



Analysis of Queueing System in an Agricultural Development Bank Using Cost Model and Shewhart Control Charts

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Abstract: The banking sectors have a major contribution to the economic growth of a country. Commercial banks play a crucial role in improving the financial status of the nation and channelizing the savings into investment priorities. As commercial banks lend their services to industries, trade, commerce, and not for Agriculture sectors. The need for Agricultural development bank (ADB) has become crucial, as the majority of the Indian economy depends on agriculture. ADB has its focus on providing credits/loans to farmers and other customers related to agriculture and production, to help them in farming, storage of food materials or goods, and transportation of products. In this article, we have analyzed various performance measures of the existing system against the proposed banking queue system in Peak and Off-Peak hours by using the Shewhart control chart technique for the study carried out in an ADB, Union Bank of India, Hassan district, India, in which have adopted the multi-server M/M/c Queueing model, Cost model and Shewhart control chart to reduce the risk and helps us in decision making. Origin pro-8 software is used for plotting of graphs. The main objective of this study is to optimize the service level by computing the cost of providing the service level for the Multi-queue Multi-server (MQMS) model (same as M/M/1 model) and Single queue Multi-server (SQMS) model (same as M/M/c model). A comparative cost analysis has been made for both models. We observe that the SQMS model is more effective than MQMS in reducing the waiting time of customers with the optimal number of servers.

Keywords: Queueing theory, M/M/c model, Shewhart control chart, cost optimization.

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1. Introduction

In India, Agriculture sector contributes significantly to India's GDP which is approximately 16% for FY24 by sustaining livelihood and food security. The sector faces lot of issues such as variations in agriculture market prices, meteorological conditions. In order to support farmers and agribusiness sectors, banks in India are providing financial assistance in order meet the demand of farmers motivating them to use innovative technologies and hence increase the yield of crops. Various financial schemes such as kisan credit card scheme, solar schemes are available in order to run daily operations, purchase agriculture machines, marketing products and to expand agribusiness. National Bank for Agriculture and Rural Development (NABARD) provide finance to cooperative banks which are responsible for credit disbursement. The ratio of institutional credit to total agricultural output is increasing year by year.

To modernize agriculture, various technologies are adopted and new challenges are being faced by banks. Long queues are common in banks which are majorly caused by credit system compared to active accounts. Long queues are caused due to inadequate staffs, illiteracy of farmers, poor digital infrastructure, complex documentation.

Banks can adopt many solutions to manage queues such as

1. Staff adjustments
2. Having different counters for agriculture loans
3. Educating farmers by proper communication
4. Digitization for loan process.

Statistical tools

Control charts: These are most effective tools to test whether the process is under control or not. They help the managers to correct the problems and understand the process patterns involved [1-3].

Queueing theory

The theory of queues plays a significant role in major and minor sectors.

In banking sector, this concept will help us to optimize work efficiency, customer needs, reduce costs.

A huge contribution is made in the area of Queueing theory, where the concept of Queueing theory is applied to various sectors [4, 5]. New approaches are made in banking sectors to arrive at effective system [6-14].

Single queue Multi-server model (M/M/c model)

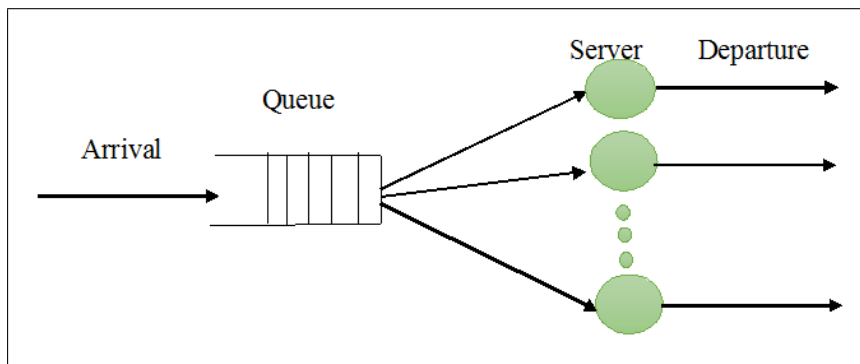


Figure 1: Single queue Multi-server queueing model

In the above model represented in Fig 1, there are c parallel servers where the service pattern is identical for all servers. This model is same as $M/M/c$ model.

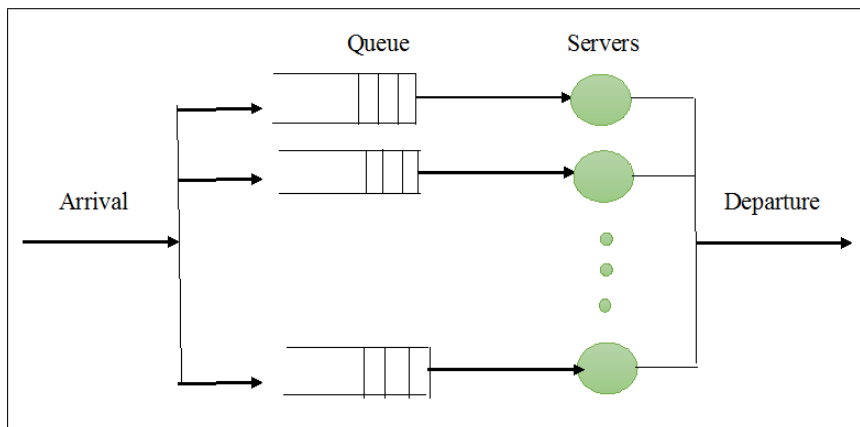


Figure 2: Multi queue Multi-server queueing model

Figure 2 represents multiple queues with c parallel servers, which is the same as the $M/M/1$ model. If λ is the arrival rate, then in this case the arrival rate reduces to λ/c as there are c individual single-server queues.

In this article, the banking system is monitored and analysed by using control chart techniques [15]. The data is collected from ADB, Union Bank of India, Hassan district, India, in which the models M/M/1 with parallel servers and M/M/c, the multi-server Queueing model are applied to the system which can help us to find optimum number of counters/servers [16 -18]. Also the cost analysis has been carried out for both the models [19, 20].

2. Objectives of the Study

- To assess various performance measures of the existing system against the proposed banking queue system in Peak and Off-Peak hours by using Shewhart control chart technique.
- To compute the cost of providing the service level for Multi-queue Multi server (MQMS) model (same as M/M/1 model) and Single queue Multi-server model (SQMS) model (same as M/M/c model) and obtain a comparative cost analysis for both the models.
- To help the banking system by choosing SQMS model which reduced the waiting time of customers over MQMS model.

3. Methodology and Model Formulation

In the banking system, the queue was analysed by using Shewhart control chart technique. These Control charts will help us to decide upon the maximum and minimum number of customers the bank can accommodate, we have also adopted models for Cost optimization in order to improve the efficiency of the bank and its profitability. Total Cost computation during Peak and Off-Peak hours will help the bank management to decide upon providing services in a better manner.

In the banking system, there are 3 types of customers and the services offered to customers of type A, B and C where type A includes withdrawal of amount, Account opening, Western union customers, type B includes enquiry, salary of customers and type C includes agricultural customers. Currently the existing number of servers in the bank is $c = 3$ and the number of servers proposed were $c = 4$, $c = 6$, $c = 7$. Based on the concept of standard normal distribution, the optimal number of servers with 95% and 99% service rate were observed as $c = 6$ and $c = 7$ respectively by using the relation $c = \rho + t\sqrt{\rho}$ where t is the critical value of standard normal variable z at 5% and 1% level of significance for one tailed test. In the study, the Queueing analysis was carried out for $c = 3$, $c = 4$, number of servers during Off-Peak and Peak hours and listed out the performance measures for these servers which are presented in Table 1.

Table 1: indicates performance analysis of Union Bank of India, Kandali branch for the Servers $c = 3$ and $c = 4$

Performance measures	λ Mean arrival rate	μ Mean service rate	ρ Server utilization	$E(\ell_s)$ Average number of customers in system	$E(\ell_q)$ Average number of customers in queue	$E(w_s)$ Average waiting time of a customer in system	$E(w_q)$ Average waiting time of a customer in queue
Off-Peak Hour							
$c = 3$	49	17	96%	26	23	0.522 hrs	0.4634 hrs
$c = 4$	49	17	72%	5	2	0.083 hrs	0.024 hrs
Peak Hour							
$c = 3$	62	21	98%	64	61	1.0178 hrs	0.9702 hrs
$c = 4$	62	21	74%	5	2	0.0699 hrs	0.0223 hrs

For the above actuals, we apply the control chart technique and analyse various control chart parameters such as Mean, Variance, Upper Control Limit (UCL) and Control Limit (CL) to arrive at an efficient model.

Mean and Variance for M/M/c Queueing model

The statistical constants for the random number of customers in the queue l_q are:

-

$$E(l_q) = \frac{\lambda\mu \left(\frac{\lambda}{\mu}\right)^c p_i}{(c-1)!(c\mu - \lambda)^2} \quad (1.1)$$

-

$$Var(l_q) = \frac{(1+\rho)}{(1-\rho)} [E(l_q)] - [E(l_q)]^2 \quad (1.2)$$

The statistical constants for the random number of customers in the system (l_s) are:

-

$$E(l_s) = E(l_q) + \left(\frac{\lambda}{\mu}\right) \quad (1.3)$$

-

$$Var(l_s) = \frac{(1+\rho)}{(1-\rho)} E(l_s) - \frac{1}{3} + \frac{p_i}{3(1-\rho)} - [E(l_s)]^2 \quad (1.4)$$

The statistical constants for the waiting time of customers in the queue (w_q) are:

-

$$E(w_q) = \frac{E(l_q)}{\lambda} \quad (1.5)$$

-

$$Var(w_q) = \frac{2E(w_q)}{c\mu(1-\rho)} - [E(w_q)]^2 \quad (1.6)$$

The statistical constants for the waiting time of customers in the system (w_s) are:

-

$$E(w_s) = E(w_q) + \frac{1}{\mu} \quad (1.7)$$

-

$$Var(w_s) = \frac{2}{\mu^2} \left[\frac{1 + (\rho c)^c (\mu(c+1) - \lambda)}{\mu c! c^2 ((1-\rho)^3)} p_i \right] - [E(w_s)]^2 \quad (1.8)$$

4. Results and Discussion

Questionnaires were provided to the customers at Union Bank of India. Out of 350 questionnaires only 250 customers responded. Out of these 250 respondents, there were 12 businessmen, 20 small farmers, 75 marginal farmers and 68 large farmers, 45 civil servants and 30 civil servants with a minimum wage of Rs.15,000 and Rs.10,000 per month respectively. 40% are not satisfied with the existing banking system, 53% are well satisfied and 7% are neutral. 30% spend 0 – 30 minutes, 18% spend 1hour and 52% spend more than 2 hours. The customer's monthly generated income along with the average customer's arrival rate during Off-Peak and Peak hours is presented in Table 2.

Discussion with bank officials reveals that congestion is common with existing servers. To avoid such situations, measures were taken by the bank management such as labour division, reducing lunch time and providing huge bonus to employees. Each server operating cost is observed to be Rs. 600 per hour.

By considering the percentage of each category, Average customer arrival per hour is divided into various classes, which would sum up to a total of 49 customers per hour and 62 customers per hour during Off-Peak and Peak hours respectively.

Based on the monthly generated income of customers, we have calculated opportunity cost of customers waiting in the system per hour, Expected Waiting Cost and Expected service operating cost per hour along with Total cost per hour and are presented for both MQMS model (same as M/M/1 model) and SQMS model (M/M/c model) in Table 3 and Table 4 respectively. We observe that the costs of having MQMS model (c number of single server) is higher than having SQMS model. Hence the control chart parameters along with UCL and CL are computed for SQMS model which is more efficient. The data considered for calculation is presented in Table 4.

Parameters of control charts such as Variance, CL, UCL values are computed by using the formulas from 1.1 to 1.8 listed above and are presented in Table 5. The banking system remains to be more stable, if the CL values are achieved for various performance measures. UCL values indicates the maximum value which the system can hold. Negative estimate of variance is an indication that variance is negligible, due to which UCL values cannot be computed in such cases. We can observe that for the existing servers, that is $c = 3$ the total cost per hour is Rs. 17139 and Rs. 31710 during Off- peak and peak hours respectively which reduces to Rs. 4839 and Rs. 4454 during Off- peak and peak hours respectively for $c = 4$ servers. The total cost has increased for $c = 6$ and $c = 7$ servers. Hence $c = 4$ can be considered as the optimal number of servers.

Table 2: indicates the Customer's monthly generated income at Union Bank of India.

Weight Category	Percentage wise (%) Distribution	Number of Customers Responded	Monthly Income (Rs.)	Total Income Generated (Rs.)	Average customer arrival per hour (λ) in Off-peak hour	Average customer arrival per hour (λ) in Peak hour
A) Business						
a)Rs.50,000	5	6	300000	1200000	2	3
b)Rs.1,00,000		3	300000			
c)Rs.2,00000		3	600000			
B) Farmers						
a)Small(1 lakh)	65	20	166680	4875036	32	40
b)Marginal(3 lakh)		75	1875000			
c)Large(5 Lakh)		68	2833356			
C)Civil Servants with minimum wage						
	30	45	675000	975000	15	19
		30	300000			
Total	100	250		7050036	49	62

Table 3: Indicates cost analysis for SQMS model at Union Bank of India

Servers (c)	Average Customers per hour based on category (λ)	Opportunity Cost per hour (C_w)(in Rs)	Average Waiting Time in the system $E(w_s)$	Expected Waiting Cost in the system per hour $E(wc_s) = (\lambda E(w_s) C_w)$ (in Rs)	Expected service operating cost per hour $E(OS_c) = c(ST)$ (in Rs)	Total Cost/hr $E(TC)$
Off-Peak hour						
$c = 3$	2	2500	1.5	7500	1800	45878
	32	635		30480		
	15	271		6097.5		
$c = 4$	2	2500	0.21	1050	2400	8571
	32	635		4267.2		
	15	271		853.65		
$c = 6$	2	2500	0.11	550	3600	6832
	32	635		2235.2		
	15	271		447.15		
$c = 7$	2	2500	0.10	500	4200	7139
	32	635		2032		
	15	271		406.5		
Peak hour						
$c = 3$	3	1667	3	15003	1800	89961
	40	508		60960		
	19	214		12198		
$c = 4$	3	1667	0.18	900.18	2400	7690
	40	508		3657.6		
	19	214		731.88		
$c = 6$	3	1667	0.09	450.09	3600	6245
	40	508		1828.8		
	19	214		365.94		
$c = 7$	3	1667	0.08	400.08	4200	6551
	40	508		1625.6		
	19	214		325.28		

Table 4: Indicates cost analysis for SQMS model at Union Bank of India.

Servers (c)	Average Customers per hour based on category (λ)	Opportunity Cost per hour (C_w)(in Rs)	Average Waiting Time in the system $E(w_s)$	Expected Waiting Cost in the system per hour $E(wc_s) = (\lambda E(w_s) C_w)$ (in Rs)	Expected service operating cost per hour $E(OS_c) = c(ST)$ (in Rs)	Total Cost/hr $E(TC)$
Off-Peak hour						
$c = 3$	2	2500	0.522 hours	2610	1800	17139
	32	635		10607.04		
	15	271		2121.93		
$c = 4$	2	2500	0.083 hours	415	2400	4839
	32	635		1686.56		
	15	271		337.395		
$c = 6$	2	2500	0.0604 hours	302	3600	5375
	32	635		1227.328		
	15	271		245.526		
$c = 7$	2	2500	0.0593 hours	296.5	4200	5943
	32	635		1204.976		
	15	271		241.0545		
Peak hour						
$c = 3$	3	1667	1.0178 hours	5090.0178	1800	31710
	40	508		20681.696		
	19	214		4138.3748		
$c = 4$	3	1667	0.0699 hours	349.5699	2400	4454
	40	508		1420.368		
	19	214		284.2134		
$c = 6$	3	1667	0.0491 hours	245.5491	3600	5043
	40	508		997.712		
	19	214		199.6406		
$c = 7$	3	1667	0.048 hours	240.048	4200	5611
	40	508		975.36		
	19	214		195.168		

Table 5: Indicates the parameters of control chart, performance measures and Total cost for various servers for SQMS model

Number of servers	Parameters of Control Chart	$E(L_q)$ Average customers in queue	$E(L_s)$ Average customers in system	$E(W_q)$ Average waiting time in queue (hrs)	$E(W_s)$ Average waiting time in system (hrs)	ρ (Server utilization factor)	p_i (System idle probability)	$E(TC)$ Total cost/hr (Rs.)
Off-Peak hour								
Existing Servers (c=3)	Variance	598	597.74	0.23957	0.21562	96%	0.911%	17139
	CL = Mean	23	26	0.4634	0.522			
	U_{CL}	96	99	1.9318	1.91505			
Proposed Servers (c=4)	Variance	8.29	5.43	0.00195	Negative	72%	4.48%	4839
	CL = Mean	2	5	0.024	0.083			
	U_{CL}	11	12	0.15631	–			
Proposed Servers (c=6)	Variance	1.85	Negative	0.00006	Negative	48%	5.53%	5375
	CL = Mean	1	3	0.0016	0.0604			
	U_{CL}	2	–	0.0244	–			
Proposed Servers (c=7)	Variance	0	Negative	0.00001	Negative	41%	5.58%	5943
	CL = Mean	0	3	0.0004	0.0593			
	U_{CL}	0	0	0.01046	–			
Peak hour								
Existing Servers (c=3)	Variance	2318	2240	0.59871	Negative	98%	0.36%	31710
	CL = Mean	61	64	0.9702	1.0178			
	UCL	205	206	3.2915	–			
Proposed Servers (c=4)	Variance	9.38	8.18	0.00155	Negative	74%	4.05%	4454
	CL = Mean	2	5	0.0223	0.0699			
	U_{CL}	11	14	0.14021	–			
Proposed Servers (c=6)	Variance	1.92	Negative	0.00004	Negative	49%	5.14%	5043
	CL = Mean	1	4	0.0015	0.0491			
	U_{CL}	5	–	0.021498	–			
Proposed Servers (c=7)	Variance	1.45	Negative	0.000009	Negative	42%	5.20%	5611
	CL = Mean	1	3	0.0004	0.048			
	U_{CL}	5	–	0.00951	–			

By cost analysis, we observe that the maintenance cost of MQMS model is much expensive compared to that of SQMS model. Also we observe that for MQMS model and SQMS model, the optimal server during both Off- Peak and Peak hours is $c = 6$ and $c = 4$ respectively. Hence the values in table 5 is computed only for SQMS model. From the Graphs, Fig 3 to Fig 6, Fig 7 depicts the line plots of the Expected Service operating cost against the level of service, Expected waiting cost against the level of service and Total expected cost against the level of service and the Expected number of customers in the system against the probability that the system is idle during Off-Peak and Peak hours respectively.

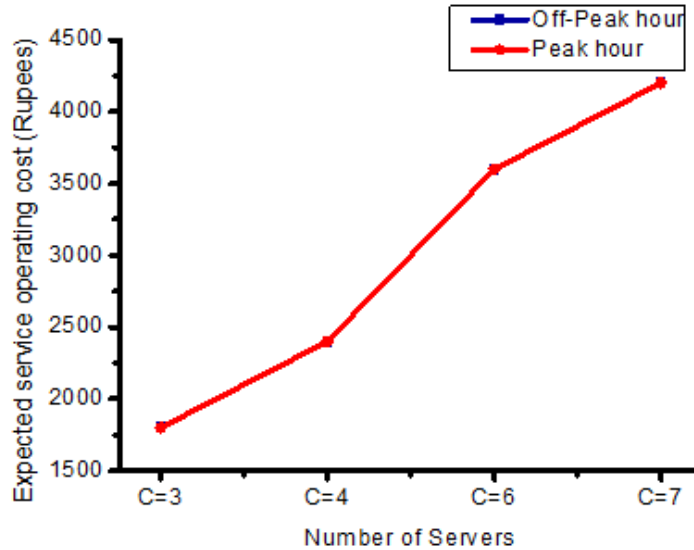


Figure 3: Expected service operating cost against the number of servers.

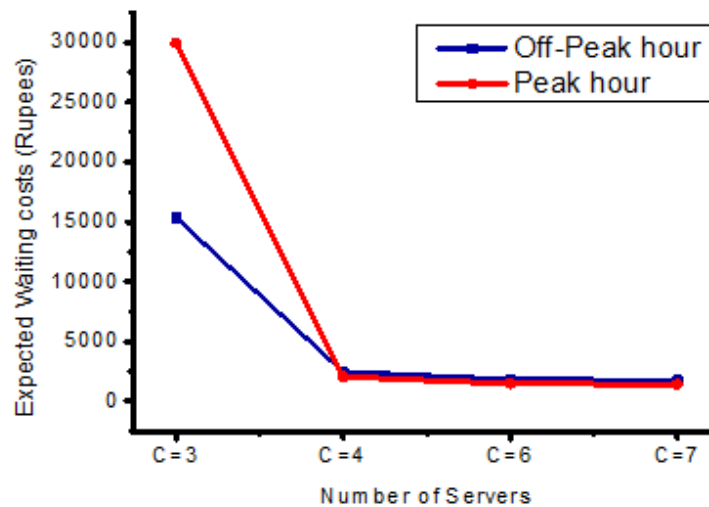


Figure 4: Expected waiting cost against the number of servers.

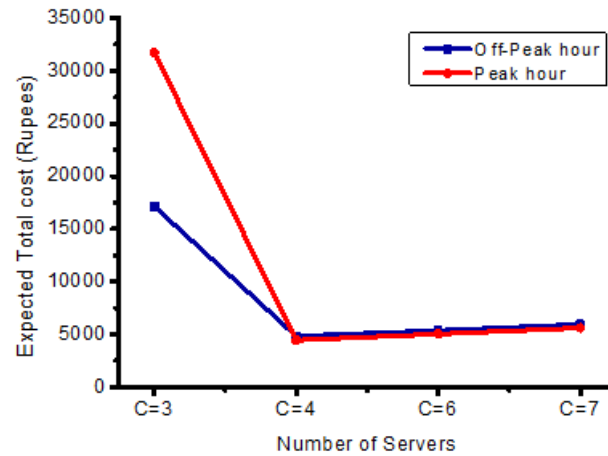


Figure 5: Expected Total cost against the number of servers.

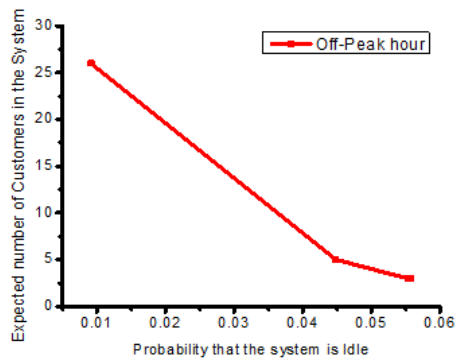


Figure 6 (a)

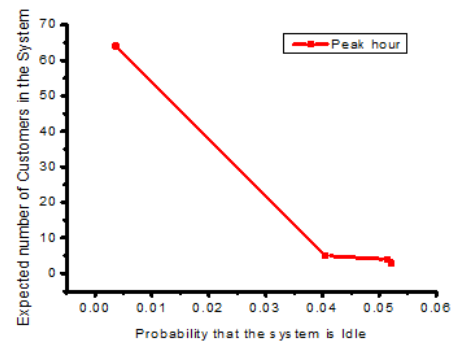


Figure 6 (b)

Figure 6: (a) & (b) Expected number of customers in the system against the probability that system is idle during Off-Peak and Peak hour.

5. Conclusion

From the above results, for existing number of servers $c = 3$ and for proposed number of servers being $c = 4$, $c = 6$, $c = 7$ the total cost per hour is Rs.8176, Rs. 3415, Rs. 4339, Rs. 4926 respectively, the UCL values for average number of customers in queue is 96, 11, 2, 0 respectively, the UCL values for average number of customers in system is 99, 12, 0, 0 respectively. The UCL values for average waiting time in queue is 1.93 hours, 0.156 hours, 0.0248 hours, 0.0099 hours respectively and the UCL values for average waiting time in queue is 1.97 hours, 0, 0, 0 respectively. From these data values, we recommend the banking system to have optimal servers which is $c = 4$ servers. Also, the bank management can decide upon the threshold values by observing the UCL values for each performance measure. Indeed the lower CL values for various servers is negligible in all cases. In order to have economic banking system, the manager needs to plan on controlling the queues which is formed during busy days. As majority of the customers are farmers who avail loans, the congestion in the system can be reduced by arranging for the availability of the field officer more often rather than visiting twice a week. An economic efficient system is achieved by applying SQMS model.

References

1. Weiss, E. N., & Tucker, C (2018) "Queue management: Elimination, expectation and enhancement", *Business Horizons*, Vol.61, Issue 5, pg.671-678.
2. Zhao X, Gilbert K (2015) "A statistical control chart for monitoring customer waiting time", *International Journal of Data Analysis Techniques and Strategies*, Vol.7, Issue 3, pg. 301-321.
3. Shore,H (2000) "General control charts for attributes", *IIE transactions*, Vol.32, pg.1149-1160.
4. Gupta, I., Zoreda, J. and Kramer, N (1971) "Hospital Manpower Planning by Use of Queuing Theory", *Health Services Research*, Vol.6, pg.76-82.
5. Sultana, M., & Islam, M. M. N (2012) "Impact of queue on customers: An analysis of some retail shops in Bangladesh", *East-West Journal of Business and Social Studies*, Vol.3, pg.95-108.
6. Abiodun, R. O (2017) "Development of Mathematical Models for Predicting Customers Satisfaction in the Banking system with a Queuing Model Using Regression Method", *American Journal of Operations Management and Information Systems*, Vol.2, Issue 3, pg.86-91.
7. Bhavin P. and Bhathawala P (2012) "Case study for bank ATM queuing model," *International Journal of Engineering Research and applications (IJERA)*, Vol.2, Issue 5, pg.1278-1284.
8. Chowdhury, M.S., M. R (2013) "Solving Of Waiting Lines Models in the Bank Using Queuing Theory Model the Practice Case: Islami Bank Bangladesh Limited, Chawkbazar Branch, Chittagong", *IOSR Journal of Business and Management (IOSR-JBM)*, pg.22-29.
9. H. Xiao and G. Zhang (2010) "The queuing theory application in bank service optimization", *International Conference on Logistics Systems and Intelligent Management (ICLSIM)*, pg.1097-1100.
10. Odirichukwu, J. C., Lekara, T., and Odii, J. N (2014) "Banking queue system in Nigeria", *Computing, Information Systems, Development Informatics & Allied Research Journal*, Vol.5, Issue 1, pg.95-106.
11. Saxena K and Soni K (2011) "A Study of Applicability of Waiting Line Model In Health Care: A Systematic Review", *IJMT*, Vol.19, Issue 1, pg.75-91.
12. Shashidhar HV and Subramanian Varadarajan (2011) "Customer Segmentation of Bank based on Data Mining Security Value based Heuristic Approach as a Replacement to K-means Segmentation", *International Journal of Computer Applications*, Vol.19, Issue 8, pg.13-18.
13. Sheikh T, Sanjay kumar singh and Anil kumar kashyap (2013) "A study of queuing model for banking system", *International Journal of Industrial Engineering and Technology*, Vol.5, pg.21-26.
14. Sheikh T, Sanjay kumar singh and Anil kumar kashyap (2013) "Application of queuing theory for the improvement of bank service", *International Journal of Advanced Computational Engineering and Networking*, Vol.1, Issue 4, pg.15-18.
15. T.Poongodi (2013) "Control chart for waiting time in system of (M / M / S):(∞ / FCFS) Queuing model", *IOSR Journal of Mathematics*, Vol.5, pp.48-53.
16. Gross, D. and Harris, C.M (1998) "*Fundamentals of queueing theory*", 3rd edition, John Wiley and sons.
17. S. K. Tiwari, Dr. V. K. Gupta, Tabi Nandan Joshi (2016) "M/M/S Queueing Theory Model to Solve Waiting Line and to Minimize Estimated Total Cost", *International Journal of Science and Research (IJSR)*, Vol.5, Issue 5, pg.1901-1904.
18. E. Berhan (2015) "Bank Service Performance Improvements using Multi-Server Queue System", *IOSR-Journal of Business and Management*, Vol.17, Issue 6, pg.65-69.
19. Kembe, M. M, Onah, E. S, Iorkegh, S (2012) "A Study of Waiting And Service Costs of a MultiServer Queuing Model in a Specialist Hospital", *International Journal of Scientific & Technology Research*, Vol.1, Issue 8, pg.19-23.
20. Olaniyi,T.A (2004) " An Appraisal of Cost of Queuing in Nigerian Banking Sector: A Case Study of First Bank of Nigeria Plc, Ilorin", *Journal of Business & Social Sciences*, Vol.9, Issues 1&2, pg.139-145.

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