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Mathematical Modeling and Revan Indices of Dominating David Derived Networks Based on Edge Partitions

Gavvala Sowmya, Vishu Kumar Mallappa

ABSTRACT: Chemical graph theory provides powerful mathematical tools for analyzing the structural and physicochemical properties of molecular systems. Among the various degree-based descriptors, the Revan indices have recently emerged as effective topological invariants for studying complex chemical and nanostructured networks. In this paper, we present a detailed mathematical investigation of the first, second, and third Revan indices of dominating David derived networks (DDDNs), a family of networks constructed from the honeycomb lattice through edge subdivision and vertex augmentation. Three variants of DDDNs $D_1(t)$, $D_2(t)$, and $D_3(t)$ are examined using systematic edge partition techniques. For each network, exact closed-form expressions for the Revan indices are derived by leveraging the structural regularities inherent to the networks. The obtained expressions highlight clear distinctions in the topological structure of the three variants, reflecting how edge partitions and degree distributions influence Revan index values. The findings contribute new insights into the mathematical behavior of dominating David derived networks and reinforce their relevance to QSPRs/QSARs modeling, nanostructure design, and theoretical chemistry.

Keywords: Dominating David derived networks, honeycomb network, Revan index, first, second and third Revan indices.

Contents

1. Introduction

Let G be a finite, simple, and connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d(u)$ represents the number of vertices adjacent to a vertex u. The parameters $\Delta(G)$ and $\delta(G)$ denote the maximum and minimum degree of the graph, respectively.The notation RE represents an Revan edge set. We refer to [1] for standard terminology and notations in graph theory. Chemical graph theory forms a significant branch of mathematical chemistry where molecular structures are modeled as graphs, with atoms represented as vertices and chemical bonds as edges [2,3]. One of the central themes in this area is the study of topological indices, numerical values derived from the structural features of molecular graphs. These indices serve as powerful tools in predicting physicochemical and biological properties of chemical compounds through QSPRs/QSARs models [2,3].

Among different graph structures used in chemical modeling, dominating David derived networks (DDDNs) have gained research attention due to their rich structural features. These networks originate from the honeycomb network through iterative edge subdivision and vertex insertion, producing highly symmetric and chemically meaningful graph families [4,5,6]. The structural variations in DDDNs allow them to effectively represent complex molecular or nanoscale systems. Earlier studies have investigated classical topological descriptors such as Zagreb indices for these networks [7,8,9], establishing foundational results linking the geometry of the networks to their combinatorial properties.

In recent years, Revan indices, introduced as degree-based invariants incorporating both global and local degree information, have emerged as promising tools for the characterization of chemical structures

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[10,11,12]. These indices—namely the first, second, and third Revan indices—are defined using the modified Revan vertex degree

$$r_G(v) = \Delta(G) + \delta(G) - d_G(v)$$

which captures variations in degree distribution more sensitively than traditional degree measures. Revan indices have demonstrated strong correlation with thermodynamic, structural, and reactivity-related properties in QSPRs/QSARs studies [10,11,12].

Despite the rich literature on classical indices, the Revan indices of dominating David derived networks have not been explored sufficiently. Since DDDNs possess distinct edge partitions and hierarchical structural patterns, analyzing their Revan indices provides deeper insight into their topological complexity and potential applications in nanoscale modeling. Motivated by this gap, the present work investigates the Revan indices R_1, R_2, and R_3 of three types of dominating David derived networks, namely $D_1(t)$, $D_2(t)$, and $D_3(t)$ using systematic edge-partition methods. The results obtained contribute to the theoretical understanding of these networks and offer new computational data that may support future applications in mathematical chemistry, molecular design, and related fields.

2. Preliminaries

Revan Vertex Degree:

[12] The Revan vertex degree is a modified vertex degree defined as

$$r_G(v) = \Delta(G) + \delta(G) - d_G(v) \tag{2.1}$$

where $\Delta(G)$ is the maximum degree of the graph, $\delta(G)$ is the minimum degree of the graph, and $d_G(v)$ is the degree of the vertex v.

Revan Indices:

The first, second and third Revan indices of a graph G are defined as:

- First Revan index:
$$R_1(G) = \sum_{u,v \in E(G)} (r_G(u) + r_G(v)) \tag{2.2}$$

- Second Revan index:
$$R_2(G) = \sum_{u,v \in E(G)} r_G(u) \times r_G(v) \tag{2.3}$$

- Third Revan index:
$$R_3(G) = \sum_{uv \in E(G)} (|r_G(u) - r_G(v)|) \tag{2.4}$$

3. Construction of Dominating David Derived Networks

Step 1: Begin with a t-dimensional honeycomb structure.
Step 2: Subdivide every original edge of the honeycomb by inserting an additional vertex at its midpoint.
Step 3: Within each hexagonal face, identify pairs of these newly inserted vertices that are exactly distance 4 apart, and join each such pair with an edge.
Step 4: Whenever these added edges intersect, introduce a new vertex at each intersection point.
Step 5: Eliminate all the original vertices and edges of the initial honeycomb network, retaining only the newly constructed structure.
Step 6: Further subdivide each horizontal edge by adding one more vertex so that each horizontal edge is split into two equal segments.

The structure of the first-type of $D_1(2)$ network is shown in figure 2.This network has maximum degree is 4 and minimum degree is 2. In this network there are six types of edges based on the Revan degree of end vertices. The edge partition of $D_1(t)$ is shown in table 1.

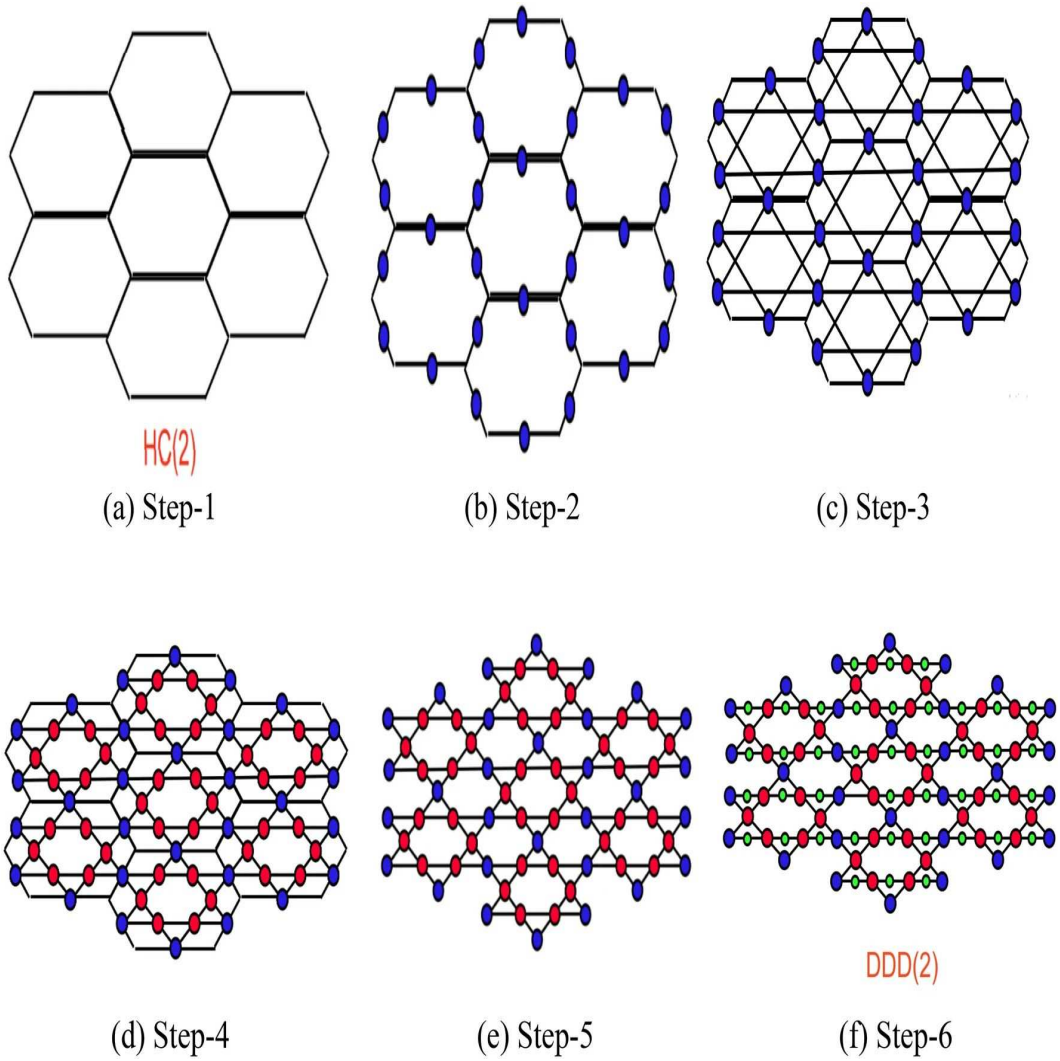

Figure 1: The steps to derive DDDN(2)

Table 1: Edges partition of $D_1(t)$

$r_G(u)$, $r_G(v)$	(2,2)	(2,3)	(2,4)	(3,3)	(3,4)	(4,4)
Frequency	$36t^2+52t+20$	$36t^2+56t+24$	$28t+16$	$9t^2+13t+5$	$4t+4$	$4t$

Table 2: Edges partition of $D_2(t)$

$r_G(u)$, $r_G(v)$	(2,2)	(2,3)	(2,4)	(3,4)	(4,4)
Frequency	$36t^2-52t+20$	$36t^2-56t+24$	$28t-16$	$18t^2-22t+6$	$4t$

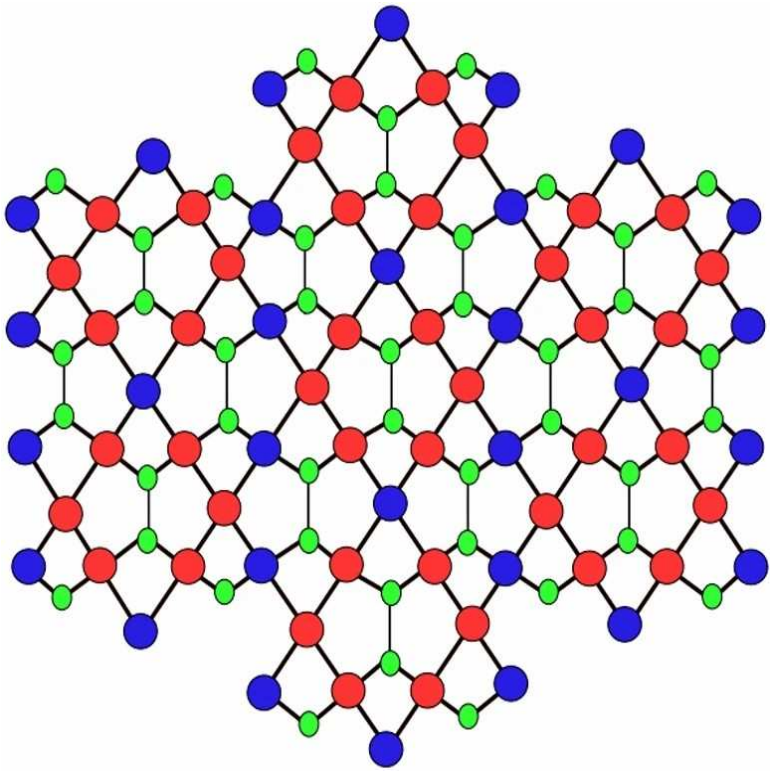

Figure 2: Second type of $D_1(2)$ network

The structure of the second-type of $D_2(2)$ network is shown in figure 3.This network has maximum degree is 4 and minimum degree is 2. In this network there are five types of edges based on the Revan degree of end vertices. The edge partition of $D_2(t)$ is shown in table 2.

The structure of the second-type of $D_3(2)$ network is shown in figure 4.This network has maximum degree is 4 and minimum degree is 2. In this network there are three types of edges based on the Revan degree of end vertices. The edge partition of $D_3(t)$ is shown in table 3.

Table 3: Edges partition of $D_3(t)$

$r_G(u)$, $r_G(v)$	(2,2)	(2,4)	(4,4)
Frequency	$72t^2 - 108t + 44$	$36t^2 - 20t$	$4t$

4. Main Results

Theorem 4.1 *Let $D_1(t), D_2(t)$ and $D_3(t)$ be the graphs. Then the first Revan index of the graphs is*

$$I.R_1[D_1(t)] = 378t^2 - 338t + 106$$
$$II.R_1[D_2(t)] = 450t^2 - 442t + 146$$
$$III.R_1[D_3(t)] = 504t^2 - 520t + 176$$

Proof: Let $D_1(t), D_2(t)$ and $D_3(t)$ be the network graphs. We use the edge-sum form of the first Revan

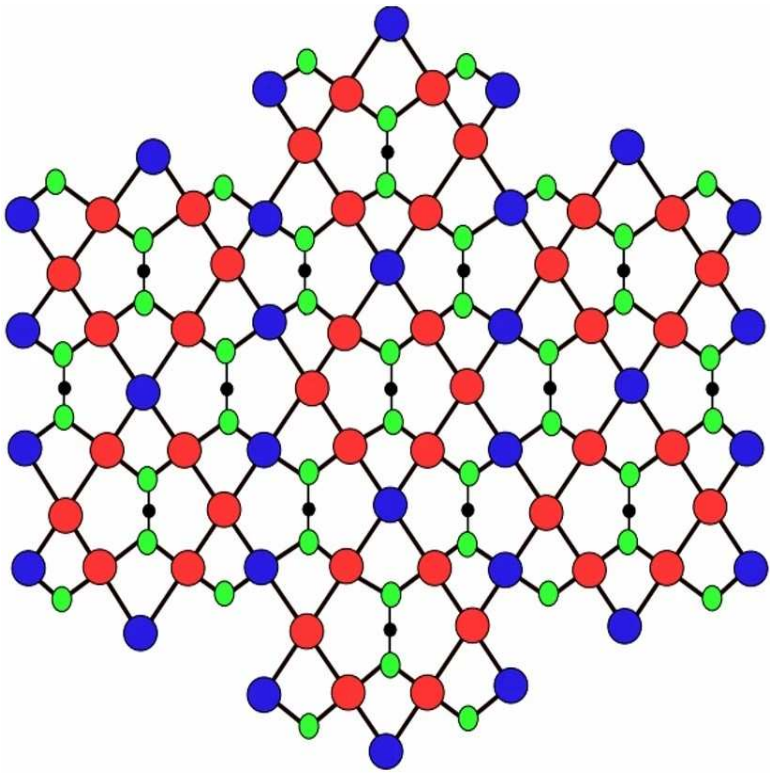

Figure 3: Second type of $D_2(2)$ network

index:

$$R_1(G) = \sum_{uv \in E(G)} \big(r_G(u) + r_G(v)\big).$$

Computation of $R_1[D_1(t)]$.
Using the edge partition from Table 1 we have

$$\begin{aligned} R_1[D_1(t)] &= |RE_{(2,2)}|(2+2) + |RE_{(2,3)}|(2+3) + |RE_{(2,4)}|(2+4) + |RE_{(3,3)}|(3+3) + |RE_{(3,4)}|(3+4) + \\ &\quad |RE_{(4,4)}|(4+4) \\ &= (36t^2 - 52t + 20)4 + (36t^2 - 56t + 24)5 + (28t - 16)6 + (9t^2 - 13t + 5)6 + (4t - 4)7 + (4t)8 \\ &= 144t^2 - 208t + 80 + 180t^2 - 280t + 120 + 168t - 96 + 54t^2 - 78t + 30 + 28t - 28 + 32t \\ &= 378t^2 - 338t + 106. \end{aligned}$$

Computation of $R_1[D_2(t)]$.
Using the edge partition from Table 2 we get

$$\begin{aligned} R_1[D_2(t)] &= |RE_{(2,2)}|(2+2) + |RE_{(2,3)}|(2+3) + |RE_{(2,4)}|(2+4) + |RE_{(3,4)}|(3+4) + |RE_{(4,4)}|(4+4) \\ &= (36t^2 - 52t + 20)4 + (36t^2 - 56t + 24)5 + (28t - 16)6 + (18t^2 - 22t + 6)7 + (4t)8 \\ &= 144t^2 - 208t + 80 + 180t^2 - 280t + 120 + 168t - 96 + 126t^2 - 154t + 42 + 32t \\ &= 450t^2 - 442t + 146. \end{aligned}$$

Computation of $R_1[D_3(t)]$.

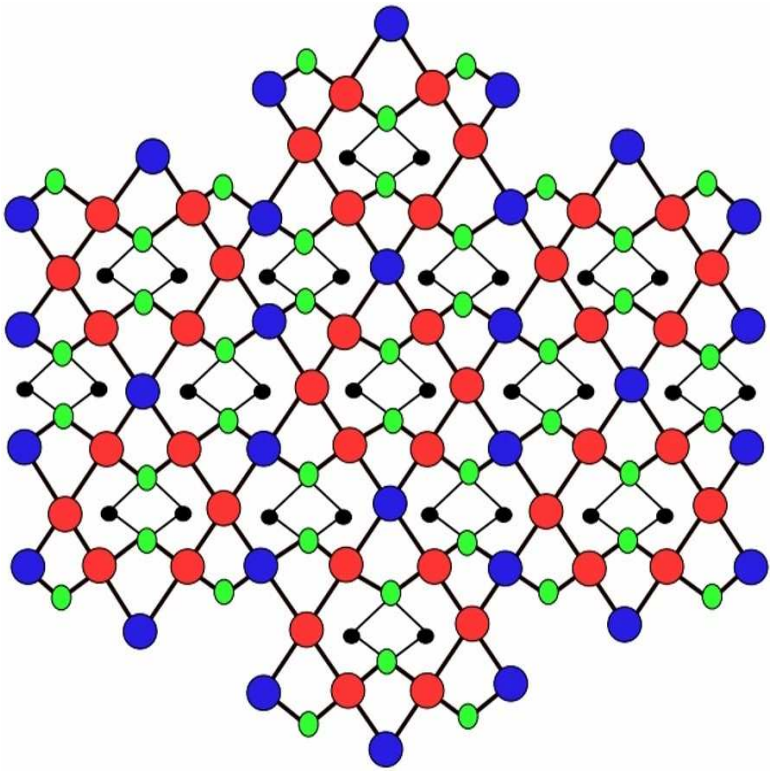

Figure 4: Second type of $D_3(2)$ network

Using the edge partition from Table 3 we obtain

$$\begin{aligned} R_1[D_3(t)] &= |RE_{(2,2)}|(2+2) + |RE_{(2,4)}|(2+4) + |RE_{(4,4)}|(4+4) \\ &= (72t^2 - 108t + 44)4 + (36t^2 - 20t)6 + (4t)8 \\ &= 288t^2 - 432t + 176 + 216t^2 - 120t + 32t \\ &= 504t^2 - 520t + 176. \end{aligned}$$

□

Theorem 4.2 *The second Revan topological indices of $D_1(t)$, $D_2(t)$ and $D_3(t)$ are given by:*

$$\begin{aligned} I.R_2[D_1(t)] &= 441t^2 - 325t + 93 \\ II.R_2[D_2(t)] &= 576t^2 - 520t + 168 \\ III.R_2[D_3(t)] &= 576t^2 - 528t + 176 \end{aligned}$$

Proof: Let $D_1(t)$, $D_2(t)$ and $D_3(t)$ be the network graphs. The second Revan index of a graph G is defined as

$$R_2(G) = \sum_{uv \in E(G)} \big(r_G(u) \cdot r_G(v)\big).$$

Computation of $R_2[D_1(t)]$

Using the edge partitioning from Table 1, we obtain

$$\begin{aligned}R_2[D_1(t)] &= |RE_{(2,2)}|(2\times 2)+|RE_{(2,3)}|(2\times 3)+|RE_{(2,4)}|(2\times 4)+|RE_{(3,3)}|(3\times 3)+|RE_{(3,4)}|(3\times 4)\\ &\quad +|RE_{(4,4)}|(4\times 4)\\ &= (36t^2-52t+20)4+(36t^2-56t+24)6+(28t-16)8+(9t^2-13t+5)9+(4t-4)12+(4t)16\\ &= 144t^2-208t+80+216t^2-336t+144+224t-128+81t^2-117t+45+48t-48+64t\\ &= 441t^2-325t+93.\end{aligned}$$

Computation of $R_2[D_2(t)]$

Using the edge partitioning from Table 2, we get

$$\begin{aligned}R_2[D_2(t)] &= |RE_{(2,2)}|(2\times 2)+|RE_{(2,3)}|(2\times 3)+|RE_{(2,4)}|(2\times 4)+|RE_{(3,4)}|(3\times 4)+|RE_{(4,4)}|(4\times 4)\\ &= (36t^2-52t+20)4+(36t^2-56t+24)6+(28t-16)8+(18t^2-22t+6)12+(4t)16\\ &= 144t^2-208t+80+216t^2-336t+144+224t-128+216t^2-264t+72+64t\\ &= 576t^2-520t+168.\end{aligned}$$

Computation of $R_2[D_3(t)]$

Using the edge partitioning from Table 3, we obtain

$$\begin{aligned}R_2[D_3(t)] &= |RE_{(2,2)}|(2\times 2)+|RE_{(2,4)}|(2\times 4)+|RE_{(4,4)}|(4\times 4)\\ &= (72t^2-108t+44)4+(36t^2-20t)8+(4t)16\\ &= 288t^2-432t+176+288t^2-160t+64t\\ &= 576t^2-528t+176.\end{aligned}$$

□

Theorem 4.3 *The third Revan topological indices of $D_1(t)$, $D_2(t)$ and $D_3(t)$ are given by:*

$$\begin{aligned}I.R_3[D_1(t)] &= 36t^2+4t-12\\ II.R_3[D_2(t)] &= 54t^2-22t-2\\ III.R_3[D_3(t)] &= 72t^2-40t\end{aligned}$$

Proof: Let $D_1(t)$, $D_2(t)$ and $D_3(t)$ be the network graphs. The third Revan index of a graph G is defined as

$$R_3(G)=\sum_{uv\in E(G)}\big|r_G(u)-r_G(v)\big|.$$

Computation of $R_3[D_1(t)]$.

Using the edge partitioning from Table 1, we obtain

$$\begin{aligned}R_3[D_1(t)] &= |RE_{(2,2)}||2-2|+|RE_{(2,3)}||2-3|+|RE_{(2,4)}||2-4|+|RE_{(3,3)}||3-3|+|RE_{(3,4)}||3-4|+|RE_{(4,4)}||4-4|\\ &= (36t^2-52t+20)0+(36t^2-56t+24)1+(28t-16)2+(9t^2-13t+5)0+(4t-4)1+(4t)0\\ &= 36t^2-56t+24+56t-32+4t-4\\ &= 36t^2+4t-12.\end{aligned}$$

Computation of $R_3[D_2(t)]$.

Using the edge partitioning from Table 2, we get

$$\begin{aligned}R_3[D_2(t)] &= |RE_{(2,2)}||(2-2)|+|RE_{(2,3)}||(2-3)|+|RE_{(2,4)}||(2-4)|+|RE_{(3,4)}||(3-4)|+|RE_{(4,4)}||(4-4)|\\ &= (36t^2-52t+20)0+(36t^2-56t+24)1+(28t-16)2+(18t^2-22t+6)1+(4t)0\\ &= 36t^2-56t+24+56t-32+18t^2-22t+6\\ &= 54t^2-22t-2.\end{aligned}$$

Computation of $R_3[D_3(t)]$:
Using the edge partitioning from Table 3, we obtain

$$\begin{aligned} R_3[D_3(t)] &= |RE_{(2,2)}||(2-2)| + |RE_{(2,4)}||(2-4)| + |RE_{(4,4)}||(4-4)| \\ &= (72t^2 - 108t + 44)0 + (36t^2 - 20t)2 + (4t)0 \\ &= 72t^2 - 40t. \end{aligned}$$

□

Comparison:
Table 4 shows the computed values of the first, second and third Revan indices for the dominating David derived networks $D_1(t), D_2(t)$ and $D_3(t)$. The graph below figure.5 compares how these values change as t grows .

Table 4: The comparison of R_1, R_2, and R_3 for $D_1(t)$, $D_2(t)$ and $D_3(t)$ graphs

t	$R_1[D_1(t)]$	$R_1[D_2(t)]$	$R_1[D_3(t)]$	$R_2[D_1(t)]$	$R_2[D_2(t)]$	$R_2[D_3(t)]$	$R_3[D_1(t)]$	$R_3[D_2(t)]$	$R_3[D_3(t)]$
1	146	154	160	209	224	224	28	30	32
2	942	1062	1152	1207	1432	1424	140	170	208
3	2494	2870	3152	3087	3792	3776	324	418	528
4	4802	5578	6160	5849	7304	7280	580	774	992
5	7866	9186	10176	9493	11968	11936	908	1238	1600
6	11686	13694	15200	14102	17784	17744	1308	1810	2352
7	16262	19102	21232	19427	24752	24704	1780	2490	3248
8	21594	25410	28272	25717	32872	32816	2324	3278	4288
9	27682	32618	36320	32889	42144	42080	2940	4174	5472
10	34526	40726	45376	40943	52568	52496	3628	5178	6800

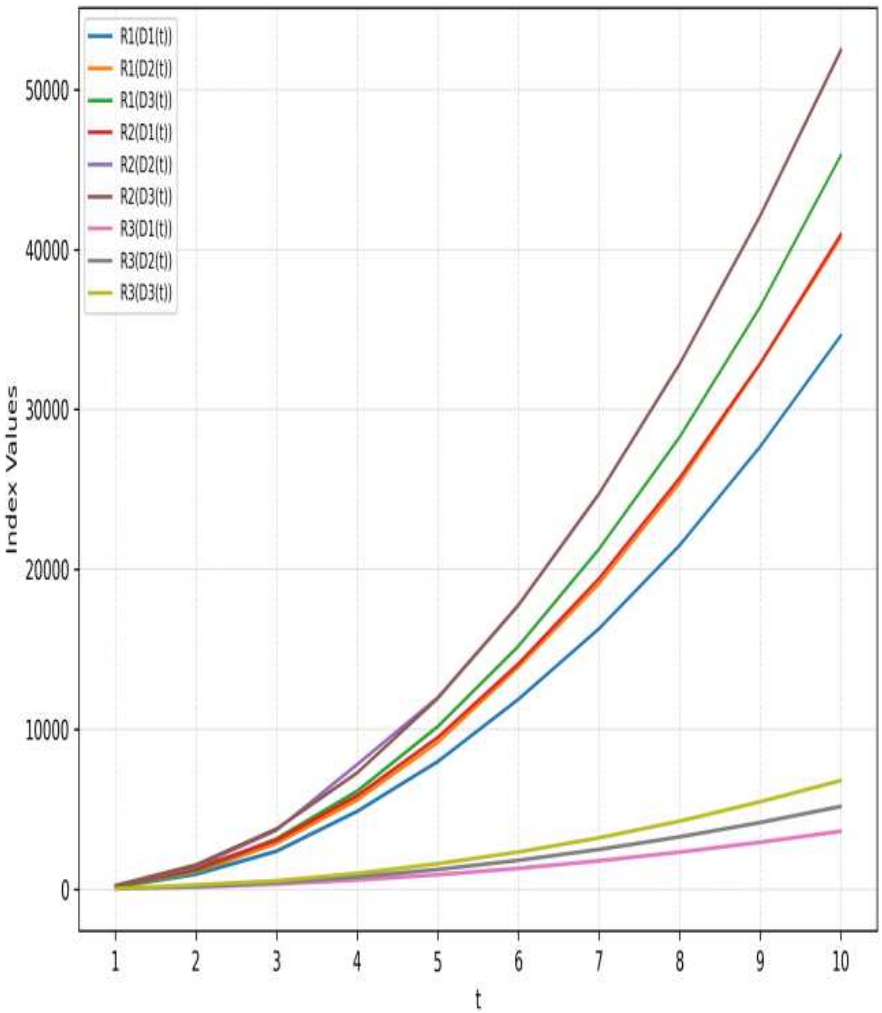

Figure 5: Comparison graph for R_1, R_2, R_3 for $D_1(t), D_2(t), D_3(t)$

5. Conclusion

In this paper, we studied the first, second, and third Revan indices of the three types of dominating David derived networks: $D_1(t)$, $D_2(t)$, and $D_3(t)$. By using edge partitions, we derived exact formulas for all three indices and compared how these values change as the parameter t increases.

From the comparison graph, we observe that all three Revan indices increase as the networks grow, showing that their structure becomes more complex with larger values of t. When comparing the three network types, the results clearly show that $D_2(t)$ has the highest Revan index values for R_1, R_2, and R_3. This indicates that $D_2(t)$ is the most complex and structurally rich network among the three.

Overall, this study demonstrates that Revan indices are useful for understanding the topological behavior of dominating David derived networks. The findings can support future research involving more advanced network models or applications in chemical graph theory and QSPRs/QSARs studies.

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References

1. Dobrynin A.A, Entringer.R, and Gutman.I, Wiener index of trees: theory and applications, Acta Applicandae Mathematica., volume.-66, pp. 211–249, 2001.
2. Todeschini.R and Consonni.V, Molecular Descriptors for Chemoinformatics, Wiley-online library, volume-1, pp. 82–97, 2009.
3. Gutman.I and Polansky.O.E, Mathematical Concepts in Organic Chemistry, Springer, Berlin, pp. 34–39, 1986.
4. Zhao.X, Ali.H, Ali.B, and Binyamin .M.A, Statistics and Calculation of Entropy of Dominating David Derived networks, Complexity, volume. 2021, pp. 1–15, 2021.
5. Kang.S.M,Nazeer.W, Gao.W, Afzal.D, and Gillani.S.N, M-polynomials and topological indices of dominating David derived networks, Open Chemistry, volume. 16, no. 1, pp. 201–213, 2018.
6. Gao.W, Muzaffar.B, and Nazeer.W, K-Banhatti and K-hyper Banhatti indices of Dominating David Derived Networks, Open Journal of Mathematical Analysis, volume. 1, no. 1, pp. 13–24, 2017.
7. Milicevic.A, Nikolic.S, and Trinajstic.N, On reformulated Zagreb indices, Molecular Diversity, volume. 8, pp. 393–399, 2004.
8. Gutman.I and Trinajstic.N, Graph theory and molecular orbitals, Total π-electron energy of alternant hydrocarbons, Chemical Physics Letters, volume. 17, pp. 535–538, 1972.
9. Kulli V.R, On K edge index of some nanostructures, Journal of Computer and Mathematical Sciences, volume. 7, pp. 373–378, 2016.
10. Siva Kumar Pathuri, Vishu Kumar M., Silvia Leera Sequeira, and Veena K., Revan Indices of Certain Graph Models, Scope, volume. 13, no. 3, 2023.
11. Kulli V.R, Computation of some topological indices of certain networks, International Journal of Mathematical Archive, volume. 8, no. 2, pp. 99–106, 2017.
12. Kulli V.R, Revan indices of oxide and honeycomb networks, International Journal of Mathematics and its Applications, volume. 5, no. 4E, pp. 663–667, 2017.

Gavvala Sowmya,
Department of Mathematics,
School of Applied Sciences, REVA University,
India.
E-mail address: sowmyagavvala8@gmail.com

and

Vishu Kumar M.,
Department of Mathematics,
School of Applied Sciences, REVA University,
India.
E-mail address: vishukumarm@reva.edu.in