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A Comparative Analysis of Probability Education in Türkiye and New Zealand through International Frameworks

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ABSTRACT: Probability education is a fundamental element of mathematics curricula, supporting reasoning under uncertainty and statistical thinking. However, the conceptualization and sequencing of probability education differ across educational systems in terms of conceptual scope, developmental coherence, and pedagogical orientation. The purpose of this study is to examine comparatively the probability education in the mathematics curricula of Türkiye and New Zealand through internationally recognized theoretical frameworks. The study adopts a qualitative comparative document analysis design, using the official mathematics curriculum documents of both countries as primary data sources. The analysis is guided by Gal's Probability Literacy Model, the Fundamental Probabilistic Ideas framework, and the GAISE II developmental levels. The results indicate that both curricula predominantly structure probability education at intuitive and experimental levels, with strong coverage of fundamental concepts such as randomness and sample space, yet demonstrate significant gaps in advanced concepts foundational to statistical inference.

Keywords: Probability education, mathematics curriculum, comparative document analysis.

Contents

1. Introduction

Probability education is an important area for the acquisition of skills necessary for making informed and rational decisions within contemporary data-driven societies. [1]. Probabilistic thinking plays a crucial role in the development of both statistical reasoning and data literacy, underpinning essential processes such as sampling, modeling, and inference. [2]. Therefore, probability is valuable not merely as a mathematical concept but as a cognitive tool that allows individuals to navigate uncertainty in real-life contexts [3]. Batanero and Álvarez-Arroyo (2023) [4] show that when probability is integrated

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with statistical research, data analysis, informal inference, and simulation processes; it transforms from a mere calculated value into an instrument for deriving meaning from data and interpreting uncertainty, thereby deepening conceptual learning. Individuals constantly encounter randomness and uncertainty when interpreting media news, making health and financial decisions, and coping with risk-laden situations [5]. Therefore, the abilities to contextualize probabilistic knowledge, employ it in inference, and critically evaluate communicated probability messages have become indispensable—both for the practical application of statistical inference and for informed decision-making at the societal level. [5], [6], [7]. In this regard, probability is viewed not simply as a set of technical concepts but as a cognitive competency essential for contemporary societies. Nevertheless, probability continues to be presented in curricula predominantly through the classical approach, relying on randomness-generating devices such as dice, coins, and card decks [8]. This historical tendency stems from the origins of probability theory in the 17th century, when Pascal and Fermat addressed problems related to games of chance [9], [10]. However, this approach fails to adequately illuminate the connection between probability and the uncertainty, risk, and inferential processes encountered in everyday life, potentially resulting in superficial instruction [11]. Thus, contemporary curricula are urged to treat probability not merely as a computation-based domain but as a learning area that integrates diverse representational forms and authentic contexts. The transformations observed in mathematics curricula worldwide reflect a growing consensus that probability education should be approached not merely as a computation-based domain but as a learning area integrated with data-driven reasoning, statistical thinking, and real-life contexts. In Türkiye, this transformation is most recently reflected in the mathematics curriculum that came into effect in 2024. In the new curriculum, probability content has been restructured under the themes of "Probability of Events" and "Data-Based Inquiry," thereby aiming to establish stronger connections between probability and statistical reasoning as well as contextual problem-solving processes (Ministry of National Education [MNE]) [12], [13]. This update necessitates a systematic examination of how probability education is organized in the Türkiye Mathematics Curriculum in alignment with contemporary theoretical approaches. Similarly, the New Zealand curriculum has been restructured under the title "Mathematics and Statistics" rather than "Mathematics" alone (Ministry of Education [MoE]) [14], [15].As a result, probability is positioned not as an independent and isolated topic but as an integral component of the statistical investigation process. New Zealand was selected as an international benchmark for probability education in this study, a decision grounded in the country's longstanding pioneering and research-based tradition in statistics and probability education [14]. The international literature on probability education emphasizes that probability should be addressed from early ages with the function of making sense of everyday uncertainty situations and supporting statistical reasoning; however, the grade levels at which curricula structure the probability education may vary across countries [16], [17]. Indeed, a large-scale curriculum review reveals that probability content is included in limited form at the primary level in many countries, with intuitive probability comparisons such as "likely–impossible–certain" representing the most common emphasis in most programs [18]. These findings suggest the need for more systematic analyses regarding the positioning of probability education within curricula. International comparative research in mathematics education plays a critical role in understanding how factors such as curriculum design and classroom instructional quality influence mathematics learning across different countries [19]. Nevertheless, since the 2010s, the concentration of comparative studies on specific areas [20] is thought to have resulted in relatively less examination of critical domains such as statistics, mathematical reasoning, and critical thinking [21], [22]. The relative scarcity of comparative curriculum analyses specific to the probability education that systematically employ internationally recognized theoretical frameworks highlights this gap in the field. Therefore, examining the mathematics curricula of Türkiye and New Zealand to reveal how they are patterned in terms of conceptual scope, progression, and pedagogical emphasis through Gal's Probability Literacy Model [3], Fundamental Probabilistic Ideas [16] and GAISE II [6] frameworks constitutes a significant problem area capable of addressing this need in the literature.

2. Theoretical Framework

Various theoretical frameworks have been developed to evaluate the quality of probability education. These frameworks provide researchers and curriculum developers with the opportunity to systematically examine different dimensions of probability education.

2.1. Gal's Probability Literacy Model

Gal (2005) [3] grounds the rationale for probability education on two complementary foundations. Firstly, he presents probability as a fundamental part of mathematics and statistics, emphasizing that it provides a theoretical basis for advanced topics such as sampling and statistical inference. Secondly, he argues that probability knowledge is essential for individuals to make sense of real-life situations, given that daily life is permeated by uncertainty and randomness. It is his position that both of these are not mutually exclusive; rather, they necessitate that instruction be structured holistically, taking into account the demands of the external world. This perspective positions probability literacy as a multidimensional construct that is not confined to cognitive knowledge alone but involves the interplay of knowledge, skills, and dispositional elements. At the core of the model lies numeracy, which refers to individuals' capacity to critically interpret quantitative information and make informed decisions in situations involving uncertainty.

Probability Literacy Model [3], relies on five knowledge elements and dispositional elements. The knowledge elements encompass: (i) *big ideas (variation, randomness, independence, and predictability–uncertainty)*, (ii) *figuring probabilities (ways to find or estimate the probability of events, including classical, frequentist, and subjective approaches)*, (iii) *language (the terms and methods used to communicate about chance)*, (iv) *context (understanding the role and implications of probabilistic issues and messages in various contexts and in personal and public discourse)*, and (v) *critical questions (issues to reflect upon when dealing with probabilities)*. These elements show that probabilistic reasoning acquires meaning not merely through mathematical symbols but through context and interpretation. The dispositional elements of the model include: (i) *critical stance*, (ii) *beliefs and attitudes*, and (iii)*personal sentiments regarding uncertainty and risk (e.g., risk aversion)*. Instructional approaches focusing exclusively on cognitive knowledge are inadequate for developing probability literacy; cognitive and dispositional elements must be supported simultaneously. [3].

2.2. Fundamental Probabilistic Ideas

Heitele (1975) [23] identified fundamental concepts that played a central role in the development of modern probability theory and that are frequently addressed with erroneous intuitions by individuals in the absence of instruction. These concepts encompass the building blocks of probabilistic reasoning, including random experiment and sample space, independence and conditional probability, combinatorial counting, random variables and distributions, convergence, and sampling and simulation. Fundamental Probabilistic Ideas [16], building upon Heitele's (1975) work [23]. This framework identifies the core areas that constitute the conceptual backbone of probability education and where students commonly exhibit intuitive difficulties [24], [25]. Fundamental Probabilistic Ideas comprise randomness, events and sample space, combinatorial enumeration and counting, independence and conditional probability, probability distribution and expectation, convergence and the law of large numbers, sampling and sampling distribution, and modeling and simulation. Although the proposed framework integrates ideas spanning a wide range from school to university level [16], the present study concentrates exclusively on the primary education. This choice finds roots in the understanding that the primary years are the critical period for the laying down of the foundation for the basic concepts of probability.

2.3. GAISE II Framework

GAISE II (Guidelines for Assessment and Instruction in Statistics Education II) is a foundational document published in 2020 by the American Statistical Association (ASA) and the National Council of Teachers of Mathematics (NCTM), and serves as a comprehensive framework to address education in statistics and data science for the PreK-12 level [6]. The framework is based on the belief that all students should develop statistical literacy and aims to develop individuals' competencies in extracting meaning from data, engaging in statistical reasoning, and demonstrating critical skepticism when necessary. GAISE II report describes probability as the quantification of randomness and claims it as a fundamental building block of statistical inference. Although probability in mathematics is often addressed through logical deductions based on assumptions, in statistics probability serves to evaluate model fit and uncertainty based on observational data. GAISE II particularly emphasizes the critical role of random sampling and random assignment in reducing bias and enhancing the reliability of statistical conclusions [6].

GAISE II [6] covers statistical literacy development at three levels: A, B, and C. These levels are not measured by the age of the learners but on the progression of individuals' statistical reasoning skills. At Level A, students informally understand probability as a measure of the chance that an event will occur and make comparisons along a continuum from impossibility to certainty. At Level B, students begin to understand that inferences about populations can be made through samples, that sample variability creates uncertainty, and the role of random assignment in comparative studies. At Level C, probability occupies a central position in statistical inference; the concepts of conditional probability and independence are emphasized, and students become capable of interpreting sampling distributions, p-values, and confidence intervals through simulations. This three-level system enables the determination of the developmental positioning of probability education within the curricula of different countries, thereby providing a robust and systematic reference framework for comparative curriculum analyses. [6].

2.4. Integrated Theoretical Framework for Curriculum Analysis

In this study, the comparative analysis of the probability education within the mathematics curricula of Türkiye and New Zealand is approached on the basis of a holistic and multidimensional theoretical framework. This framework is grounded in the combined use of Gal's Probability Literacy Model [3], Fundamental Probabilistic Ideas [16] approach and GAISE II [6]. These frameworks address probability education from different yet complementary perspectives, enabling a comprehensive evaluation of curricula. The combined use of these three frameworks reflects an approach defined in research methodology as theoretical triangulation. Theoretical triangulation aims to transcend the limitations of adhering to a single theoretical perspective and to reveal the multifaceted nature of probability education in a more holistic manner [26], [27], [28]. This approach allows for the simultaneous assessment of curricula's literacy objectives, developmental coherence, and conceptual structure, thereby strengthening the validity and explanatory power of the findings. The holistic theoretical framework developed in this context renders visible the strengths, gaps, and areas requiring improvement in the probability curricula of Türkiye and New Zealand through multiple analytical lenses, offering evidence-based and structured implications for curriculum development efforts.

2.5. Purpose of the Research

This study aims to comparatively analyze the probability education within the mathematics curricula of Türkiye and New Zealand in light of Gal's Probability Literacy Model [3], Fundamental Probabilistic Ideas [16] approach and GAISE II [6]. Accordingly, the study examines how probability education is structured across grade levels in both countries, which conceptual and pedagogical emphases are salient, and also to what extent to which the developmental progression of probability education aligns with international frameworks. By revealing the similarities and differences in the scope and emphases of probability education across disparate educational systems, this research is thus expected to make a fresh contribution to the probability education literature, as well as yield evidence-based implications for curriculum development efforts.

3. Methodology

3.1. Research Design

This study was designed employing a comparative document analysis with a qualitative research approach. Comparative document analysis can be defined as a useful method for examining the curriculum structures, content, sequence, and emphasis of different countries [29], [30]. Within the scope of this study, the probability education in the mathematics curricula of Türkiye and New Zealand is systematically analyzed in light of internationally recognized theoretical frameworks.

3.2. Data Sources

The primary data sources of the research consist of the Türkiye Mathematics Curriculum (TMC) [12], [13] and the New Zealand Mathematics and Statistics Curriculum (NZMSC) [15]. The probability education in the NZMSC is analyzed along two fundamental dimensions in accordance with the content structure of the program: "Knowledge (K)" (factual knowledge, concepts, principles, and theories) and "Practices (P)" (skills, strategies, and applications). The examination covers Years 4–6 and Years 7–8 levels [15]. Within the scope of the TMC, grades 4–8 at the primary education level were systematically examined. The analysis process was conducted along three dimensions in accordance with the structural characteristics of the program: (i) learning outcomes that holistically express the concepts, methods, and skills students are expected to achieve, (ii) conceptual and mathematical domain skills as well as process components that constitute these outcomes, and (iii) learning-teaching experiences (LTE) that emphasize the sequential and hierarchical structure of mathematical knowledge [12], [13]. The probability-related curriculum elements at the primary education level in both countries, along with explanations, thematic structure, conceptual emphases, and pedagogical guidance, were examined in detail.

3.3. Data Analysis Procedure

Directed content analysis was employed in the research. This approach allows for the systematic analysis of data in accordance with predetermined theoretical frameworks [31]. The coding scheme was developed based on Gal's Probability Literacy Model [3], Fundamental Probabilistic Ideas [16] approach and GAISE II developmental levels (A–B–C) [6].

In the first stage of the analysis, the probability literacy components (big ideas, figuring probabilities, language, context, critical questions and dispositional elements) within the curricula were examined in accordance with Gal's model. The extent to which each component of the probability education in the curricula addressed the relevant elements was coded as "Present (P) / Partially Present (PP) / Absent (A)"; thus, the holistic approaches of the curricula toward Probability Literacy Model [3] were comparatively evaluated. The coding system was operationalized using explicit criteria. A component was coded as Present when addressed directly and coherently through explicit curriculum statements, Partially Present when addressed implicitly or in a restricted manner, and Absent when no identifiable reference was found. In the second stage, using the Fundamental Probabilistic Ideas framework [16], the extent to which fundamental concepts such as randomness, events and sample space, combinatorial enumeration and counting, probability distribution and expectation, convergence and laws of large numbers, sampling and sampling distribution, modelling and simulation, independence and conditional probability were included in the curricula was analyzed. At this stage, not only was the presence of concepts considered, but also how they were structured within conceptual coherence; the findings were again recorded using the "P/PP/A" coding scheme. In the third stage, the probability education components of both countries were aligned with GAISE II's A, B, and C developmental levels [6]. Since GAISE II does not provide probability-related curriculum elements specific to probability directly, the components were interpretively matched according to emphases such as informal understanding of probability, sampling and sample variability, conditional reasoning, and simulation-based reasoning. At this point, no quantitative coding was performed; findings were reported descriptively. The findings obtained based on the three theoretical frameworks were addressed within a holistic framework through the synthesis tables that enabled a comparative study between the two countries with supporting evidence. These tables were organized to systematically reveal the alignment levels of both countries' probability education with the theoretical frameworks.

3.4. Trustworthiness

To strengthen the internal validity and reliability of the research, the coding process was conducted by two independent domain experts. Inter-coder agreement was calculated using the reliability formula [32], which is widely used in qualitative research. The calculation revealed that the reliability coefficient exceeded .90, indicating a high level of agreement between coders. The theoretical validity of the study was supported through a theoretical triangulation approach based on the combined use of three different international frameworks [27], [28].

3.5. Limitations

A limitation of the research is that the data are based solely on the official curriculum document. The results for the probability-related curriculum elements within the curriculum document may not reflect the same in-class, as it depends on the way the teachers treat the topic and the actions they do with the students [33], [34]. Therefore, the findings reflect the intended curriculum structures of the countries rather than the implemented curriculum. Additionally, the research was limited to the primary education level. While the theoretical triangulation employed in the study enhanced analytical depth, conceptual differences among the frameworks necessitated researcher interpretation in some instances. In this regard, the findings are generalizable only in terms of the content and structural characteristics of the curricula.

4. Findings

The tables presented in the findings section summarize each curriculum component through a limited number of representative pieces of evidence. Due to space constraints, not all curriculum components and explanatory statements are included in this section; the detailed evidence set is provided in the Appendix.

4.1. Analysis through Gal's Probability Literacy Model

Table 1 presents a comparative overview of how probability education are positioned in the mathematics curricula of Türkiye and New Zealand in accordance with the components of Gal's Probability Literacy Model, along with selected evidence and explanations for each component.

When evaluated in the context of Gal's Probability Literacy Model, the NZMSC is seen to address the cognitive dimensions of probabilistic reasoning in a robust and systematic manner. Core "big ideas" such as randomness, uncertainty, variation, and the law of large numbers are structured in a gradual progression across Years 4–8; students are guided to recognize both the unpredictability in individual trials and the regularity that emerges over the long run. Concerning calculations of probability, there is a balance between the classical (equiprobable events) and frequentist (experimental/relative) probabilities, where students are encouraged to calculate probabilities theoretically as well as estimate probabilities from experimental data. Within the language component, expressing probabilities through fractions, decimals, percentages, the 0–1 scale, and verbal terms such as 'impossible', 'unlikely', 'even chance', 'likely' and 'certain' aims to develop students' proficiency in shifting between representational forms. The critical and contextual understanding of this language-for example, the vagueness of verbal probability expressions or how they can have different meanings under different contexts-is mostly implicit in the curriculum. In terms of context, probability is predominantly associated with in-school activities such as games and repeated chance experiments; generalizations to important life contexts are scant, such as health, economics, media, or social decision-making. Concerning critical questions, while reflection on methodological reliability and variability is supported through the comparison of experimental and theoretical probabilities, broader critical inquiries into the source, purpose, and meaning of probability messages remain relatively weak. Finally, since no dispositional elements were found in the curriculum, it follows that the NZMSC curriculum organizes probability literacy largely through cognitive knowledge and processes. Overall, the curriculum exhibits a strong alignment with Gal's components that are knowledge-based. Nonetheless, a more holistic probability literacy can be developed by improving contextual diversity, critical interpretation, and attitudinal components.

The probability within the TMC is seen in a multi-dimensional and gradual structure for grades 4-8. Big ideas such as randomness, uncertainty, the probability continuum, equiprobability, variation, and the convergence of experimental results toward theoretical values are structured consistently as grade level progresses; this indicates that the curriculum approaches probability not merely as procedure-oriented but with a process-based and conceptual understanding. In terms of figuring probabilities, the frequentist approach is prominently emphasized; it enables the students to interpret probability via data, make experiments, compute relative frequencies, compare prediction and experiment, and question the effect of changing the number of trials on outcome. The classical approach is systematically addressed in the context of sample space and equiprobability at the middle school level, while the subjective approach allows students to generate predictions based on personal experience and judgment.

Table 1: Examination of the Mathematics Curricula of Türkiye (T) and New Zealand (NZ) According to Gal's Probability Literacy Model

Country	P/PP/A	Evidence from the Curriculum	Explanation
Knowledge Elements			
Big Ideas			
NZ	P	- "*Situations that involve chance, uncertainty, and randomness are called chance-based situations.*" (K-Y5,6). - "*The Law of Large Numbers states that...*" (K-Y7,8).	Fundamental big ideas such as uncertainty, randomness, variation, and convergence (law of large numbers) are addressed.
T	P	- "*Based on the given examples, students are expected to make predictions about the probabilities of events involving uncertainty.*" (MAT4-LTE). - "*Students are expected to recognize that experimental results may differ and that they may encounter different outcomes in each experiment, and to discuss these differences around the idea of randomness.*" (MAT.6.6.1 LTE).	Fundamental big ideas such as uncertainty, randomness, variation, and convergence (law of large numbers) are addressed.
Figuring probabilities			
NZ	P	- "*Calculating probabilities using a spinner, where each event is a fraction or combination of fractions on the spinner.*" (P-Y6). - "*Carrying out a chance experiment and calculating the experimental probability of each outcome.*" (P-Y7,8). - "*Calculating probabilities for complementary events.*" (P-Y7,8).	Classical and frequentist probability approaches are seen to be addressed.
T	P	- "*b) Converts predictions about the probability of an event into different numerical representations.*" (MAT.5.6.1.). - "*a) Finds the mathematical relationship to calculate the probability of an event and its complement.*" (MAT.7.7.1.). - "*Students are presented with a sample event for which they can estimate the probability and subsequently conduct the experiment.*" (MAT.8.7.1 LTE).	The frequentist, classical, and subjective approaches are all supported.
Language			
NZ	P	- "*Likelihood can be visualised using a number line from 0 to 1.*" (K-Y5,6). - "*Identifying the likelihood of an everyday event as being impossible, unlikely, even-chance, likely, or certain.*" (P-Y5). - "*Probabilities can be expressed as a fraction or decimal between 0 and 1, or as a percentage between 0% and 100%.*" (K-Y7,8).	Probability language and representations are systematically addressed.
T	P	- "*MAT.4.4.1. Being able to determine the probability of any event from daily life as 'impossible, possible, certain'.*" - "*MAT.5.6.1. Being able to interpret that the probability of any event is between 0 and 1 (probability spectrum).*" - "*Based on the example addressed, students are expected to describe their expectations using expressions such as 'very likely, more than 50%, highly probable'.*" (MAT.8.7.1 LTE).	Probability language is addressed through verbal and numerical forms.
Context			
NZ	P	- "*Conducting repeated chance experiments or games, identifying the outcomes, and describing differences between them using likelihood vocabulary.*" (P-Y5). - "*Identifying the likelihood of an everyday event as being impossible, unlikely, even-chance, likely, or certain.*" (P-Y5).	Probability is addressed within interactive contexts.
T	P	- "*Students are provided with appropriate examples of uncertain situations that address probability from daily life.*" (MAT.4.4.1 LTE). - "*A discussion environment is created in the classroom based on sample events.*" (MAT.5 LTE).	Probability is linked to daily-life and interdisciplinary contexts.
Critical questions			
NZ	P	- "*Answering questions about the probability of combinations of outcomes, including checking that the sum of all probabilities is 1.*" (P-Y6).	Reflection on uncertainty and variability is supported.
T	P	- "*Students are asked to comment on probabilities using words that convey uncertainty.*" (MAT4-LTE). - "*During this process, questions such as 'Why is the probability of drawing a red ball high or low?' are posed.*" (MAT.5.6.2 LTE).	Inquiry into probability reasoning is supported.
Dispositional elements			
NZ	A	-	Dispositional elements are not explicitly present.
T	P	- "*Students review their own experiences and personal judgments about probability.*" (MAT.5.6.2 LTE). - "*Students are encouraged to be open and willing to change in the face of changing situations.*" (MAT.8.7.1 LTE).	Critical stance and awareness of subjectivity are supported.

Nevertheless, the integration of these approaches and explicit discussion of the judgmental nature of probability remain limited; calculation and verification processes play a more dominant role. The curriculum is strong in terms of the language component: students are expected to transition between different representational forms through verbal probability expressions, the 0–1 interval, fractions, percentages, relative frequencies, and visual representations such as tree diagrams and graphs. However, the dimension of developing critical awareness regarding context-dependent meanings and potential ambiguities of verbal probability language remains relatively implicit. In terms of context, probability is addressed through uncertain situations from daily life, games, and interdisciplinary examples; this supports the structuring of probability within meaningful contexts for students. Within the critical questions component, students are encouraged to justify their predictions, question experimental results, and discuss theoretical–experimental differences; however, broader critical dimensions regarding the source, purpose, and real-life interpretation of probability messages are not systematically addressed. Regarding dispositional elements, students are encouraged to question their own judgments and reflect on their decision-making processes, with a particular emphasis on critical stance at the upper grade levels. However, aspects such as risk perception and emotional responses to uncertainty are not articulated through explicit probability-related curriculum elements. Overall, the TMC demonstrates strong alignment with Gal's cognitive components. However, more explicit and contextual strengthening of critical and affective dimensions would make a significant contribution to the holistic development of probability literacy.

4.2. Analysis through Fundamental Probabilistic Ideas

The probability education in the mathematics curricula of Türkiye and New Zealand was examined in accordance with the Fundamental Probabilistic Ideas framework [16] and a comparative overview of the status in terms of theoretical components is presented in Table 2.

Table 2: Examination of the Mathematics Curricula of Türkiye and New Zealand According to the Fundamental Probabilistic Ideas (FPI)

Country	FPI	P/PP/A	Evidence from the Curriculum	Explanation
NZ	Randomness	P	- "*Situations that involve chance, uncertainty, and randomness are called chance-based situations.*" (K-Y5,6).	It positions randomness not merely as intuitive but as a fundamental component of probabilistic reasoning.
T		P	- "*Students are expected to recognize that experimental results may differ and that they may encounter different outcomes in each experiment, and to discuss these differences around the idea of randomness.*" (MAT.6.6.1 LTE).	The concept of randomness is explicitly named, it is addressed through experimental activities.
NZ	Events and sample space	P	- "*The sample space is the set of all possible outcomes of an experiment.*" (K-Y5,6). - "*An event is a subset of the sample space and thus can be a single outcome or a combination of outcomes.*" (K-Y7,8).	The concepts of sample space and events are addressed explicitly and systematically.
T		P	- "*Students are enabled to express that the totality of all possible outcomes that can be obtained from any experiment constitutes the sample space, and that all subsets that can be derived from the sample space are events.*" (MAT.7.7.1. LTE).	The concepts of sample space and events are addressed explicitly and systematically.
NZ	Combinatorial enumeration and counting	PP	- "*Lists, tables, and tree diagrams are useful systematic methods for generating all possible outcomes.*" (K-Y7,8).	There is a foundation for combinatorial thinking, but it is not explicitly present.
T		PP	- "*Students are asked to display all outcomes in the examined events using different representations such as the listing method or tree diagrams.*" (MAT.7.7.1. LTE). - "*Drawing experiments from a bag containing balls of different colors, comparison of equiprobable and non-equiprobable situations.*" (MAT.7.7.2 LTE).	There is a foundation for combinatorial thinking, but it is not explicitly present.
NZ	Independence and conditional probability	A	-	Independence and conditional probability component is not explicitly addressed.
T		A	-	Independence and conditional probability component is not explicitly addressed.
NZ	Probability distribution and expectation	A	-	Probability distribution and expectation component is not explicitly addressed.
T		A	-	Probability distribution and expectation component is not explicitly addressed.
NZ	Convergence and laws of large numbers	P	- "*The Law of Large Numbers states that as the number of trials in a chance experiment increases, the experimental probability will approach the event's theoretical probability.*" (K-Y7,8).	The law of large numbers is addressed explicitly and at a conceptual level.
T		P	- "*Consequently, students are expected to recognize that as the number of trials increases in the conducted experiment, experimental probability values vary less and gradually approach the theoretical probability value.*" (MAT.8.7.1 LTE).	Convergence and laws of large numbers are addressed at a conceptual level; the relationship between the number of trials and convergence is structured through experimental observations without using the name of the law.
NZ	Sampling and sampling distribution	A	-	Sampling and sampling distribution component is not explicitly addressed.
T		A	-	Sampling and sampling distribution component is not explicitly addressed.
NZ	Modelling and simulation	PP	- "*Conducting repeated chance experiments or games, identifying the outcomes, and describing differences between them using likelihood vocabulary.*" (P-Y5).	Modeling of probabilistic situations is included through chance experiments and games. Simulations are limited to physical and simple experiments.
T		P	- "*The simulations featured in this theme are carried out through online applications and statistical software. Students can create simulations as a digital task. They determine and decide which tools to use and how to use them for simulations.*" (MAT.8.7.1 LTE).	The simulation and modeling component is explicitly and strongly present.

NZMSC addresses fundamental probabilistic ideas with explicit and conceptual coherence, particularly along the axes of randomness, sample space and events, convergence, and the law of large numbers. Randomness, uncertainty, and variation are introduced from Year 5 onward through chance situations and repeated experiments. It is explicitly emphasized that outcomes may vary across repetitions of the same experiment. Students are expected to systematically list all possible outcomes, construct the sample space using tables and tree diagrams, and interpret events as subsets of the sample space. The law of large numbers is addressed at both conceptual and applied levels; the convergence of experimental probability toward theoretical probability as the number of trials increases is explicitly observed and justified. In contrast, concepts such as independence and conditional probability as well as probability distribution and sampling distributions are not included at the primary education level; combinatorial reasoning is indirectly supported through tools such as tree diagrams and listing, but is not positioned as an explicit learning objective. The modeling and simulation component is included to a limited extent through repeated experiments and games, remaining largely at an intuitive and semi-concrete level. Overall, NZMSC structures fundamental probabilistic ideas with conceptual clarity and pedagogical coherence, while deliberately leaving certain advanced ideas outside the scope of primary education.

In the TMC, fundamental probabilistic ideas are addressed within a gradual and practice-based structure across grades 4–8. Randomness and variation are emphasized through students' recognition that experimental outcomes may differ and their discussion of these differences; probability is largely made meaningful through experimental activities and relative frequency. The concepts of sample space and events are explicitly defined; students are expected to list all possible outcomes, construct the sample space, and distinguish events derived from this space. The law of large lumbers is addressed without being named; students are guided to observe that as the number of experimental trials increases, experimental probability becomes more stable and approaches the theoretical value. Combinatorial thinking is indirectly supported through tree diagrams, listing, and comparison of equiprobable and non-equiprobable situations, but is not defined as an explicit reasoning objective. Independence, conditional probability, probability distribution and sampling distributions are not included; this parallels the situation in NZMSC. In contrast, the modeling and simulation component is relatively stronger in the TMC. In addition to physical experiments, students are expected to model probability situations through digital tools, online simulations, and interdisciplinary contexts.

4.3. Analysis through GAISE II Developmental Levels

The probability education in the mathematics curricula of Türkiye and New Zealand was examined in accordance with Levels A, B, and C of the GAISE II [6], and a comparative overview of how both countries position probability within the statistical problem-solving process is presented in Table 2.

Table 3: Examination of the Mathematics Curricula of Türkiye and New Zealand According to the GAISE II Framework

Country	**GAISE II Level**	**Evidence from the Curriculum**	**Explanation**
NZ	Level A	- "*Identifying the likelihood of an everyday event as being impossible, unlikely, even-chance, likely, or certain.*" (P-Y5). - "*Likelihood can be visualised using a number line from 0 to 1.*" (K-Y5,6).	Probability is addressed intuitively in daily life contexts, and the comparison of events along a continuum from impossibility to certainty is explicitly emphasized.
T		- "*Being able to determine the probability of any event from daily life as 'impossible, possible, certain.'* " (MAT.4.4.1). - "*Being able to interpret that the probability of any event is between 0 (impossible) and 1 (certain).*" (MAT.5.6.1).	Through the language of probability, students are expected to make intuitive comparisons.
NZ	Level B	- "*Results from sets of repeated trials for the same experiment may vary.*" (K-Y7,8). - "*Comparing experimental probability (using at least 30 trials) to theoretical probability, and explaining why they differ.*" (P-Y7,8).	Emphasis is placed on students calculating probability based on experimental data, observing variability through repetitions, and making experimental–theoretical comparisons.
T		- "*Being able to estimate the probability of an event based on observation.*" (MAT.6.6.1). - "*b) Makes inferences regarding the relationship between the number of trials and relative frequencies in an experiment.*" (MAT.6.6.1.).	Experiment, relative frequency, and the number of trials are emphasized. Students are expected to recognize variability through experimentation and to interpret probability in a data-based manner.
NZ	Level C	-	Level C characteristics are not included in the curriculum at the Years 5–8 levels.
T		-	Level C characteristics are not included at the grades 4–8.

The analysis conducted according to the GAISE II framework indicates that NZMSC demonstrates strong alignment predominantly with Level A and Level B competencies in probability education. Students' intuitive comparison of probability along a continuum from impossibility to certainty in everyday life contexts, and their expression of these comparisons through representations such as the number line, meet the fundamental expectations of Level A. At Level B, probability-related curriculum elements such as calculating relative frequency through repeated experiments, recognizing the variability of outcomes in the same experiment, and comparing experimental probability with theoretical probability are prominent. In particular, the emphasis on the law of large numbers allows students to experimentally discover the idea of long-term regularity and supports their interpretation of probability from a data-based perspective. However, concepts aligned with Level C, such as conditional probability, independence, p-values, or sampling distributions, are not explicitly present.

When the TMC is evaluated in terms of the GAISE II framework, it also exhibits a structure that predominantly aligns with Level A and Level B competencies. Through probability language, the probability spectrum, and verbal expressions such as "impossible–possible–certain," students are expected to make intuitive comparisons; in this respect, the fundamental characteristics of Level A are met. In the context of Level B, concepts such as conducting experiments, calculating relative frequency, discussing the effect of the number of trials on outcomes, and making inferences based on experimental probability are prominent. Students' generation of observation-based predictions and their transformation of experimental results into numerical representations support the interpretation of probability in relation to data. Finally, concepts aligned with Level C, such as conditional probability, independence, p-values, or sampling distributions, are not explicitly present.

5. Discussion and Conclusion

This study comparatively investigates the contents of the primary education mathematics curricula of Türkiye and New Zealand based on the Probability Literacy Model, Fundamental Probabilistic Ideas, and GAISE II developmental levels. The results show that both countries structure probability gradually from intuitive comparisons toward experimental approaches, yet exhibit limitations in terms of theoretical aspects, higher-level inference, and critical dimensions. Based on the Probability Literacy Model [3], both curricula strongly incorporate the knowledge components of probability literacy, including big ideas, figuring probabilities, and language. In terms of the context dimension, both curricula address probability through games, experiments, and examples from daily life. It is emphasized that probability education is a field with applications across different scientific disciplines [4]. By comparison, the TMC allows a relatively greater range in terms of contextual diversity by more explicitly incorporating interdisciplinary contexts of science and social studies, whereas in NZMSC, contexts remain largely limited to experimental and school-based situations. Yet, the interpretation of multiple probabilistic situations in daily life, such as medical test results, is of crucial importance [36]. NZMSC enhances the theoretical part of the probabilistic reasoning component by covering the key ideas of randomness, variation, and the law of large numbers at the specific and conceptual level. On the other hand, the TMC focuses more on the frequentist approach and experimental tasks, supporting students' data-based interpretation of probability through the prediction–experiment–comparison cycle. However, in both curricula, the critical questions component remains largely limited to mathematical consistency and computation accuracy; broader critical dimensions emphasized by Gal—such as the source, purpose, context, and societal consequences of probability messages—are not explicitly addressed. More subjective contexts, which rely on personal judgments and experiences in assigning probabilities to potential outcomes, are rarely considered in the school curriculum [37]. Regarding dispositional elements, this dimension is not present at the level of explicit probability-related curriculum elements in the case of NZMSC, whereas in the case of TMC, critical stance and awareness of subjective judgments are partially supported; however, risk perception and attitudes toward uncertainty remain at an implicit level. The international literature also emphasizes that probability levels should not remain restricted to the cognitive components alone but also to its critical and contextual components [3]. In this context, while both countries' curricula are aligned with probability literacy objectives, they have aspects requiring improvement in terms of contextual knowledge and critical stance.

In the context of the Fundamental Probabilistic Ideas framework [16], the concepts of randomness, sample space and events, and convergence/law of large numbers are clearly present in both NZMSC and TMC; students observe experimental variability and are guided to recognize the emergence of regularity over the long run. Although combinatorial thinking is used at an instrumental level (lists, tables and tree diagrams) in both countries, it is restricted to an explicit reasoning objective. The fact that concepts like independence, conditional probability, sampling, and probability distributions, which build a basis for statistical inference, do not exist at grades 4-8 in both curricula means that students get very limited opportunities for connecting probability with more advanced statistical ideas. Probability models such as the binomial or normal distribution are tools that enable the structuring of reality and play a central role in problem solving [38]. In this context, as Batanero et al. (2016) [16] emphasized, probability should be considered not merely as an experimental calculation tool but as a mediator for statistical inference. It is suggested that statistics may lose its capacity for meaning-making without its connection to probability [16]. In addition, Pfannkuch (2005) [39] stated that establishing the connection between ideas of variability and probability plays a crucial role in students' transition to statistical inference. The aspect of modeling and simulation practices has a stronger presence in TMC using technology, while in the case of NZMSC, it largely remains limited to physically and semi-concretely executed experiments. According to Martins (2018) [40], simulation-based approaches in modern probability education not only helps to improve the conceptual understanding but also enhance their interest and enjoyment in learning probability. Additionally, the use of simulation in the informal statistical inference process facilitates certain aspects of computation and visualizes sampling variability; however, it also carries the risk of reducing probability to a frequentist view [41]. This issue again reveals the significance of addressing different interpretations of probability (classical, frequentist, subjective) with equal emphasis. The analysis from the framework of the GAISE II shows that the curricula of both countries mainly align with Level A and Level B competencies. Students engage in intuitive comparisons along the probability continuum ranging from impossibility to certainty (Level A) and develop a data-oriented interpretation of probability by recognizing variability through experimental data, relative frequencies, and the number of trials (Level B). In NZMSC, experimental–theoretical comparisons and the emphasis on the Law of Large Numbers, and in TMC, observation-based prediction and calculation through relative frequency, are strong indicators of these levels. By contrast, neither curriculum includes explicit probability-related curriculum elements addressing core Level C concepts, such as conditional probability, independence, sampling distributions, confidence intervals, and p-values. Pfannkuch (2005) [39] highlights the historically late integration of probability and inference within the field of statistics, noting that the development of instructional pathways to formal inference—and particularly the deliberate connection of variability ideas with probability—constitutes a crucial element of this process. In this context, the limited inclusion of Level C competencies in both countries' curricula makes it difficult for students to develop an understanding that positions probability at the center of statistical inference. It is assumed that in most countries, the statistics curriculum prior to grade 10 remains largely limited to descriptive statistics; countries such as New Zealand, which systematically incorporate informal statistical inference from primary school onward, constitute rare exceptions [42]. Estrella et al. (2023) [43], in a study they carried out among students of K-4 levels (ages 5-9), developed a learning trajectory for informal statistical inference, targeting the concepts of sampling, frequency distribution, randomness, and sampling variability. These results show that inferential thinking can be supported at earlier ages and that existing curricula can be enriched in this manner. These studies indicate that the mathematics curricula of Türkiye and New Zealand could also be developed through similar approaches. Based on these results, it is suggested that curricula be restructured to position probability not merely as a domain of experimental calculation but as a basic building block of statistical inference. Particularly, addressing conditional probability, independence, sampling distributions, and simulation-based inference approaches at earlier levels could support the transition toward Level C competencies of GAISE II [6]. For future research, design-based studies could be conducted to investigate how critical questioning, risk perception, and attitudes toward uncertainty, which are not addressed in curricula but are emphasized by Gal (2005) [3] and GAISE II (2020) [6], can be addressed in classroom practices. Moreover, research may be carried out on how advanced components of the Fundamental Probabilistic Ideas, such as conditional probability, independence, and probability distributions, can be integrated through pedagogical approaches suitable for the primary education level. Fischbein (1975) [44] advocates for early probability teaching; the study by Estrella et al. (2023) [43] on informal statistical inference at the K-4 level shows that such integrations are possible. Mixed-method research that investigates curriculum analyses alongside classroom practices and teacher beliefs could provide substantial contributions regarding evaluating the alignment between the intended curriculum and the implemented curriculum. In this study, only the "intended curriculum" was examined based on official curriculum documents. However, curricula are framework documents that set forth the goals ultimately intended to be achieved; how these goals are operationalized in textbooks and how they are enacted through teachers' classroom practices may significantly influence the quality of concepts and attitudes imparted to students. For that reason, future research should be expanded to encompass the implemented curriculum by examining textbooks, instructional practices.

References

1. Steen, L. A. (Ed.), *Mathematics and democracy: The case for quantitative literacy*, National Council on Education and the Disciplines, Washington, DC, (2001).

2. Garfield, J., and Ben-Zvi, D., *Developing students' statistical reasoning: Connecting research and teaching practice*, Springer, (2008).

3. Gal, I., *Towards "probability literacy" for all citizens: Building blocks and instructional dilemmas*, In G. A. Jones (Ed.), Exploring probability in school: Challenges for teaching and learning, 39-63, Springer, Boston, MA, (2005).

4. Batanero, C., and Álvarez-Arroyo, R., *Teaching and learning of probability*, ZDM–Mathematics Education 56(1), 5-17, (2023).

5. Álvarez-Arroyo, R., Batanero, C., and Gea, M. M., *Probabilistic literacy and reasoning of prospective secondary school teachers when interpreting media news*, ZDM–Mathematics Education 56(6), 1045-1058, (2024).

6. Bargagliotti, A., Franklin, C., Arnold, P., Gould, R., Johnson, S., Perez, L., and Spangler, D., *Pre-K-12 guidelines for assessment and instruction in statistics education II (GAISE II)*, 2nd ed., American Statistical Association, (2020).

7. Gigerenzer, G., *Calculated risks: How to know when numbers deceive you*, Simon and Schuster, (2015).

8. Ingram, J., *Randomness and probability: exploring student teachers' conceptions*, Mathematical Thinking and Learning 26(1), 1-19, (2024).

9. Feldman, D. P., *Chaos and fractals: an elementary introduction*, Oxford University Press, (2012).

10. Grinstead, C. M., and Snell, J. L., *Introduction to probability*, American Mathematical Society, (2012).

11. Utts, J., *What educated citizens should know about statistics and probability*, The American Statistician 57(2), 74-79, (2003).

12. Ministry of National Education, *Primary school mathematics curriculum (grades 1, 2, 3, and 4)*, MNE Publications, Ankara, Türkiye, (2024).

13. Ministry of National Education, *Middle school mathematics curriculum (grades 5, 6, 7, and 8)*, MNE Publications, Ankara, Türkiye, (2024).

14. Forbes, S., *The coming of age of statistics education in New Zealand, and its influence internationally*, Journal of Statistics Education 22(2), 1-19, (2014).

15. Ministry of Education, *New Zealand curriculum*, New Zealand Ministry of Education, Wellington, New Zealand, (2022).

16. Batanero, C., Chernoff, E. J., Engel, J., Lee, H. S., and Sánchez, E., *Research on teaching and learning probability*, ICME-13 Topical Surveys, Springer, Cham, (2016a).

17. Zieffler, A., Garfield, J., and Fry, E., *What is statistics education?*, In International handbook of research in statistics education, 37-70, Springer, Cham, (2018).

18. López, C., and Gómez, P., *Probability in different countries of the world: teaching probability in primary education*, Educación matemática 34(3), 42-64, (2022).

19. Cai, J., Mok, I. A., Reddy, V., and Stacey, K., *International Comparative Studies in Mathematics: Lessons for Improving Students' Learning*, Springer, Cham, (2016).

20. Wang, J., and Lin, E., *A meta-analysis of comparative studies on Chinese and US students' mathematics performance: Implications for mathematics education reform and research*, Educational Research Review 4(3), 177-195, (2009).

21. Cai, J., Watson, A., and Xu, B., *Beyond Asian and Western traditions of mathematics education*, In Fourth international handbook of mathematics education, 167-193, Springer Nature, Cham, (2024).

22. Yang, X., Chan, M. C. E., Kaur, B., Deng, J., Luo, J., and Wen, Y., *International comparative studies in mathematics education: a scoping review of the literature from 2014 to 2023*, ZDM–Mathematics Education 57(4), 1-17, (2025).

23. Heitele, D., *An epistemological view on fundamental stochastic ideas*, Educational Studies in Mathematics 6(2), 187-205, (1975).

24. Batanero, C., Henry, M., and Parzysz, B., *The Nature of Chance and Probability*, In G. A. Jones (Ed.), Exploring Probability in School, Mathematics Education Library 40, 15-37, Springer, Boston, MA, (2005).

25. Batanero, C., Biehler, R., Maxara, C., Engel, J., and Vogel, M., *Using simulation to bridge teachers content and pedagogical knowledge in probability*, In 15th ICMI Study Conference: The Professional education and development of teachers of mathematics, 1-6, State University of São Paulo, São Paulo, Brazil, (2005).

26. Denzin, N. K., *The research act: A theoretical introduction to sociological methods*, McGraw Hill, (1978).

27. Denzin, N. K., *Triangulation 2.0*, Journal of Mixed Methods Research 6(2), 80-88, (2012).

28. Flick, U., *Designing qualitative research*, SAGE, London, England, (2007).

29. Bowen, G. A., *Document analysis as a qualitative research method*, Qualitative Research Journal 9(2), 27-40, (2009).

30. Creswell, J. W., and Poth, C. N., *Qualitative Inquiry and Research Design: Choosing Among Five Approaches*, SAGE Publications, (2017).

31. Hsieh, H. F., and Shannon, S. E., *Three approaches to qualitative content analysis*, Qualitative Health Research 15(9), 1277-1288, (2005).

32. Miles, M. B., and Huberman, A. M., *Qualitative data analysis: An expanded sourcebook*, Sage Publications, Thousand Oaks, CA, (1994).

33. Remillard, J. T., *Examining key concepts in research on teachers' use of mathematics curricula*, Review of Educational Research 75(2), 211-246, (2005).

34. Pepin, B., and Haggarty, L., *Mathematics textbooks and their use in English, French and German classrooms: a way to understand teaching and learning cultures*, Zentralblatt für didaktik der Mathematik 33(5), 158-175, (2001).

35. Gal, I., *Adult statistical literacy: Meanings, components, responsibilities*, International Statistical Review 70(1), 1-25, (2002).

36. Borovcnik, M., *A plea for a relatively strong role for probability within stochastic curricula*, Joint ICMI/IASE Study: Teaching Statistics in School Mathematics, (2008).

37. Eichler, A., *Statistics teaching in German secondary high schools*, Joint ICMI/IASE study: teaching statistics in school mathematics. Challenges for teaching and teacher education. Proceedings of the ICMI study 18, (2008).

38. Martignon, L., *Fostering children's probabilistic reasoning and first elements of risk evaluation*, In Probabilistic thinking: Presenting plural perspectives, 149-160, Springer, Dordrecht, (2014).

39. Pfannkuch, M., *Probability and Statistical Inference: How Can Teachers Enable Learners to Make the Connection?*, In G. A. Jones (Ed.), Exploring Probability in School, Mathematics Education Library 40, 267-294, Springer, Boston, MA, (2005).

40. Martins, R. M. D. C., *Learning the principles of simulation using the birthday problem*, Teaching Statistics 40(3), 108-111, (2018).

41. Batanero, C., and Borovcnik, M., *Educational Principles for Statistics and Probability. Statistics and Probability in High School*, SensePublishers, Rotterdam, (2016).

42. van Dijke-Droogers, M., Drijvers, P., and Bakker, A., *Introducing statistical inference: Design of a theoretically and empirically based learning trajectory*, International Journal of Science and Mathematics Education 20(8), 1743-1766, (2022).

43. Estrella, S., Méndez-Reina, M., and Vidal-Szabó, P., *Exploring informal statistical inference in early statistics: a learning trajectory for third-grade students*, Statistics Education Research Journal 22(2), 1-16, (2023).

44. Fischbein, E., *The intuitive source of probability thinking in children*, Reidel, (1975).

Appendix 1. Analysis through Gal's Probability Literacy Model

Country	Evidence from the Curriculum
	Knowledge Elements
	Big Ideas
NZ	- "*Situations that involve chance, uncertainty, and randomness are called chance-based situations.*" (K-Y5,6). - "*The Law of Large Numbers states that as the number of trials in a chance experiment increases, the experimental probability will approach the experiment's theoretical probability.*" (K-Y7,8). - "*Results from sets of repeated trials for the same experiment may vary.*" (K-Y7,8). - "*Carrying out chance experiments of at least 100 trials and comparing the experimental probability of each individual outcome to its theoretical probability, in order to demonstrate the Law of Large Numbers.*" (P-Y7,8).
T	- "*Based on the given examples, students are expected to make predictions about the probabilities of events involving uncertainty...*" (MAT4-LTE). - "*For example, a student is asked to make a prediction about the probability of an event according to their own judgment, use randomness, and analyze the result.*" (MAT4-LTE). - "*a) Establishes causal or logical relationships regarding the probabilities of events.*" (MAT.5.6.2.). - "*Students are expected to recognize that experimental results may differ and that they may encounter different outcomes in each experiment, and to discuss these differences around the idea of randomness*" (MAT.6.6.1 LTE). - "*MAT.7.7.2. Being able to evaluate whether events belonging to the same experiment are equiprobable.*" - "*. . . students are expected to recognize that as the number of trials increases in the conducted experiment, experimental probability values vary less and gradually approach the theoretical probability value.*" (MAT.8.7.1 LTE).
	Figuring probabilities
NZ	- "*Calculating the probabilities of individual outcomes.*" (P-Y6). - "*Calculating probabilities using a spinner, where each event is a fraction or combination of fractions on the spinner.*" (P-Y6). - "*The estimated probability of an event from an experiment is the number of times the event happens divided by the total number of trials in the experiment (i.e. the relative frequency for that event).*" (K-Y7,8). - "*If all possible outcomes are assumed to be equally likely, the probability of an event is number of ways the event can happen/total number of possible outcomes.*" (K-Y7,8). - "*Carrying out a chance experiment and calculating the experimental probability of each outcome.*" (P-Y7,8). - "*Calculating probabilities for events as decimals, fractions, and percentages.*" (P-Y7,8). - "*Calculating probabilities for complementary events.*" (P-Y7,8).
T	- "*b) Converts predictions about the probability of an event into different numerical representations.*" (MAT.5.6.1.). - "*c) Makes judgments regarding the use of relative frequency to calculate probability value based on inferences.*" (MAT.6.6.1.). - "*b) Finds the mathematical relationship to calculate the probability of an event and its complement.*" (MAT.7.7.1.). - "*b) Performs calculations regarding the probability of events being equiprobable or not.*" (MAT.7.7.2.). - "*Students are expected to first be able to estimate the probabilities of given events. Students are presented with a sample event for which they can estimate the probability and subsequently conduct the experiment.*" (MAT.8.7.1 LTE).
	Language
NZ	- "*Likelihood can be visualised using a number line from 0 to 1.*" (K-Y5,6). - "*Conducting repeated chance experiments or games, identifying the outcomes, and describing differences between them using likelihood vocabulary.*" (P-Y5). - "*Identifying the likelihood of an everyday event as being impossible, unlikely, even-chance, likely, or certain (e.g. the event 'the sun will rise tomorrow' is certain).*" (P-Y5). - "*Placing everyday events on a number line according to their likelihood.*" (P-Y5). - "*Lists, tables, and tree diagrams are useful systematic methods for generating all possible outcomes.*" (K-Y7,8). - "*Probabilities can be expressed as a fraction or decimal between 0 and 1, or as a percentage between 0% and 100%.*" (K-Y7,8). - "*Calculating probabilities for events as decimals, fractions, and percentages.*" (P-Y7,8).
T	- "*MAT.4.4.1. Being able to determine the probability of any event from daily life as 'impossible, possible, certain'.*" - "*MAT.5.6.1. Being able to interpret that the probability of any event is between 0 (impossible) and 1 (certain), including 0 and 1 (probability spectrum).*" - "*b) Converts predictions about the probability of an event into different numerical representations.*" (MAT.5.6.1.). - "*c) Expresses in their own words that the probability of any event that will occur is between 0 and 1 (including 0 and 1).*" (MAT.5.6.1.). - "*MAT.5.6.2. Being able to structure events as less or more probable.*" - "*Students are asked to comment on the probabilities of events using words from daily life that indicate uncertainty (such as possible, not possible, might happen, certain, impossible, not sure, unlikely, chance, never, most likely, probably, definitely)*" (MAT5-LTE). - "*In this theme, the probability of an event is interpreted experimentally. Students are explained that each result obtained from any experiment is expressed as an outcome.*" (MAT.6.6.1. LTE). - "*At the end of the experiment, they are asked to compare the observed frequencies in the experiments (each experiment consisting of 20 trials) with the expected frequencies. They are asked to express probability numerically regarding their comparison (for example, recognizing and stating that the expression "6 out of 20" indicates a fraction*" (MAT.6.6.1 LTE). - "*Students are asked to display all outcomes in the examined events using different representations such as the listing method or tree diagrams.*" (MAT.7.7.1. LTE). - "*c) Classifies mutually exclusive and non-mutually exclusive events.*" (MAT.7.7.3.). - "*By asking students about their expectations regarding the probability of the given event based on the example addressed, they are expected to describe their expectations using expressions such as 'very likely, more than 50%, highly probable, higher chance.*" (MAT.8.7.1 LTE). - "*Students are asked to record the experimental results for the probability of the selected event using tally tables, etc., to visualize the data obtained at the end of the experiment with an appropriate graph, and to calculate relative frequencies.*" (MAT.8.7.1 LTE).
	Context
NZ	- "*Conducting repeated chance experiments or games, identifying the outcomes, and describing differences between them using likelihood vocabulary.*" (P-Y5). - "*Identifying the likelihood of an everyday event as being impossible, unlikely, even-chance, likely, or certain (e.g. the event 'the sun will rise tomorrow' is certain).*" (P-Y5). - "*Comparing the likelihood of different events.*" (P-Y7,8).
T	- "*Students are provided with appropriate examples of uncertain situations that address the probability of any event from daily life and that arouse interest and curiosity (such as "This summer will be dry," "Tomorrow birds will talk to us," etc.). Subsequently, well-analyzed different materials (spinner, glass jar or colored cubes drawn from a transparent bag, etc.) are used for students to estimate the probability of an event occurring*" (MAT.4.4.1 LTE). - "*A discussion environment is created in the classroom based on various sample events selected from subjects such as social studies (such as whether it will rain tomorrow, whether a cat will enter the classroom)*" (MAT.5 LTE). - "*In addition to experiments such as tossing coins, number cubes and picture cubes, spinning spinners, and drawing balls or number chips from a bag, contexts related to science (such as the blood types of students in the class) or social studies (such as a spinner game showing the birthplaces of students in the class) are selected.*" (MAT.6.6.1 LTE). - "*By selecting contexts related to science (such as blood type compatibility for blood donation), equiprobable and non-equiprobable events are examined by students.*" (MAT.7.7.1 LTE). - "*MAT.8.7.1. Being able to make decisions by determining the appropriate probability approach (subjective, experimental, theoretical) regarding the probability of an event that may be encountered in real life.*" - "*Examples from real-life situations that can be related to physical education or social studies courses are questioned by students. For instance, situations regarding the appropriateness of probability approaches in a student's decision about the elective course they will choose or the sports activity they will participate in are discussed.*" (MAT.8.7.1. LTE).
	Critical questions
NZ	- "*Answering questions about the probability of combinations of outcomes, including checking that the sum of all the probabilities is 1.*" (P-Y6). - "*Comparing experimental probability (using at least 30 trials) to theoretical probability, and explaining why they differ and how increasing the number of trials reduces this difference.*" (P-Y7,8).
T	- "*Students are asked to comment on the probabilities of events using words that convey uncertainty about events that occur or may occur in daily life.*" (MAT.4.4.1). - "*a) Establishes causal or logical relationships regarding the probabilities of events.*" (MAT.5.6.2.). - "*During this process, questions such as "Why is the probability of drawing a red ball high or low?" are posed.*" (MAT.5.6.2. LTE). - "*c) Makes judgments regarding the use of relative frequency to calculate probability value based on inferences.*" (MAT.6.6.1.). - "*Subsequently, students are asked questions such as how they can record the experimental results and how many times they need to repeat this experiment at minimum to verify their predictions.*" (MAT6-LTE). - "*Students are enabled to discover the recurring structure for calculating the probability of the selected event based on their predictions. From this point, students are enabled to make a generalization that the probability value of an event is the ratio of the number of outcomes belonging to the event to the number of all possible outcomes, and to relate theoretical probability to ratio.*" (MAT.7.7.1 LTE). - "*A discussion environment is created in the classroom based on the given sample event, enabling students to inquire about probability interpretations of this event according to different probability approaches.*" (MAT.8.7.1. LTE).
	Dispositional elements
	1. Critical stance. 2. Beliefs and attitudes. (Gal, 2002) 3. Personal sentiments regarding uncertainty and risk.
NZ	-
T	- "*To attract students' attention to the topic, examples are provided related to daily life situations involving probability that they have encountered.*" (MAT4-LTE). - "*A sample event is provided for students to review their own experiences and personal judgments about the probability of any event and to establish causal or logical relationships.*" (MAT.5.6.2 LTE). - "*By mentioning the works of Turkish scientist Salih Zeki, students are enabled to conduct an examination of his studies on probability. By supporting students' curiosity tendencies, they are expected to appreciate and respect Salih Zeki's effort, success, and dedication in his probability studies.*" (MAT.7.7.3 LTE). - "*Students are asked to provide justifications for their predictions and to explain the reasoning they used to reach their judgments, thereby supporting original thinking tendencies.*" (MAT.8.7.1 LTE). - "*In particular, it is recommended that students re-examine their personal judgments when estimating probability through the subjective approach, and they are encouraged to be open and willing to change in the face of changing situations.*" (MAT.8.7.1 LTE).

Appendix 2. Analysis through Fundamental Probabilistic Ideas

Country	FPI	Evidence from the Curriculum
NZ	Randomness	- "*Situations that involve chance, uncertainty, and randomness are called chance-based situations.*" (K-Y5,6). - "*Results from sets of repeated trials for the same experiment may vary.*" (K-Y7,8).
T		- "*For example, a student is asked to make a prediction about the probability of an event according to their own judgment, use randomness, and analyze the result.*" (MAT4-LTE). - "*Students are expected to recognize that experimental results may differ and that they may encounter different outcomes in each experiment, and to discuss these differences around the idea of randomness.*" (MAT.6.6.1 LTE).
NZ	Events and sample space	- "*The possible outcomes for a chance-based situation can be arranged into events.*" (K-Y5,6). - "*The sample space is the set of all possible outcomes of an experiment.*" (K-Y5,6). - "*Listing the sample space of an event.*" (P-Y6). - "*Lists, tables, and tree diagrams are useful systematic methods for generating all possible outcomes.*" (K-Y7,8). - "*An event is a subset of the sample space and thus can be a single outcome or a combination of outcomes.*" (K-Y7,8).
T		- "*Students are explained that each result obtained from any experiment is expressed as an outcome.*" (MAT.6.6.1 LTE). - "*Students are enabled to express that the totality of all possible outcomes that can be obtained from any experiment constitutes the sample space, and that all subsets that can be derived from the sample space are events.*" (MAT.7.7.1. LTE). - "*Students are presented with a sample event for which they can estimate the probability and subsequently conduct the experiment.*" (MAT.8.7.1 LTE).
NZ	Combinatorial enumeration and counting	- "*Lists, tables, and tree diagrams are useful systematic methods for generating all possible outcomes.*" (K-Y7,8).
T		- "*Students are asked to display all outcomes in the examined events using different representations such as the listing method or tree diagrams.*" (MAT.7.7.1. LTE). - "*Drawing experiments from a bag containing balls of different colors, comparison of equiprobable and non-equiprobable situations.*" (MAT.7.7.2 LTE).
NZ	Independence and conditional probability	-
T		-
NZ	Probability distribution and expectation	-
T		-
NZ	Convergence and laws of large numbers	- "*The Law of Large Numbers states that as the number of trials in a chance experiment increases, the experimental probability will approach the experiment's theoretical probability.*" (K-Y7,8) - "*Carrying out chance experiments of at least 100 trials and comparing the experimental probability of each individual outcome to its theoretical probability, in order to demonstrate the Law of Large Numbers.*" (P-Y7,8). - "*Comparing experimental probability (using at least 30 trials) to theoretical probability, and explaining why they differ and how increasing the number of trials reduces this difference.*" (P-Y7,8).
T		- "*Thus, students are expected to recognize the relationship between experimental and theoretical probability. Consequently, students are expected to recognize that as the number of trials increases in the conducted experiment, experimental probability values vary less and gradually approach the theoretical probability value.*" (MAT.8.7.1 LTE).
NZ	Sampling and sampling distribution	-
T		-
NZ	Modelling and simulation	- "*Conducting repeated chance experiments or games, identifying the outcomes, and describing differences between them using likelihood vocabulary.*" (P-Y5).
T		- "*While the experiments presented to students are feasible, all these experiments are carried out through in-class and out-of-class activities. In addition to experiments such as tossing coins, number cubes and picture cubes, spinning spinners, and drawing balls or number chips from a bag, contexts related to science (such as the blood types of students in the class) or social studies (such as a spinner game showing the birthplaces of students in the class) are selected.*" (MAT.6.6.1 LTE). - "*The simulations featured in this theme are carried out through online applications and statistical software. Students can create simulations as a digital task. They determine and decide which tools to use and how to use them for simulations. Thus, students update their own competencies in accordance with the selected digital tool.*" (MAT.8.7.1 LTE).

Appendix 3. Analysis through GAISE II Framework

Country	GAISE II Level	Evidence from the Curriculum
NZ	Level A	- "*Situations that involve chance, uncertainty, and randomness are called chance-based situations.*" (K-Y5,6). - "*Identifying the likelihood of an everyday event as being impossible, unlikely, even-chance, likely, or certain.*" (P-Y5). - "*Likelihood can be visualised using a number line from 0 to 1.*" (K-Y5,6).
T		- "*Being able to determine the probability of any event from daily life as 'impossible, possible, certain'.*" (MAT.4.4.1.). - "*Being able to interpret that the probability of any event is between 0 (impossible) and 1 (certain).*" (MAT.5.6.1). - "*MAT.5.6.2. Being able to structure events as less or more probable.*".
NZ	Level B	- "*Results from sets of repeated trials for the same experiment may vary.*" (K-Y7,8). - "*The estimated probability of an event from an experiment is the number of times the event happens divided by the total number of trials.*" (K-Y7,8). - "*Comparing experimental probability (using at least 30 trials) to theoretical probability, and explaining why they differ.*" (P-Y7,8).
T		- "*Being able to estimate the probability of an event based on observation.*" (MAT.6.6.1). - "*b) Makes inferences regarding the relationship between the number of trials and relative frequencies in an experiment.*" (MAT.6.6.1.). - "*Students are expected to recognize that experimental results may differ and that they may encounter different outcomes in each experiment, and to discuss these differences around the idea of randomness.*" (MAT.6.6.1 LTE).
NZ	Level C	-
T		-

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