Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2026 (44) 2** : 1–9.
ISSN-0037-8712
doi:10.5269/bspm.80514

Dynamics of a Delayed Eco-Epidemic Model with Disease in the Prey

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ABSTRACT: This paper presents the mathematical analysis of a delayed eco-epidemic model that incorporates the effect of disease within the prey population. The stability of the model, both with and without delay, is analyzed. The Hopf bifurcation of the model is discussed by considering the time delay as a bifurcation parameter. Moreover, a behavioral change is observed in the system as it moves from a stable to an unstable state when the delay parameter crosses the threshold value, leading to a Hopf bifurcation from co-existence state. In addition, the stochastic stability of the model at the co-existence state is investigated. To demonstrate the validity of the theoretical analysis, some numerical simulations are presented.

Keywords: Prey-predator model, time delay, Hopf bifurcation, stochastic stability.

Contents

1. Introduction

Since the introduction of Lotka-Volterra model, many experts have looked into how prey and predator species interact with each other. The interaction between the susceptible, infected, and recovered populations is also becoming an interesting area of study after the ground-breaking work of Kermack and McKindrick [1]. An important part of eco-epidemiology study is looking at how diseases change over time in ecological systems. It was Anderson and May [2], who first talked about studies that combined these two systems.Chattopadhyay and Arino [3] were the first to introduce the term eco-epidemiology for these kinds of models.In the past few decades, many scientists and researchers have looked into the relationships between prey and predators using a wide range of biological factors [6,8,14]. In epidemiology, various mathematical models have been developed with the help of different forms of incidence rates [12,15]. We develop a mathematical model to investigate the interactions and population dynamics within a prey-predator system, where some of the preys have a spreading disease. The stability behavior of the model at the equilibrium states is investigated, and the analytical outcomes are substantiated by numerical simulations.

2. Mathematical Model

In this model, the prey population is categorized into two compartments: the susceptible prey population density at time t is shown by $x(t)$, the infected prey population density at time t is shown by $y(t)$. The predator population density at time t is shown by $z(t)$.

Model Assumptions:

2020 *Mathematics Subject Classification:* 92D25.
Submitted November 30, 2025. Published March 14, 2026

(i) The susceptible prey population grows logistically with intrinsic growth rate $r > 0$.

(ii) According to a transmission rate $\beta > 0$ the susceptible prey population gets sick when it comes into touch with the infected prey. Only prey animals get the sickness, and it's not passed down through genes.

(iii) Assuming that $\mu_1 > 0$ and $\mu_2 > 0$, we can say that the predator has an advantage over the susceptible prey and an advantage over the infected prey.

(iv) δ_1 shows the diseased death rate of infected prey, while δ_2 shows the normal death rate of predators when they don't have any prey.

(v) When prey populations are turned into predator populations, the constant $\alpha \in (0,1)$ shows how much they change. It seems likely that a predator's reproduction will not happen right away after it has eaten its food, but will be delayed by the time it takes for the predator to get pregnant [5,9].

Based on the above assumptions, the model equations are given by

$$\begin{aligned} \frac{dx}{dt} &= rx - \frac{\beta xy}{1+y} - \mu_1 xz \\ \frac{dy}{dt} &= -\delta_1 y + \frac{\beta xy}{1+y} - \mu_2 yz \\ \frac{dz}{dt} &= -\delta_2 z + \alpha\mu_1 x(t-\tau)z(t-\tau) + \alpha\mu_2 y(t-\tau)z(t-\tau) \end{aligned} \tag{2.1}$$

where $\tau > 0$ is the amount of time needed for the gestation of the predator.

3. Stability Analysis

The system (2.1) exhibits four equilibrium states as follows:

(i) Fully-washed state: $E_1 = (0,0,0)$

(ii) Infected prey washed state: $E_2 = \left(\frac{\delta_2}{\alpha\mu_1}, 0, \frac{r}{\mu_1}\right)$

(iii) Predator washed state: $E_3 = \left(\frac{\delta_1}{\beta - r}, \frac{r}{\beta - r}, 0\right), (\beta > r)$

(iv) Co-existence state: $E_4 = (x^*, y^*, z^*)$, where

$$\begin{aligned} x^* &= \frac{\mu_2\alpha(\mu_1\delta_1 + r\sigma_2) + \mu_1\delta_1\delta_2 + (r-\beta)\mu_2\delta_2}{\mu_1\alpha(\mu_1\delta_1 + r\mu_2)} \\ y^* &= \frac{\beta\delta_2 - \alpha(\mu_1\delta_1 + r\mu_2)}{\alpha(\mu_1\delta_1 + r\mu_2)} \\ z^* &= \frac{\alpha(\mu_1\delta_1 + r\mu_2) + (r-\beta)\mu_2\delta_2}{\mu_1\delta_2} \end{aligned}$$

The co-existence state E_4 exists if $\frac{\alpha(\mu_1\delta_1 + r\mu_2)}{\delta_2} < \beta < r$.

The Jacobian matrix of system (2.1) at the equilibrium state is given by

$$J = \begin{bmatrix} r - \frac{\beta y}{1+y} - \mu_1 z & -\frac{\beta x}{(1+y)^2} & -\mu_1 x \\ \frac{\beta y}{1+y} & -\delta_1 + \frac{\beta x}{(1+y)^2} & -\mu_2 y \\ \alpha\mu_1 z e^{-\lambda\tau} & \alpha\mu_2 z e^{-\lambda\tau} & -\delta_2 + \alpha(\mu_1 x + \mu_2 y)e^{-\lambda\tau} \end{bmatrix} \tag{3.1}$$

3.1. Stability of co-existence state

The local stability of the coexistence state is analysed using the Jacobian matrix of the system (2.1), which is given by

$$J^* = \begin{bmatrix} r - \frac{\beta y^*}{1+y^*} - \mu_1 z^* & -\frac{\beta x^*}{(1+y^*)^2} & -\mu_1 x^* \\ \frac{\beta y^*}{1+y^*} & -\delta_1 + \frac{\beta x^*}{(1+y^*)^2} & -\mu_2 y^* \\ \alpha\mu_1 z^* e^{-\lambda\tau} & \alpha\mu_2 z^* e^{-\lambda\tau} & -\delta_2 + \alpha\left(\mu_1 x^* + \mu_2 y^*\right) e^{-\lambda\tau} \end{bmatrix} \tag{3.2}$$

The characteristic equation of (3.2) is given by

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 + \left[b_1\lambda^2 + b_2\lambda + b_3\right] e^{-\lambda\tau} = 0 \tag{3.3}$$

where

$$a_1 = \frac{\beta y^*}{1+y^*} - \frac{\beta x^*}{(1+y^*)^2} + \mu_1 z^* + \delta_1 + \delta_2 - r$$

$$a_2 = \delta_1\delta_2 + (\delta_1+\delta_2)(\mu_1 z^* - r) + \frac{\beta(\delta_1+\delta_2)y^*}{1+y^*} - \frac{\beta(\mu_1 z^* - r + \delta_2)x^*}{(1+y^*)^2}$$

$$a_3 = (\mu_1 z^* - r)\delta_1\delta_2 + \frac{\beta\delta_1\delta_2 y^*}{1+y^*} - \frac{\beta\delta_2(\mu_1 z^* - r)x^*}{(1+y^*)^2}$$

$$b_1 = -\alpha(\mu_1 x^* + \mu_2 y^*)$$

$$b_2 = \frac{\alpha\beta(\mu_1 x^* + \mu_2 y^*)x^*}{(1+y^*)^2} - \frac{\alpha\beta(\mu_1 x^* + \mu_2 y^*)y^*}{1+y^*} + \alpha(\mu_1 x^* + \mu_2 y^*)z^* - \alpha(\mu_1 z^* - r + \delta_1)(\mu_1 x^* + \mu_2 y^*)$$

$$b_3 = \frac{\alpha\beta(\mu_1 z^* - r)(\mu_1 x^* + \mu_2 y^*)x^*}{(1+y^*)^2} + \frac{\alpha\beta\mu_1(\mu_2 y^{*2} - x^* z^*)x^*}{(1+y^*)^2} + \frac{\alpha\beta[\mu_2 y^{*2} z^* - \delta_1 x^*(\mu_1 x^* + \mu_2 y^*)]y^*}{1+y^*}$$
$$+ \alpha\mu_1^2\delta_2 x^* z^* + \alpha\mu_2(\mu_1 z^* - r)y^* z^* + \alpha\delta_1(r - \delta_1 z^*)(\mu_1 x^* + \mu_2 y^*)$$

For $\tau = 0$, equation (3.3) becomes

$$\lambda^3 + (a_1+b_1)\lambda^2 + (a_2+b_2)\lambda + (a_3+b_3) = 0 \tag{3.4}$$

It can be easily verified that $(a_1+b_1)(a_2+b_2) - (a_3+b_3) > 0$. We can say that the system (2.1) is locally asymptotically stable at the co-existence state $E_4 = (x^*, y^*, z^*)$ using the Routh-Hurwitz criterion.

3.2. Hopf Bifurcation Analysis

For $\tau > 0$, assume that equation (3.3) has a complex root of the form $i\omega(\omega > 0)$. Substituting this complex root into equation (3.3) and separating real and imaginary parts, we obtain

$$\begin{aligned} b_2\omega\cos\omega\tau - (b_3 - b_1\omega^2)\sin\omega\tau &= \omega^3 - a_2\omega \\ (b_3 - b_1\omega^2)\cos\omega\tau + b_2\omega\sin\omega\tau &= a_1\omega^2 - a_3 \end{aligned} \tag{3.5}$$

Elimination of the delay parameter τ from (3.4) gives

$$\omega^6 + P_1\omega^4 + P_2\omega^2 + P_3 = 0 \tag{3.6}$$

Where

$$P_1 = {a_1}^2 - 2a_2 - {b_1}^2$$
$$P_2 = \left({a_2}^2 - 2a_1a_3 + 2b_1b_3 - {b_2}^2\right)^2$$
$$P_3 = {a_3}^2 - {b_3}^2$$

It can be easily verified that $P_1 = \left({a_1}^2 - 2a_2 - {b_1}^2\right) > 0$ and $P_3 = \left({a_3}^2 - {b_3}^2\right) < 0$. Therefore, according to the theorem's criteria, there must be a singular positive ω_0 that satisfies equation (3.6). In other words, equation (3.3) has a pair of purely imaginary roots of the form $\pm i\omega_0$.

From (3.5), we get

$$\tau_j = \frac{1}{\omega_0}\left\{\arccos\left[\frac{\left(a_1{\omega_0}^2 - a_3\right)\left(b_3 - b_1{\omega_0}^2\right) + \left({\omega_0}^3 - a_2\omega_0\right)b_2\omega_0}{\left(b_3 - b_1{\omega_0}^2\right)^2 + {b_2}^2{\omega_0}^2}\right] + 2j\pi\right\}, j = 0, 1, 2, ... \tag{3.7}$$

For $\tau = 0$, E_4 is stable. Hence by Butler's lemma, E_4 remains stable for $\tau < \tau_0$. We have to show that $\left.\frac{d\,\mathrm{Re}(\lambda)}{d\tau}\right|_{\lambda = i\omega_0} > 0$

This proves that for every $\tau > \tau_0$, there exists at least one eigenvalue with a positive real part. In addition, the necessary periodic solution is obtained when the requirements for Hopf bifurcation [13] are satisfied. Now differentiating (3.3) w.r.t. τ we get,

$$\left[3\lambda^2 + 2a_1\lambda + a_2 + (2b_1\lambda + b_2)e^{-\lambda\tau} - (b_1\lambda^2 + b_2\lambda + b_3)\tau e^{-\lambda\tau}\right]\frac{d\lambda}{d\tau} = (b_1\lambda^2 + b_2\lambda + b_3)\lambda e^{-\lambda\tau}$$

$$\left(\frac{d\lambda}{d\tau}\right)^{-1} = \frac{3\lambda^2 + 2a_1\lambda + a_2}{(b_1\lambda^2 + b_2\lambda + b_3)\lambda e^{-\lambda\tau}} + \frac{2b_1\lambda + b_2}{(b_1\lambda^2 + b_2\lambda + b_3)\lambda} - \frac{\tau}{\lambda}$$

Substituting $\lambda = i\omega_0$ we get

$$\left.\left(\frac{d\lambda}{d\tau}\right)^{-1}\right|_{\lambda = i\omega_0} = \frac{-3{\omega_0}^2 + 2ia_1\omega_0 + a_2}{i\omega_0(-b_1{\omega_0}^2 + ib_2\omega_0 + b_3)e^{-\lambda\tau}} + \frac{2ib_1\omega_0 + b_2}{i\omega_0(-b_1{\omega_0}^2 + ib_2\omega_0 + b_3)} - \frac{\tau}{i\omega_0}$$

We have $\left.\frac{d\,\mathrm{Re}(\lambda)}{d\tau}\right|_{\lambda = i\omega_0} = \mathrm{Re}\left.\left(\frac{d\lambda}{d\tau}\right)^{-1}\right|_{\lambda = i\omega_0}$

$$= \frac{2a_1\left[\cos(\omega_0\tau)\left(b_3 - b_1{\omega_0}^2\right) + \sin(\omega_0\tau)b_2\omega_0\right] - 2{b_1}^2{\omega_0}^2 + 2b_1b_3 - {b_2}^2}{(P^2 + Q^2)\omega_0}$$
$$+ \frac{\left(3{\omega_0}^2 - a_2\right)\left[\cos(\omega_0\tau)b_2\omega_0 - \sin(\omega_0\tau)\left(b_3 - b_1{\omega_0}^2\right)\right]}{(P^2 + Q^2)\omega_0}$$

Together with (3.5), it follows that

$$\mathrm{Re}\left.\left(\frac{d\lambda}{d\tau}\right)^{-1}\right|_{\lambda = i\omega_0} = \frac{3{\omega_0}^4 - 2\left(2a_2 - {a_1}^2 + {b_1}^2\right){\omega_0}^2 + \left({a_2}^2 + 2b_1b_3 - {b_2}^2\right)}{P^2 + Q^2}$$

where

$$P = \sin(\omega_o\tau)\left(b_3 - b_1\omega_o^2\right) - \cos(\omega_o\tau)b_2\omega_o$$

$$Q = \cos(\omega_o\tau)\left(b_3 - b_1\omega_o^2\right) + \sin(\omega_o\tau)b_2\omega_o$$

Clearly

$$\left.\frac{d\,\mathrm{Re}(\lambda)}{d\tau}\right|_{\lambda=i\omega_0} > 0 \text{ if } \left(2a_2 - a_1^2 + b_1^2\right) < 3\left(a_2^2 + 2b_1b_3 - b_2^2\right)$$

Based on above analysis and the results of Hale [4], the following conclusion is reached:

Theorem 3.1 *If $\left(2a_2 - a_1^2 + b_1^2\right) < 3\left(a_2^2 + 2b_1b_3 - b_2^2\right)$ then the equilibrium state E_4 of system (2.1) is unstable for $\tau \geq \tau_0$ and is asymptotically stable for $0 \leq \tau < \tau_0$ and system (2.1) go through Hopf bifurcation at E_4 when $\tau = \tau_0$*

3.3. Stochastic behaviour of the system at co-existence state

This section examines the dynamical behaviour of the stochastic version of system (2.1) at co-existence state. We agree that the random changes are of the white noise variety [13] and they are added to the susceptible prey, infected prey and predator individuals to integrate the effects of environmental fluctuations into the model (2.1). Then the system (2.1) takes the following form:

$$\begin{aligned} \frac{dx}{dt} &= rx - \frac{\beta xy}{1+y} - \mu_1 xz + \sigma_1(x - x*)d\xi_t^1 \\ \frac{dy}{dt} &= -\delta_1 y + \frac{\beta xy}{1+y} - \mu_2 yz + \sigma_2(y - y*)d\xi_t^2 \\ \frac{dz}{dt} &= -\delta_2 z + \alpha\mu_1 xz + \alpha\mu_2 yz + \sigma_3(z - z*)d\xi_t^3 \end{aligned} \tag{3.8}$$

where $\xi_t^i = \xi_i(t)$, $i = 1, 2, 3$ are conventional Wiener processes that are independent of each other, and $\sigma_1, \sigma_2, \sigma_3$ are constants that are referred to as the forces of ecological vacillations.

The stochastic differential system (3.8) can be cantered at E_4 with the change of variables, given by $u_1 = x - x*, u_2 = y - y*,\ u_3 = z - z*$

The linearized stochastic DEs at the state E_4 are given by

$$du\,(t) = f\,(u(t))\,dt + g\,(u(t))\,d\xi\,(t) \tag{3.9}$$

Where

$$\begin{aligned} u\,(t) &= (u_1(t), u_2(t), u_3(t))^T \\ f\,(u(t)) &= J^* \\ g\,(u) &= \begin{bmatrix} \sigma_1 u_1 & 0 & 0 \\ 0 & \sigma_2 u_2 & 0 \\ 0 & 0 & \sigma_3 u_3 \end{bmatrix} \end{aligned} \tag{3.10}$$

Let $W\,(t, u)$ be a continuously differentiable function defined on $[0, +\infty) \times \mathbb{R}^2$. The differential operator L for a function $W\,(t, u)$ is given by

$$\begin{aligned} LW(t,u) &= \frac{\partial W\,(t,u)}{\partial t} + f^T\,(u)\,\frac{\partial W\,(t,u)}{\partial u} + \frac{1}{2} Tr\left[g^T\,(u)\,\frac{\partial^2 W\,(t,u)}{\partial u^2} g\,(u)\right] \\ \frac{\partial W}{\partial u} &= col\left(\frac{\partial W}{\partial u_1}, \frac{\partial W}{\partial u_2}, \frac{\partial W}{\partial u_3}\right), \frac{\partial^2 W\,(t,u)}{\partial u^2} = \left(\frac{\partial^2 W}{\partial u_j \partial u_i}\right)\ ;\ i, j = 1, 2, 3 \end{aligned} \tag{3.11}$$

where T denotes transposition.

It can be easily verified that, the co-existence state of model (3.8) is stable if and only if the trivial solution of (3.9) is stable. As established in the work of Afanas'ev et al. [11], the subsequent theorem is valid.

Theorem 3.2 *Suppose there exists a function* $W(t,u) \in C^{1,2}\left([0, +\infty) \times \mathbb{R}^2, \mathbb{R}^+\right)$ *that satisfies the following inequalities:*

$$K_1|u|^p \le W\,(t,u) \le K_2|u|^p,\ LW\,(t,u) \le -K_3|u|^p \tag{3.12}$$

where K_1, K_2, K_3 *and* p *are all positive constants.*

Then the trivial solution of (3.9) is exponentially $p-$ *stable for* $t \ge 0$.

Theorem 3.3 *Assume that* $\sigma_1^2 \le 2\left(\mu_1 z^* + \frac{\beta y^*}{1+y^*} - r\right)$, $\sigma_1^2 \le 2\left(\delta_1 - \frac{\beta x^*}{(1+y^*)^2}\right)$, $\sigma_3^2 \le 2\delta_2$ *hold. Then, the trivial solution of (3.9) is asymptotically mean square stable.*

Proof: Consider the Lyapunov function as

$$W(u) = \frac{1}{2}\left[w_1 u_1^2 + w_2 u_2^2 + w_3 u_3^2\right] \tag{3.13}$$

Where w_1, w_2, w_3 are non-negative constants to be chosen later. With $p = 2$, it is easy to verify that inequalities (3.13) are true.

$$\begin{aligned} LW(u) = & w_1\left[\left(r - \frac{\beta y^*}{1+y^*} - \mu_1 z^*\right)u_1 - \frac{\beta x^*}{(1+y^*)^2}u_2 - \mu_1 x^* u_3\right]u_1 \\ & + w_2\left[\frac{\beta y^*}{1+y^*}u_1 + \left(-\delta_2 + \frac{\beta x^*}{(1+y^*)^2}\right)u_2 - \mu_2 y^* u_3\right]u_2 \\ & + w_3\left(\alpha\mu_1 z^* u_1 + \alpha\mu_2 z^* u_2 - \delta_2 u_3\right)u_3 + \frac{1}{2}Tr\left[g^T(u)\frac{\partial^2 W(t,u)}{\partial u^2}g(u)\right] \end{aligned} \tag{3.14}$$

with $\frac{1}{2}Tr\left[g^T(u)\frac{\partial^2 W(t,u)}{\partial u^2}g(u)\right] = \frac{1}{2}\left[w_1 u_1^2 + w_2 u_2^2 + w_3 u_3^2\right]$

Choose $\frac{\beta x^*}{(1+y^*)^2}w_1 = \frac{\beta y^*}{1+y^*}w_2$, $\mu_1 x^* w_1 = \alpha\mu_1 z^* w_3$ & $\mu_1 y^* w_2 = \alpha\mu_2 z^* w_3$, then

$$LW(u) = -\left[\left(\mu_1 z^* + \frac{\beta y^*}{1+y^*} - r\right) - \frac{1}{2}\sigma_1^2\right]w_1 u_1^2 - \left[\left(\delta_1 - \frac{\beta x^*}{(1+y^*)^2}\right) - \frac{1}{2}\sigma_2^2\right]w_2 u_2^2 - \left[\delta_2 - \frac{1}{2}\sigma_3^2\right]w_3 u_3^2$$

Based on Theorem 3.2, the proof is completed. □

4. Numerical Simulations

The section deals with some numerical simulations for supporting the analytical results.

Example 4.1 *For the parameter values* $r = 1.5, \beta = 0.8, \delta_1 = 0.964, \delta_2 = 0.47$, $\mu_1 = 0.102$, $\mu_2 = 0.068$, $\alpha = 0.25$. *We can ascertain from (3.7) whenever the time delay* (τ) *goes past the threshold value* $\tau_0 = 0.23$, E_4 *reports loss in stability and a family comprising periodic solutions bifurcate from* E_4 *(Figure 1 - Figure 3). However,* E_4 *is asymptotically stable for* $0 \le \tau < \tau_0$

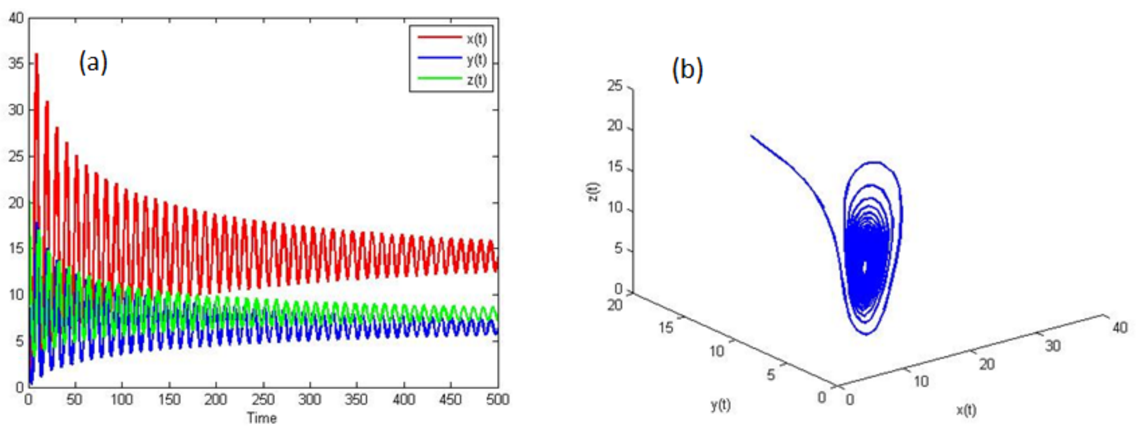

Figure 1: (a) Trajectories and (b) Phase portraits of the system (2.1) at E_4 when $\tau = 0.2 < \tau_0 = 0.23$

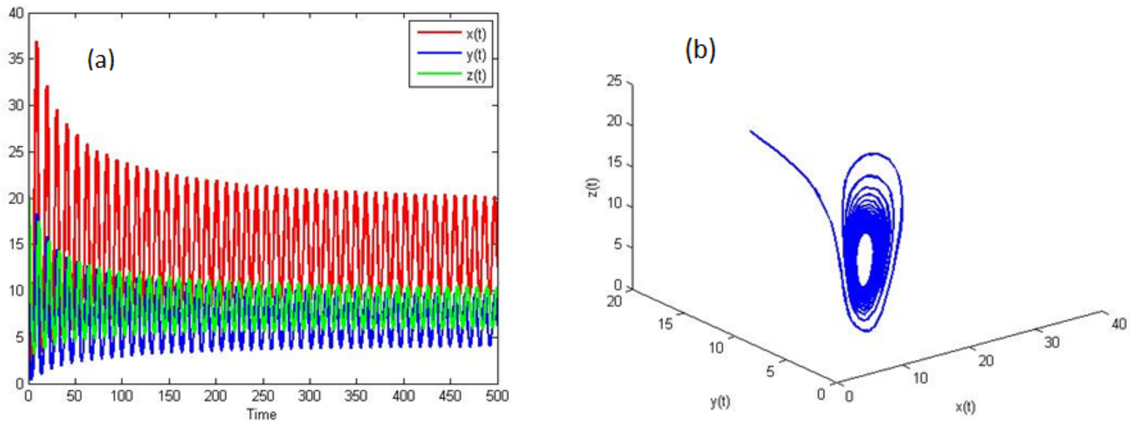

Figure 2: (a) Trajectories and (b) Phase portraits of the system (2.1) at E_4 when $\tau = \tau_0 = 0.23$

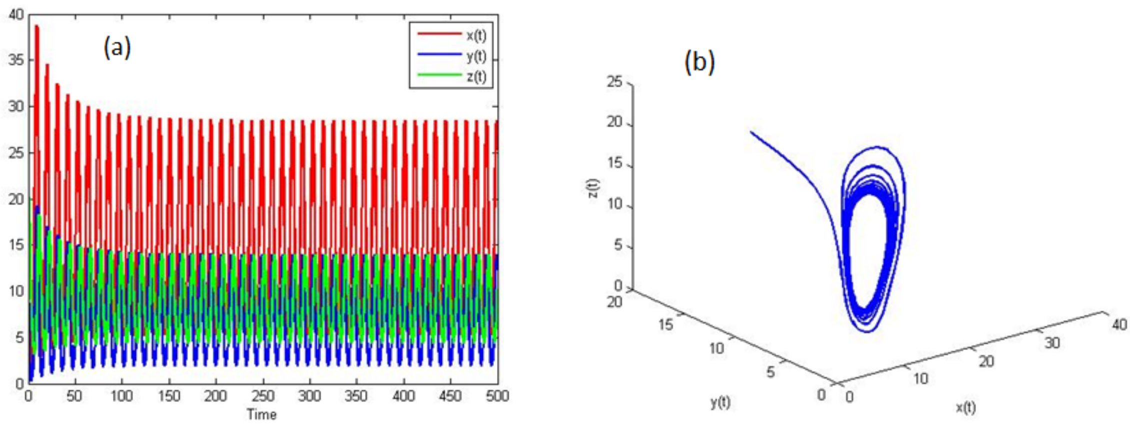

Figure 3: (a) Trajectories and (b) Phase portraits of the system (2.1) at E_4 when $\tau = 0.29 > \tau_0 = 0.23$

5. Conclusion

This paper looked in some detail at effect of delay [7,10] on eco-epidemic model with disease in the prey population. The co-existence state E_4 of the system (2.1) is locally asymptotically stable if $\frac{(r\mu_2 + \mu_1\delta_1)\,\alpha}{\delta_2} < \beta < \gamma$. Employing Lyapunov functional technique, the global stability pertaining to the co-existence state E_4 is made clear. Moreover, a behavioral change is observed in the system as it moves from a stable to an unstable state when the delay parameter crosses the threshold value τ_0 leading to a Hopf bifurcation from E_4. Further, the stochastic system remains globally asymptotically stable provided that the intensities of the white noise stay below specific threshold values. To conclude, numerical simulations were arrived at as a way of validating analytical results.

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