



A Study on Radio Labeling of Some Triangle-Free Graphs

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ABSTRACT: Java program pseudo code is used to determine the radio number and radio gracefulness of certain triangle-free graphs viz., n -crown, Andrasfai graph, Grötzsch graph and Heawood graph.

Keywords: Radio graceful labeling, radio number, Grötzsch graph, Heawood graph, Andrasfai graph, n -crown graph.

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1. Introduction

The co-channel interference are crucial when designing radio networks in a telecommunications system. We try to assign radio network transmitters channels so that the network satisfies all interference constraints. Ale introduced the channel assignment problem, which involves assigning channels to the transmitters. Refer to [1, 4] for further information on radio channel assignment. In radio networks, the bandwidth allotted for the assignments determined by the frequency span, assuming that the frequencies are evenly distributed throughout the spectrum. In this instance, there is a strong correlation between the geographic location of the transmitters and the interference between them. In the past, radio network designers only took into account the two levels of interference—major and minor. If there is significant interference between two transmitters, they are categorized as very close transmitters; if there is little interference, they are classified as close transmitters. The interference graph is created and channel assignment is transformed into graph labeling in order to solve the channel assignment problem. The vertices in an interference graph represent the transmitters, and two vertices are connected by an edge if the corresponding transmitters have major interference, two vertices have minor interference, and corresponding vertices are at distance two if the transmitters are at distance three or farther away. Put differently, vertices that are two distances apart represent close transmitters, while adjacent vertices represent very close transmitters. Indeed, Roberts [2] suggested that two transmitters with minor interference will receive different channels, and two transmitters with major interference will receive channels that are at least two apart.

2. Literature Survey

Radio-labeling was introduced by Griggs and Yeh [3], where in channels are associated with non-negative integers. Motivated by these issues [2, 3], we calculate the radio number and radio gracefulness of a few triangle-free graphs in this paper. Finite, simple, undirected, and connected graphs are all taken into consideration in this paper. Let $E(G)$ and $V(G)$ represent the vertex and edge sets of G , respectively. The channel assignment problem for radio transmitters serves as the driving force behind

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radio labeling, also known as multilevel distance labeling. Radio labeling itself is a well-explored area with extensive research focusing on minimizing interference in network communications. For instance, further results on radio numbers in specialized graph constructions were provided by Baskar Mari et al. (2024), advancing theoretical bounds. Gallian's dynamic surveys consolidate these developments and discuss the landscape of graph labeling problems, including radio and Pell labeling variants.

3. Preliminaries

This section is devoted to recalling some basic notions on the concepts dealt with in the paper.

Definition 3.1 A function f from $V(G)$ to the set of positive integers which satisfies the criteria $|f(u) - f(v)| \geq 2$ if $d(u, v) = 1$ and $|f(u) - f(v)| \geq 1$ if $d(u, v) = 2$ is known as distance two labeling or $(L(2, 1)$ -labeling) of a graph G . The upper limit of $\{|f(u) - f(v)| : u, v \in V(G)\}$ is the span of f .

Definition 3.2 A function f from $V(G)$ to the set of positive integers that satisfies the following criteria is a distance -3 labeling of a graph G : $|f(u) - f(v)| \geq 3$ if $d(u, v) = 1$. $|f(u) - f(v)| \geq 2$ if $d(u, v) = 2$ and $|f(u) - f(v)| \geq 1$ if $d(u, v) = 3$.

Definition 3.3 A function $f: V(G) \rightarrow \{1, 2, \dots\}$ is an antipodal labeling of G with diameter $\text{diam}(G)$ if it follows the criteria: for every $u, v \in V(G)$, $\exists d(u, v) + |f(u) - f(v)| \geq \text{diam}(G)$. The maximum value of $\{|f(u) - f(v)| : u, v \in V(G)\}$ is the span of antipodal labeling. $\text{an}(G)$ represents the antipodal number for G , which is the minimum span of all antipodal labelings of G .

Definition 3.4 For a graph the number of edges defines its length.

Definition 3.5 The number of edges in the shortest or least path between any two vertices of a graph gives distance between them. There may be multiple shortest paths connecting two vertices.

Definition 3.6 The maximum distance between any pair of vertices in graph gives its diameter.

Definition 3.7 An undirected graph which does not contain a cycle C_3 as a sub graph is known as a triangle-free graph.

Definition 3.8 A radio labeling f of G assigns positive integers to the vertices of G in such a way that $|f(u) - f(v)| \geq \text{diam}(G) + 1 - d(u, v) \forall u, v \in V(G)$. The radio number $\text{rn}(G)$ is the smallest span of a radio labeling for G . When $\text{diam}(G)$ is two, the radio labeling and distance two labeling are the same. if $\text{rn}(G) = \text{number of vertices of } G$ then G is radio graceful.

Remark 3.1 let f be an optimum radio labeling of graph G . we can associate to f an ordering of the vertices of G , increasing by their labels. Denote $v_1, v_2, v_3, \dots, v_n$ the vertices of G in this order: $f(v_1) < f(v_2) < \dots < f(v_n)$. we have $f(v_1) = 1$ $\text{rn}(G) = \text{span}(f) = 1 + \sum_{i=1}^{n-1} (f(v_{i+1}) - f(v_i))$ if $f(v_{i+1}) - f(v_i) = 1$, then we must have $d(v_i, v_{i+1}) = \text{diam}(G)$.

4. Main Results

Most research papers in this field usually try to find the upper limits of the radio number or the exact value of the radio number that makes the graph work. As we start the valuation at 1, the usual lower bound for each of the graphs we look at here is 1. It is obvious that any modification of the lower bound will affect the optimality $\text{rn}(G)$ of the labeling. To demonstrate the radio graceful labeling of Grötzsch and Heawood graphs, we examined two conditions for the upper and lower limits. It is then clear that those limits are the same as the number of vertices. Additionally, to verify the radio labeling and antipodal labeling of the n -Crown graph, as well as the radio graceful labeling of the Andrásfai graph, we employed Java-based implementations.

Definition 4.1 The n -crown graph for an integer $n \geq 3$ is a complete bipartite graph $K_{n,n}$ with horizontal edges removed. Note that the term "crown graph" has also been used to refer to a sunlet graph $C_n \odot K_1$.

Theorem 4.1 For $n \geq 3$ in n -crown graph, $rn(G) = 4n - 2$.

Proof:

Let the diameter of crown graph is 3. And $f: V(G) \rightarrow \{1, 2, 3, \dots, 4n - 2\}$ where $f(u_i) = 4i - 3$ and $f(v_i) = 4i - 2$ for $1 \leq i \leq n$ such that $d(u, v) + |f(u) - f(v)| \geq 1 + diam(G)$ for all (u, v) .

then one can observe that the following results for $1 \leq i, j \leq n$

$d(u_i, u_j) = 2$ and minimum of $|f(u_i) - f(u_j)| = 4$ if $i \neq j$;

$d(v_i, v_j) = 2$ and minimum of $|f(v_i) - f(v_j)| = 4$ if $i \neq j$;

$d(u_i, v_j) = 1$ and minimum of $|f(u_i) - f(v_j)| = 3$ if $i \neq j$

and $d(u_i, v_j) = 3$ and minimum of $|f(u_i) - f(v_j)| = 1$ if $i = j$.

□

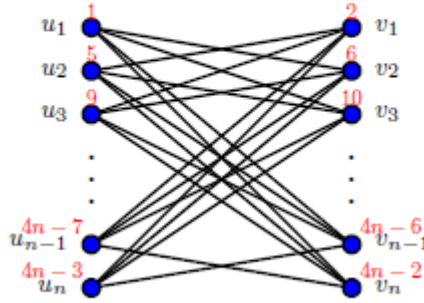


Figure 1: $rnH(n,n) = 4n-2$

Theorem 4.2 For $n \geq 3$ the radio antipodal number of n -Crown graph $2n$.

Proof:

Let $f: V(G) \rightarrow \{1, 2, \dots, 2n\}$ where $f(u_i) = i$ and $f(v_i) = 2n - i + 1$ for $1 \leq i \leq n$ such that, for all (u, v) $d(u, v) + |f(u) - f(v)| \geq diam(G)$. Thus, for $1 \leq i, j \leq n$, one observes the following results:

$d(u_i, u_j) = 2$ and $\min |f(u_i) - f(u_j)| = 1$ if $i \neq j$;

$d(v_i, v_j) = 2$ and $\min |f(v_i) - f(v_j)| = 1$ if $i \neq j$;

$d(u_i, v_j) = 1$ and $\min |f(u_i) - f(v_j)| = 2$ if $i \neq j$

and $d(u_i, v_j) = 3$ and $\min |f(u_i) - f(v_j)| = 1$ if $i = j$.

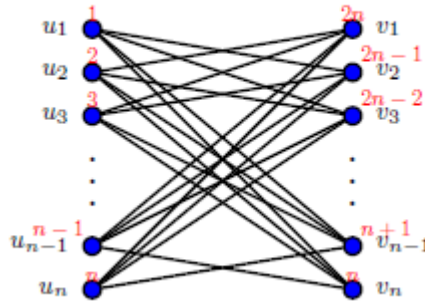
□

The following graphs represents the radio and antipodal labelings of n -crown graph.

Corollary 4.1 The n -crown graph satisfies the distance -3 labeling with the above labelings

Definition 4.2 For any natural number $n \geq 1$, the Andrásfai graph is a circulant graph on $3n - 1$ vertices and $(n(3n - 1))/2$ edges. $V(And(n)) = \{v_1, v_2, \dots, v_{3n-1}\}$, and $v_i, v_j \in E(And(n))$ if and only if $|i - j| \equiv 1 \pmod{3}$. So, $And(1) = K_2$, $And(2) = C_5$ and $And(3)$ is the Mobius ladder graph with 8 vertices or Wagner graph.

Remark 4.1 $And(1)$ is K_2 is radio graceful. $diam(K_2) = 1$. If $f(v_1) = 1$ and $f(v_2) = 2$, then f is an injective function and satisfies the relation $|f(v_1) - f(v_2)| \geq diam(K_2) + 1 - d(v_1, v_2)$. Hence, $And(1)$ is radio graceful.

Figure 2: $anH(n,n) = 2n$

Theorem 4.3 For $n \geq 2$, $And(n)$ is radio graceful and $rn(And(n)) = 3n - 1$.

Proof: The vertex and edge set of $And(n)$ is as follows:

$V(And(n)) = \{v_1, v_2, v_3, \dots, v_{3n-1}\}$ and $E(And(n)) = \{v_i v_j \text{ where } |i - j| \equiv 1 \pmod{3} \text{ for all } i, j\}$.

For $n \geq 2$ $diam(And(n)) = 2$, let $f: V(And(n)) \rightarrow \{1, 2, 3, \dots, 3n-1\}$. Then for $1 \leq i \leq 3n-1$ we come across with two cases:

Case (i): For odd n , one obtains $f(v_{2i-1}) = i$, for $1 \leq i \leq \frac{3n-1}{2}$ and $f(v_{2i}) = \frac{3n-1}{2} + i$, for $1 \leq i \leq \frac{3n-1}{2}$.

Case (ii): For even n one obtains $f(v_{2i-1}) = i$, for $1 \leq i \leq \frac{3n}{2}$ and $f(v_{2i}) = \frac{3n}{2} + i$, for $1 \leq i \leq \frac{3n}{2} - 1$.

To claim that the radio labeling of f is a valid one has to satisfy the radio condition $d(u, v) + |f(u) - f(v)| \geq 1 + diam(And(n)) = 3$. that holds for all pairs of vertices (u, v) where $u \neq v$. We now consider the pairs (v_i, v_j) to examine the label difference.

If $d(v_i, v_j) = 1$ then $|f(v_i) - f(v_j)| \geq 2$ and also if $d(v_i, v_j) = 2$ then $|f(v_i) - f(v_j)| \geq 1$ where $i \neq j$ and $1 \leq d(v_i, v_j) \leq 2$. Hence, the radio condition (1) is satisfied. \square

Theorem 4.4 a) The Grötzsch graph is radio graceful. b) The Heawood graph is radio graceful.

Proof:

To demonstrate that the values articulated in the theorem are lower bounds for the radio number, we will employ the concept from remark 2.1. Let f be an optimal radio labeling, and let $v_1, v_2, v_3, \dots, v_n$ are the vertices of the graph in the order of their labels. We examine the greatest quantity of pairs (v_i, v_{i+1}) such that $f(v_{i+1}) - f(v_i) = 1$. These pairs must have the property that $d(v_i, v_{i+1}) = diam(G)$ because of the radio condition. We will show that the claimed values are upper bounds for the radio numbers of the graphs whose radio labelings have spans equal to these values.

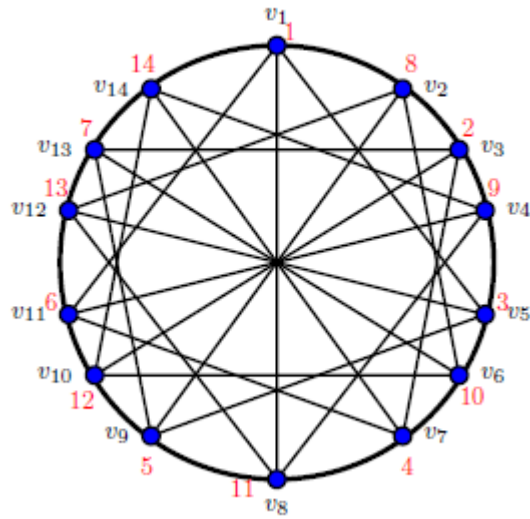


Figure 3: $And(5)$ is radio graceful

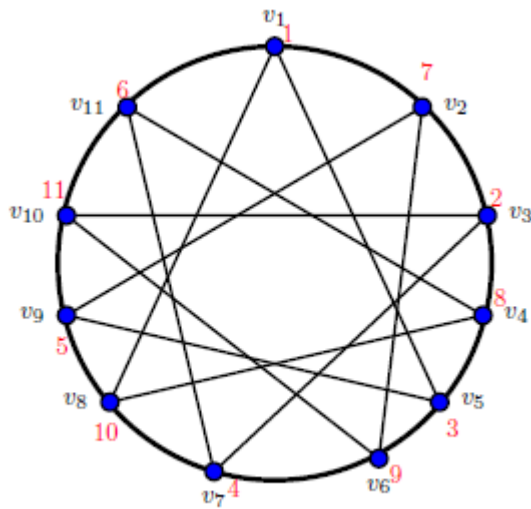


Figure 4: $And(4)$ is radio graceful

a) The Grötzsch graph, is triangle-free graph consisting of 11 vertices, 20 edges. Also, $rn(G) \geq |V(G)| = 11$. The following fig, shows that $rn(G) \leq 11$ for Grötzsch graph which implies that $rn(G) = 11$ and hence is radio graceful.

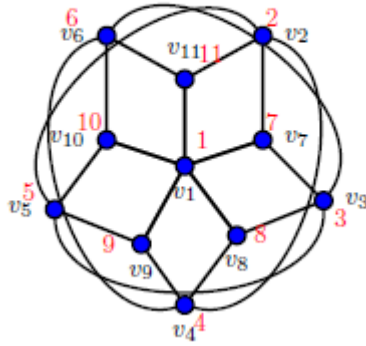


Figure 5: radio graceful labeling of Grötzsch graph

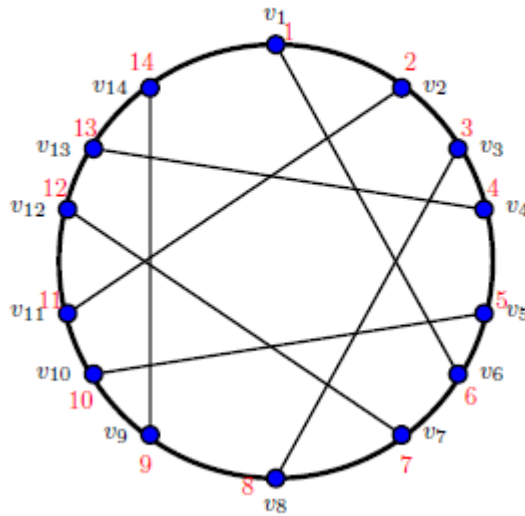


Figure 6: radio graceful labeling of Heawood graph

b) The Heawood graph, is a triangle-free graph consisting of 14 vertices, 20 edges. Also, $rn(G) \geq |V(G)| = 14$. The following fig, shows that $rn(G) \leq 14$ for Heawood graph which implies that $rn(G) = 14$ and hence is radio graceful.

□

5. Conclusion

In this paper the radio graceful labeling and radio number of selected triangle-free graphs is inspected through algorithmic analysis using pseudo code. For the Grötzsch and Heawood graphs, It is shown that both upper and lower bounds of radio labeling correspond to the number of vertices. The study also verifies radio and antipodal labeling for n-Crown graphs, and confirms radio graceful labeling for

the Andrásfai graph using Java-based implementations. These results contribute to the broader understanding of optimal labeling strategies in triangle-free graph classes and provide a foundation for further computational exploration. We can apply the concept of radio-labeling for different families of graphs.

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