Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2026 (44) 7** : 1–19.
ISSN-0037-8712
doi:10.5269/bspm.80473

Controlling Inflation and Deterioration in Single-Channel Supply Chains: A Ramp-Type Demand Model with Partial Backlogging*

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ABSTRACT: In today's dynamic and inflation-sensitive markets, new product firms commonly face an unpredictable customer demand problem, increasing operational costs, and risks associated with product deterioration. The paper develops a supply chain inventory model that captures these real-world challenges within a single-manufacturer–single-retailer framework. Customer demand is modelled using a ramp-type, time-dependent function that reflects the phased growth typical of new products in sectors such as electronics, fashion, and consumer durables. The production rate is considered as a demand-dependent, scalable operation. The model allows for constant deterioration, inflation in cost parameters, and partial backlogging, assuming only a portion of unmet demand to be backlogged and the rest to be lost in sales. A total cost function is developed for a finite planning horizon, and its convexity is established. On this basis, an optimization capacity is suggested as a way of finding the supply chain replenishment cycle that will reduce supply chain expenses. Sensitivity analysis and numerical examples are offered to evaluate the practical relevance of the model. The results give management information in order to formulate an adaptive and responsive policy of replenishment to inflation depending on the customer dynamics, product lifeways, and supply chain limitations.

Key Words: Two echelon supply chain, deterioration, backlogging, ramp type demand inflation.

Contents

* The project is partially supported by None.
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2020 *Mathematics Subject Classification:* 34B05, 65L06, 90-10.
Submitted November 29, 2025. Published April 11, 2026

1. Introduction

With the fast, highly competitive global sales market place of today, the supply chains are constantly put to the challenge of responding swiftly, correctly, and with low cost. This is no doubt felt more so in businesses which continuously release new products - consumer electronics, fashion, pharmaceuticals and fast-moving consumer goods (FMCG). The current new product markets are marked by brief product life cycle, ever shifting customer tastes and preferences, and needs that vary considerably with time. This is because the performance of the supply chain is largely dictated by how well a supply chain can anticipate and react effectively to these changes yet, it is one of the most challenging issues that the operations managers are grappling with currently. The demand of the new introduced products is likely to follow the shape of the ramp where the demand is low at the start but after gaining acceptance in the market the demand rises sharply and levels off or even decreases. This kind of two-step growth pattern is extremely widespread in the practice, such as the launch of the newest smartphone model, a new seasonal fashion line of dress, or a new over the counter pharmaceutical product. Lack of alignment between production and inventory decisions and these moving demand stages can be very costly and the effects of this may be in the form of the stockouts and even lost sales or even high inventory and product obsolescence. Another complication is the fact that the product is deteriorated. The degradation can be either of a physical-spoilage of the food or chemicals, such as that of perishable or fragile electronics or pharmaceuticals where the speed of obsolescence or expiry is very high. The costs related to deterioration effects may severely deform the estimates of the relevant costs and result in inappropriate managerial decisions. To these strains, supply chains must now more than ever undertake the operation under the pressure of inflation that distorts their total cost structures. Inflation impacts on all the key components of costs, which include raw materials, labour, warehousing, transportation, and capital expenditures as well and complicates costs control significantly. Longer or inadequately calculated inventory cycles in an inflationary environment will translate to very high procurement and holding costs and will eventually reduce the profitability margins and deteriorated the competitive standings of a company. Conventional inventory models commonly make use of either the assumption of a fixed or deflationary cost which are ineffective to account these reality cost dynamics and thus result in optimal decision making. Besides, customer responses to the shortages of products should be realistic. Actually not every customer would wait until the items that have been ordered back come; others will go to the competitors and others will merely submit their purchase orders to be cancelled. This reality can be reported only by partial backlogging, in which case only a fraction of the unsatisfied demand is backlogged and the rest is lost. Sale of products in high demand markets with a short selling period may induce a great loss in demand. The likelihood of loss of revenues where the time to sell a product is minimal also heightens customer impatience as well as competition alternatives.

1.1. Research Motivation and Contribution

This environment is becoming more complex and to operate efficiently in it, firms have to be able to use inventory models that extend beyond the simplifying assumptions posited in classical models. They must have the ability to include various realistic features-like ramp-type demand patterns, product deterioration, cost adjustments (inflation) and some backlogging simultaneously to obtain operation strategy to achieve the minimum total cost and bearable service level. The research is a response to this requirement by formulating a detailed and analytically manageable inventory model that integrates the following operational realities. The main features of the proposed model are as follows: the model combines a ramp-type time-dependent demand model that portrays the two-phase pattern of growth that is commonly observed in the introduction of a new product. A single-manufacture, single-retailer structure, representing many-world supply chains, especially in centralized or vertically integrated settings. A demand-dependent production rate, where production scales with demand, enabling flexible and responsive manufacturing. A Constant rate of deterioration, modelling items that lose value or usability over time, such as perishables, medicines, and electronics. Inflation-indexed cost parameters that consider increasing costs of production, storage, and shortages over the planning horizon. Partial backlogging to account for realistic customer behaviour in stockouts. These are incorporated in a finite planning horizon model with multiple deliveries per order cycle for finer and more realistic delivery scheduling.

1.2. Research Objective

The main aim of this study is to formulate a mathematical framework that calculates the ideal replenishment cycle time that optimizes the total supply chain cost. The model accounts for a finite planning horizon and continuous planning cycles. The total cost function encompasses procurement costs, holding charges, deterioration losses, inflationary impacts, and shortage and lost sales penalties. We suggest an optimization algorithm for determining the optimum cycle length that balances responsiveness with cost management. Numerical examples are given to confirm the model, and sensitivity analysis is performed to analyse how alterations in major parameters- e.g., inflation rate, deterioration rate, or demand ramp-up rate- impact the optimal solution. The research is effective in several practical and managerial contributions by considering the ramp-like demand, inflationary effect of costs, product degradation, and partial backlogging under one unified model. First, it provides a powerful planning instrument in the determination of optimal replenishment and delivery timeframes in settings that have inflation and product degradation. Second, the model assists managers in capacity planning and inventory management by taking into consideration nonlinear demand trend and time dependent demand as it applies to new product markets. Third, it provides an understanding of prices and procurement in a scenario of inflation where cost increases are radical and transform the plans of operations. Fourthly, realistic backlogging behaviour helps firms to maximize the level of customer service as well as reducing the revenue loss caused by stockouts. Lastly, the study will add to overall supply chain agility and affordability in the volatile market settings and help towards closing the gap in knowledge between theoretical modelling and practical implementation by offering a holistic framework of managing uncertainty, complexity, and financial variability throughout the supply chain.

2. Literature Review

Conventionally, the two key participants in a supply chain-manufacturers and retailers-have been rather independent of each other with the big inventories being viewed as a buffer against uncertainty. Nonetheless, with the increase in competition and the reduction in profitability, companies have been working towards having a coordinated and responsive supply chain. The existing studies determine a higher alignment of the manufacturing and retail processes in a retail sector particularly in regard to the establishment of consistency in the production rates, replacement policies, and inventory decisions to the changes in customer demand. All this reflects the general movement toward agility-focused supply chains, cost efficiency, and coordinated decision making to attain maximum overall performance. Coordinated decision-making across the supply chain allows firms to synchronize deliveries in a way that minimizes total integrated costs. **Clark and Scarf (1960)** were among the first to explore inventory management in a multi echelon supply chain context. They assumed a constant demand rate, a condition that typically holds during a product's mature stage. In contrast, during the growth or decline phases of a product's life cycle, demand is better represented by a linear function. Early contributions to this area include the works of **Resh et al. (1976)** and Donaldson (1977), who developed models incorporating linearly varying demand over time. Most literature on time-dependent demand in inventory systems explores two main patterns: (i) a linear increase or decrease in demand and (ii) exponential growth or decline, however, perpetual growth in demand is unrealistic. To address this, Hill (1995) introduced a model where demand increases initially and then stabilizes. Since then, several researchers have extended time-dependent demand analysis within EOQ/EPQ frameworks and multi-echelon systems. For example, Goyal and Gunasekaran (1995) have suggested an integrated framework for coordinating production, inventory and marketing model to find the optimal level of production and ordering of raw materials in a multi-echelon supply chain. But in actual situations- especially in developing economies where inflation is high- the influence of inflation and the time value of money become decisive forces that need to be factored into the decision-making process. Consequently, omitting these monetary considerations may result in unrealistic inventory choices. To meet this need, a number of researchers have extended classic models to include inflationary impacts. These early attempts in this direction include the publications of Buzacott (1975) and Mishra (1975), who separately proposed EOQ models assuming a uniform inflation rate on all cost items. Based on this, an EOQ model that included the discount rate and inflation was proposed by Bierman and Thomas (1977). This model was later improved upon by Mishra (1979) by assuming varied rates of inflation for individual cost components. Most recently, Chern et al. (2008) proposed inventory models with partial backlogging for deteriorating items under both fluctuating demand and inflationary environments. Research on inventory models that consider deterioration and shortages of products has also been getting more attention as they have significant applications in real life. Most products such as medicines, fruits and vegetables deteriorate over time and thus these considerations have become important for proper inventory management. Many researchers have contributed in this area over the past several decades. The original work of Ghare and Schrader (1963) presented an inventory model with exponential deterioration under conditions of constant demand. This was subsequently generalized by Covert and Philip (1973) previously a Weibull distribution to improve the deterioration process. Some of the subsequent developments include Wu's (2001) model, which combined ramp-type demand, Weibull distributed deterioration and Partial backlogging. Khanra and Chaudhuri (2003) used a quadratic time-dependent pattern of demand in order to prevent the unrealistic peaks in demand seen with exponential models. Manna and Chaudhuri (2006) formulated a production-inventory model with a ramp-type, two period demand pattern in which the rate of production is based on demand levels. Zhou et al. (2008) investigated coordination in a two echelon supply chain with one manufacturer and one retailer, where demand at the retail level was dependent on the inventory available. Skouri et al. (2009) provided a general inventory model covering ramp-type demand, Weibull deterioration and partial backlogging. They considered two cases: one initiating from a shortages and the other without. Singh and Singh (2010) studied a supply chain model with uncertain lead times, fuzzy ramp-type demand, and partial backlogging for perishable items. He et al. (2010) introduced a two -echelon inventory model for deteriorating products, in which the manufacturer ships products to multiple markets with different selling seasons. Taleizadah et al. (2012) more recently studied a multi-product, multi-buyer inventory model with a single vendor under chance constraints, uniform demand distribution, lot-size dependent lead times, and partial backlogging. Additional advancements are Singh et al. (2012), who studied shortages in an economic lot-size production model with rework operations and flexibility in operations. Sarkar (2012) generalized the traditional EOQ model by incorporating time-dependent demand, deteriorating materials, procurement discounts and delayed payment opportunities. Goyal et al. (2013) designed a production plan for products that get better or worse over time, under a ramp-type demand situation. Jaggi et al. (2018) presented an inventory model of deteriorating items of imperfect quality with the assumption of exponentially decreasing demand, trade credit, and partial backlogging. Based on this, **Vishnoi et al. (2025)** suggested an optimization-based model incorporating strategies for sustainable items with deteriorating items, including market price, stock-dependent demand, varying production rates, and controllable carbon emissions. The model presented considers maximizing coordination between the manufacturer and the retailer in a two-echelon supply chain context. One of the salient aspects of the model is that it allows for different deterioration rates for various products, knowing that deterioration doesn't take place at the same level for all products. The production and demand rates are also assumed to be ramp-type representing a more practical scenario where these rates change gradually over time. The model also captures the inflation effects and includes partial backlogging of unsatisfied demand, and this improves the applicability of the model in dynamic and inflation-prone market conditions.

3. Assumptions and Notations

To develop the model, the following assumptions and notation are considered.

3.1. Assumptions

1. The model entails one manufacturer supplying to one retailer.

2. Production rate *P(t)* is demand dependent and demand rate *d(t)* is a ramp type function of time given by

$$d(t) = \begin{cases} f(t), & t < \mu, \\ f(\mu), & t \geq \mu. \end{cases}$$

where *f(t)* is a positive continuous function of

$$t \in (0, T]$$

and defined as

$$f(t) = ae^{bt}$$

and *P=kd(t)* where

$$a,\, b > 0, \quad k > 1$$

.

3. The process of deterioration happens at a steady rate, and no deteriorated unit is repaired or replaced in the same cycle.
4. The store may partially backorder unsatisfied demand, which is satisfied during the next delivery cycle.
5. The model incorporates the impact of inflation over time.
6. In each ordering cycle, there are several deliveries and a finite planning horizon with constant cycles. Only one cycle is modelled, so the first delivery is taken to be from the previous cycle.
7. The process of supplying is taken to be instantaneous, and the system only handles one kind of item.

3.2. Notation

Following table represent the notations

Table 1: Notations

Symbol	**Description**
B	Ratio of retailer's demand back-ordered
r	Inflation rate
Q_m	Amount of finished product manufactured by the manufacturer per production cycle
Q_r	Amount received by the retailer from the manufacturer per delivery
θ_2	Deterioration rate at manufacturer
θ_3	Rate at which the retailer's inventory deteriorates
n	Number of deliveries per order
$q_m i(t1)$	Manufacturer's finished product inventory level at any point in time t
$q_r i(t1)$	Level of the retailer's finished goods inventory at any particular time t
C_{1m}	Cost to manufacturer for placing and setting up an order per cycle
C_{2m}	Manufacturer's cost per unit of finished goods per unit time for holding
C_{1r}	Retailer's cost to set up an order per cycle
C_{2r}	Retailer's cost per unit of finished goods per unit time for holding
C_3	Cost per unit time for the retailer to back up demand
C_4	Cost to retailer per unit of unmet demand due to lost sales
C_m	Manufacturer's unit cost of finished goods
C_r	Retailer unit cost of finished goods
MI_m	Manufacturer's maximum capacity for finished goods in inventory
MI_r	Retailer's maximum capacity for finished goods in inventory
TC_m	Manufacturer's total cost per unit time, present value adjusted
TC_r	Retailer's total cost per unit time, present value adjusted
TC	Total cost per unit time, present value adjusted

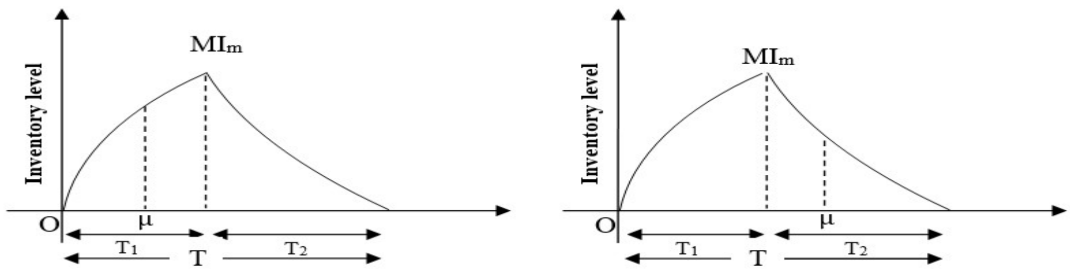

Figure 1 (a) -Manufacture Production inventory system Case -I **Figure 1(b)-Manufacture Production Inventory system Case-II**

Figure 1: Manufacturer Inventory Model

4. Model Development

The research is based on cooperation between manufacturer and retailer, simulated as a two-stage supply chain. The first stage refers to the manufacturer's production system. The raw materials from outside suppliers are procured by the manufacturer, who then converts them into finished goods, which are delivered in fixed-size batches to the retailer at fixed intervals.

4.1. Manufacturer's finished goods inventory system

The manufacture's inventory system, as illustrated in Fig. 1(a) and Fig.1 (b), can be divided into two independent phases, denoted by T_1 and T_2 . This decomposition simplifies the problem, as well as its analytical derivation and evaluation. Each phase operates within its own time frame t_1 , which begins at the start of the respective phase T_1. During the time period inventory builds up, and as a result, deterioration becomes effective. At the $t_1 = T_1$ the production stops, and the inventory level reaches its maximum, denoted by MI_m . During the subsequent time period T_2 , no production occurs; the inventory level gradually decreases due to both customer demand and deterioration, eventually reaching zero at the end of $t_2 = T_2$. The manufacturer's finished goods inventory system, as shown in Fig. 1 (a) and Fig. 1 (b), can be described at any time t by the following differential equations

$$\frac{dq_{mi}(t_1)}{dt_1} = P(t_1) - d(t_1) - \theta_2\, q_{mi}(t_1) \quad where \quad 0 \le t_1 \le T_1 \tag{4.1}$$

$$\frac{dq_{mi}(t_2)}{dt_2} = -d(t_2) - \theta_2\, q_{mi}(t_2) \quad where \quad 0 \le t_2 \le T_2 \tag{4.2}$$

with boundary conditions $q_{mi}(0) = 0$ and $q_{mi}(T_2) = 0$. There are two possible relations between parameters T_1 and μ;

1. $0 \le \mu \le T_1$
2. $T_1 \le \mu \le T$

Each case results in different ordering, holding cost, and deterioration cost. The details of each case are discussed separately below:

Case I $0 \le \mu \le T_1$

$$\frac{dq_{m_1}(t_1)}{dt_1} = (k-1)ae^{bt_1} - \theta_2 q_{m_1}(t_1) \quad where \quad 0 \le t_1 \le \mu \tag{4.3}$$

$$\frac{dq_{m_2}(t_1)}{dt_1} = (k-1)ae^{b\mu} - \theta_2 q_{m_2}(t_1) \quad where \quad \mu \le t_1 \le T_1 \tag{4.4}$$

$$\frac{dq_{m_3}(t_2)}{dt_2} = -ae^{b\mu} - \theta_2 q_{m_3}(t_2) \quad where \quad 0 \le t_2 \le T_2 \tag{4.5}$$

with boundary conditions $q_{m_1}(0) = 0$, $q_{m_1}(\mu_-) = q_{m_2}(\mu_+)$, and $q_{m_3}(T_2) = 0$ the solutions of equations (3) to (5) are

$$q_{m1}(t_1) = \frac{(k-1)a}{(b+\theta_2)}\left(e^{bt_1} - e^{-\theta_2 t_1}\right) \quad where \quad 0 \le t_1 \le \mu \tag{4.6}$$

$$q_{m2}(t-1) = \frac{(k-1)ae^{\mu b}}{\theta_2} - \frac{(k-1)a}{\theta_2(b+\theta_2)}\left(b\,e^{-(b+\theta_2)t_1} - \theta_2\right)e^{-\theta_2 t} \quad where \quad \mu \le t_1 \le \ T_1 \tag{4.7}$$

$$q_{m3}(t_2) = \frac{ae^{\mu b}}{\theta_2}\left(e^{\theta_2(T_2-t)} - 1\right) \quad where \quad 0 \le t_2 \le \ T_2 \tag{4.8}$$

Based on Fig. 1 (a), the maximum inventory level of finished goods is$Mq_{m1} = q_{m3}(0)$

$$Mq_{m1} = \frac{ae^{\mu b}}{\theta_2}\left(e^{aT_2} - 1\right) \approx ae^{\mu b}\left(T_2 + \frac{1}{2}\theta_2 T_2^2\right) \tag{4.9}$$

The production quantities are

$$Q_{m1} = \int_0^{T_1} P(t)\,dt = \int_0^{\mu} P(t)\,dt + \int_{\mu}^{T_1} P(t)\,dt$$

$$Q_{m1} = ka\left(T_1 + bT_1\mu - \frac{1}{2}b\mu^2 + \frac{1}{2}b^2T_1\mu^2 - \frac{1}{2}b^2\mu^3\right) \tag{4.10}$$

1. A set up cost of production, $c1m$ is incurred at the start of each production cycle. As this expense is already paid in the beginning of the cycle, its current value is the same. The current value of set-up cost therefore is:

$$SE = c_{1m} \tag{4.11}$$

2. There is inventory at time period T1 and T2. Here, the cost of inventory of the retailer should not be considered by the system because the holding cost of manufacturers will be regarded only. Maximum present value of the holding cost per cycle is found by considering the inventory holding costs at T1 and T2, including the deterioration and present value of money. The whole of the holding costs is associated with the manufacture; this is the first and second term in equations (12) and (13). In case this system takes into account the retailers, the holding cost of the items that are shipped to the retailer goes to the retailer. It ought to be deducted on manufacturer. It is the last term in eq. (12) and eq. (13). The current value holding cost is (when $0 \le \mu_1 \le \ T_3$)

$$HD_{m1}^1 = C_{2m}\int_0^{T_1} q_{m1}(t_1)e^{-rt_1}\,dt_1 + C_{2m}\int_0^{T_2} q_{m2}(t_2)e^{-r(T_1+t_2)}\,dt_2$$
$$-\left\{C_{2m}\int_0^{T_3} q_r(t_3)e^{-rt_3}\,dt_3\right\}\sum_{j=0}^{(n-1)} e^{-irT_5}$$

(From sec. 4.2, case-III, eq. 40)

$$HD_{m1}^1 = C_{2m}\left[\frac{1}{2}(k-1)a\mu^2 + \frac{1}{2}(k-1)a\left(T_1^2 - \mu^2\right)e^{-r_1\mu} + aT_1^2e^{-r_1\mu}\right]$$
$$-nC_{2m}\left[a\mu_1\{T_3 + \mu_1(b+\theta_3)(T_3-\mu_1)\} + aT_3(T_3-\mu_1)(1+b\theta_1)\right] \tag{4.12}$$

when $T_5 \ge \mu_1 \ge T_3$ (Third term is from sec. 3.2, case-IV and eq. 54)

$$HD^2_{m1} = C_{2m}\left[\int_0^{t_1} q_{m1}(t_1)e^{-rt_1}\,dt_1 + \int_{t_1}^{\infty} q_{m1}(t_1)e^{-r(t_1+\tau)}\,dt_1\right]$$

$$+C_{2m}\int_0^{t_2} q_{m2}(t_2)e^{-r(T_1+t_2)}\,dt_2 - \left\{C_{2m}\int_0^{T_3} q_r(t_3)e^{-rt_3}\,dt_3\right\}\sum_{i=0}^{n-1} e^{-irT_5}$$

$$HD^2_{m1} = C_{2m}\left[\frac{1}{2}(k-1)a\mu^2 + \frac{1}{2}(k-1)a\left(T_1^2-\mu^2\right)e^(b-r)\mu + aT_2^2e^{b\mu}e^{-rT_1}\right] \tag{4.13}$$

3. The item cost accounts for both the cost of items sold and the loss due to deterioration. Since production is initiated at $t_1 = 0$, the present worth of the item cost is not affected by discounting. Therefore, the present worth of the item cost can be expressed as

$$IT_{m1} = c_m Q_{m1} \approx c_m ka(T_1 + b\mu T_1 - b\mu^2) \tag{4.14}$$

Consequently, the current value of the total expenditure incurred at each one cycle is the set- up cost (SE), holding cost (HDm), and item cost (ITm). Therefore, the current value total cost per cycle is obtained as:

$$TC^1_{m1} = \frac{1}{T}\left(SE + HD^1_{m1} + IT_{m1}\right) \quad when \quad 0 \le \mu \le T_1 \quad and \quad 0 < \mu_1 \le T_3 \tag{4.15}$$

$$TC^2_{m1} = \frac{1}{T}\left(SE + HD^2_{m1} + IT_{m1}\right) \quad when \quad 0 \le \mu \le T_1 \quad \text{and} \quad T_3 \le \mu_1 \le T_5 \tag{4.16}$$

Case II ($T_1 \le \mu \le T$)
In this case, the eq. (1) and (2) become;

$$\frac{dq_{m_1}(t_1)}{dt_1} = (k-1)ae^{bt_1} - \theta_2 q_{m_1}(t_1) \quad where \quad 0 \le t_1 \le\ T_1 \tag{4.17}$$

$$\frac{dq_{m2}(t_2)}{dt_2} = -ae^{bt_2} - \theta_2\, q_{m2}(t_2) \quad where \quad T_1 \le t_2 \le \mu \tag{4.18}$$

$$\frac{dq_{m_3}(t_2)}{dt_2} = -ae^{b\mu} - \theta_2 q_{m_3}(t_2) \quad where \quad \mu \le t_2 \le T \tag{4.19}$$

with boundary conditions $q_{m_1}(0) = 0$, $q_{m_2}(\mu_-) = q_{m_3}(\mu_+)$ and $q_{m_3}(T) = 0$ the solutions of equations (17) to (19) are

$$q_{m1}(t) = \frac{(k-1)a}{(b+\theta_2)}\left(e^{bt_1} - e^{\theta_2 t_1}\right) \quad where \quad 0 \le t_1 \le T_1 \tag{4.20}$$

$$q_{m2}(t_1) = \frac{ae^{(b+\theta_2)\mu}}{\theta_2(b+\theta_2)}\left\{(b+\theta_2)e^{(T_2-\mu)\theta_2} - b\right\}e^{-\theta_2 t_1} - \frac{ae^{bt_2}}{(b+\theta_2)} \quad where \quad T_1 \le t_2 \le \mu \tag{4.21}$$

$$q_{m3}(t_2) = \frac{ae^{b\mu}}{\theta_2}\{e^{\theta_2(T_2-t_2)} - 1\} \quad where \quad \mu \le t_2 \le\ T \tag{4.22}$$

Based on Fig. 1(b), the maximum inventory level of finished goods occurs at the end of the production period T_1, just before production stops. This level can be expressed as: $Mq_{m2} = q_{m1}(T_1)$

$$M_{q_2} = \frac{(k-1)a}{(b+\theta_2)}\left\{e^{bT_1} - e^{-\theta_2 T_1}\right\} \approx (k-1)a\left(T_1 + \frac{1}{2}(b-\theta_2)T_1^2\right) \tag{4.23}$$

The production quantities are

$$Q_{m_2} = \int_0^{T_1} P(t)\,dt = \int_0^{T_1} kae^{bt}\,dt \approx ka\left(T_1 + \frac{1}{2}bT_1^2\right) \tag{4.24}$$

(when $0 \leq \mu_1 \leq T_3$) (Third term is from sec. 4.2, case-III and eq.40)

$$HD^1_{m_2} = c_{2m}\int_0^{T_1} q_{m_1}(t_1)e^{-rt_1}\,dt_1 + c_{2m}\int_0^{T_2} q_{m_2}(t_2)e^{-r(T_1-t_2)}\,dt_2 - \left\{c_{2m}\int_0^{T_3} q_r(t_3)e^{-rt_3}\,dt_3\right\}\sum_{i=0}^{n-1} e^{-irT_5}$$

$$\begin{aligned} HD^1_{m_2} &= c_{2m}\left[\frac{1}{2}(k-1)aT_1^2 + a(\mu - T_1)e^{-rT_1}\left\{T_2 + \mu(b+\theta_1)(T-\mu)\right\} + aT_2(T-\mu)e^{\mu(b-r)}\right] \\ &\quad - nc_{2m}\left[a\mu_1\left\{T_3 + \mu(b+\theta_3)(T_3-\mu_1)\right\} + aT_3(T_3-\mu)(1+b\mu_1)\right] \end{aligned} \tag{4.25}$$

(When $T_3 \leq \mu_1 \leq T_5$) (Third term is from sec. 4.2, case-IV and eq. (54))

$$HD^1_{m_2} = c_{2m}\int_0^{T_1} q_{m_1}(t_1)e^{-rt_1}\,dt_1 + c_{2m}\int_0^{T_2} q_{m_2}(t_2)e^{-r(T_1+t_2)}\,dt - \left\{c_{2m}\int_0^{T_3} q_r(t_3)e^{-rt_3}\,dt_3\right\}\sum_{i=0}^{n-1} e^{-irT_5}$$

$$\begin{aligned} HD^2_{m_2} &= c_{2m}\left[\frac{1}{2}(k-1)aT_1^2 + a(\mu - T_1)e^{-rT_1}\left\{T_2 + \mu(b+\theta_2)(T-\mu)\right\} + aT_2(T-\mu)e^{\mu(b-r)}\right] \\ &\quad - \pi ac_{2m}\left[T_3^2 + \frac{1}{2}(b+\theta_3)T_3^2\right] \end{aligned} \tag{4.26}$$

$$IT_{m2} = c_m \cdot Q_{m2} \approx c_m k\alpha\left(T_1 + \frac{1}{2}bT_1^2\right) \tag{4.27}$$

$$TC^1_{m2} = \frac{1}{T}\left(SE + HD^1_{m2} + IT_{m2}\right) \quad when \quad T_1 \leq \mu \leq T \quad and \quad 0 \leq \mu_1 \leq T_3 \tag{4.28}$$

$$TC^2_{m2} = \frac{1}{T}\left(SE + HD^2_{m2} + IT_{m2}\right) \quad when \quad T_1 \leq \mu \leq T \quad and \quad T_3 \leq \mu_1 \leq T_5 \tag{4.29}$$

4.2. Retailer's finished goods inventory system (for $0 \leq \mu_1 \leq T_3$)

Fig. 2(a) and Fig 2(b) show the change in the inventory level of the retailer. Since $P > d$, It is assumed that the first delivery is made at the start of the inventory cycle of the retailer since. Some amount of the delivered stock is consumed in serving the already received back-orders leaving a balance of MI_r units as the opening inventory. The inventory quantity of the retailer reduces within the time period T_3, because of retailer demand and item degradation. When inventory level is zero, then the end level is equal to zero. This is followed by shortages in the later period T_4 -some of the unmet demand is stocked up and the rest is lost sales. The backlogged demand is only met within the subsequent replenishment cycle. n deliveries are carried out in the overall time $T = T_1 + T_2$ of retailer cycle.
The retailer's inventory system from Fig. 2 (a) and Fig. 2 (b) at any time t can be represented by the following differential equation; 2.

$$\frac{dq_{ri}(t)}{dt} = -d(t) - \theta_3 q_{ri}(t) \quad when \quad 0 \leq t \leq T_5 \tag{4.30}$$

with boundary condition There are two possible relationships among the parameters T_3, T_5 and μ:

1. $0 \leq \mu_1 \leq T_3$
2. $T_3 \leq mu_1 \leq T_5$

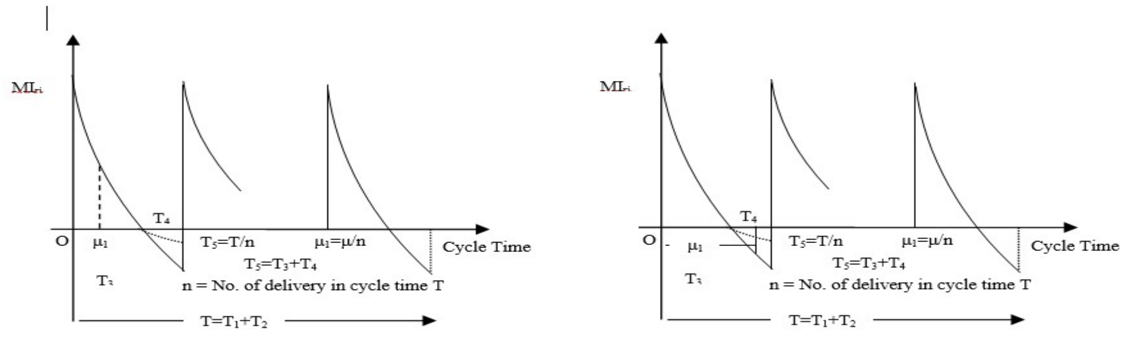

Figure 2 (a) -Consumption inventory system **Figure 2(b)- Consumption inventory system**

Figure 2: Retailer Inventory Model

Each case results in different expressions for ordering cost, holding cost and deterioration cost in the retailer's inventory system. These scenarios are analysed separately in the following sections:

Case III $(0 \le \mu_1 \le T_3)$
In this case, Eq. (30) becomes

$$\frac{dq_{r1}(t_3)}{dt_3} = -ae^{bt_3} - \theta_3 q_{r1}(t_3) \quad when \quad 0 \le t_3 \le \mu_1 \tag{4.31}$$

$$\frac{dq_{r2}(t_3)}{dt_3} = -ae^{b\mu_1} - \theta_3 q_{r2}(t_3) \quad when \quad \mu_1 \le t_3 \le T_3 \tag{4.32}$$

$$\frac{dq_{r3}(t_4)}{dt_4} = -Bae^{b\mu_1} \quad when \quad 0 \le t_4 \le T_4 \tag{4.33}$$

with boundary conditions $q_{r2}(T_3) = 0$ and $q_{r3}(0) = 0$; The solutions of equations (31) to (33) are

$$q_{r1}(t_3) = \frac{ae^{(b+\theta_3)\mu_1}}{\theta_3(b+\theta_3)} \left\{(b+\theta_3)e^{(T_3-\mu_1)\theta_3} - b\right\} e^{-\theta_3 t_3} - \frac{ae^{bt_3}}{(b+\theta_3)} \quad when \quad 0 \le t_3 \le \mu_1 \tag{4.34}$$

$$q_{r2}(t_3) = \frac{ae^{b\mu_1}}{\theta_3} \left\{e^{\theta_3(T_3-t_3)} - 1\right\} \quad when \quad \mu_1 \le t_3 \le T_3 \tag{4.35}$$

$$q_{r3}(t_4) = -Bae^{b\mu_1} t_4 \quad when \quad 0 \le t_4 \le T_4 \tag{4.36}$$

Based on Fig. 2 (a), the retailer's maximum inventory level is $Mq_{r1} = q_{r1}(0)$

$$Mq_{r2} = \frac{ae^{(b+\theta_3)\mu_1}}{\theta_3(b+\theta_3)} \left\{(b+\theta_3)e^{(T_3-\mu_1)\theta_3} - b\right\} e^{-\theta_3 t_3} - \frac{a}{(b+\theta_3)} \tag{4.37}$$

The quantity per delivery to the retailer is

$$Q_{r1} = Mq_{r1} + Bae^{b\mu_1} T_4 \tag{4.38}$$

1. Each delivery incurs an initial ordering cost, c_{1r}, at the start of the delivery cycle. Since this cost is paid upfront, its present worth remains unchanged. Therefore, the present worth of the ordering cost is:

$$OR = c_{1r} \tag{4.39}$$

2. Inventory is held during the time period T_3. The present worth of the holding cost during this period accounts for the time value of money and deterioration of items. It is given by:

$$HD_{r1} = c_{2r} \int_0^{T_3} q_{ri}(t_3)e^{-rt_3}dt_3 = c_{2r}\left[\int_0^{\mu_1} q_{r1}(t_3)e^{-rt_3}dt_3 + \int_{\mu_1}^{T_3} q_{r2}(t_3)e^{-rt_3}dt_3\right]$$

$$HD_{r1} = c_{2r}\left[a\mu_1\{T_3 + \mu_1(b+\theta_3)(T_3-\mu_1)\} + ae^{b\mu_1}(T_3-\mu_1)\left(T_3 + \frac{1}{2}\theta_3 T_3^2\right)\right] \tag{4.40}$$

3. Shortage occurs during time period T_4. The present worth backlog cost is

$$BA_1 = c_3 \int_0^{T_4} \left[-q_{r3}(t_4)\right]e^{-r(T_3-t_4)}\,dt_4 \approx Bae^{b\mu_1}c_3\left[\frac{1}{2}\left(1-rT_3\right)T_4^2 - \frac{1}{3}rT_4^3\right]. \tag{4.41}$$

4. Lost sale occurs during the time period T_4. In this period, the total shortage is denoted by $\int_0^{T_4} d(t)\,dt$ and a portion $\int_0^{T_4} Bd(t)\,dt$ of this shortage is backlogged. The difference between the total shortage and the backlogged portion during this time period, The present worth lost sale cost is

$$LS_1 = c_4 \int_0^{T_4} \left\{d(t) - Bd(t)\right\}e^{-r(T_4+t_4)}\,dt_4 \approx (1-B)\,ae^{b\mu_1}\,c_4\left[(1-rT_4)T_4 - \frac{1}{2}rT_4^2\right]. \tag{4.42}$$

5. The item cost includes both the cost of items sold and the loss due to deterioration. Since orders are placed at $t=0$ and $t = T_3 + T_4$, the present worth of the item cost must account for the time value of money at these two ordering points. Thus, the present worth of the item cost is:

$$q_{T_r1} = c_r M q_{r_1} + c_r BaT_4 e^{b\mu_1} e^{-r(T_3+t_4)}.$$

$$q_{T_r1} \approx c_r\left[a\left\{T_3 + \mu(b+\theta_3)(T_3-\mu_1)\right\} + Bae^{b\mu_1}T_4\left(1 - r(T_3+T_4)\right)\right]. \tag{4.43}$$

The present worth total cost incurred during a single delivery cycle is the sum of the ordering cost (OR), the holding cost (HD_r), the backlog cost (BA), lost sale cost (LS) and the item cost (IT_r). therefore, the present worth total cost per delivery is given by:

$$TC_r' = \frac{1}{T}\left(OR_r + HD_{r1} + BA_1 + LS_1 + IT_{r1}\right) \tag{4.44}$$

There are n deliveries per cycle, with a fixed time interval between deliveries is given by $T_5 = T/n$. Since the costs of each delivery are incurred at different times, their present worth must be discounted accordingly. Let TC_r be the present worth cost of a single delivery. Then, the present worth total cost per cycle at $t=0$ is the sum of the discounted costs of all n deliveries:

$$TC_{r1} = TC_r' \sum_{i=0}^{n-1} e^{-irT_5} = \left(\frac{OR_r + HD_{r1} + BA_1 + LS_1 + IT_{r1}}{T}\right)\left(\frac{1-e^{-rT}}{1-e^{-rT_5}}\right) \tag{4.45}$$

Case IV $\quad (T_3 \le \mu_1 \le T_5)$
In this case, Eq. (30) becomes

$$\frac{dq_{r1}(t_3)}{dt_3} = -ae^{bt_3} - \theta_3 q_{r1}(t_3) \quad when \quad 0 \le t_3 \le T_3 \tag{4.46}$$

$$\frac{dq_{r2}(t_4)}{dt_4} = -Bae^{bt_4} \quad when \quad T_3 \le t_3 \le \mu_1 \tag{4.47}$$

$$\frac{dq_{r3}(t_4)}{dt_4} = -Bae^{b\mu_1} \quad when \quad \mu_1 \le t_4 \le T_5 \tag{4.48}$$

with boundary conditions $q_{r1}(T_3) = 0$ and $q_{r2}(0) = 0$ and $q_{r2}(\mu_1) = q_{r3}(\mu_1)$; The solutions of equations (46) to (48) are

$$q_1(t_3) = \frac{ae^{(b+\theta_3)T_3}}{\theta_3(b+\theta_3)}e^{-\theta_3 t_3} - \frac{ae^{bt_3}}{(b+\theta_3)} \quad when \quad 0 \le t_3 \le T_3 \tag{4.49}$$

$$q_{r2}(t_4) = \frac{Ba}{b}\left(1 - e^{bt_4}\right) \quad when \quad T_3 \le t_3 \le \mu_1 \tag{4.50}$$

$$q_{r3}(t_4) = Bab\,\mu_1^2 - Bat_4\,e^{\beta\mu_1} \quad when \quad \mu_1 \le t_4 \le T_5 \tag{4.51}$$

Based on Fig. 2 (b), the retailer's maximum inventory level is $Mq_{r2} = q_{r1}(0)$

$$M_{q_2} = a\left[T_3 + \frac{1}{2}(b+\theta_3)\,T_3^2\right] \tag{4.52}$$

The quantity per delivery to the retailer is

$$Q_{r2} = Mq_{r2} + Ba\left[(\mu_1 - T_3) + \frac{1}{2}\,b\left(\mu_1^2 - T_3^2\right) + e^{b\mu_1}\,(T_5 - \mu_1)\right] \tag{4.53}$$

$$HD_{r2} = c_2 r\int_0^{T_3} q_{ri}(t_3)\,e^{-rt_3}\,dt_3 = ac_{2r}\left[T_3^2 + \frac{1}{2}(b+\theta_3)\,T_3^2\right] \tag{4.54}$$

$$BA_2 = c_3\int_0^{T_4}\left[-q_{r3}(t_4)\right]e^{-r(T_3+t_4)}\,dt_4 = c_3\int_{T_3}^{\mu_1}\left[-q_{r3}(t_4)\right]e^{-r(T_3+t_4)}\,dt_4 + c_3\int_{\mu_1}^{T_5}\left[-q_{r3}(t_4)\right]e^{-r(T_3+t_4)}\,dt_4$$

$$\begin{aligned} BA_2 \approx c_3\Bigg[&Bae^{-rT_3}\left(\mu_1^2 - T_3^2\right) + Bae^{b\mu_1}e^{-rT_3}\left\{\frac{1}{2}\left(T_5^2 - \mu_1^2\right) - \frac{1}{3}r\left(T_5^3 - \mu_1^3\right)\right\} \\ &-Bab\mu_1^2(T_5 - \mu_1)\,(1 - rT_5 - r\mu_1)\Bigg] \end{aligned} \tag{4.55}$$

$$LS_2 = c_4\int_0^{T_4}\{d(t) - Bd(t)\}\,e^{-r(T_3+t_4)}\,dt_4 \approx (1-B)\,ae^{bT_3}\,c_4\left[T_5 - T_4 + b\mu_1(T_5 - \mu_1)\right] \tag{4.56}$$

$$qT_{r2} = c_r Mq_{r2} + c_r e^{-r(T_3+t_4)}\int_0^{T_4} Bd(t)\,dt$$

$$qT_{r2} \approx c_r\left[a\left\{T_3 + \frac{1}{2}(b+\theta_3)T_3^2\right\} + Ba\{T_5 - T_3 + b\mu_1(T_5 - \mu_1)\}\{1 - r(T_3 + T_4)\}\right] \tag{4.57}$$

The total cost of present worth incurred every time an order is delivered consists of the following variables: The ordering cost (OR), the holding cost (HD_r), the backlog cost (BA), the lost sale cost (LS) and the item cost (IT_r). The current value of the overall cost per delivery is indicated by the presence cost as:

$$TC_r'' = \frac{1}{T}(OR_r + HD_{r2} + BA_2 + LS_2 + IT_{r2}) \tag{4.58}$$

The number of deliveries per cycle is n and a constant time period between deliveries has been specified as: $T_5 = T/n$. Consequently at $t = 0$ the present value of the total cycle cost is.

$$TC_{r2} = TC_r''\sum_{i=0}^{n-1} e^{-irT_s} = \left(\frac{OR_r + HD_{r2} + BA_2 + LS_2 + IT_{r2}}{T}\right)\cdot\left(\frac{1 - e^{-rT}}{1 - e^{-rT_s}}\right) \tag{4.59}$$

It is evident that the results presented in Section 3 are insufficient to derive the total system cost function for this case. Therefore, determining the complete system cost requires a more detailed examination of the relationships among the key time parameters: T_1, T_2, T_3, T_4, T_5, T, μ_1, μ. By analyzing these interdependence, we ensure an accurate formulation of the system dynamics.

5. The Optimal Replenishment Policy

The results in previous sub-section lead to the following total system cost function over time [0, T];
Case A: $0 \leq \mu_1 \leq T_3$

$$TC_h = \begin{cases} TC_1 & 0 < \mu \leq T_1 \quad (a) \\ TC_2 & T_1 \leq \mu < T \quad (b) \quad whwre \quad TC_1 = TC^1_{m1} + TC_{r1} \quad and \quad TC_2 = TC^1_{m2} + TC_{r2} \end{cases} \tag{5.1}$$

Case B: $0 \leq \mu_1 \leq T_3$

$$TC_g = \begin{cases} TC_3 & 0 < \mu \leq T_1 \quad (a) \\ TC_4 & T_1 \leq \mu < T \quad (b) \quad whwre \quad TC_3 = TC^2_{m1} + TC_{r1} \quad and \quad TC_4 = TC^2_{m2} + TC_{r2} \end{cases} \tag{5.2}$$

and the problem is $MinTC(T_1^*)$
We have approximated the exponential function by retaining terms up to the second degree, allowing for a tractable analytical model. An approximate model involving a single retailer and single manufacturer is developed to derive the optimal production policy and lot size. Given the condition $T = T_1 + T_2$; $T_s = T/n$ and assumed that $T_4 = \alpha T_3$ where $0 < \alpha < 1$. The solution requires analysing each branch of the function separately. The results are then combined to determine the optimal policy over the entire domain. It can be verified that the total cost function $TC(T_1)$, is continuous at the point μ. To find the optima value of T_1, The first order condition for a minimum of the cost function in the first case $TC_1(T_1)$ is $\frac{dTC_1(T_1)}{dT_1} = 0$

$$are^{b\mu}c_{2m}T_1^2 + \left[a(k-1)c_{2m}e^{\mu(b-r)} - 2ac_{2m}e^{b\mu}\right]T_1 + \left[kac_m(1+b\mu) + 2ac_{2m}e^{b\mu}T - ac_{2m}e^{b\mu}T^2\right] = 0$$

Suppose derivative $\frac{dTC_1(T_1)}{dT_1}$ is vanishes at $T_{1.1}$ with $0 \leq T_{1.1} \leq T$ For this $T_{1.1}$ we have
$\left(\frac{d^2TC_1(T_1)}{dT_1^2}\right)_{T=T_{1,1}} = 2are^{b\mu}c_{2m}T_1 + [a(k-1)c_{2m}e^{\mu(b-r)} - 2ac_{2m}e^{b\mu}] > 0$ shows the convexity of the function TC.

6. Solution Procedure

Since $T = T_1 + T_2$, $T_5 = T/n$ and $T_5 = T_3 + T_4$ and suppose that $T_4 = \alpha T_3$, $(0 < \alpha \ll 1)$ this implies that $T_5 = (1+\alpha)T_3$, that is, $T_3 = T/n(1+\alpha)$ and $T_4 = \alpha T/n(1+\alpha)$ Hence T_2 is replaced by $(T - T_1)$, T_3 by $T/n(1+\alpha)$ and T_4 by $\alpha T/n(1+\alpha)$ We find the total system cost function in terms of T, T_1 and n Given a fix cycle time T and some number of delivery n we are required to determine the value of decision variable T_1 which will give a minimum TC. As the number of deliveries per order n is a discrete variable. Next, a plan of action is suggested to achieve the best policy of production:
Step 1. By the fact that n is an integer value, we begin by picking an integer value of n where $n \geq 1$.
Step 2 Find the first derivative of $TC_i(i = 1, 2, 3, 4)$ with respect to T_1 and set them equal to zero.
Step 3 Optimal value of T_1 at each n denoted by T_1^*.
STEP 4 Repeat the process 1-3 with all possible values of n until the minimum TC is found where

$$TC((n^*-1), T_1(n^*-1)) \geq TC(n^*, T_1^*) \quad and \quad TC(n^*, T_1^*) \leq TC((n^*+1), T_1(n^*+1))$$

In above equations, $T_1(n^* \pm 1)$ is the optimal T_1 when the number of delivery is $n^* \pm 1$

7. Numerical Examples and Sensitivity Analysis

In this section, we present several numerical examples to illustrate the theoretical results derived in the previous sections. These examples demonstrate how the model behaves under various parameters settings and validate the analytical findings. In addition, a sensitivity analysis is conducted to examine the impact of key parameters on the optimal order quantity and the total system cost. This analysis provides insights into the robustness of the proposed model and highlights the influence of factors such as demand rate, deterioration rate, holding cost, inflation rate, and discount rate on the overall system performance.

Example-1 The input parameters are:
c_{1m} = \$90 per order, c_{2m} = \$5 per unit per week, c_{1r} = \$50 per order,c_{2r} = \$6 per unit per week c_m = \$15 per unit, c_r = \$20 per unit, $B = 0.8$, $r = 0.06$, c_3 = \$15 per unit, c_4 = \$35 per unit, $k = 3$, $a = 1$, $b = 2$, $T = 20$ weeks, $\theta_2 = 0.06$, $\theta_3 = 0.09$, $\mu = 1$ and $\alpha = 0.2$.
Using eq. $(60a)$ we found the value of $T_1 = 5.10 week$ and $T_2 = 16.48$ week for $n = 4$, , $\mu_1 = 0.2$ $T_3 = 3.33$ week, $T_4 = 0.67$ week and the best cost of the overall system cost TC_1 = \$389.56. We get to observe that the findings of this analysis met the condition of convexity and the conditions of the eq. (60 a) like $0 \leq \mu_1 \leq T_3$, $0 \leq \mu \leq T_1$
Using eq. $(60b)$, we found the value of $T_1 = 3.52$ week and $T_2 = 14.90$ week for $n = 5$, $\mu_1 = 2$, $\mu = 8$ week, $T_3 = 4.16$ week, $T_4 = 0.83$ week and the optimal value of the total system cost TC_1 = \$317.17. Here $0 \leq \mu_1 \leq T_3$, $T_1 \leq \mu \leq T$.

Example 2 $T_3 \leq \mu \leq T_5$
Where the parameters are the same as those of ex. 1,With the help of eq. (61a), we obtained the value of $T_1 = 4.11$ week and $T_2 = 15.89$ week for $n = 5$, $\mu = 2week$, $\mu_1 = 0.2$ $T_3 = 3.33$ week, $T_4 = 0.67$ week and the best cost of the overall system cost TC_1 = \$244.10. The point is that there is no viable solution to this case that would have answered both the condition conditions $T_3 \leq \mu_1 \leq T_5$ and $0 \leq \mu \leq T_1$

Based on the information of example 1 above, a sensitivity analysis is done in trying to investigate how the change impacts some of the fundamental parameters (a, b, μ, T) of the basic model upon the optimum policy (i. e. Optimal ordering quantity and optimal total system cost). The tables 2 display the results and some interesting findings include the following:

1. Optimal changes in the demands parameter a have no effect on the optimal production time whereas changes in the system cost in a positive direction are influenced by changes of the demand parameter a, and changes in the system cost change are influenced in a negative way by changes of the demand parameter b.
2. Time parameter μ changes have negative implication and T_1 have positive implication on the production time and the cost of the total system.
3. The changes in the optimal total system cost indicate that the model is highly sensitive to the changes on $a, b, \mu and T_1$.

Table 2: Sensitivity Analysis

Parameter Percentage of changes	T1	TC1	Parameter Percentage of changes	T1	TC1
μ = -50	+8.03	-29.91	a = -50	0.00	-48.77
-25	+4.11	-15.16	-25	0.00	-24.59
+25	-4.31	+14.75	+25	0.00	+24.18
+50	-8.62	+29.09	+50	0.00	+48.36
T = -50	-73.72	-29.91	b = -50	+8.23	-31.1
-25	-36.47	-13.11	-25	+4.41	-15.98
+25	+36.07	+9.42	+25	-4.31	+16.93
+50	+71.76	+15.98	+50	-8.62	+32.78

8. Observation

Ramp-Type Demand Accurately Simulates Early Product Launch Behavior Ramp-type demand curves are specifically best suited to simulate the sales trend of recently introduced products. For most realistic cases, when introducing a new product in the marketplace, customer awareness is minimal and demand begins slowly. With the passage of time, as promotional efforts gain momentum, world-of-mouth gets around, and early customers publicize their experiences, demand growth becomes more accelerated. This phenomenon is particularly striking across industries such as consumer electronics (e.g., smartphones, smartwatches), fashion wear (e.g., new season collections), and technology-based products (e.g., wearables or electric cars). Demand would eventually stabilize or slow down as the product becomes mature or overtaken by newer models. Such modeling of the ramp-type demand function will enable the supply chain manager to forecast the inventory requirements much more realistically and modify the production rates in a dynamic way, rather than assuming constant or linearly increasing demand by conventional models. The method will also aid in strategic decision-making in regards to timing to make promotions and capacity management in considerations of the increased phase in demand.

Inflation puts a very important dimension to the supply chain planning especially in the high cost and volatile environments The increase in inflation levels rises the main elements of the cost structure such as those of raw material, labor, warehousing, transportation, and capital costs linearly and hence increases the financial pressure on the firms. These increased costs will force the need to make inventory and replacement decisions to ensure economic feasibility and strategic success. Even minor procurement or inventory turnover delays in high-inflation economies or in times of global economic turmoil (e.g., post-pandemic, geopolitical tensions) can lead to much higher overall costs. Static cost parameters are usually assumed in traditional inventory models, but such models do not work well in situations where cost elements are increasing over time. Subsidiarizing inflation into the supply chain model infuses fiscal realism and puts decision-makers in the position of being able to forecast and hedge against cost increase. For instance, purchasing in large quantities at the beginning of the planning horizon, or optimizing delivery frequency and timing, can reduce the effect of inflation. Modelling the time value of money and inflation base costs allows companies to create more robust and cost-efficient inventory strategies.

Backlogging and Lost Sales Represent Realistic Consumer Behavior Not all customers in actual settings react to a product's unavailability in the same manner. Certain customers are eager to wait for the next shipment-particularly if the product is viewed as high-value or non-substitutable (e.g., luxury items, particular drugs). These are the customers that lead to the demand that is backlogged, which is fulfilled later in the cycles. However, the customers can not be so understanding. Stockouts in highly competitive markets with other substitute products in abundance, typically cause the lost sales due to customers shifting to substitute brands or not making purchases at all. This trend is highly differentiated in high-speed retail, technology and FMCG environment. To model this duality of the demand in the case of stockouts, it is important to include the concept of partial backlogging in the model, in order to more realistically plan the level of inventory and the service rate. This is a strategy that enables companies to balance between low holding costs and high levels of customer satisfaction.

Deterioration and Inventory Value Decline The aspect of obsolescence is a significant consideration in inventories that comprise of products that are perishable, degradable, or those with a very limited product life. In this type of industries food and beverages, pharmaceuticals, electronic products and fashion goods, among other examples- the bullshit inventory does not hold up to their full value through a time span. Products can go bad, wear out or be over with or out of fashion. Constant rate deterioration models are simple to examine and reflect the gradual deterioration or decreased usefulness or quality or market value over some time. Absence of deterioration tends to create excessive optimistic inventory policies which tend to create a lot of waste, markdowns, or disposal expenses. Deterioration, conversely, makes more realistically estimations of holding cost, and underlies more stringent control of inventories and shorter replenishment cycle or increased turnovers, which are impractical perishable and high-technology products inventories.

Finite Planning Horizon Mirrors Campaign-Based and Seasonal Operations Majority of products roll-outs and promotional exercise activities are structured with constraints of a finite planning horizon. Be it in terms of limited-stock, seasonal lines of clothing, pharmacy batches expiring, etc, the supply chain decision making is most likely constrained by timeframes, marketing cycles or even legally binding duties. Inventory and production decisions made over a finite horizon are modelled such that the operations of firms could be optimized to work together with campaign goals and product life-cycle limitations. This is specifically helpful in those industries, where the turnover in stocks and the sales windows are so time-sensitive to make it in time like the consumer electronics or retail sphere. The strategic clearance of end of cycle inventory is also aided in clearance by using Finite-horizon modelling to ensure that unsellable or outdated stock is not accumulated.

9. Managerial Insights

His studies give several practical managerial implications on decision-makers who have to work in uncertainty of demand, inflationary pressure, inventory losses but limited capacity. The conclusions are very applicable to companies that have high volatility on demand, have low product life cycles and those that have sensitive cost structures.

- **Launch Planning: Aligning Capacity with Demand Phases** The two-stage type of ramp-type of demand pattern is also important when it comes to effective capacity planning especially when launching products. The slow-growth phase is only used as a buffer in establishing supply chain objects, having favourable contracts with the suppliers and balancing the production lines. The manufacturing firms are required to rapidly increase their production and logistics when the demand reaches a high to avoid stockouts. This will see the firms balance the production to the cost-efficiency area of the services by aligning production timelines, labor demands, and the acquisition of raw materials to predict the timing of the ramp-up of demand within a firm.

- **Inventory Buffer Strategy: Intelligent Use of Partial Backlogging** Partial backlogging is realistic, as it represents the behavior of consumers that, in their case, this type of delay will only result in part of the customers waiting. Such understanding will enable the managers to employ differentiated service strategies, giving preference to high-margin or high-loyalty customers so that essential demand will be fulfilled even when there is a shortage. The awareness that demand will not be captured will make managers not build backlogs unrealistically; this aids in improving inventory turnover, hence. Partial backlogging is particularly applicable in places that have low consumer tolerance towards delays, and low switching costs.

- **Inflation-Aware Procurement: Strategic Timing of Purchases** The use of inflation in cost parameters explains the advantages of future planning in procurement. Companies can be able to prepare stocks ahead by racing to procure the raw materials or long lead time materials before prices hike, when the inflationary pressures are just beginning. This is especially relevant in the case when the parts are imported or need special machinery since the cost will only rise because of the form of delays. Managers are able to capitalize on multi-period contracts featuring price guarantees and embed inflation projections in budgets and models of costs. Procurement that is inflation conscious assists in protecting their profit margins and cash flows in unpredictable economic situations.

- **Deterioration Management: Inventory Discipline & Responsiveness** A significant cost and an operation risk is deterioration that affected perishable goods or technology products that are subject to obsolescence. The addition of the constant deterioration rates makes the model highlight the necessity to reduce holding times and eliminate the slow moving inventory. The shorter replenishment cycles, the delivery systems through JIT and real-time monitoring of the inventory shelf-life could help reduce these risks. To preserve value, investing in a tracking technology (RFID or IoT) will allow timely making interventions such as discounts, offers or redistribution.

- **Multi-Delivery Advantage: Flexibility and Demand Responsiveness** The ability to allow multiple deliveries per order that comes out of the model will provide a powerful instrument in the optimization of inventory and logistics. Instead of using single bulk shipments, companies are able to schedule deliveries so as to better react to a varying demand. It has been workable in the fast-moving consumer goods section, fashion as well as in electronics, where the consumer preferences are rapidly evolving, and retail space is a luxury. The multi-delivery system will minimize the overstocking process, financial impact of overcapacity inventory, enabling leaner operations as well as enabling the producer to modify the mid-cycle production plans using the real-time demand information.

Overall Contributions to Supply Chain Management The model introduced here gives us an amalgamated framework, whereby, the supply chain and operations managers would be in a position to: more precisely forecast and react to time-sensitive set needs. Capacity and schedule of deliveries according to the market variations and cost dynamics. Limit the risks of the price increase through the well-timed purchases. Reduce inventory wastage through spoilage, expiration or obsolescence. Enhance the service provision to customers by segmented backlogging plans. Increase agility of supply chains by integrating multi-delivery and demand-based operation. Such insights are highly theoretical but practically applicable, which offer a valid foundation in making decisions in dynamic and high stakes markets.

10. Conclusion

Companies ought to take into account the current trends, which are defined by fast innovation, market instability, and the unpredictability of the economy and implement novel and dynamic supply chain strategies, which reflect the nuances of the contemporary operations. In addressing this shortcoming, the present study has produced an omnivorous, analytically sound inventory model that could be able to accommodate a number of subtle, issues that are usually encountered in new product releases or highly demand, short-life cycle markets. The given model combines four main dimensions necessary to make decisions in a supply chain practically: Ramp-type demand refers to the behavioral tendency in the market introduction and consumption saturation of a new product that is also observable in numerous occasions in the introduction of new products in industries such as electronics, fashion, and pharmaceuticals. Partial Backlogging: It represents the realistic customer behaviour; some customers would wait till the product became available, and other customers would switch to the services of other competitors or even cancel their orders. The Inventory Deterioration: This concerns the depreciation of the value or the product utility over time because of spoilage, obsolescence, or expiration- of particular importance to perishable goods, high-technology products, and seasonal merchandise. Inflation: Is economic realistic in its consideration that the cost of procurement, holding and shortage is not constant and tends to increase due to macroeconomic pressure. This study can serve as an effective and realistic inventory management instrument, in that it entails placing these dimensions into a single-manufacturer, single-retailer system and using the system to examine over a finite planning horizon. The fact that it takes several deliveries in the cycle increases the flexibility of the operations and the model is more appropriate in sectors where responsiveness and agility are the most significant competitive benefits. The establishment of the optimal replenishment cycle to incur the least expenses of the total supply chain (procurement expenses, holding expenses, deterioration expenses, shortage expenses, and inflation expenses) can be done through an optimization strategy that is embraced in this paper. The relevance of the model is evidenced by numerical examples and sensitivity analysis that indicate that the different key parameters will influence the supply chain performance based on their differences. In the management perspective, the model offers practical information regarding the buffering of inventory, demand planning, timing of procurement, and delivery timing. Such insights also help organizations to make information-based, factual distribution of resources as well as develop tactical oriented decision making within the context of real-world conditions, and improve profitability and customer service accordingly.

11. Future Research Directions

Despite the fact that the existing model offers a solid base to perceive the behaviour of supply chain under deterministic and inflation-impacted scenarios, it has multiple directions of future research: Stochastic Demand Modelling: Uncertainties with customer demand in the future can be modelled by use of probability it functions or a simulation model in order to describe any unpredictable behaviour of a market place. Multi-Echelon Extensions: It is possible to add more manufacturers, retailers, or distribution centres to the model to have a more suitable perspective regarding supply chain coordination and research decentralized decision-making. Dynamic Pricing: Integrating demand functions with pricing strategies could be seen as especially valuable to demonstrate how the firms can affect demand through time in controlling inventories. Costs of Carbon Emissions and Sustainability: Because the concept of sustainability has currently gained strategic imperative, incorporation of the environmental cost parameters in the model, such as emissions or wastes, may serve to augment the relevance of the model to green supply chain management. Behavioural and AI-Driven Forecasting: Real-time adaptability and responsiveness can be further improved by the application of machine learning to more accurately predict demand upon product launch, using the past periods around product releases. The work is an addition to the literature of operations research and supply chain management through an inventory model, an idea that is realistic, flexible, and cost-sensitive in its approach, thereby being able to rise up to some of the most challenging situations in operations caused by the nature of goods and market demands, as witnessed in the modern business environment. The model combines ramp-type demand and partial backlogging and inventory depreciation and inflation in a finite planning horizon to provide a theoretically valid and practical model. It can be utilized in academic research and managerial decision making that allows firms to venture into dynamic markets, make efficient use of their costs and offer strong levels of customer service.

References

1. Buzakot, J.A., *Economic order quantities with inflation*, Operational Research Quarterly. 26, 533-558, (1975).
2. Bierman, H. and Thomas, J., *Inventory decision under inflationary conditions*, Decision Sciences, 8 (1), 151-155, (1977).
3. Cleark A.J. and Scarf, H., *Optimal policy for a multi-echelon inventory problem*, Management Science, 6, 475-490,(1960).
4. Covert, R.P. and Philip, G.C., *An EOQ model for items with weibull distribution deterioration*, AIIE Transaction, 5, 323-326, (1973).
5. Chern, M., Yang H., Teng, J., Papachristos, S., *Partial backlogging inventory lot-size models for deteriorating items with fluctuating demand under inflation*, European Journal of Operational Research, 191, 127-141, (2008).
6. Donaldson, W.A., *Inventory replenishment policy for a linear trend in demand: an analytic solution*, Operational Research quarterly, 28, 663-670, (1977).
7. Goyal, S.K. and Gunasekaran, A., *An integrated production–inventory–marketing model for deteriorating items*, Computers and Industrial Engineering, 28, 755–762, (1995).
8. Ghare, P.M. and Schrader, S.F., *A model for exponentially decaying inventory*, Journal of Industrial Engineering, 14, 238-243, (1963).
9. Goyal, S.K., Singh, S.R. and Dem, H., *Production policy for ameliorating/deteriorating items with ramp type demand*, International Journal of Procurement Management, 6 (4), 444-465, (2013).
10. Hill, R.M., *Inventory model for increasing demand followed level demand*, Journal of the Operational Research Society, 46, 1250-1259, (1995).
11. He, Y., Wang, S.Y., Lai, K.K., *An optimal production-inventory model for deteriorating items with multiple-market demand*, European Journal of Operational Research, 203, 593–600, (2010).
12. Jaggi, C.K., Gautam, P., Khanna, A., *Inventory decisions for imperfect quality deteriorating items with exponential declining demand under trade credit and partially backlogged shortages, in: Quality, IT and Business Operations*, Springer, Singapore, 213–229, (2018).
13. Khanra, S. and Chaudhuri, K.S., *A note on an order level inventory model for a deteriorating item with time dependent quadratic demand*, Computer and Operations Research, 30, 1901-1906, (2003).
14. Manna, S.K. and Chandhuri, K.S., *An EOQ model with ramp type demand, time dependent deterioration rate, unit production cost and shortages*, European Journal of Operational Research, 171, 557-566, (2006).
15. Mishra, R. B., *An optimum production lot size model for a system with deteriorating inventory*, International Journal of Production Economics, 15, 495-505, (1975).
16. Mishra, R.B., *A note on optimal inventory management under inflation*, Naval Research Logistics Quarterly 26, 161-165, (1979).
17. Resh, M. Friedman, M. and Barbosa, L.C., *On a general solution of the deterministic lot size problem with time proportional demand*, Operations Research, 24, 718-725, (1976).
18. Skouri, K., Konstantaras, I., Papachristos, S. and Ganas, I., *Inventory models with ramp type demand rate partial backlogging and weibull deterioration rate*, European Journal of Operational Research, 192, 79-92, (2009).
19. Singh, S.R. and Singh, C., *Supply chain model with stochastic lead time under imprecise partially backlogging and fuzzy ramp-type demand for expiring items*, International Journal of Operational Research, 8 (4),511-522, (2010).
20. Singh, N., Vaish, B. and Singh, S. R., *An economic production lot-size (EPLS) model with rework and flexibility under allowable shortages*, International Journal of Procurement Management, 5(1), 104-122, (2012).
21. Singh, S.R., Vishnoi, M. *Two storage inventory model for perishable items with trapezoidal type demand under conditionally permissible delay in payment*, Proceedings of the Second International Conference on Soft Computing for Problem Solving (SocProS 2012), 1369-1385, (2012).

22. Sarkar, B., *An EOQ model with delay in payments and time varying deterioration rate*, Mathematical and Computer Modeling, 55, (2012), 367–377, (2012).

23. Taleizadeh, A.A., Niaki, S.T.A. and Makui, A., *Multi-product multiple-buyer single vendor supply chain problem with stochastic demand, variable lead-time, and multi-chance constraint*, Expert Systems with Applications, 39, 5338–5348, (2012).

24. Wu, K. S., *An EOQ inventory model with ramp type demand rate for items with weibull deterioration ramp type demand and partial backlogging*, Production Planning and Control, 12 (8), 787-793, (2001).

25. Vishnoi, M., Singhal, S., Singh, S.R. *Optimization of a sustainable integrated inventory model for decaying items with multi-variate demand and controllable emission*, Sustainable Inventory Management: Perspectives from India, 125-146, (2025).

26. Zhau, Y.W., Min, J. and Goyal, S.K., *Supply chain coordination under an inventory level dependent demand rate*, International Journal of Production Economics, 113 (2), 518-527, (2008).

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