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Topological Indices of Eccentricity on Double and Strong Double Graphs of Cycloalkanes

Sreekrupa Bagepalli Krishnamurthy, Vishu Kumar Mallappa, Gavvala Sowmya

ABSTRACT: In the theory of chemical graphs, To evaluate the relationship between the structure of molecules with the physicochemical behavior of chemical species, topological indices are quantitative descriptor that are frequently used. The eccentricity connectivity index, the total eccentricity index, the average eccentricity index, the Zagreb eccentricity indices, and the Zagreb degree–eccentricity indices are among the eccentricity-driven topological measures that we assess in this work using a thorough distance-based analysis. These indices are calculated for the double and strong double graph constructions corresponding to cycloalkane structures, which are particularly important because of their frequent occurrence in molecules that are relevant to pharmaceuticals and their wide range of therapeutic uses.

Keywords: Eccentricity, cycloalkanes, double graph, strong double.

Contents

1. Introduction

A chemical graph is a mathematical graph that is constructed from the molecular structure of a chemical compound, with bonds acting as edges and atoms acting as vertices[1]. A molecular graph's topology can be described numerically using an index of topology.It is described as a function $f : \phi \to \mathbb{R}$, Here is the collection of finite simple graphs, and is a collection of real numbers that meet the condition $f(G) = f(H)$ when G is isomorphic to H [2],[3]. Harold Wiener [4] established the first topological index of a chemical graph in 1947 to help predict the point at which it boils of chemical compounds. The Weiner index is the name of this statics. In chemistry, topological indices are employed extensively to identify the physical characteristics, chemical reactions, and biological activities of chemical compounds [5]. These have a broad use in establishing correlation between the physicochemical characteristics and structure of molecules.

Let be a connected, finite, simple, undirected graph, where, and stand for G's vertex set and edge set, respectively. [6]. In the domains of chemistry, biochemistry, and nanotechnology, chemical graph theory offers a variety of distance-based topological indices of a graph that are helpful in isomer discrimination, structure property, and structure activity correlations. [7], [8]. Vikas Sharma et al associates. [9] presented a novel distance-based topological index called the eccentricity the connection index is described as

$$\xi^c(G) = \sum_{v \in V(G)} d_G(v) e_G(v) \tag{1.1}$$

It has been demonstrated that the eccentricity connectivity index predicts pharmacological properties and can lead creation of efficient anti-HIV drugs [10]. Buckley F and Harary F [11] introduced the eccentric mean, which is the average eccentricity found between the radius and diameter of a graph.

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Further studied by Dankelmann P, Goddard WE [12] as the the definition of an average eccentricity index is

$$avec(G) = \frac{1}{n}\sum_{v\in V(G)} e_G(v), \quad n = |V(G)| \tag{1.2}$$

K. Fathilahikani et al.[13] introduced the definition of the total eccentricity index is

$$\zeta(G) = \sum_{v\in V(G)} e_G(v) \tag{1.3}$$

M. Ghorbani et al.[15] and D. Vukicevic et al.[14] presents a new idea version of Zagreb indices called the Zagreb eccentricity indices described as

$$M_1^*(G) = \sum_{uv\in E(G)} [e_G(u) + e_G(v)] \tag{1.4}$$

$$M_1^{**}(G) = \sum_{v\in V(G)} [e_G(v)^2] \tag{1.5}$$

$$M_2^*(G) = \sum_{uv\in E(G)} e_G(u)e_G(v) \tag{1.6}$$

Padmapriya P, Veena Mathad [16] introduced another version of the Zagreb indices based on eccentricity called the Zagreb degree eccentricity indices defined as

$$DE_1(G) = \sum_{v\in V(G)} (d_G(v) + e_G(v))^2 \tag{1.7}$$

$$DE_2(G) = \sum_{uv\in E(G)} [d_G(u) + e_G(u)][d_G(v) + e_G(v)] \tag{1.8}$$

2. Preliminaries

Definition 2.1 *Cycloalkanes, commonly known as saturated hydrocarbons, have ring like structure due to their saturated nature. They have at least three alkane molecules in their structure which aids in their ability to form a ring. They have a generic formula $C_tH_{(2t)}$, t is the number of carbon atoms present in the organic compound, which decides the structure of cycloalkane. Cycloalkanes have many applications in medical, cosmetic, food, perfume manufacturing, and petroleum industries [17].*

Definition 2.2 *A graph's double graph,represented by $D(G)$ was defined by E. Munarini et al. [18]. To construct it, take two instances of, in where a vertex u_i in the initial copy is next to u_j in a second copy if u_i, u_j are adjacent in G .*

Definition 2.3 *Strong double graph [19] of the graph G is the double graph, where a vertex u_i in initial copy is adjacent to u_j in second copy if i=j. It is denoted by SD(G).*

Figure 1: Cycloalkanes (C_3H_6,C_4H_8,C_5H_{10})

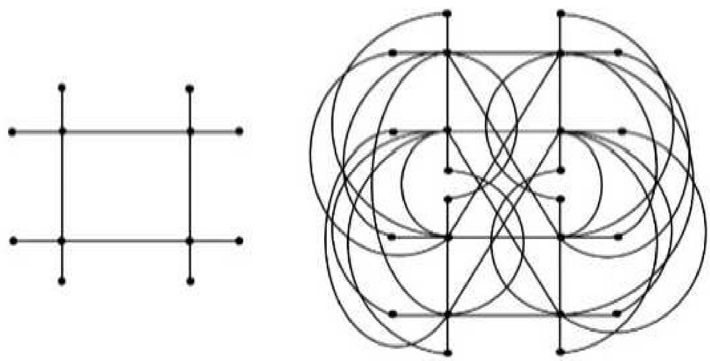

Figure 2: Cyclobutane (C_4H_8) and its double graph ($D(C_4H_8)$.

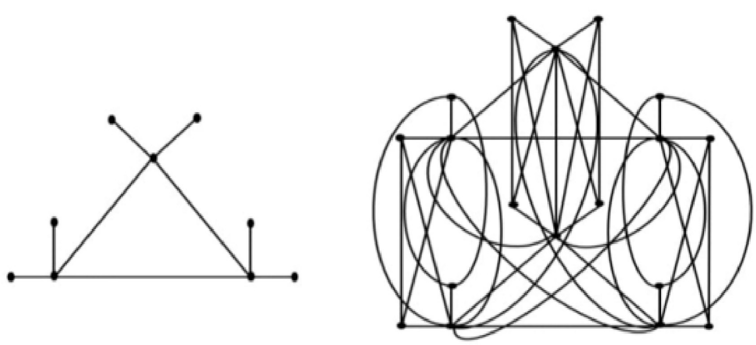

Figure 3: Cyclopropane (C_3H_6) with its powerful double graph ($SD(C_3H_6)$)

3. Topological Double Indices and Strong Double Graphs of Cycloalkanes

This section will establish the eccentricity-based topological indices of the strong and double graphs of cycloalkanes.

Theorem 3.1 *Let $D(C_tH_{2t})$ be the two graphs of cycloalkane C_tH_{2t}. Then ∀ t≥ 3*

1. $\xi^c\left(D(C_tH_{2t})\right) = 24\left\lfloor \frac{t}{2} \right\rfloor + 32t$
2. $avec\left(D(C_tH_{2t})\right) = \left\lfloor \frac{t}{2} \right\rfloor + \frac{5}{3}$
3. $\zeta\left(D(C_tH_{2t})\right) = 6t\left\lfloor \frac{t}{2} \right\rfloor + 10t$
4. $M_1^*\left(D(C_tH_{2t})\right) = 24t\left\lfloor \frac{t}{2} \right\rfloor + 32t$
5. $M_1^{**}\left(D(C_tH_{2t})\right) = 6t\left\lfloor \frac{t}{2} \right\rfloor^2 + 20t\left\lfloor \frac{t}{2} \right\rfloor + 18t$
6. $M_2^*\left(D(C_tH_{2t})\right) = 12t\left\lfloor \frac{t}{2} \right\rfloor^2 + 32t\left\lfloor \frac{t}{2} \right\rfloor + 20t$
7. $DE_1\left(D(C_tH_{2t})\right) = 6t\left\lfloor \frac{t}{2} \right\rfloor^2 + 68t\left\lfloor \frac{t}{2} \right\rfloor + 226t$
8. $DE_2\left(D(C_tH_{2t})\right) = 12t\left\lfloor \frac{t}{2} \right\rfloor^2 + 176t\left\lfloor \frac{t}{2} \right\rfloor + 612t$

Proof: Let $G(V, E)$ be the double graph $D(C_tH_{2t})$ of cycloalkane C_tH_{2t}. Then $\forall t \geq 3$ in G, $|V(G)| =$ 6t and $|E(G)| = 12t$. There are two kinds using the algebraic method By algebraic method, there are two types of vertices (V_2, V_8) and two types of edges $(E_{(2,8)}, E_{(8,8)})$, as follows:

$$V_2 = \{v \in V(G)/d_G(v) = 2\},$$

$$V_8 = \{v \in V(G)/d_G(v) = 8\},$$

$$E_{(2,8)} = \{uv \in E(G)/d_G(u) = 2 \ \& \ d_G(v) = 8\},$$

$$E_{(8,8)} = \{uv \in E(G)/d_G(u) = d_G(v) = 8\}.$$

Table 1: Cardinality of the Vertex and Edge Partitions of $D(C_tH_{2t})$.

$\|V_2\|$	$\|V_8\|$	$\|E_{(2,8)}\|$	$\|E_{(8,8)}\|$
$4t$	$2t$	$8t$	$4t$

Also, we have

$$e_G(V_2) = \{\ e_G(v)/v \in V_2\} = \left\lfloor \frac{t}{2} \right\rfloor + 2$$
$$e_G(V_8) = \{\ e_G(v)/v \in V_8\} = \left\lfloor \frac{t}{2} \right\rfloor + 1$$

1. To compute $\xi^c\left(D(C_tH_{2t})\right)$ using(1),we have

$$\begin{aligned}
\xi^c(G) &= \sum_{v\in V(G)} d_G(v)e_G(v)\\
&= \sum_{v\in V_2} d_G(v)e_G(v) + \sum_{v\in V_8} d_G(v)e_G(v)\\
&= (|V_2|\times 2\times e_G(V_2)) + (|V_8|\times 8\times e_G(V_8))\\
&= [4t\times 2\times(\lfloor\tfrac{t}{2}\rfloor+2)] + [2t\times 8\times(\lfloor\tfrac{t}{2}\rfloor+1)]\\
\Rightarrow \xi^c(G) &= 24t\left\lfloor\frac{t}{2}\right\rfloor + 32t\\
\therefore\ \xi^c\left(D(C_tH_{2t})\right) &= 24t\left\lfloor\frac{t}{2}\right\rfloor + 32t
\end{aligned}$$

2. To compute $avec\left(D(C_tH_{2t})\right)$ using (2), we have

$$\begin{aligned}
avec(G) &= = \frac{1}{n}\sum_{v\in V(G)} e_G(v), \quad n = |V(G)|\\
&= \frac{1}{|v(G)|}\left[\sum_{v\in V_2} e_G(v) + \sum_{v\in V_8} e_G(v)\right]\\
&= \frac{1}{6t}\left[\sum_{v\in V_2} e_G(v) + \sum_{v\in V_8} e_G(v)\right]\\
&= \frac{1}{6t}\left[(|V_2|\times e_G(V_2)) + (|V_8|\times e_G(V_8))\right]\\
&= \frac{1}{6t}\left[4t(\lfloor\tfrac{t}{2}\rfloor+2) + 2t(\lfloor\tfrac{t}{2}\rfloor+1)\right]\\
\Rightarrow avec(G) &= \left\lfloor\frac{t}{2}\right\rfloor + \frac{5}{3}\\
\therefore\ avec\left(D(C_tH_{2t})\right) &= \left\lfloor\frac{t}{2}\right\rfloor + \frac{5}{3}
\end{aligned}$$

3. To compute $\zeta\left(D(C_tH_{2t})\right)$ using (3),

$$\begin{aligned}
\zeta(G) &= \sum_{v\in V(G)} e_G(v)\\
&= \sum_{v\in V_2} e_G(v) + \sum_{v\in V_8} e_G(v)\\
&= (|V_2|\times e_G(V_2)) + (|V_8|\times e_G(V_8))\\
&= 4t(\lfloor\tfrac{t}{2}\rfloor+2) + 2t(\lfloor\tfrac{t}{2}\rfloor+1)\\
\Rightarrow \zeta(G) &= 6t\left\lfloor\frac{t}{2}\right\rfloor + 10t\\
\therefore\ \zeta\left(D(C_tH_{2t})\right) &= 6t\left\lfloor\frac{t}{2}\right\rfloor + 10t
\end{aligned}$$

4. To compute $M_1^*\left(D(C_tH_{2t})\right)$ using (4), we have

$$
\begin{aligned}
M_1^*(G) &= \sum_{uv\in E(G)} \left(e_G(u)+e_G(v)\right)\\
&= \sum_{uv\in E_{(2,8)}} \left(e_G(u)+e_G(v)\right) + \sum_{uv\in E_{(8,8)}} \left(e_G(u)+e_G(v)\right)\\
&= [|E_{(2,8)}| \times (e_G(V_2)+e_G(V_8))] + [|E_{(8,8)}| \times (e_G(V_8)+e_G(V_8))]\\
&= [8t(\lfloor \tfrac{t}{2} \rfloor + 2 + \lfloor \tfrac{t}{2} \rfloor + 1)] + 4t(\lfloor \tfrac{t}{2} \rfloor + 1 + \lfloor \tfrac{t}{2} \rfloor + 1)\\
\Rightarrow M_1^*(G) &= 24t\left\lfloor \frac{t}{2} \right\rfloor + 32t\\
\therefore\ M_1^*\left(D(C_tH_{2t})\right) &= 24t\left\lfloor \frac{t}{2} \right\rfloor + 32t
\end{aligned}
$$

5. To compute $M_1^{**}\left(D(C_tH_{2t})\right)$ using (5), we have

$$
\begin{aligned}
M_1^{**}(G) &= \sum_{v\in V(G)} e_G(v)^2\\
&= \sum_{v\in V_2} e_G(v)^2 + \sum_{v\in V_8} e_G(v)^2\\
&= (|V_2| \times e_G(V_2)^2) + (|V_8| \times e_G(V_8)^2)\\
&= 4t\left(\lfloor \tfrac{t}{2} \rfloor + 2\right)^2 + 2t\left(\lfloor \tfrac{t}{2} \rfloor + 1\right)^2\\
\Rightarrow M_1^{**}(G) &= 6t\left\lfloor \tfrac{t}{2} \right\rfloor^2 + 20t\left\lfloor \tfrac{t}{2} \right\rfloor + 18t\\
\therefore\ M_1^{**}\left(D(C_tH_{2t})\right) &= 6t\left\lfloor \tfrac{t}{2} \right\rfloor^2 + 20t\left\lfloor \tfrac{t}{2} \right\rfloor + 18t
\end{aligned}
$$

6. To compute $M_2^*\left(D(C_tH_{2t})\right)$ using (6), we have

$$
\begin{aligned}
M_2^*(G) &= \sum_{uv\in E(G)} e_G(u)e_G(v)\\
&= \sum_{uv\in E_{(2,8)}} e_G(u)e_G(v) + \sum_{uv\in E_{(8,8)}} e_G(u)e_G(v)\\
&= [|E_{(2,8)}| \times e_G(V_2) \times e_G(V_8)] + [|E_{(8,8)}| \times e_G(V_8) \times e_G(V_8)]\\
&= 8t\left(\lfloor \tfrac{t}{2} \rfloor + 2\right)\left(\lfloor \tfrac{t}{2} \rfloor + 1\right) + 4t\left(\lfloor \tfrac{t}{2} \rfloor + 1\right)^2\\
\Rightarrow M_2^*(G) &= 12t\left\lfloor \tfrac{t}{2} \right\rfloor^2 + 32t\left\lfloor \tfrac{t}{2} \right\rfloor + 20t\\
\therefore\ M_2^*\left(D(C_tH_{2t})\right) &= 12t\left\lfloor \tfrac{t}{2} \right\rfloor^2 + 32t\left\lfloor \tfrac{t}{2} \right\rfloor + 20t
\end{aligned}
$$

7. To compute $DE_1\left(D(C_tH_{2t})\right)$ using (7), we have

$$
\begin{aligned}
DE_1(G) &= \sum_{v\in V(G)} \left(d_G(v)+e_G(v)\right)^2\\
&= \sum_{v\in V_2} \left(d_G(v)+e_G(v)\right)^2 + \sum_{v\in V_8} \left(d_G(v)+e_G(v)\right)^2\\
&= \left[|V_2| \times (2+e_G(V_2))^2) + (|V_8| \times (8+e_G(V_8))^2\right]\\
&= \left[4t\left(2+\lfloor \tfrac{t}{2} \rfloor + 2\right)^2\right] + \left[2t \times \left(8+\lfloor \tfrac{t}{2} \rfloor + 1\right)^2\right]\\
\Rightarrow DE_1(G) &= 6t\left\lfloor \tfrac{t}{2} \right\rfloor^2 + 68t\left\lfloor \tfrac{t}{2} \right\rfloor + 226t\\
\therefore\ DE_1\left(D(C_tH_{2t})\right) &= 6t\left\lfloor \tfrac{t}{2} \right\rfloor^2 + 68t\left\lfloor \tfrac{t}{2} \right\rfloor + 226t
\end{aligned}
$$

8. To compute $DE_2\left(D(C_tH_{2t})\right)$ using (8), we have

$$\begin{aligned}
DE_2(G) &= \sum_{uv\in E(G)} (d_G(u)+e_G(u))(d_G(v)+e_G(v)) \\
&= \sum_{uv\in E_{(2,8)}} (d_G(u)+e_G(u))(d_G(v)+e_G(v)) + \sum_{uv\in E_{(8,8)}} (d_G(u)+e_G(u))(d_G(v)+e_G(v)) \\
&= [|E_{(2,8)}|\times(2+e_G(V_2))(8+e_G(V_8))] + [|E_{(8,8)}|\times(8+e_G(V_8))^2] \\
&= 8t\left(2+\left\lfloor \tfrac{t}{2}\right\rfloor+2\right)\left(8+\left\lfloor \tfrac{t}{2}\right\rfloor+1\right)+4t\left(8+\left\lfloor \tfrac{t}{2}\right\rfloor+1\right)^2 \\
\Rightarrow DE_2(G) &= 12t\left\lfloor \tfrac{t}{2}\right\rfloor^2+176t\left\lfloor \tfrac{t}{2}\right\rfloor+612t \\
\therefore\ DE_2\left(D(C_tH_{2t})\right) &= 12t\left\lfloor \tfrac{t}{2}\right\rfloor^2+176t\left\lfloor \tfrac{t}{2}\right\rfloor+612t
\end{aligned}$$

□

Theorem 3.2 *Let $SD(C_tH_{2t})$ be the strong double graphs of cycloalkane C_tH_{2t}. Then $\forall t\geq 3$*

1. $\xi^c(SD(C_tH_{2t})) = 30t\left\lfloor \frac{t}{2}\right\rfloor + 42t$
2. $avec(SD(C_tH_{2t})) = \left\lfloor \frac{t}{2}\right\rfloor + \frac{5}{3}$
3. $\zeta(SD(C_tH_{2t})) = 6t\left\lfloor \frac{t}{2}\right\rfloor + 10t$
4. $M_1^*(SD(C_tH_{2t})) = 30t\left\lfloor \frac{t}{2}\right\rfloor + 42t$
5. $M_1^{**}(SD(C_tH_{2t})) = 6t\left\lfloor \frac{t}{2}\right\rfloor^2 + 20t\left\lfloor \frac{t}{2}\right\rfloor + 18t$
6. $M_2^*(SD(C_tH_{2t})) = 15t\left\lfloor \frac{t}{2}\right\rfloor^2 + 42t\left\lfloor \frac{t}{2}\right\rfloor + 29t$
7. $DE_1(SD(C_tH_{2t})) = 6t\left\lfloor \frac{t}{2}\right\rfloor^2 + 80t\left\lfloor \frac{t}{2}\right\rfloor + 300t$
8. $DE_2(SD(C_tH_{2t})) = 15t\left\lfloor \frac{t}{2}\right\rfloor^2 + 240t\left\lfloor \frac{t}{2}\right\rfloor + 950t$

Proof: Let $G(V,E)$ be the strong double graph $SD(C_tH_{2t})$ of cycloalkane C_tH_{2t}. Then $\forall t\geq 3$ in G, we have $|V(G)|=6t$ and $|E(G)|=12t$. Using algebraic approach, there are two kinds of vertices (V_3,V_9) and three types of edges $(E_{\{3,3\}},E_{\{3,9\}},E_{\{9,9\}})$ as follows:

$$V_3=\{v\in V(G)/d_G(v)=3\},$$

$$V_9=\{v\in V(G)/d_G(v)=9\},$$

$$E_{(3,9)}=\{uv\in E(G)/d_G(u)=3\ \&\ d_G(v)=9\}$$

$$E_{(9,9)}=\{uv\in E(G)/d_G(u)=9=d_G(v)\}$$

,

$$E_{(3,3)}=\{uv\in E(G)/d_G(u)=3=d_G(v)\}.$$

Also, we have

Table 2: Cardinality of the Vertex and Edge Partitions of $SD(C_tH_{2t})$

$\|v_3\|$	$\|v_9\|$	$\|E_{(3,9)}\|$	$\|E_{(9,9)}\|$	$\|E_{(3,3)}\|$
$4t$	$2t$	$8t$	$5t$	$2t$

$$e_G(V_3) = e_G(v)/v \in V_3 = \left\lfloor \frac{t}{2} \right\rfloor + 2,$$
$$e_G(V_9) = e_G(v)/v \in V_9 = \left\lfloor \frac{t}{2} \right\rfloor + 1.$$

1. To compute $\xi^c(SD(C_tH_{2t}))$ using (1), we have

$$\begin{aligned}
\xi^c(G) &= \sum_{v \in V(G)} d_G(v) e_G(v) \\
&= \sum_{v \in V_3} d_G(v) e_G(v) + \sum_{v \in V_9} d_G(v) e_G(v) \\
&= (|V_3| \times 3 \times e_G(V_3)) + (|V_9| \times 9 \times e_G(V_9)) \\
&= [4t \times 3(\lfloor \tfrac{t}{2} \rfloor + 2)] + [2t \times 9(\lfloor \tfrac{t}{2} \rfloor + 1)] \\
\Rightarrow \xi^c(G) &= 30t \left\lfloor \frac{t}{2} \right\rfloor + 42t. \\
\therefore\ \xi^c(SD(C_tH_{2t})) &= 30t \left\lfloor \frac{t}{2} \right\rfloor + 42t.
\end{aligned}$$

2. To compute $avec\,(SD(C_tH_{2t}))$ using (2), we have

$$\begin{aligned}
avec(G) &= \frac{1}{n} \sum_{v \in V(G)} e_G(v), \quad n = |V(G)| \\
&= \frac{1}{|V(G)|} \left[\sum_{v \in V_3} e_G(V) + \sum_{v \in V_9} e_G(V) \right] \\
&= \frac{1}{6t} \left[(|V_3| \times e_G(V_3)) + (|V_9| \times e_G(V_9)) \right] \\
&= \frac{1}{6t} \left[4t \left(\lfloor \tfrac{t}{2} \rfloor + 2 \right) + 2t \left(\lfloor \tfrac{t}{2} \rfloor + 1 \right) \right] \\
\Rightarrow avec(G) &= \lfloor \tfrac{t}{2} \rfloor + \tfrac{5}{3} \\
\therefore\ avec\,(SD(C_tH_{2t})) &= \lfloor \tfrac{t}{2} \rfloor + \tfrac{5}{3}
\end{aligned}$$

3.To compute $\zeta\,(SD(C_tH_{2t}))$ using (3), we have

$$\begin{aligned}
\zeta(G) &= \sum_{v \in V(G)} e_G(v) \\
&= \sum_{v \in V_3} e_G(v) + \sum_{v \in V_9} e_G(v) \\
&= (|V_3| \times e_G(V_3)) + (|V_9| \times e_G(V_9)) \\
&= 4t \left(\lfloor \tfrac{t}{2} \rfloor + 2 \right) + 2t \left(\lfloor \tfrac{t}{2} \rfloor + 1 \right) \\
\Rightarrow \zeta(G) &= 6t \lfloor \tfrac{t}{2} \rfloor + 10t \\
\therefore\ \zeta\,(SD(C_tH_{2t})) &= 6t \lfloor \tfrac{t}{2} \rfloor + 10t
\end{aligned}$$

4. To compute $M_1^*\left(SD(C_tH_{2t})\right)$ using (4), we have

$$\begin{aligned}
M_1^*(G) &= \sum_{uv\in E(G)} (e_G(u)+e_G(v)) \\
&= \sum_{uv\in E_{(3,9)}} (e_G(u)+e_G(v)) + \sum_{uv\in E_{(9,9)}} (e_G(u)+e_G(v)) \\
&\quad + \sum_{uv\in E_{(3,3)}} (e_G(u)+e_G(v)) \\
&= [|E_{(3,9)}|\times(e_G(V_3)+e_G(V_9))] + [|E_{|9,9)}|\times(e_G(V_9)+e_G(V_9))] \\
&\quad + [|E_{(3,3)}|\times(e_G(V_3)+e_G(V_3))] \\
&= 8t\left(\left\lfloor \tfrac{t}{2}\right\rfloor+2+\left\lfloor \tfrac{t}{2}\right\rfloor+1\right) + 5t\left(\left\lfloor \tfrac{t}{2}\right\rfloor+1+\left\lfloor \tfrac{t}{2}\right\rfloor+1\right) + 2t\left(\left\lfloor \tfrac{t}{2}\right\rfloor+2+\left\lfloor \tfrac{t}{2}\right\rfloor+2\right) \\
\Rightarrow M_1^*(G) &= 30t\left\lfloor \tfrac{t}{2}\right\rfloor + 42t \\
\therefore\ M_1^*\left(SD(C_tH_{2t})\right) &= 30t\left\lfloor \tfrac{t}{2}\right\rfloor + 42t
\end{aligned}$$

5. To compute $M_1^{**}\left(SD(C_tH_{2t})\right)$ using (5), we have

$$\begin{aligned}
M_1^{**}(G) &= \sum_{v\in V(G)} e_G(v)^2 \\
&= \sum_{v\in V_3} e_G(v)^2 + \sum_{v\in V_9} e_G(v)^2 \\
&= (|V_3|\times e_G(V_3)^2) + (|V_9|\times e_G(V_9)^2) \\
&= 4t\left(\left\lfloor \tfrac{t}{2}\right\rfloor+2\right)^2 + 2t\left(\left\lfloor \tfrac{t}{2}\right\rfloor+1\right)^2 \\
\Rightarrow M_1^{**}(G) &= 6t\left\lfloor \tfrac{t}{2}\right\rfloor^2 + 20t\left\lfloor \tfrac{t}{2}\right\rfloor + 18t \\
\therefore\ M_1^{**}\left(SD(C_tH_{2t})\right) &= 6t\left\lfloor \tfrac{t}{2}\right\rfloor^2 + 20t\left\lfloor \tfrac{t}{2}\right\rfloor + 18t
\end{aligned}$$

6. To compute $M_2^*\left(SD(C_tH_{2t})\right)$ using (6), we have

$$\begin{aligned}
M_2^*(G) &= \sum_{uv\in E(G)} e_G(u)e_G(v) \\
&= \sum_{uv\in E_{(3,9)}} e_G(u)e_G(v) + \sum_{uv\in E_{(9,9)}} e_G(u)e_G(v) + \sum_{uv\in E_{(3,3)}} e_G(u)e_G(v) \\
&= [|E_{(3,9)}|\times e_G(V_3)e_G(V_9)] + [|E_{(9,9)}|\times e_G(V_9)e_G(V_9)] + [|E_{(3,3)}|\times e_G(V_3)e_G(V)_3)] \\
&= 8t\left(\left\lfloor \tfrac{t}{2}\right\rfloor+2\right)\left(\left\lfloor \tfrac{t}{2}\right\rfloor+1\right) + 5t\left(\left\lfloor \tfrac{t}{2}\right\rfloor+1\right)^2 + 2t\left(\left\lfloor \tfrac{t}{2}\right\rfloor+2\right)^2 \\
\Rightarrow M_2^*(G) &= 15t\left\lfloor \tfrac{t}{2}\right\rfloor^2 + 42t\left\lfloor \tfrac{t}{2}\right\rfloor + 29t \\
\therefore\ M_2^*\left(SD(C_tH_{2t})\right) &= 15t\left\lfloor \tfrac{t}{2}\right\rfloor^2 + 42t\left\lfloor \tfrac{t}{2}\right\rfloor + 29t
\end{aligned}$$

7. To compute $DE_1\left(SD(C_tH_{2t})\right)$ using (7), we have

$$
\begin{aligned}
DE_1(G) &= \sum_{v\in V(G)} (d_G(v)+e_G(v))^2 \\
&= \sum_{v\in V_3}(d_G(v)+e_G(v))^2 + \sum_{v\in V_9}(d_G(v)+e_G(v))^2 \\
&= (|V_3|\times(3+e_G(V_3))^2) + (|V_9|\times(9+e_G(V_9))^2) \\
&= 4t\left(3+\left\lfloor \tfrac{t}{2}\right\rfloor+2\right)^2 + 2t\left(9+\left\lfloor \tfrac{t}{2}\right\rfloor+1\right)^2 \\
\Rightarrow DE_1(G) &= 6t\left\lfloor \tfrac{t}{2}\right\rfloor^2 + 80t\left\lfloor \tfrac{t}{2}\right\rfloor + 300t \\
\therefore\ DE_1\left(SD(C_tH_{2t})\right) &= 6t\left\lfloor \tfrac{t}{2}\right\rfloor^2 + 80t\left\lfloor \tfrac{t}{2}\right\rfloor + 300t
\end{aligned}
$$

8. To compute $DE_2\left(SD(C_tH_{2t})\right)$ using (8), we have

$$
\begin{aligned}
DE_2(G) &= \sum_{uv\in E(G)} (d_G(u)+e_G(u))(d_G(v)+e_G(v)) \\
&= \sum_{uv\in E_{(3,9)}} (d_G(u)+e_G(u))(d_G(v)+e_G(v)) + \sum_{uv\in E_{(9,9)}} (d_G(u)+e_G(u))(d_G(v)+e_G(v)) \\
&\quad + \sum_{uv\in E_{(3,3)}} (d_G(u)+e_G(u))(d_G(v)+e_G(v)) \\
&= [|E_{(3,9)}|\times(3+e_G(V_3))(9+e_G(V_9))] + [|E_{(9,9)}|\times(9+e_G(V_9))^2] \\
&\quad + [|E_{(3,3)}|\times(3+e_G(V_3))^2] \\
&= 8t\left(3+\left\lfloor \tfrac{t}{2}\right\rfloor+2\right)\left(9+\left\lfloor \tfrac{t}{2}\right\rfloor+1\right) + 5t\left(9+\left\lfloor \tfrac{t}{2}\right\rfloor+1\right)^2 + 2t\left(3+\left\lfloor \tfrac{t}{2}\right\rfloor+2\right)^2 \\
\Rightarrow DE_2(G) &= 15t\left\lfloor \tfrac{t}{2}\right\rfloor^2 + 240t\left\lfloor \tfrac{t}{2}\right\rfloor + 950t \\
\therefore\ DE_2\left(SD(C_tH_{2t})\right) &= 15t\left\lfloor \tfrac{t}{2}\right\rfloor^2 + 240t\left\lfloor \tfrac{t}{2}\right\rfloor + 950t
\end{aligned}
$$

□

Comparison This section displays the topological indices based on eccentricity of double and strong graphs of cycloalkanes numerically and graphically as follows:

Table 3: Numerical Representation of Double Graph of Cycloalkanes $D(C_tH_{2t})$, $3 \le t \le 12$

t	ξ^c	$avec$	ζ	M_1^*	M_1^{**}	M_2^*	DE_1	DE_2
3	168	2.667	48	168	132	192	900	2400
4	320	3.667	88	320	328	528	1544	4048
5	400	3.667	110	400	410	660	1930	5060
6	624	4.667	168	624	792	1344	2904	7488
7	728	4.667	196	728	924	1568	3388	8736
8	1024	5.667	272	1024	1552	2720	4752	12064
9	1152	5.667	306	1152	1746	3060	5346	13572
10	1520	6.667	400	1520	2680	4800	7160	17920
11	1672	6.667	440	1672	2948	5280	7876	19712
12	2112	7.667	552	2112	4248	7728	10200	25200

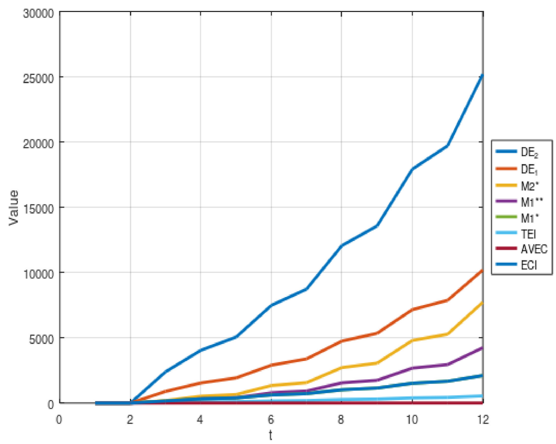

Figure 4: Graphical Representation of Double Graph of Cycloalkanes $D(C_tH_{2t})$, $(3 \leq t \geq 12)$

Table 4: Numerical Representation of Strong Double Graph of Cycloalkanes $SD(C_tH_{2t})$, $3 \leq t \leq 12$

t	ξ^c	$avec$	ζ'	M_1^*	M_1^{**}	M_2^*	DE_1	DE_2
3	216	2.667	48	216	132	258	1158	3615
4	408	3.667	88	408	328	692	1936	5960
5	510	3.667	110	510	410	865	2420	7450
6	792	4.667	168	792	792	1740	3564	10830
7	924	4.667	196	924	924	2030	4158	12635
8	1296	5.667	272	1296	1552	3496	5728	17200
9	1458	5.667	306	1458	1746	3933	6444	19350
10	1920	6.667	400	1920	2680	6140	8500	25250
11	2112	6.667	440	2112	2948	6754	9350	27775
12	2664	7.667	552	2664	4248	9852	11952	35160

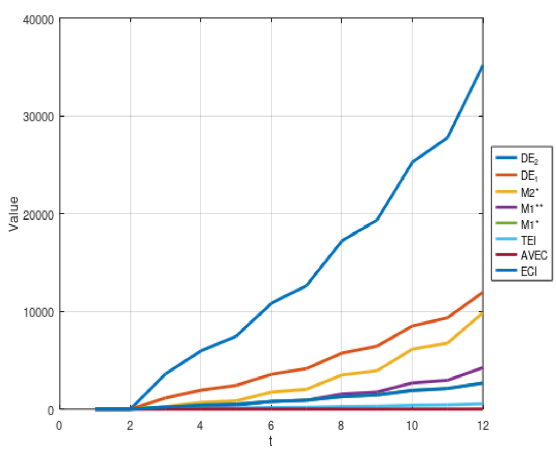

Figure 5: Graphical Representation of Strong Double Graph of Cycloalkanes $SD(C_tH_{2t})$, $3 \leq t \leq 12$

4. Conclusion

In this work, we aim to construct two new types of graphs from cycloalkanes utilising the double graph and strong double graph graph procedures. We have derived generalized explicit formulae to determine the eccentricity connectivity index ξ^c ,indicator of total eccentricity ζ , average eccentricity index (avec) , Zagreb eccentricity indices $(M_1^c, M_1^{n''}, M_2^c)$, Zagreb degree eccentricity indices (DE_1, DE_2) of the newly constructed graphs. The geometric arrangement and comparability of the acquired the outcomes are illustrated visually and quantitatively. These results are convenient for further analysis of the diverse properties of chemical compounds.

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Sreekrupa B. K.,
Department of Mathematics,
School of applied sciences,
REVA University, Bengalore
India.
E-mail address: krupa.maths@gmail.com

and

Vishu Kumar M.,
Department of Mathematics,
School of applied sciences,
REVA University ,Bangalore
India.
E-mail address: vishukumar@reva.edu.in

and

Gavvala Sowmya,
Department of Mathematics,
School of applied sciences,
REVA University, Bangalore
India.
E-mail address: sowmyagavvala@gmail.com