R-functions involved in the statement converge completely.

Likewise by considering Table 1, the image formula for incomplete R-functions under the Elzaki, Aboodh, Pourreza, Mohand, Sawi, Kamal, and G transforms, as well as the α-integer Laplace transform, may also be obtained. Additionally, if we use the relation (1.17) and put $\tau=\frac{\nu}{2}-\frac{1}{4},\mu=1$, and $\kappa=0$, the main results in Theorems 2.3 and 2.4 reduce to the Hankel transforms of the incomplete R-functions. For the sake of simplicity, we have omitted specifics of these kinds of findings.

Finally, we note that we have performed a thorough analysis of integral transformations using the incomplete R-functions and other related generalized special functions. For incomplete R-functions, it has created certain image formulas using two new integral transforms: the general integral transform and the generalized Bessel-Maitland integral transform. The universality of generalized integral transforms, combined with the flexibility of the family of incomplete R-functions analyzed in this study, enables the derivation of numerous image formulas under various integral transforms. These transforms establish connections with a wide range of special functions, allowing for the systematic evaluation of integral representations and properties across different mathematical frameworks. By leveraging this approach, it becomes possible to unify and extend known results while also discovering new relationships among special functions arising in applied mathematics, physics, and engineering.

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Table 1: Relationships between the general integral transform and certain integral transforms.

Specialization	**Integral Transforms**	**Definition**
$\phi(\wp) = 1, \sigma(\wp) = \wp$	Laplace transform	$\mathcal{L}\{f(z)\} = \int_0^\infty e^{-\wp z} f(z)dz$
$\phi(\wp) = 1/\wp, \sigma(\wp) = 1/\wp$	Sumudu transform	$\mathcal{S}\{f(z)\} = \frac{1}{\wp} \int_0^\infty e^{-\frac{z}{\wp}} f(z)dz$
$\phi(\wp) = \wp, \sigma(\wp) = 1/\wp$	Elzaki transform	$E\{f(z)\} = \wp \int_0^\infty e^{-\frac{z}{\wp}} f(z)dz$
$\phi(\wp) = 1/\wp, \sigma(\wp) = \wp$	Aboodh transform	$A\{f(z)\} = \frac{1}{\wp} \int_0^\infty e^{-\wp z} f(z)dz$
$\phi(\wp) = 1, \sigma(\wp) = \wp^{1/\alpha}$	α-integer Laplace transform	$\mathcal{L}_\alpha\{f(z)\} = \int_0^\infty e^{-\wp^{1/\alpha} z} f(z)dz, \quad \alpha \in \mathbb{R}_0^+$
$\phi(\wp) = \wp, \sigma(\wp) = \wp^2$	Pourreza transform	$HJ\{f(z)\} = \wp \int_0^\infty e^{-\wp^2 z} f(z)dz$
$\phi(\wp) = \wp^2, \sigma(\wp) = \wp$	Mohand transform	$M\{f(z)\} = \wp^2 \int_0^\infty e^{-\wp z} f(z)dz$
$\phi(\wp) = 1/\wp^2, \sigma(\wp) = 1/\wp$	Sawi transform	$Sa\{f(z)\} = \frac{1}{\wp^2} \int_0^\infty e^{-\frac{z}{\wp}} f(z)dz$
$\phi(\wp) = 1, \sigma(\wp) = 1/\wp$	Kamal transform	$K\{f(z)\} = \int_0^\infty e^{-\frac{z}{\wp}} f(z)dz$
$\phi(\wp) = \wp^\alpha, \sigma(\wp) = 1/\wp$	G-transform	$G\{f(z)\} = \wp^\alpha \int_0^\infty e^{-\frac{z}{\wp}} f(z)dz$, α is integer

Priti Purohit,
School of Science and Technology,
Vardhman Mahaveer Open University, Kota.
E-mail address: `pritipurohitmath@vmou.ac.in`

and

Ravi Gupta,
School of Science and Technology,
Vardhman Mahaveer Open University, Kota.
E-mail address: `rgupta@vmou.ac.in`