Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2026 (44) 7** : 1–14.
ISSN-0037-8712
doi:10.5269/bspm.80497

Optimizing Production Strategies for Deteriorating Items in Two-Level Manufacturing

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ABSTRACT: A modern business enterprise should develop a sound operational system of organization to meet the challenges of an ever-changing environment where conditions are somewhat too unpredictable. The need to increase production is determined by the demand for specific goods rising with time, which in many cases is done because of the growth in popularity and the value associated with the product. Thus, the organization faces pressure to increase its manufacturing rates to meet the increased demand. The proposed model optimizes a deteriorating goods inventory system with two different manufacturing rates and exponentially formulable demand. Production initially starts off at a constant rate, how is that production, demand, inherent deterioration and variable manufacturing rates combine to build up the inventory level progressively. The more the demand rate is increased, the more the production rate is increased by the factor known as a so that it is managed according to the market. The feedback system is important to ensure that supply and demand is balanced in a dynamic marketplace. The empirical results only further show that a 10% increase in production costs is equivalent to a 5% increase in total costs, which shows the non-linear relationship between cost input and the overall financial performance. To promote the reliability of the model, a sensitive analysis has been carefully conducted, and thus a robustness check of the model and predictive reliability of its operational results has been verified.

Key Words: EPQ Model, deterioration, exponential demand, two levels of manufacturing rate.

Contents

1. Introduction

Robust management systems are an imperative to decreasing costs and protecting a company's reputation. Scholarly investigation has mostly concentrated on the administration of perishable assets which by their nature deteriorate with time - examples of such assets are food, pharmaceutical products and radioactive materials. This inherent deterioration demands special mechanisms for stocking inventory so as to keep the wastage at an optimum. The field of inventory management for deteriorating items has been comprehensively explored. Prominent contributions include those by [35] and [6] who developed inventory models incorporating demand proportional to time.

[39] developed a deterministic model of inventory of deteriorating commodities stored in a multi-facility replenishment system. [12] and [32] considered deteriorating items with limited replenishment capacities for which optimal policies have been developed for the two-warehouse case. [56] reviewed deterministic lot-size inventory models which are based on assumptions of market shrinkage and shortages

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2020 *Mathematics Subject Classification:* 90B05.
Submitted November 30, 2025. Published February 21, 2026

which are relevant for deteriorating products. [2] has proposed a production model where there is deterioration dynamics, optimal replacement strategy and shortages under stochastic demand and variable production situations. [40] analyzed the interrelation between cash flow management and inventory deterioration and explicitly stated delayed payment, if any, that is permissible. [57] presented a demand-based ramp-type EOQ model for partial backlogs with time-reliant deterioration. [4] considered product deterioration in trapezoidal demand distributions. [10] projected an EPQ model for the fading products with anticipated back orders. [53] have developed a production economy model for non-instantially perishable products with exponentially increasing demand and time-dependent holding costs. [44] and [15] made production-inventory models for the two and three-echelon systems and with the consideration of shortages and deterioration respectively. [24] proposed an EPQ model with the incorporation of two-stage credit financing, demand fluctuation and system unreliability. [41]describe the optimal replenishment decision within a trade-credit strategy for retail outlets experiencing variable demand for deteriorating products. [42] revisit the question of whether the automation approach in an innovative production system effectively helps control defective manufacturing. [13] examined problems for item deterioration and customer retrials in the context of inventory management problems.

[16] have compared the use of exponential and Verhulst demand functions in an (EPQ) model. [21] developed the concept of progressive sustainable inventory management, which involves advertising strategies, wherein trade-credit policy governs demand patterns and stock levels. [7] discuss the effect of the inflation rate on deteriorating items in three-echelon supply management systems. [25] have suggested an EPQ model that includes linear holding cost, exponential demand, but with staged production intervals for the items with delayed depreciation. [27] considered an EPQ model that includes green technology investment, preservation scheme, carbon emission, stock demand, and price sensitivity. [45] suggested an EPQ model to soften the items with a delay in payments and shortages. [46] developed a model for deteriorating elements that depends upon stock levels and time-dependent demand. [47] developed models which deal with policies about return and sustainable EOQ/EPQ in different classes of products. Similar work was undertaken in the work by [28,29,30,31] and [54]. [26] considered an EPQ model that is used to account for production, deterioration, stock, time, and lead time-driven demand. [17] proposed a fuzzy inventory model for a seasonal product that is subject to deterioration. [48,49,38], [1] optimized the EOQ model of perishable products with the additional consideration of inflation and carbon emissions. In [18] developed an inventory model that includes a fuzzy environment and incorporates inflation, CO2 emission, and partial backordering. [50] revisited the EOQ model for defective goods, considering demand to be variable over time, especially in the presence of substitutes. [22] investigated production and inventory systems optimization with deterioration, shortages, and incorporated the effect of carbon emission policy along with green technologies. [8] again evaluate the impact of green attributes on retail industry profitability by analyzing how to optimize profit through variable production rates and unit costs, while accounting for the selling price and demand relative to the product's green level. [14] proposed a sustainable production and inventory system for perishable products using the Markdown Method and investments for decreasing carbon emissions. [9] again optimized strategies for production and remanufacturing in a complex, innovative single-stage system operating under uncertainty and waste elimination. [43]again provide Various non-linear programming models that apply to a multi-cycle innovative production system designed for a multi-stage process. This system integrates autonomation and remanufacturing within the same or different cycles to minimize waste. [36] have used bacterial foraging optimization within a two-warehouse inventory model, accounting for generalized exponential demand characteristics of the items, partial backlogging and inflation. [19] proposed a model based on flower pollination optimization to control the product's demand, which is time- and price-dependent. [37] Revisiting carbon emissions, partial backlog, and price-dependent demand for a stock model. [52] studied a just-in-time purchasing with slotted backorders in a two-warehouse deterministic inventory system for perishable products. [55] present a case study in the agri-food industry to represent a sustainable production-inventory system, which considers CO2 emissions, energy consumption, fuel utilization, and the issues related to quality degradation under stochastic demand. [58] focus on optimisation of an inventory model for deteriorating items considering green technology investments involving time-sensitive demand, carbon emission and selling prices. [11] model, which uses Maclaurin series approximations, Weibull distributed EPQ inventory framework, and different time-dependent holding costs across different demand scenarios. [3] consider the demand mod-

eling with preservation technology investments in a multi-period inventory model with price, time and service-level parameters. [33] present an inventory model for a retailer dealing with deteriorating items, power-pattern demand, and a partial backordering problem for both fuzzy and non-fuzzy cases. In this research, [23] study the integration of RFID technology and scarcity management with a multi-objective sustainable inventory model, which operates under a stochastic environment. [34] use a parametric model for perishable goods inventory, based on price interval demand and advance payment policies, using particle swarm optimization. Eco-friendly packaging and green investments are essential considerations for building the multi-phase sustainable production and inventory model for seasonal commodities, as noted by [5].

Research Gap Although the literature on deteriorating-item inventory models is extensive, the specific combination of (i) continuously exponentially increasing demand, (ii) a two-level (piecewise) manufacturing rate where production accelerates once demand reaches a threshold, and (iii) an analytic convexity proof together with low-deterioration reduced-form expressions has not been addressed in prior studies. Existing works focus on different, essential dimensions—for example, advertising and trade-credit policies in retailer problems [21], integrated vendor–buyer models with discrete setup costs [41]. These papers do not develop a continuous EPQ framework that simultaneously models exponential demand growth and a two-stage production-rate policy, prove the convexity of the resulting total cost function, and derive closed-form reduced relations for low deterioration rates. The present study fills that gap by (1) formulating a continuous two-level EPQ model with exponential demand, (2) proving cost convexity to guarantee a unique optimum, (3) deriving simplified expressions for the low-deterioration limit, and (4) performing comprehensive sensitivity analysis to inform managerial decisions.

Recent studies have extended these directions toward smart production and sustainability. [41] explored autonomation and defect control in multi-stage production systems. [8] focused on environmentally sustainable production policies incorporating green technology and carbon emissions considerations. [9] investigated smart single-stage production and re-manufacturing decisions.

Despite these valuable contributions, two research gaps remain evident. First, while exponential or ramp-type demand and preservation investments are separately considered, no study has concurrently modeled exponentially increasing demand with a deliberately designed two-level manufacturing policy. Second, limited research provides an explicit analytical convexity proof of the total cost in a continuous EPQ framework with production-rate transitions and closed-form expressions for the low-deterioration case. The present study bridges these gaps by developing a two-level EPQ model with exponential demand, establishing convexity to confirm a unique optimal solution, and deriving simplified reduced forms that yield managerial insights and computational efficiency.

This study develops a biographical inventory management model for perishable commodities stocked continuously, in which the production rate is variable, and the structural dynamics of the system are taken into account. The model's cost function is confirmed to be convex thus guaranteeing the existence of a unique optimal solution. The results show in particular that the optimal inventory levels decrease, while the cycle length is not reduced; furthermore, the deterioration is very low. Reduced form equations for production cycle time, inventory and optimum average cost are derived for low deterioration rates and confirmed by numerical studies and sensitivity analysis on the parameters affecting the total cost. Table -1 provides an overview of the current work compared to similar studies.

Table 1: Comparison of previous work with our study

Researcher	Exponential demand	Deterioration	Increasing Production Rate
[1]	✓	✓	×
[2]	×	✓	×
[3]	×	×	×
[4]	×	✓	×
[5]	×	×	×
[6]	×	✓	×
[7]	×	✓	×
[8]	×	✓	×
[9]	×	✓	×
[10]	×	✓	×
[11]	×	✓	×
[12]	×	✓	×
[13]	×	✓	×
[14]	×	✓	×
[17]	×	✓	×
[18]	×	✓	×
[19]	×	✓	×
[20]	×	✓	×
[21]	×	✓	×
[22]	×	✓	×
[23]	×	×	×
[25]	×	✓	×
[26]	×	✓	×
[32]	×	✓	×
[33]	×	×	×
[35]	✓	✓	×
[36]	×	✓	×
[37]	×	×	×
[41]	×	✓	×
[42]	×	×	×
[43]	×	×	×
[51]	×	✓	×
[52]	✓	✓	×
[54]	×	×	×
[57]	×	✓	×
This Paper	✓	✓	✓

2. Assumptions and Notations in this Model

The suggested inventory model is developed based on the following assumptions:

(a) Exponentially increasing variable demand over time, i.e. $D = ae^{bt}$

(b) The production rate is maintained above the demand rate to ensure that stock shortages do not occur.

(c) Initially, production started with a production rate P after some time of T_c production rate increases by a multiple of a constant $\alpha > 1$ and becomes αP

(d) Shortage is not allowed in this model.

(e) Constant deterioration rate.

(f) Replacement is instantaneous.

Table 2: **Notations**

Inventory Parameters	
P	Production Rate
b	Markup b, $b > 0$
a	Scale demand, $a > 0$
θ	Deterioration rate
C_p	Cost of production(in dollars)/unit
Q_1	The amount of inventory present at the given moment $t = T_c$
Q_2	The amount of stock available ath that specific moment $t = T_d$
C_d	Cost of Deterioration (in dollars)/unit
C_h	Cost of product holding (in dollars)/unit
C_0	Cost of ordering the product (in dollars)/unit
T	Total Cycle time (in months)
$I_i(t)$	Level of inventory at any given time $t(i = \beta, \gamma, and \delta)$
Decision variables	
T_i	Unit times, $i(i = c \quad and \quad d)$
Function	
TC	Total cost (in dollars)/unit

3. Mathematical Formulation

3.1 Two-Level Manufacturing Inventory Models

Initially, the model starts with a zero inventory level, and then the inventory (Stock) level changes over time. At $t = 0$, there is no inventory in the original manufacturing setup. Inventory levels rise due to increasing demand over time, $t = 0$ to $t = T_c$ and reaches the level Q_1 . The popularity of the product and increased demand, and some effect of deterioration, force the manufacturers to speed up production, and as a result, the production rate increases to αP where $\alpha > 1$ is a constant, during time $T_c < t < T_d$. At this stage, the inventory level reaches Q_2, the highest level of inventory. After that, the inventory declines due to corrosion and weak demand, reaching 0 at the time T. Let $I_1(t)$, where $i(i = \beta, \gamma \quad \text{and} \quad \delta)$ denotes the inventory levels.

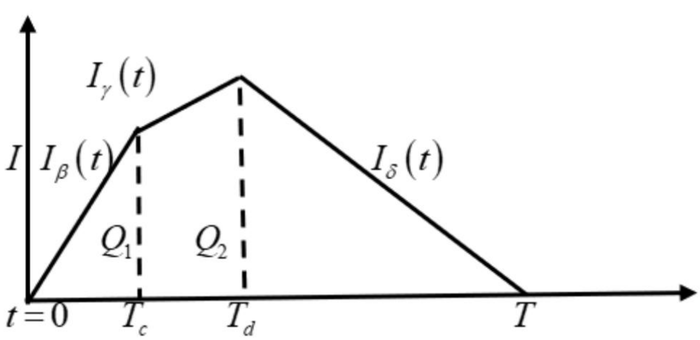


Figure 1: Level of Inventories at various time intervals

In the interval $(0, T)$, the mathematical equations are described as follows. We consider the following differential equation:

$$I'_\beta(t) + \theta I_\beta(t) = P - ae^{bt} \quad t \in [0, T_c], \tag{3.1}$$

$$I'_\gamma(t) + \theta I_\gamma(t) = \alpha P - ae^{bt} \quad t \in [T_c, T_d] \tag{3.2}$$

$$I'_\delta(t) + \theta I_\delta(t) = -ae^{bt} \quad t \in [T_d, T]. \tag{3.3}$$

The following boundary conditions are applied to solve these mathematical equations.

$$I_\beta(t) = 0, \quad \text{when} \quad t = 0, \quad \text{and} \quad I_\delta(t) = 0 \quad \text{when} \quad t = T \tag{3.4}$$

Now, using the given boundary conditions, we obtain the solutions of equations 3.1, 3.2 and 3.3 . Hence, from equation(3.1)

$$I_\beta(t) = \frac{a}{\theta + b})(-e^{bt} + e^{-\theta t}) + \frac{P}{\theta}(1 - e^{-\theta t}); \quad t \in [0, T_c] \tag{3.5}$$

Hence, from equation(3.2)

$$I_\gamma(t) = \frac{\alpha P}{\theta}(1 - e^{\theta(T_c - t)}) - \frac{a}{(\theta + b)}(e^{bt} - e^{bT_c}e^{\theta(T_c - t)}) + e^{\theta(T_c - t)}Q_1; \quad t \in [T_c, T_d] \tag{3.6}$$

Hence, from equation(3.3)

$$I_\delta(t) = \frac{a}{(\theta + b)}(e^{bt}e^{\theta(T - t)} - e^{bt}); \quad t \in [T_d, T] \tag{3.7}$$

Now using the condition $I_\gamma(T_c) = I_\beta(T_c)$ and $I_\gamma(T_c) = Q_1$ in equation (3.5) and (3.6), we get

$$Q_1 = \frac{a}{(\theta + b)}(e^{-\theta T_c} - e^{bT_c}) - \frac{P}{\theta}(e^{-\theta T_c} - 1) \tag{3.8}$$

Now using the condition $I_\gamma(T_d) = I_\delta(T_d)$ in equation (3.6) and (3.7), we get

$$T = \frac{1}{(\theta + b)} \log[e^{T_d(\theta + b)} + \frac{\theta + b}{a}Q_1 e^{\theta T_c} + \frac{(\theta + b)\alpha P}{\theta a}(-e^{\theta T_c} + e^{\theta T_d}) - (e^{T_d(\theta + b)} - e^{T_c(\theta + b)})] \tag{3.9}$$

Total Cost(TC): The totality of ordering, production , deterioration, and holding costs. after reviewing each expense separately, these expenses are combined.

(i) **Production Cost**

$$PC = \frac{C_P}{T}\int_0^T Pdt = \frac{C_P}{T}(\alpha PT_d - (\alpha - 1)PT_c) \tag{3.10}$$

(ii) **Cost of ordering the products**

$$OC = \frac{C_0}{T} \tag{3.11}$$

(iii) **Cost of holding the products**

$$\begin{aligned}
HC &= \frac{C_h}{T}\left[\int_0^{T_c} I_\beta(t)dt + \int_{T_c}^{T_d} I_\gamma(t)dt + \int_{T_d}^{T} I_\delta(t)dt\right] \\
&= \frac{C_h}{T}\left[\frac{P}{\theta}\left(T_c - \frac{1}{\theta}(1 - e^{-\theta T_c})\right) - \frac{a}{b + \theta}\left((e^{-bT_c} + 1)\frac{1}{b} + \frac{1}{\theta}(-1 + e^{-\theta T_c})\right) + (1 - e^{-\theta(-T_c + T_d)})\frac{Q_1}{\theta}\right. \\
&\quad + \frac{\alpha P}{\theta}\left((T_d - T_c) + \frac{1}{\theta}(e^{\theta(T_c - T_d)} - 1)\right) + \frac{a}{\theta + b}\left(\frac{e^{bT_c}}{\theta}(1 - e^{-\theta(T_c - T_d)}) + \frac{1}{b}(-e^{bT_d} + e^{bT_c})\right) \\
&\quad \left. + \frac{a}{\theta + b}\left(\frac{e^{bT}}{\theta}(e^{\theta(T - T_d)} - 1) + \frac{1}{b}(e^{bT_d} - e^{bT})\right)\right]
\end{aligned} \tag{3.12}$$

(iv) **Deteriorating Cost:**

$$DC = \frac{\theta C_d}{T}\left[\frac{T_c P}{\theta} - \frac{1}{\theta}(1-e^{-\theta T_c})\left(\frac{P}{(\theta)^2} - \frac{a}{\theta(\theta+b)}\right) + \frac{1}{b(\theta+b)}(1-e^{-bT_c}) - \frac{Q_1}{\theta}(e^{\theta(T_c-T_d)}-1)\right.$$
$$+\left((T_d-T_c)+(e^{\theta(T_c-T_d)}-1)\frac{1}{\theta}\right)\frac{\alpha P}{\theta} - \left(e^{bT_c}\left((e^{-\theta(T_c-T_d)}-1)\frac{1}{\theta}\right) - (e^{bT_c}-e^{bT_d})\frac{1}{b}\right)\frac{a}{\theta+b}$$
$$\left.+\frac{a}{\theta+b}\left(e^{bT}\left(\frac{1}{\theta}(e^{\theta(T-T_d)}-1)\right) + \frac{1}{b}(e^{bT_d}-e^{bT})\right)\right] \tag{3.13}$$

Total Cost: The sum of deterioration, production, ordering, and holding costs is equal the total cost. **TC= PC + OC + HC + DC**

$$TC = \frac{1}{T}\left[C_p(\alpha P T_d + P T_c(1-\alpha)) + C_0 + (C_h+\theta C_d)\left(\frac{P}{\theta}\left(T_c + \frac{1}{\theta}(e^{-\theta T_c}-1)\right)\right.\right.$$
$$\left.+\frac{a}{\theta+b}\left(\frac{1}{b}(1-e^{-bT_c}) - (e^{-\theta T_c}-1)\frac{1}{\theta}\right) + (1-e^{-\theta(T_c-T_d)})\right)\frac{Q_1}{\theta}$$
$$+\left(\frac{1}{\theta}(1-e^{-\theta(T_c-T_d)}) + (T_c-T_d)\right)\frac{\alpha P}{\theta} + \frac{a}{\theta+b}\left(-\frac{e^{bT_c}}{\theta}\left(e^{-\theta(T_c-T_d)}-1\right) - (e^{bT_d}-e^{bT_c})\frac{1}{b}\right)$$
$$\left.+\frac{a}{\theta+b}\left(\frac{e^{bT}}{\theta}\left(e^{\theta(T-T_d)}-1\right) + \frac{1}{b}(e^{bT_d}-e^{bT})\right)\right] \tag{3.14}$$

Optimization Function: Z =Min(TC)

subject to $T_c > 0$ and $T_d > 0$

Solution Methodology: The objective of the proposed model is to minimize the total cost, which comprises production, holding, ordering, and deterioration costs. Here the decision variables are T_c and T_d. Therefore the solution process in this model has been carried out with the help of the Hessian matrix. To determine their optimal values, the total cost function was partially differentiated with respect to each decision variable, and the resulting first-order conditions were equated to zero. The simultaneous solution of these equations provides the stationary points corresponding to the minimum total cost.

$$\frac{\partial TC}{\partial T_c} = 0, \frac{\partial TC}{\partial T_d} = 0$$

To verify the convexity of the total cost function and to ensure the existence of a unique optimal solution, the **Hessian matrix** was evaluated at the stationary point.

$$H = \begin{bmatrix} \frac{\partial^2 TC}{\partial^2 T_c} & \frac{\partial^2 TC}{\partial T_c T_d} \\ \frac{\partial^2 TC}{\partial T_d T_c} & \frac{\partial^2 TC}{\partial^2 T_d} \end{bmatrix}$$

Here the principal minors $H_1 = \frac{\partial^2 TC}{\partial^2 T_c} > 0, H_2 = \frac{\partial^2 TC}{\partial^2 T_d} > 0, H_3 = \frac{\partial^2 TC}{\partial^2 T_c}\frac{\partial^2 TC}{\partial^2 T_d} - (\frac{\partial^2 TC}{\partial T_d T_c})^2 > 0$

and their signs confirmed that the matrix is positive definite, which indicates the convex nature of the cost function. Hence, the obtained solution corresponds to the global minimum.

4. Numerical Example

The optimization and sensitivity analysis were carried out using Mathematica 12.0. A gradient-based optimization technique was applied to determine the optimal production rate, cycle length, and total cost under varying parameter settings. The convex behavior of the total cost function, as illustrated in Figure 2, supports the analytical findings and demonstrates the robustness of the proposed solution approach. Random data has been sourced from the production inventory models for deteriorative items with three levels of production and shortages to illustrate the model.

P = \$200, C_h = \$0.25 per unit, C_0 = \$600 per unit, C_p = \$4 per unit, C_d = \$0.5 per unit, $\alpha = 1.5$, $b = 0.3$, $a = 25$, $\theta = 3\%$

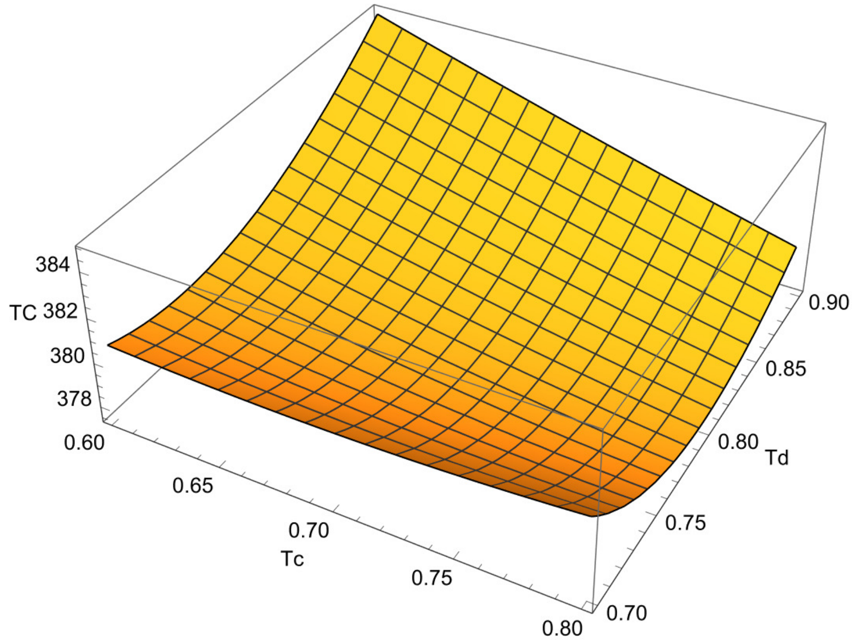


Figure 2: Convexity of the overall cost function

Optimal Solution: From equations (3.9) and (3.14) $T_c = 0.630521$ month and $T_d = 0.793781$ month, Total cycle time $T = 3.62419$ month, and Total Cost $TC = \$380.022$

5. Sensitivity Analysis

This paper examines the impact of variations in some variable parameters (specifically, and) on the optimal inventory cycle time and the related total cost. Sensitivity analysis was performed by independently varying each parameter in the range of -20% to 20% while all other parameters were assumed to remain constant. The results are summarized in detail in Tables 3 and 4.

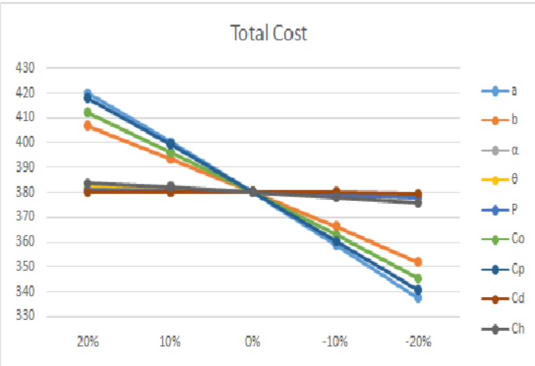
Table 3: Variation in Cycle time concerning change in various parameter values

	a	b	α	θ	P	C_0	C_p	C_d	C_h
20%	3.425	3.276	3.632	3.595	3.602	3.857	3.445	3.621	3.575
10%	3.519	3.439	3.628	3.609	3.612	3.744	3.530	3.622	3.599
0%	3.624	3.624	3.624	3.624	3.624	3.624	3.624	3.624	3.624
−10%	3.743	3.834	3.620	3.638	3.639	3.494.	3.727	3.625	3.649
−20%	3.880	4.077	3.616	3.653	3.657	3.353	3.842	3.627	3.675

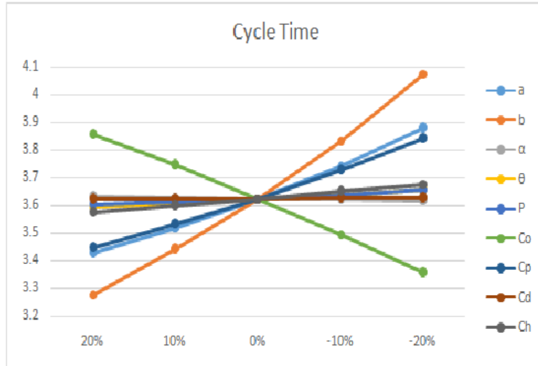
Table 4: Variation in Total Cost concerning change in various parameter values

	a	b	α	θ	P	C_0	C_p	C_d	C_h
20%	420.37	406.73	381.79	382.31	381.24	412.09	418.01	380.26	383.98
10%	400.45	393.54	380.90	381.17	380.69	396.30	399.17	380.14	382.01
0%	380.02	380.02	380.02	380.02	380.02	380.02	380.02	380.02	380.02
−10%	358.99	366.12	379.13	378.87	379.20	363.16	360.51	379.90	378.00
−20%	337.26	351.80	378.24	377.71	378.19	345.64	340.60	379.78	375.96

Figure 3: The visual depiction of the sensitivity analysis is presented in figure(3) and figure(4)



(a) Variation in total cost



(b) Variation in total cycle time

6. Observation

The sensitivity analysis provides detailed information on the effects of the change in some of the most critical parameters on the optimum cycle time and total cost of the proposed inventory model. As shown in Table 3, some parameters such as demand scale, markup, production speed, production rate have an inverse relationship with cycle time. When these parameters rise $10\% - 20\%-$ wise, the best cycle time decreases, which means faster production cycle to meet higher market demand or increased production capacity. On the contrary, reduction in these parameters means slower production process causing longer cycle durations. The rate of deterioration has been found to have a similar negative relationship with cycle time.

As the level of deterioration grows, so do the products depreciate more rapidly, forcing the system to reduce the production cycle in an effort to reduce wastage. Meanwhile, cost parameters, such as order, production, deterioration and holding costs, show a mixed influence on cycle time. For example, a rise in production or deterioration costs usually causes cycles to be shorter and vice versa, but if the production or deterioration costs fall, then the cycle time increases as there is less financial loss for companies to hold on to inventory longer.

The data in Table 4 further reveal that total cost is very sensitive to changes in costs in the production and ordering costs. When these costs rise by 20%, the overall cost increases dramatically, but if these costs decrease by 20% the overall cost decreases dramatically. Among all the parameters, production cost, ordering cost and deterioration cost have the greatest impact on the total cost, proving the fact that cost management in terms of these is very important for profitability. The convex shape of the total cost curve (Figure 2) justifies that the cost function is such to permit a unique optimum where the balance of the costs of production, holding and deterioration will provide the lowest total cost.

In summary, the results validate the fact that, (i) the higher rates of deterioration and production tend to speed-up the inventory turnover without affecting the total cost, (ii) lessening of production and ordering costs can contribute significantly to minimize the overall expenditure, and (iii) convex cost behavior of the model guarantees stable and predictable operational policy to optimize it.

7. Conclusion

This paper is a statistical analysis of perishable products production and stock control during exponentially growing demand and two-stage production levels. The proposed model is useful in combining the dynamics of demand, deterioration and cost interactions in order to reduce the total system cost. The existence of the unique optimal solution of the decision variables- the rate of production and cycle time was established by using optimization methods through the Hessian matrix establishing the convexity of the total cost of the full non-linear equation.

Some of the key relationships are highlighted by the numerical illustration and sensitivity analysis. To start with, a rise in decline rates and production rates reduces the optimum period of the cycle and the enterprises have to speed up production process and sales activities to prevent losses that can be attributed to spoils. Secondly, the decreased costs of production and ordering affect the profitability positively because it will allow companies to extend the production cycle and use inventory more efficiently. The model also exposes a non linear cost structure in such a way that a 10 per cent change in the cost of production results in about a 5 percent increase in the overall costs thus exhibiting negative marginal effects.

In general, the suggested model offers operationally viable and mathematically balanced form of production decision-making. It provides both a compromise of the overall cost and product availability and is particularly applicable in those industries that deal with perishable goods including food, pharmaceuticals, agro based goods.

8. Managerial Insight and Future Scope of Research

The implications about this study on the part of the manager are critical. The deterioration rate versus cycle time would imply that money to be invested in preservation technologies, e.g., temperature regulation, enhanced packaging, or replenishment in a timely manner, to increase shelf life and minimize deterioration-associated expenses, should be a top priority of the managers. The adoption of such strategies helps companies to run with a longer production cycle, hence reducing the total cost of operation.

Total cost is highly sensitive to both production and ordering costs, the fact that shows the necessity of production planning and order consolidation to prepare strategies crucial to cost optimization. The managers are encouraged to schedule production runs based on the demand intensity and not to save a consistent production rate. This dynamism accommodates unnecessary inventory and shortages. Moreover, optimization of markup and scaling demand will maximize profit generation capacity as well as inventory turnover which will maintain steady profit margins.

The convexity of the total cost curve ensures that the optimal policy to produce has one unique optimal policy that provides managers with a definite instrument that helps to establish the most economical combination of production rate and cycle time. The structure of the model also highlights the advantage of two-level manufacturing control, which can be transferred to actual situation when the different manufacturing stages work with a variable speed taking into consideration machine capacity or labor efficiency.

In practice, this model serves as a decision-support model to maximize the production level, flow replenishment and cost management of the perishable goods in the agriculture, pharmaceutical, and fast-moving consumer goods (FMCG) industries. It helps firms to achieve an optimal balance between the cost efficiency and service level in the case of dynamic demand.

Future Research Scope: The current model can be developed in a number of ways in future research. With stochastic or fuzzy demand environments, trade -credit financing and carbon -emission constraints, it would increase applicability to uncertainty and sustainability-oriented operations. Moreover, the model can be further elaborated with the extension to the multi-echelon chain of supply, or the minimization of costs of green technology implementation and preservation can also be regarded as a way of fine-tuning the decision-making framework, and enhancing the environmental performance.

Author Contribution

All authors contributed to the study's conception, design, data preparation, and analysis.

Declarations

Ethics approval and consent to participate

The submitted work is original and has not been published elsewhere in any form or language (partially or entirely). All authors have contributed significantly to the conception, design, or the collection, analysis, or interpretation of data.

Data availability

All necessary data generated or analyzed during this study are included in this article.

Conflict of interest

The authors declare no conflict of interests.

Funding

No funding was received to conduct this study.

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