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Extensions in UP-Algebras and Related Properties*

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ABSTRACT: In this work, we present and explore the idea of extensions of UP-algebras, inspired by similar constructions in KU-algebras. We begin by recalling fundamental notions related to UP-algebras and proposea formal definition of an extension in this context. Several illustrative examples are provided to demonstrate the structure and behavior of these extensions. We examine the relationship between extensions and homomorphic images, ideals, and congruences, and we establish some necessary and sufficient conditions under which a UP-algebra extension exists. Furthermore, we compare our results with those obtained in the theory of KU-algebra extensions to highlight similarities and differences. This study opens new directions in the algebraic analysis of UP-structures.

Keywords: Extended UP−algebra, ideals, isomorphic extended UP−algebra.

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1. Introduction

Algebraic structures of logical origin have been extensively investigated due to their potential applications in mathematics, logic, and computer science. One of the important non-classical algebraic systems is the class of KU-algebras, first studied by Hu and Li [4], and further developed by several authors in various directions. For instance, Akram et al. [3] examined cubic KU-subalgebras, while Ansari and Koam [12] studied rough approximations in KU-algebras. More recently, Ali, Haider, and Ansari [1] proposed an extension of KU-algebras that revealed new structural properties and enriched the theoretical framework. In 2017, Iampan [2] introduced UP-algebras as a new branch of logical algebras, which generalize certain aspects of KU-algebras while exhibiting independent characteristics. Since their inception, UP-algebras have attracted significant attention, and numerous generalizations and extensions have been established. For example, Sawika et al. [7] investigated derivations of UP-algebras, while Satirad et al. [9] introduced generalized power UP-algebras. Poungsumpao et al. [10] studied fuzzy UP-ideals and fuzzy UP-subalgebras, extending the theory in the context of fuzzy set theory. Additionally, Iampan [11] considered fully UP-semigroups, and Iampan [5] developed UP-isomorphism theorems, thereby deepening the structural understanding of these algebras. Further contributions include the study of independent UP-algebras by Iampana et al. [6], and the exploration of algebraic graphs associated with UP-algebras by Ansari et al. [8]. These results illustrate the growing importance of UP-algebras in both pure and applied mathematics. The present work is motivated by the parallel development of KU- and UP-algebras and, in particular, by the extensions of KU-algebras considered in [1,12]. Inspired by these studies, we aim to investigate analogous extensions in the setting of UP-algebras, thereby establishing new structural insights and broadening the scope of research in this area.

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2. Preliminaries

This section provides definitions and associated terms related to UP-algebras, UP-subalgebras, and UP-ideals, supported by examples and relevant results.

Definition 2.1 ([2]) *A structure $(X;\cdot,1)$ of type $(2,0)$ is called a* UP-algebra *if the operation "$\cdot$" and the constant 1 satisfy the following axioms for all $a,b,c\in X$:*

(UP-1) $(b\cdot c)\cdot\big((a\cdot b)\cdot(a\cdot c)\big)=1,$

(UP-2) $1\cdot a=a,$

(UP-3) $a\cdot 1=1,$

(UP-4) *If $a\cdot b=1$ and $b\cdot a=1$, then $a=b$.*

Definition 2.2 ([1]) *An algebra structure $(X;\cdot,1)$ of type $(2,0)$ is called a* KU-algebra *if the following axioms hold for all $a,b,c\in X$:*

(KU-1) $(b\cdot a)\cdot\big((a\cdot c)\cdot(b\cdot c)\big)=1,$

(KU-2) $1\cdot a=a,$

(KU-3) $a\cdot 1=1,$

(KU-4) *If $a\cdot b=1$ and $b\cdot a=1$, then $a=b$.*

Definition 2.3 ([1]) *Assume that X is a KU-algebra. A nonempty subset $J\subseteq X$ is called a* KU-ideal *of X if it satisfies the following conditions:*

1. *$1\in J$;*
2. *For any $a,b\in X$, whenever $a\in J$ and $a\cdot b\in J$, it follows that $b\in J$.*

Lemma 2.1 ([?]) *In every KU-algebra X, the following identity holds for all $a,b,c\in X$:*

$$c\cdot(b\cdot a)=b\cdot(c\cdot a).$$

Theorem 2.1 ([2]) *Every KU-algebra is a UP-algebra. However, the converse does not always hold; that is, a UP-algebra need not be a KU-algebra.*

Example 2.1 ([2]) *Let $X=\{1,a,b,c,d\}$ and define the binary operation "$\cdot$" on X by the following Cayley table:*

$\cdot$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	1	1	1
b	1	b	1	1	1
c	1	b	b	1	1
d	1	b	b	d	1

A direct verification shows that $(X;\cdot,1)$ satisfies every axiom of a UP-algebra. On the other hand, condition (KU-1) *fails. Indeed,*

$$(1\cdot c)\cdot\big((c\cdot a)\cdot(1\cdot a)\big)=c\cdot(b\cdot a)=c\cdot b=b,$$

which is not equal to 1. Hence, $(X;\cdot,1)$ cannot be a KU-algebra.

Definition 2.4 ([1]) *Assume that X is a nonempty set and that $K\subseteq X$ is a nonempty subset. An extended KU-algebra associated with K is an algebra $(X_K;\cdot,K)$, the following axioms are satisfied for all $a,b,c\in X$, where " $\cdot$ " is a binary operation onX_K:*

(KUE- 1) $(b \cdot a) \cdot \big((a \cdot c) \cdot (b \cdot c)\big) \in K$,

(KUE- 2) $a \cdot K = \{a \cdot k : k \in K\} \subseteq K$,

(KUE- 3) $K \cdot a = \{k \cdot a : k \in K\} = \{a\}$,

(KUE- 4) If $a \cdot b \in K$ *and* $b \cdot a \in K$, *then either* $a = b$ *or both* a *and* b *lie in* K.

Example 2.2 ([1]) *Let* $X = \{1, 2, 3, 4\}$ *and* $K = \{1, 2\}$. *The following Cayley table defines a binary operation "$\cdot$" on* X_K, *showing that* $(X_K; \cdot, K)$ *forms an extended KU-algebra:*

$\cdot$	1	2	3	4
1	1	2	3	4
2	1	2	3	4
3	2	1	2	2
4	1	2	4	1

A direct verification confirms that all axioms of an extended KU-algebra are satisfied with respect to the subset K.

Definition 2.5 ([1]) *Let* X_K *be an extended KU-algebra. A subset* $J \subseteq X_K$ *is called an ideal of* X_K *if it satisfies the following conditions:*

(1) $K \subseteq J$;

(2) For any $x, y \in X_K$, *if* $x \in J$ *and* $x \cdot y \in J$, *then* $y \in J$.

Proposition 2.1 ([2]) *The following identities hold in every UP-algebra.* $\forall a, b, c \in X$,

(1) $a \cdot a = 1$,

(2) If $a \cdot b = 1$ *and* $b \cdot c = 1$, *then* $a \cdot c = 1$,

(3) From $a \cdot b = 1$ *it follows that* $(c \cdot a) \cdot (c \cdot b) = 1$,

(4) From $a \cdot b = 1$ *it also follows that* $(b \cdot c) \cdot (a \cdot c) = 1$,

(5) $a \cdot (b \cdot a) = 1$,

(6) $(b \cdot a) \cdot a = 1$,

(7) $a \cdot (b \cdot b) = 1$.

3. Extended UP-Algebras

This section introduces concept of extension of UP-algebras and presents several related results.

Definition 3.1 *Let* X *be a non-empty set, and let* $\emptyset \neq U \subseteq X$. *An extended UP-algebra associated with* U *is defined as the algebra* $(X_U; \cdot, U)$, *where "$\cdot$" is a binary operation on* X_U *satisfying the following axioms:*

(UPE- 1) $(b \cdot c) \cdot \big((a \cdot b) \cdot (a \cdot c)\big) \in U$,

(UPE- 2) $a \cdot U = \{a \cdot u : u \in U\} \subseteq U$,

(UPE- 3) $U \cdot a = \{u \cdot a : u \in U\} = \{a\}$,

(UPE- 4) If $a \cdot b \in U$ *and* $b \cdot a \in U \Rightarrow a = b$ *or both* a *and* b *lie in* U. *for all* $a, b, c \in X$

For convenience, we shall refer to X_U *simply as the extended UP-algebra* $(X_U; \cdot, U)$.

Definition 3.2 *A binary relation $\leq$ on the extended UP-algebra X_U is defined as follows: for $a, b \in X_U$, $a \leq b$ iff $a = b$ or $(a \cdot b \in U$ and $b \notin U)$.*

Example 3.1 *Let $X = \{a, b, c, d\}$ and $U = \{a, b\}$. As demonstrated in the table below:*

$\cdot$	a	b	c	d
a	a	b	c	d
b	a	b	c	d
c	a	a	b	b
d	a	b	d	a

The structure X_U satisfies the axioms of an extended UP$-$algebra.

Proposition 3.1 *Let X_U be an extended UP-algebra. Then, for all $a, b, c \in X$, the following properties hold:*

(1) $a \cdot a \in U$,

(2) $a \cdot (b \cdot a) \in U$,

(3) $a \cdot \big((a \cdot b) \cdot b\big) \in U$,

(4) If $a \cdot b \in U$ *and* $b \cdot c \in U$, *then* $a \cdot c \in U$,

(5) If $a \cdot b \in U$ *then* $(b \cdot c) \cdot (a \cdot c) \in U$,

(6) If $a \cdot b \in U$ *then* $(c \cdot a) \cdot (c \cdot b) \in U$,

(7) If $u \cdot a \in U$, *and* $u \in U$ *then* $a \in U$.

Proof: (1) Let $a \in X$ and $u \in U$. By axiom (UPE - 2), we have $a \cdot u = a$ and $u \cdot u = u$. Applying (UPE-1) $a \cdot a = u \cdot (a \cdot a) = (u \cdot u) \cdot \big((a \cdot u) \cdot (a \cdot u)\big) \in U$.
(2) and (3) proof directly follow from the Definition 3.2
(4) Assume that $a \cdot b \in U$ and $b \cdot c \in U$. By Definition 3.2, $a \leq b$ means that either $a = b$ or $a \cdot b \in U$ with $b \notin U$. If $a = b$, then $a = c$. In this case, since $b \cdot c \in U$, we have $b \cdot c = a \cdot c \in U$.
(5) Assume that $a \cdot b \in U$. Then, by Definition 3.2, we have $a \leq b$, and therefor $a \cdot c \leq b \cdot c$. By the definition of the order, this implies $(a \cdot c) \cdot (b \cdot c) \in U$.
(6) Can be proved similarly to (3).
(7) Suppose that $u \cdot a \in U$ and $u \in U$. By axiom (UPE-3), we have $a = u \cdot a$. Since $u \cdot a \in U$, it follows that $a \in U$.

Proposition 3.2 *Let X_U be an extended UP- algebra.Then $\forall a, b, c \in X$ and for every $u \in U$.*

(1) $a \leq a$,

(2) $a \leq b$ *and* $b \leq a$, *then* $a = b$,

(3) If $a \leq u$, *then* $a = u$

(4) If $a \leq b$ *and* $b \leq c$, *then* $a \leq c$.

Proof: (1) By the definition of the relation $\leq$ we have $a \leq a \Leftrightarrow a \cdot a \in U$. Since Proposition3.1 (1), states that $a \cdot a \in U$. For every $a \in X$, it follows that $a \leq a$.
(2) Assume that $a \leq b$ and $b \leq a$. Suppose, toward a contradiction, that $a \neq b$. By Definition 3.2, the assumptions $a \leq b$ and $b \leq a$ imply $a \cdot b \in U, b \cdot a \in U, a, b \notin U$. By axiom (UPE-4), it follows that $a = b$, which is a contradiction.
(3) Is similar to (1), *the proof follows immediately by definition of $\leq$.*
(4) Suppose that $a \leq b$ and $b \leq c$. If $a = b$ or $b = c$, the conclusion $a \leq c$ is immediate. Now suppose $a \neq b$ and $b \neq c$. By Definition 3.2, we have $a \cdot b \in U$, $b \cdot c \in U$ and $b, c \notin U$. Using (UPE-1), with y replaced by a and c replaced by c, we obtain $(b \cdot c) \cdot \big((a \cdot y) \cdot (a \cdot c)\big) \in U$. Since $b \cdot c \in U$ and U is closed under left multiplication by (UPE-2), this simplifies to $(a \cdot c) \in U$. Because $c \notin U$, the definition of $\leq$ implies $a \leq c$.

Definition 3.3 *A non-empty subset U of a UP-algebra X is called the minimal set of the ordered structure $(X_U, \leq)$ if, whenever $a \leq b$, it follows that $a = u$ for some $u \in U$, for all $a, b \in X$.*

Lemma 3.1 *An extended UP-algebra X_U with binary relation $\leq$ is a partial ordered set with a minimal set. Proof: From the definition of $\leq$ and Proposition 3.1 (1), we have $a \leq a$ for all $x \in X$. Thus $\leq$ is reflexive. Now assume that $a \leq b$ and $b \leq a$. If $a = b$, there is nothing to prove. Otherwise, by definition $\leq$, $b \cdot a \in U$ and $a \cdot b \in U$. Using axiom (UPIE-4), this implies $a = b$. Hence, $\leq$ is antisymmetric. Next, to show transitivity, assume $a \leq b$ and $b \leq c$. If $a = b$ or $b = c$, then clearly $a \leq c$. Otherwise, by definition of $\leq$, $a \cdot b \in U$ and $b \cdot c \in U$. Applying axioms (UPE-1) and (UPE-3), we obtain $(b \cdot c) \cdot \big((a \cdot b) \cdot (a \cdot c)\big) \in U$, which forces $a \cdot c \in U$. Hence $a \leq c$, and so $\leq$ is transitive. Finally, for any $u \in U$ and any $a \in X$, we have $a \leq u$ by Definition 3.2. Thus, if $a \leq u$ for some $u \in U$, then Definition 3.2 implies $a = u$. Therefore, every element of U is minimal, and U is the minimal set in $(X_U, \leq)$. Thus $(X_U, \leq)$ is a partially ordered set with a minimal set U.*

Theorem 3.1 *Each UP-algebra is an extended UP-algebra; conversely, an extended UP-algebra is a UP-algebra iff U consists of a single element.*

Proof: Clearly, any UP-algebra $(X; \cdot, 1)$ can be regarded as an extended UP-algebra X_U by taking $U = \{1\}$. Conversely, suppose X_U is an extended UP-algebra with $U = \{u\}$. Then the structure $(X_u; \cdot, 1 = u)$ forms a UP-algebra. Now, assume that an extended UP-algebra X_U is itself a UP-algebra. Let $u_1, u_2 \in U$. By axiom (UPE-3), we have $u_1 \cdot u_1 = u_1$ and $u_2 \cdot u_2 = u_2$ However, since X_U is a UP-algebra, Proposition 2.1 (1) implies that $u_1 \cdot u_1 = u_2 \cdot u_2 = 1$. Therefore $u_1 = u_2 = 1$, which leads to the conclusion that $U = \{1\}$.

Example 3.2 *Consider the set $X = \{a, b, c, d, e\}$ and $U = \{a, b\}$. The binary operation "$\cdot$" is defined on X according to the table that follows.*

$\cdot$	a	b	c	d	e
a	a	b	c	d	e
b	a	b	c	d	e
c	a	b	a	a	e
d	a	a	d	a	e
e	a	a	b	a	a

The table shows that X_U is an extended UP-algebra, but not UP-algebra since $(d \cdot b) \cdot \big((c \cdot d) \cdot (c \cdot b)\big) = b \in U \neq (c \cdot b) \cdot \big((d \cdot c) \cdot (d \cdot b)\big) = a \in U$ because U is not single element.

Theorem 3.2 *Two extended UP-algebras $X_{(U_1)}$ and $X_{(U_2)}$ having the same operation must satisfy $U_1 = U_2$. Proof: Suppose $x \in U_1$. By axiom (UPE-3) we obtain $x = x$. Then using Proposition 3.2 (1), it follows that $x \in U_2$, so $_1 \subseteq U_2$. Applying the same argument in the opposite direction yields $U_2 \subseteq U_1$. Therefore, we conclude that $U_1 = U_2$.*

Definition 3.4 *Let $Y \subseteq X$ and $L \subseteq U$. If the structure $(Y; \cdot, L)$ froms an extended UP-algebra on its own, then $(Y; \cdot, L)$ is called an extended sub-algebra of X_U.*

Example 3.3 *Let $Y = \{a, b, c\} \subseteq X = \{a, b, c, d\}$ and $U = \{a, b\}$.*

$\cdot$	a	b	c	d
a	a	b	c	d
b	a	b	c	d
c	c	a	b	b
d	a	b	d	a

Then X_U an extended UP-algebra, and Y_U is an extended sub-algebra of X_U.

Proposition 3.3 *Given a family $\{(X_i; \cdot; U)\}_{(i \in \Delta)}$ of extended UP-subalgebras of an extended UP-algebra $(X_U; \cdot; U)$, the intersection $\bigcap_{(i \in \Delta)} (X_u; \cdot;)$ is itself an extended UP-algebra.*

Theorem 3.3 *Let X_U be an extended UP-algebra. A structure Y_M is an extended sub-algebra of X_U iff the following cicrumstances are met:*

(1) $a \cdot b \in Y$ for all $a, b \in Y$;

(2) $M = U \cap Y$.

Proof: Assume that Y_M is a sub-algebra of the extended UP-algebra X_U. Then for all $a, b \in Y$, we have $a \cdot b \in Y$. Let $M = U \cap Y$. Since clearly $M \subseteq U$, it follows that Y_M is also a sub-algebra of X_U. By Theorem 3.2, we obtain $M = L = U \cap Y$. The converse direction follows immediately.

Corollary 3.1 *If X_L is sub-algebra of X_U, then it follows that $L = U$. Proof: Since X_L is a sub-algebra of X_U, by definition we must have $L = U \cap X$. But X is the entire algebra's underlying set, therefor $U \cap [X = U$. Thus, $L = U$.*

4. Ideals in extended UP-Algebras

This section studies ideals of extended UP-algebras and their main properties.

Definition 4.1 *Let X_U be an extended UP-algebra. A subset $J \subseteq X$ is called an ideal of X_U if the following conditions hold:*

(1) $U \subseteq J$;

(2) For all $a, b \in X$, if $a \in J$ and $a \cdot b \in J$, then $b \in J$.

It is evident that the trivial ideals of X_U are U and X_U itself.

Example 4.1 *Let $X = \{a, b, c, d, e\}$ and $U = \{a, b\}$. The following table illustrates that X_U forms an extended UP-algebra.*

$\cdot$	a	b	c	d	e
a	a	b	c	d	e
b	a	b	c	d	e
c	a	b	a	a	e
d	a	a	d	a	e
e	a	a	b	a	a

Then $\{a, b, c, d\} = J \subseteq X$ is an ideal of the extended UP-algebra X_U.

Theorem 4.1 *For every ideal J of the extended UP-algebra X_U. If $a \in J$ and $a \leq b$, then $b \in J$. Proof: Assume that J is an ideal of an extended UP -algebra X_U. By Definition 3.2 if $b \leq a$ then either $a = b$ or $a \cdot b \in U$ and $b \notin U$. By Definition 4.1 $U \subset J$ so $a \cdot b \in J$ and J is an ideal of X_U then $b \in J$.*

Lemma 4.1 *Let $\{J_i : i \in N\}$ be an ideal family of X_U. Then $\bigcap_{(i \in N)} J_i$ is also ideal of X_U. Proof: Since $U \subseteq J_i \forall i \in N$, we have $U \subseteq \bigcap_{(i \in N)} J_i$. Let $a, b \in X$ with $a \in \bigcap_{(i \in N)} J_i$ and $a \cdot b \in \bigcap_{(i \in N)} J_i$. Then $a, a \cdot b \in J_i$ for each i. Because each J_i is an ideal, it follows that $b \in J_i \forall i$, and hence $b \in \bigcap_{(i \in N)} J_i$. Thus, $\bigcap_{(i \in N)} J_i$ is an ideal of X_U.*

Theorem 4.2 *Let $(X, \cdot, U)$ be an extended UP-algebra, and define $(X_1, *, 1)$ as a UP-algebra where*

$$X_1 = (X \setminus U) \cup \{1\}.$$

Then, for any ideal J of the extended UP-algebra X_U, the set

$$I_1 = (J \setminus U) \cup \{1\}$$

is an ideal of the UP-algebra X_1.

Proof: Clearly, $1 \in I_1$. Let $a, b \in X_1$ and suppose $b \in I_1$ with $a * b \in I_1$.
If $b = 1$, then $a * 1 = a \in I_1$, so the condition is satisfied.
Assume now that $a, b \neq 1$. Then

$$a \in X_1 \setminus \{1\} = X \setminus U, \qquad b \in I_1 \setminus \{1\} = J \setminus U.$$

If $a * b = 1$, then by the definition of the UP-algebra operation, $a \cdot b \in U$. Since J is an ideal, $a \cdot b \in J$ and $b \in J$ imply $a \in J$. Moreover, $a \notin U$, so $a \in J \setminus U \subseteq I_1$.
If $a * b \neq 1$, then $a \cdot b \notin U$ and

$$a * b = a \cdot b \in I_1.$$

Because J is an ideal and $b \in J$, we deduce $a \in J$, hence $a \in J \setminus U \subseteq I_1$.
In all cases, $a \in I_1$, proving that I_1 is an ideal of X_1.

Example 4.2 *Let $X = \{a, b, c, d, e\}$ and $U = \{a, b\}$. The operation table below shows that X_U forms an extended UP-algebra.*

$\cdot$	a	b	c	d	e
a	a	b	c	d	e
b	a	b	c	d	e
c	a	a	a	a	e
d	a	a	d	a	e
e	a	a	b	a	a

Now, let $X_1 = \{1, c, d, e\}$ with the operation defined by the table below.

$\cdot$	1	c	d	e
1	1	c	d	e
c	1	1	1	1
d	1	c	1	e
e	1	c	d	1

Then X_1 forms a UP-algebra. It can be observed that $J = \{a, b, c, d\}$ is an ideal of X_U, and $I_1 = (J \setminus U) \cup \{1\} = \{1, c, d\}$ is an ideal of X_1.

Definition 4.2 *Let $(X_1, \cdot_1, U_1)$ and $(X_2, \cdot_2, U_2)$ be two extended UP-algebras. A map $\tau : (X_1, \cdot_1, U_1) \to (X_2, \cdot_2, U_2)$ is called an isomorphism if:*

(1) τ is bijective, and;

(2) For all $a, b \in X_1, \tau(a \cdot_1 b) = \tau(a) \cdot_2 \tau(b)$.

If such a map τ exists, then $(X_1, \cdot_1, U_1)$ is said to be isomorphic to $(X_2, \cdot_2, U_2)$, and we write $X_{1_{U_1}} \cong X_{2_{U_2}}$.

Theorem 4.3 *Let $\tau : (X_1, \cdot_1, U_1) \to (X_2, \cdot_2, U_2)$ be an isomorphism between two extended UP-algebras. Then $\tau(U_1) = U_2$. Proof: By Definition 4.2, the structure $(\tau(X_1), \cdot_2, \tau(U_1))$ is itself an extended UP-algebra. Since $\tau(X_1) = X_2$, it follws that $(X_2, \cdot_2, \tau(U_1))$ is an extended UP-algebra. Therefor, by Theorem 3.1 we conclude that $\tau(U_1) = U_2$.*

Theorem 4.4 *Let $\tau : (X_1, \cdot_1, U_1) \longrightarrow (X_2, \cdot_2, U_2)$ be an isomorphism of extended UP-algebras, and let I be an ideal of $X_{1(U_1)}$. Then*

$$J = \tau(I)$$

is an ideal of $X_{2(U_2)}$.

Proof: Since τ is bijective and I is an ideal of $X_{1(U_1)}$, we have

$$U_1 \subseteq I \implies \tau(U_1) \subseteq \tau(I).$$

By Theorem 4.3, $\tau(U_1) = U_2$, so

$$U_2 \subseteq J = \tau(I).$$

The remaining conditions for J to be an ideal follow immediately from the fact that τ is an isomorphism.

5. Conclusion

In this paper, we introduced an extension of UP-algebras, called extended UP-algebras X_U, defined with respect to a non-empty subset $U \subset X$. We showed that every UP-algebra can be viewed as an extended UP-algebra, and that an extended UP-algebra X_U coincides with the original UP-algebra X precisely when U is a singleton set. Several structural properties of extended UP-algebras were established, including results describing the behaviour of ideals and the preservation of ideal structure under isomorphisms. We also provided examples illustrating how extensions enlarge the algebra while maintaining the UP-axioms and supporting the transfer of ideal-related properties. As possible directions for future research, one may study similar extension techniques on other implication-based or logical algebras. Further investigations might also consider identities and applications involving fuzzification, rough sets, soft sets, coding theory, and related structures within the framework of extended UP-algebras.

References

1. K. Ali NA, A. Haider, and M. A. Ansari. "On an extension of KU-algebras. *AIMS Mathematics* 6, no. 2 : 1249-1257 (2021).
2. A. Iampan, A new branch of the logical algebra: UP-algebras, *Journal of Algebra and Related Topics* 5, no. 1: 35-54 (2017).
3. M. Akram, N. Yaqoob, and M. Gulistan, Cubic KU-subalgebras, *Int. J. Pure Appl. Math.*, (5) 89 , 659-665 (2013).
4. Q. P. Hu and X. Li, On BCH-algebras, *Math. Semin. Notes, Kobe Univ.*, 11, 313-320, (1983).
5. A. Iampan, THE UP-ISOMORPHISM THEOREMS FOR UP-ALGEBRAS, *Discussiones Mathematicae General Algebra and Applications* , 3 113–123 , 9, (2019).
6. A. Iampana, P. Julathab, P. Khamrotc, D. A. Romano, Independent UP -algebras, *Journal of Mathematics and computer science*, 65 - 76, 27,(2022).
7. K. Sawika, R. Intasan, A. Kaewwasri,and A. Iampan, DERIVATIONS OF UP-ALGEBRAS, *Korean J. Math.* No. 3, pp. 345–367, 24 (2016)
8. Ansari, Moin A., Azeem Haidar, and Ali NA Koam., On a graph associated to UP-algebras , *Mathematical and Computational Applications* 23, no. 4 :61, (2018).
9. A. Satirad, P. Mosrijai, A. Iampan, Generalized power UP-algebras, *Int. J. Math. Comput. Sci.*, no. 1, 17–25, , 14 (2019)
10. P. Poungsumpao, W. Kaijae, S. Arayarangsi, A. Iampan, Fuzzy UP-ideals and fuzzy UP-subalgebras of UP-algebras in term of level subsets, *International Journal of Mathematics and Computer Science*, no. 3, 647–674, 14, (2019).
11. A. Iampan, Introducing fully UP-semigroups, *Discuss. Math., Gen. Algebra Appl*, no. 2, 297–306 38, (2018).
12. M.A.Ansari, A. N. A. Koam, Roughapproximations in KU-Algebras, *Italian J. Pure Appl. Math.*, 679–691, 40 (2018).

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